TESTS FOR COVARIANCE STATIONARITY AND WHITE NOISE, WITH AN APPLICATION TO EURO/US DOLLAR EXCHANGE RATE

An approach based on Time-Frequency domain

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Abstract

This paper proposes two non parametric tests for stationarity and white noise against the alternative of time-varying covariance structure with an application to euro/us dollar exchange rate . These tests are based on stability of evolutionary spectral density of the process. Graphical methods using the size and power, con…rm the e¢ciency of our approach when compared with other stationarity tests, especially when data are non stationary with an approximately constant variance. If the null hypothesis of the stationarity is rejected, i.e., possible structural break in the series, our approach can be used for an estimation of the break point. When the method is applied to euro/us dollar exchange rate, it reveals a break point and a short-term instability while the long-term stability is not rejected by the tests.

Keywords: Evolutionary spectral density; Stationarity; White noise; Break point; Long-term stability; Short-term stability; Size-Power Curves; P value discrepancy plots

JEL classi…cation: C12; C22

1. Introduction

Stationarity hypothesis is often required to prove asymptotic properties of many estimators used in econometrics. Many stationarity tests exist in the literature. Pagan and Schwert (1990) proposed several non parametric tests to examine covariance stationarity in the stock market data, but their approaches examine essentially the homogeneity of the variance and this can be insu¢cient to detect the non stationarity of the processes which have a constant

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variance and a time-varying covariance (see section 4.3, relation (4.7)). Kwiatkowski et al. (1992) proposed the well known KPSS test but it is only concerned by non stationary data with possible unit roots. This paper proposes a non parametric test for covariance stationarity and another one for the white noise, based on the stability of evolutionary spectral density (see, for example, Priestley (1969 and 1988), Dahlhaus (1996), Adak (1998), Ombao et al.(2001-2002)). While the Sachs and Neumann's stationarity test (2000) uses the wavelets theory framework, our approach is based on local Fourier transform. It improves the Priestley and Rao's $\hat{A}^2$ test (1969) and it detects other forms of non stationarity that the Pagan and Schwert's test (1990) can do. Since the covariance function is the Fourier transform of the spectral density, our tests are sensitive to many kinds of instability on the covariance structure. Another advantage to use the evolutionary spectral density is to detect the structural change simultaneously in the frequency and time domain. In this way, Artis et al.(1992) studied the long and short run (low and high frequency) instability of the velocity of money in European countries. Section 2 gives an estimator of the evolutionary spectral density, section 3 defines the tests, section 4 proposes a graphical comparison with other tests and also an application to euro/us dollar exchange rate.

2. Theory of the evolutionary spectrum

2.1. Definition

The theory of the evolutionary spectrum (Priestley, 1965) is concerned by oscillatory processes, i.e., processes $\{X_t\}$ defined as follows:

$$X_t = \int_{-\pi}^{\pi} A_t(w) e^{iwt} dZ(w), \tag{2.1}$$

where, for each w, the sequence $\{A_t(w)\}$, as function of t, has a generalized Fourier transform whose modulus has an absolute maximum at the origin. $\{dZ(w)\}$ is an orthogonal process on $[-\pi;\pi]$ with $E[dZ(w)] = 0$[1], $E[|dZ(w)|^2] = d\mu(w)$ where $\mu(w)$ is a measure. Without loss of generality, the evolutionary spectral density of the process $\{X_t\}$ is defined by $h_t(w)$ as follows:

[1] This condition implies that $E(X_t) = 0$.

$$h_t(w) = \frac{dH_t(w)}{dw}, \quad -\pi \leq w \leq \pi, \tag{2.2}$$

where $dH_t(w) = |A_t(w)|^2 d\mu(w)$. The Priestley's evolutionary spectrum theory is a particularly attractive concept, since it has a physical interpretation. It encompasses most other approaches as special cases and it includes many types of non stationary processes. The instantaneous variance of $\{X_t\}$ is given by:

$$\sigma_t^2 = \mathrm{var}(X_t) = \int_{-\pi}^{\pi} h_t(w)dw. \tag{2.3}$$

2.2. Estimation of the evolutionary spectrum

An estimator for $h_t(w)$ at time t and frequency w, can be obtained by using two windows $\{g_u\}$ and $\{w_v\}$. Without loss of generality, $\hat{h}_t(w)$ is constructed as follows:

$$\hat{h}_t(w) = \sum_{v \in Z} w_v |U_{t-v}(w)|^2, \tag{2.4}$$

where $U_t(w) = \sum_{u \in Z} g_u X_{t-u} e^{-iw(t-u)}$. We choose the following windows $\{g_u\}$ and $\{w_v\}$:

$$g_u = \begin{cases} 1/(2\sqrt{h\pi}) & \text{if } |u| \leq h \\ 0 & \text{if } |u| > h \end{cases} \quad \text{and } w_v = \begin{cases} 1/T' & \text{if } |v| \leq T'/2 \\ 0 & \text{if } |v| > T'/2 \end{cases}. \tag{2.5}$$

Here we take $h = 7$ and[2] $T' = 20$. From Priestley (1988) we have, $E(\hat{h}_t(w)) \approx h_t(w)$, $\mathrm{var}(\hat{h}_t(w))$ decreases when T' increases and: $\forall (t_1; t_2), \forall (w_1; w_2)$, $\mathrm{cov}[\hat{h}_{t_1}(w_1)\, \hat{h}_{t_2}(w_2)] \approx 0$ if at least one of the following conditions (i) or (ii) is satisfied[3]:

$$\text{(i)} \quad |t_1 - t_2| \geq T', \qquad\qquad \text{(ii)} \quad |w_1 \pm w_2| \geq \frac{\pi}{h}. \tag{2.6}$$

3. Definition of the tests

Let $\{X_t\}_{t=1}^{T}$ be data from a discrete process $\{X_t\}$ with evolutionary spectral density $h_t(w)$. We consider the set of times $\{t_i = 20i\}_{i=1}^{I}$, where $I = [\frac{T}{20}]$ ([:] denotes the integer part of argu-

[2] This is the choice adopted by Artis et al. (1992)

[3] For more details about relations (i) and (ii) of (2.6) and the choice of h and T', see Priestley (1969 and 1988).

ment) and the set of frequencies $\{w_j = \frac{\pi}{20}(1+3(j-1)\}_{j=1}^{7}$, this implies that $\{t_i\}$ and $\{w_j\}$ satisfy the conditions (i) and (ii) of (2.6). Let $Y_{ij} = \log \hat{h}_{t_i}(w_j)$, $h_{ij} = \log h_{t_i}(w_j)$, $Y_{i.} = \frac{1}{7}\sum_{j=1}^{7} Y_{ij}$, $Y_{.j} = \frac{1}{I}\sum_{i=1}^{I} Y_{ij}$, $\mu_T = \frac{1}{7I}\sum_{i=1}^{I}\sum_{j=1}^{7} Y_{ij}$, $\hat{\sigma}^2 = \frac{1}{I}\sum_{i=1}^{I}(Y_{i.}-\mu_T)^2$ and $S_r = \frac{1}{\hat{\sigma}\sqrt{I}}\sum_{i=1}^{r}(Y_{i.}-\mu_T)$, for $r = 1,\ldots,I$, $i = 1,\ldots,I$ and $j = 1,\ldots,7$. From Priestley (1988), we have:

$$Y_{ij} = h_{ij} + e_{ij}\ , \tag{3.1}$$

where the sequence $\{e_{ij}\}$ is approximately normal, uncorrelated and identically distributed.

Theorem 3.1. Let $T_1 = \sup_{r=1,\ldots,I}|S_r|$. Then, under the null hypothesis of stationarity of $\{X_t\}$, the limiting distribution of T_1 is given by:

$$F_1(a) = 1 - 2\sum_{k=1}^{\infty}(-1)^{k+1}\exp(-2k^2a^2). \tag{3.2}$$

From (3.2), some useful critical values C_α, i.e. $Pr(T_1 > C_\alpha) = \alpha$, are $C_{0.1} = 1.22$, $C_{0.05} = 1.36$ and $C_{0.01} = 1.63$.

Proposition 3.2. Let $T_2 = \dfrac{7(I-1)I\sum_{j=1}^{7}(Y_{.j}-\mu_T)^2}{6\sum_{j=1}^{7}\sum_{i=1}^{I}(Y_{ij}-Y_{.j})^2}$.

Suppose that $\{X_t\}$ is stationary. Then under the null hypothesis that $\{X_t\}$ is a white noise, T_2 has a Fisher-Snedecor distribution with 6 and $7(I-1)$ degrees of freedom.

See appendix for the proofs of theorem 3.1 and proposition 3.2.

Remark 1. Theorem 3.1 investigates the null hypothesis H_o: $h_{ij} = h_j$, i.e., the spectrum is independent of the set of times $\{t_i\}$.

Remark 2. The statistic T_1 is based on cumulative sums (cusum) and this method is usually used to detect structural break in times series. Hence if the stationarity hypothesis is rejected (i.e. possible break point in the times series), the point $r_0 = \arg T_1$ (i.e., r_0 is the point where the sup of $|S_r|$, $r = 1,\ldots,I$ is realized) is an estimation of the important break point of the process.

Remark 3. For the stationarity test, we have constructed the statistic T_1 on a grid of frequencies that recovers approximately the interval $(0;\pi)$ as $\{w_j = \frac{\pi}{20}(1 + 3(j - 1)\}_{j=1}^{7}$. But it can be useful to know if a specific frequency is stationary or not. This information enables us to know the behaviours of the series for the long-term (low frequency) or for the short-term (high frequency). We define w_0 as a stable frequency if the spectral density on w_0 is independent of time, i.e., $h_t(w_0) = C_{w_0}\ \forall t$ (C_{w_0} is a constant depending only to w_0). If w_0 is stable and close to zero there is a stability of long-term significance and if w_0 is stable and close to π there is a stability of short-term significance. There is an instability of long-term or short-term significance if respectively w_0 is a low or a high no stable frequency. For the test of the stability of the frequency w_0, we apply the statistic T_1 by using only the frequency w_0 instead of the set $\{w_j = \frac{\pi}{20}(1 + 3(j - 1)\}_{j=1}^{7}$.

Remark 4. Proposition 3.2 tests the null hypothesis of the constancy of the spectrum, H_o: $h_{ij} = h$.

4. Comparison with other tests by graphical methods[4]

We use the P value discrepancy plots and the size-power curves, defined by Davidson and Mackinnon (1993 and 1994), to give some comparisons and analysis for the powers and the sizes of our tests. Following the authors, the graphs convey much more information, in a more easily assimilated form, than classical tables can do.

4.1. P value discrepancy Plots

Consider a test statistic τ with asymptotic distribution function $F(\tau)$. Denote by $\{\tau_j\}_{j=1}^{N}$, N realizations of the statistic τ, generated by using a DGP[5] which satisfies the null hypothesis. The P value of each τ_j is the value p_j, where:

$$p_j = 1 - F(\tau_j) = \Pr(\tau > \tau_j). \tag{4.1}$$

[4] For more details, see Davidson and Mackinnon (1993 and 1994).
[5] Data Generating Process.

Consider now $\hat{F}_0$, the empirical distribution function of $\{p_j\}_{j=1}^N$, defined at any point x_i in the (0,1) interval as follows:

$$\hat{F}_0(x_i) = \frac{1}{N}\sum_{j=1}^{N} I(p_j \leq x_i), \tag{4.2}$$

where $I(p_j \leq x_i)$ is an indicator function taking the value 1 if the argument is true and 0 otherwise. Davidson and MacKinnon suggest the following choice of $\{x_i\}_{i=1}^m$:

$$x_i = .001, .002, \ldots, .010, .015, \ldots, .990, .991, \ldots, .999 \quad (m = 215). \tag{4.3}$$

The P value discrepancy plot is the graph of $\hat{F}_0(x_i) - x_i$ against x_i. If the distribution of τ used to compute the p_j is correct, each of the p_j should be distributed as uniform (0,1). Therefore, when $\hat{F}_0(x_i) - x_i$ is plotted against x_i, the resulting graph should be close to the horizontal axis $y = 0$.

4.2. Size-Power Curves

The size-power curves of the statistic test τ is constructed with two empirical distribution functions $\hat{F}_0$ and $\hat{F}_1$. $\hat{F}_0$ is given by (4.2), $\hat{F}_1$ is constructed in the same way of $\hat{F}_0$ but instead of τ_j we use N realizations $\{\tau'_j\}_{j=1}^N$ generated by using a DGP which satisfies a given alternative hypothesis. The size-power curve of the statistic τ, is the locus of points $(\hat{F}_0(x_i), \hat{F}_1(x_i))$ when x_i describes the (0,1) interval. Given two tests $\tau^{(1)}$ and $\tau^{(2)}$, if the test $\tau^{(1)}$ has a good power than the test $\tau^{(2)}$ then the size-power curve of $\tau^{(1)}$ converges more quickly to the horizontal line $y = 1$ than the one of $\tau^{(2)}$.

4.3. Comparison of the stationarity tests

We compare, by graphical methods, the statistic T_1 of theorem 3.1 with both Pagan and Schwert's test (PS) and the Priestley and Rao's test (PR). Given the data $\{X_t\}_{t=1}^T$, the PS statistic is defined as follows:

$$PS = \frac{\sqrt{\frac{T}{2}}(\hat{\sigma}_1 - \hat{\sigma}_2)}{\sqrt{2\hat{\sigma}}}, \tag{4.4}$$

where $\hat\sigma_1 = \frac{2}{T}\sum_{j=1}^{T/2} X_j^2$, $\hat\sigma_2 = \frac{2}{T}\sum_{j=\frac{T}{2}+1}^{T} X_j^2$, $\hat\sigma = \hat\gamma_0 + 2\sum_{j=1}^{8}\hat\gamma_j(1-(j/9))$ and $\{\hat\gamma_j\}$ is a consistent estimator of the autocovariance function of $\{X_t^2\}$. Under the null hypothesis of stationarity, the limiting distribution of PS is $N(0,1)$. The PR statistic is defined as:

$$PR = \frac{21T'}{4h}\sum_{i=1} (Y_{i\cdot} - \mu_T)^2, \quad (T' = 20,\ h = 7), \tag{4.5}$$

with the same notations as section 3. Under the null hypothesis of stationarity, the PR is distributed as $\chi^2_{(I-1)}$. The DGP under the null and the alternative hypothesis are respectively $\{y_t\}$ and $\{v_t\}$ defined as follows:

$$y_t = \rho y_{t-1} + \varepsilon_t,\ \rho = 0.2,\ \varepsilon_t \sim \text{i.i.n}(0,1),\quad t = 1,\dots,400., \tag{4.6}$$

$$v_t = c_t u_t,\quad c_t = \sqrt{1-\rho_t^2},\quad t = 1,\dots,400., \tag{4.7}$$

where $u_t = \rho_t u_{t-1} + e_t$, $e_t \sim$ i.i.n(0,1), $\rho_t = 0.7$ if $t \le 200$ and $\rho_t = 0.4$ if $t > 200$. The evolutionary spectral densities of $\{u_t\}$ and $\{v_t\}$ are given respectively by $h_t^{(u)}(w) = \frac{1}{2\pi}|1-\rho_t e^{-iw}|^{-2}$ and $h_t^{(v)}(w) = c_t^2 h_t^{(u)}(w)$. We can easily prove that: $var(u_t) = \int_{-\pi}^{\pi} h_t^{(u)}(w)dw = c_t^{-2}$ and then $var(v_t) = c_t^2 var(u_t) = 1$; despite a constant variance, the process $\{v_t\}$ is non stationary because its spectrum is time dependent. The graphs are constructed from $N = 2500$ runs. Figure 1 shows that the PS is a biased test, it has a low power against the alternative given by (4.7) and T_1 has a good power. Figure 2 confirms the correct sizes of the PS and the T_1 statistics (their P value discrepancy plots are close to the horizontal axis, $\sup|\hat F_0(x_i) - x_i| = 0.0430$ for PS and $\sup|\hat F_0(x_i) - x_i| = 0.0530$ for T_1). In Figure 3 we can see that the P value discrepancy plot of the PR statistic is far from the horizontal axis $y = 0$ ($\sup|\hat F_0(x_i) - x_i| = 0.969$), i.e., the sizes of PS and T_1 are more satisfying than the one of PR.

Figure 1

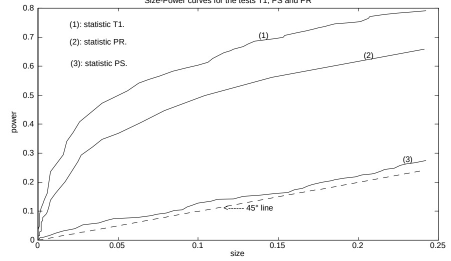


Figure 2

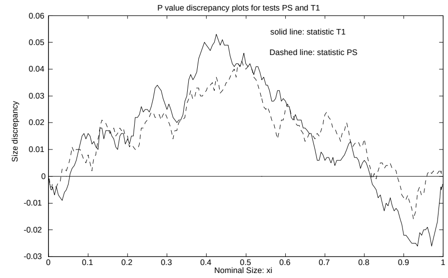
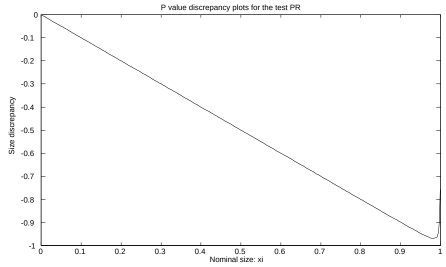


Figure 3



4.4. Size and Power of the white noise test

We compare, by graphical methods, the statistic T_2 of proposition 3.2 with both the well known Bera and Jarque's test (bj) and a cumulative sums of squares test (cumusq). The Bera and Jarque's test is applied under the null hypothesis that data are independents and identically distributed as normal distribution, hence it' is can be used both for testing normality and gaussian white noise. Given the data $\{X_t\}_{t=1}^{T}$, the statistic (bj) is defined as follows:

$$bj = \frac{T}{6}\beta_1 + \frac{T}{24}(\beta_2 - 3)^2 \qquad (4.8)$$

where $\beta_1^{1/2} = \frac{\mu_3}{(\mu_2)^{3/2}}$ (Skewness), $\beta_2 = \frac{\mu_4}{(\mu_2)^2}$ (Kurtosis), $\mu_k = \frac{1}{T}\sum_{t=1}^{3}\left(X_t - \bar{X}\right)^k$ and $\bar{X} = \frac{1}{T}\sum_{t=1} X_t$. Under the null hypothesis that $\{X_t\}_{t=1}^{T}$ come from gaussian white noise, it's well known that the distribution function of (bj) is $\chi^2(2)$. The cumulative sums of squares test (cumsq) is defined as follows:

$$cumsq = \sqrt{n/2}\max_{r=1,\ldots,n} |D_r| \qquad (4.9)$$

where $D_r = \frac{\sum_{t=1} X_t^2}{\sum_{t=1} X_t^2} - \frac{r}{T}$ with $r = 1, \ldots, T$. The distribution function of the statistic cumsq, under the null hypothesis of gaussian white noise, is given by the following proposition

Proposition 4.1. Let $\{X_t\}_{t=1}^{T}$ be a sequence of independent, identically distributed Normal $(0, \sigma^2)$ random variables (Gaussian white noise). Then the distribution function of the statistic cumsq (4.9) is given by the function F_1 of the theorem 3.1.
See appendix for the proof proposition (4.1).

Now, since the distributions functions of the (bj) and cumsq tests are known, it's possible to use graphical methods described in section 4 for the comparisons with the statistic T_2. The DGP of the null hypothesis is the white noise f"$_t$ s i.i.n(0;1); t = 1;:::;400g and for the alternative hypothesis, we take the stationary process fy_tg given by (4.6). The graphs are constructed from N = 2500 runs. Figure 4 indicates a good power for T_2 its size-power curve is very high and converges more quickly to the horizontal line y = 1 while for the statistics (bj) and cumsq the curves are closes to 45$^{\pm}$ line, hence it seems that tests (bj) and cumsq are biased because their powers are closes to their sizes. Finally the Figure 5 indicates a correct size for the three tests T_2; (bj) and cusumsq since the P value discrepancy plot of each test is close to the horizontal axis y = 0.

Figure 4

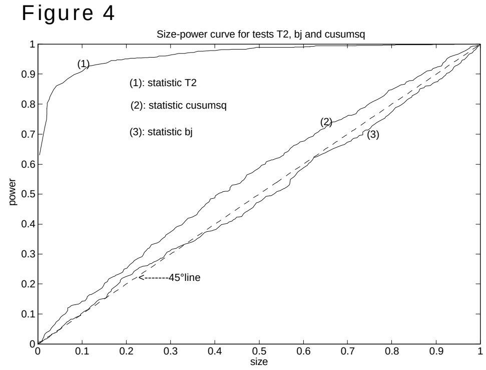
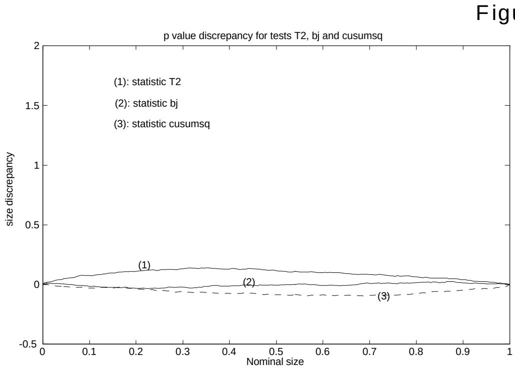


Figure 5



4.5. Euro/us dollar exchange rate

We apply the tests T_1, PR and PS to log-returns of euronus dollar exchange rate X_t = log(p_t=$p_{t\ ¡\ 1}$) (see Figure 6), with fp_tg representing the daily exchange rate from 01/01/1999 to 30/04/2001. The rejection of stationarity hypothesis of X_t is con…rmed by T_1 = 1:4053516 (the critical value of T_1 is given by 1:36 for ® = 0:05) and PS = ¡ 3:8372449 (we reject the null hypothesis if jPSj > 1:96 for ® = 0:05) while the PR statistic do not allow to reject the stationarity hypothesis. Since the stationarity is rejected by T_1, there is a possible break point which can be estimated by r_0 = arg T_1 (see section 3, remark 2). We …nd r_0 around the number of order 200 of the times series. For the frequency domain, we apply the test T_1 (as it's indicated to section 3, remark 3) to speci…cs frequencies: for w_0 = ¼=20 (low frequency or long-term), the statistic T_1 = 1:183 and we cannot reject the stability of the long-term.

For w_0 = 13¼=20 corresponding to a short period of 2¼=w_0 ¼ 3 days (we use daily data), the statistic T_1 = 1:429 rejects the stability of the short-term component 13¼=20 (the critical value of T_1 is given by 1:36 for ® = 0:05). The estimation of evolutionary spectral density (Figure 7) indicates several movements in the data (short and long run) which can be explained by the peaks on high and low frequencies. We can also observe that the locations of the peaks are approximately stable in the frequencies axis but their amplitudes change with time. This means that X_t has some characteristics of uniformly modulated process, i.e.: X_t = ¾(t)Z_t where Z_t is a stationary process and ¾(t) a deterministic function depending only on time.

Figure 6

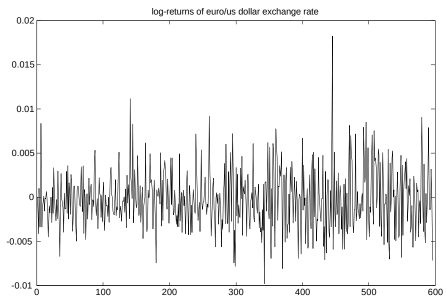
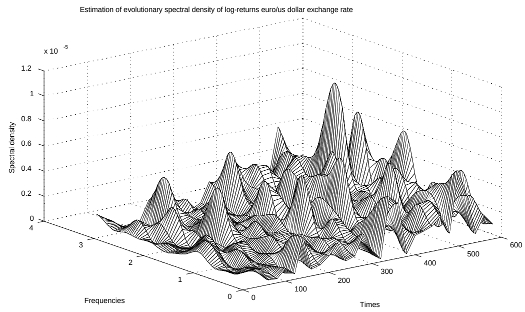


Figure 7



5. Conclusion

We have used the evolutionary spectral density to construct a test for stationarity and another one for white noise. While other stationarity tests are concerned by particularly non stationary processes, our approach detects many types of instability in the covariance structure. Since the asymptotic distribution functions of our tests are obtained with $l = [\frac{T}{20}]$ (instead of T on the usual cases), our approach is particularly useful with large samples of high-frequency data (like …nancial data).

Appendix

Proof of theorem 3.1.

Under the null hypothesis of stationarity, the spectrum is constant over time, i.e., $h_{i:} = h$. From the

Priestley's relation (3.1), we can write the following standard linear regression model:

$$Y_{i\cdot} = h + e_{i\cdot},\ i = 1, \dots, I = [\frac{T}{20}], \tag{5.1}$$

where $\{e_{i\cdot}\}$ is approximately uncorrelated and identically distributed since $\{e_{ij}\}$ is an i.i.d. sequence. The ols estimate of h is given by $\hat{h} = \frac{1}{I}\sum_{i=1}^{I} Y_{i\cdot} = \frac{1}{7I}\sum_{i=1}^{I}\sum_{j=1}^{7} Y_{ij} = {}^{1}{}_{T}$ and the ols residuals are $\hat{e}_{i\cdot} = Y_{i\cdot} - \hat{h}$, thus S_r are the cumulated sums of ols residuals. Let $B^{(I)}(z) = \frac{1}{\hat{\sigma}\sqrt{I}}\sum_{i=1}^{[Iz]} \hat{e}_{i\cdot}$. Since the assumptions of theorem 1 of Ploberger and Kramer (1992) are abviously satisfied, the limiting distribution of $B^{(I)}(z)$ is the standard Brownian bridge $B(z)$, hence the limiting distribution of $\sup_{0 \le z \le 1} \left|B^{(I)}(z)\right|$ is $\sup_{0 \le z \le 1} |B(z)|$. From Billingsley (1968),

$$\Pr(\sup_{0 \le z \le 1} |B(z)| > a) = 2\sum_{k=1}^{\infty} (-1)^{k+1} \exp(-2k^2a^2);$$

the desired conclusion (3.2) holds since $T_1 = \sup_{r=1,\dots,I} |S_r| = \sup_{0 \le z \le 1} \left|B^{(I)}(z)\right|$.

Proof of proposition 3.2.

The null hypothesis of white noise is true if the spectrum is simultaneously independent over the set of time and the set of frequency. Under the sationarity assumption, the constancy of the spectrum over time is satisfied, i.e., $h_{ij} = h_j$ and the model (3.1) becomes,

$$Y_{ij} = h_j + e_{ij}\ ;\ i = 1, \dots, I;\ j = 1, \dots, 7. \tag{5.2}$$

The model (5.2) can also be written as follows,

$$
\underbrace{\begin{pmatrix} Y_{11} \\ \vdots \\ Y_{17} \\ \vdots \\ \vdots \\ \vdots \\ Y_{i1} \\ \vdots \\ Y_{i7} \\ \vdots \\ \vdots \\ \vdots \\ Y_{l1} \\ \vdots \\ Y_{l7} \end{pmatrix}}_{Y_{(7l;1)}}
=
\underbrace{\begin{pmatrix} 1 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 1 & \vdots & \vdots \\ 0 & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ \vdots & 0 & \vdots \\ \vdots & 1 & \vdots \\ \vdots & \vdots & \vdots \\ \vdots & 1 & 0 \\ \vdots & \vdots & 0 \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & 0 \\ \vdots & \vdots & 1 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 1 \end{pmatrix}
\begin{pmatrix} h_1 \\ \vdots \\ h_i \\ \vdots \\ h_7 \end{pmatrix}}_{X_{(7l;7)}b_{(7;1)}}
+
\underbrace{\begin{pmatrix} e_{11} \\ \vdots \\ e_{17} \\ \vdots \\ \vdots \\ \vdots \\ e_{i1} \\ \vdots \\ e_{i7} \\ \vdots \\ \vdots \\ \vdots \\ e_{l1} \\ \vdots \\ e_{l7} \end{pmatrix}}_{U_{(7l;1)}}
\tag{5.3}
$$

Now we can apply the classical Fisher-Snedecor's test in the regression model (5.3) to test the null hypothesis of the constancy of h_j, i.e. $h_j = h$, against the alternative of frequency-varying of the spectrum. The Fisher's test is given by T_2 (see for example Davidson and Mackinnon (1993)).

Proof of proposition 4.2

From the theorem.1 of Inclan and Tiao (1994); the limiting distribution of $\sqrt{n/2}\,|D_r|$ is the standard Brownian bridge $B(z)$. Hence similar arguments likes the proof of theorem 1 completes the demonstration.

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