Price Bids and Capacity Choice in Electricity Markets[¤]

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Abstract

This paper analyses o¤er prices and capacity choice by two asymmetric and capacity-constrained …rms competing in a spot market for electricity with uniform auctions. We …nd that the scope for Bertrand competition is limited to the case of very low demand levels. For higher levels of demand, market clears at the highest possible price level, the exogenous price-cap. In the capacity choice stage of the game, …rms can endogenously switch from Betrand competition to those regimes that guarantee higher pro…ts. As a consequence, generators may strategically restrain capacity. Strategic withholding is more likely when demand is known at the time where operators choose their available capacity, and is less likely when demand is a stochastic variable and values close to peak-hours occur with a high probability. When the demand distribution function is strongly skewed leftward and …rms' asymmetries are smoothed, voluntary shortage may occur.

Keywords: electricity markets, capacity choice, uniform auctions
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1 Introduction

Since the 90's an increasing number of countries have organized wholesale markets for electricity. Most of these markets are based on uniform price auction mechanisms, that is a system where every active producer receives the same price for every unit of output he is called for, as long as his bids were lower than the clearing price computed by the market operator[1]. With this mechanism, low bidders take advantage of the high bidders' proposals. This paper analyses how competition can be distorted by this mechanism where, because of capacity constraints, large generators are left with residual demand on which they have strong incentives to charge high prices.

Auctions in power markets have already been studied by several economists[2]. Von der Fehr and Harbord (1993) have developed a sealed-bid multiple-unit auction model with particular reference to the UK framework that was operating during the 90's. They show that inefficient pricing is the most likely outcome even if there is no collusive behavior. In their model, demand is determined as a random variable independent of price, each generator operates several plants for which he can submit separate bids, and generation capacity is exogenously given.

Otto López (2000) considers a market where generators can submit different bids for each next day hourly market, like in Spain since 1998. As opposed to the former British system that obliged generators to submit a single bid for the 48 thirty-minute markets, therefore facing a fluctuating and random demand, she considers that the possibility to submit 24 different hourly bids allows Spanish generators to face a quasi certain demand on each market. But they face some uncertainty on the generation cost of their competitors. Otto López compares the bidding strategies under capacity constraint and without capacity constraint and shows that for a certain range of low costs, the firms bid strictly less than in the unconstrained case. But the expected

[1] The new system installed in the UK is an exception: each active producer is paid his own bid. Also note that in California, there is a momentum in favour of price-differentiated mechanisms.

[2] We do not refer here to the set of studies that use standard Industrial Organization models of competition based on continuous and differentiable cost functions like Green and Newbery (1992). In fact, different reasons lead us to prefer stepwise offer functions: first, system operators restrict generators to choose bids in a menu of discrete prices, imposing a "tick" (that is the increment between two feasible bids) for feasibility reasons; secondly, generators face severe capacity constraints and possess a finite number of units, so these technological constraints impose discrete jumps in their supply functions.

equilibrium price will not necessarily be close to the marginal cost because of capacity constraints.

García-Martín (1999) refers to the same type of model to analyze the effects of the mechanism that has been settled in Spain to help the incumbent firms to recover stranded investments. He shows that the Spanish mechanism designed for this recovery acts as a countervailing force to market power and high prices.

All these papers have in common to consider that firms compete in prices with limited generation capacity, which is a variation around the Bertrand-Edgeworth model[3]. However, the existing models do not endogenise capacity choice, which is an important feature of spot trading. In fact, for technical reasons, such as equipment maintenance or failures, the installed capacity may not work at the maximum operating level and the spot market rules oblige generators to announce which plants they are willing to use together with their offer prices. However, beyond the technical reasons, the so-called "capacity declarations" also offer a strategic instrument for firms: by restricting capacity, operators can benefit from scarcity rents. Evidences of capacity withholding have been exhibited by several econometric analyses, among which Wolak and Patrick (1997) for the UK market, and more recently Harvey and Hogan (2001), and Joskow and Kahn (2001) for the Californian market, but an analytical model showing the incentives to withhold capacity has not yet been developed in the literature on electricity markets.

The objective of this paper is to emphasize the implicit collusion generated by the uniform price mechanism where capacity is constrained and to show that capacity withholding can be used strategically to enforce this collusion. Our model allows to characterize structural differences of electricity markets, mainly depending on installed capacity asymmetry and demand level forecast. The main result is that the uniform auction procedure actually gives strong incentives to tacit collusion and capacity withholding. Nevertheless, the risk of power shortage can be made very low, even when demand is random, by making generators jointly responsible and allowing them only cost recovery (i.e. zero profit) when, although installed capacity could match demand, shortage is voluntary provoked.

The paper is organized as follows. Section 2 is devoted to model setting. Section 3 and 4 explore, respectively, the price game and the capacity choice

[3] For recent contributions on Bertrand-Edgeworth competition, see Kreps and Scheinkman (1983), Allen and Hellwig (1986,1993) and Deneckere and Kovenock (1996).

game. Section 4.1 considers the case of deterministic demand, while Section 4.2 focuses on stochastic demand. Section 5 concludes.

2 Model setting

To analyse duopolistic competition in electricity markets, we need assumptions on technology, demand, and regulatory instruments such as price-cap, shortage penalties and rationing rules. In what follows, we detail our hypotheses. Section 2.1 explains market clearing with uniform auction rules and Section 2.2 is devoted to the timing of the game.

H_1) Technology

There are two generators labeled a and b; whose production technology is characterized by asymmetrical marginal costs ($c_a < c_b$). Marginal costs are common knowledge, which is broadly the case in wholesale electricity markets, on aggregate. Firm i's available capacity is $\overline{K}_i$ ($i = a; b$) and $\overline{K}_a > \overline{K}_b$: the low generation cost …rm is also the one with the highest installed capacity. This is a reasonable assumption even if the CCGT technology is on the verge of changing things.

Each generator is not obliged to declare capacity as totally available. One reason is that it is costly to prepare and to operate a generation plant. When …rm i announces that $K_i(\quad \overline{K}_i)$ is available, it must be ready to produce up to K_i if the market operator dispatches it, which means maintenance, warming up and monitoring expenses. The second reason is that generators can consider that retaining some capacity is a good way to increase pro…ts. In order to emphasize this second reason, we assume that …rm i incurs no cost in declaring the availability of K_i. Only generation costs will be paid for the output e¤ectively produced.

We also assume that power shortage can occur only if provoked by …rms: the installed capacity is su¢cient to provide the whole demand (even in peak hours).

H_2) Demand

Demand D is assumed totally inelastic: this mainly re‡ects the fact that, while the technology to meter consumption on an hourly basis is available, "no electricity market in operation today makes substantial use of real-time prices, i.e. charges a customer time-varying prices that re‡ect the time-varying cost of producing electricity at the wholesale level" (Borenstein, 2001). Like costs, demand is common knowledge to all players.

For a given demand D, supply can appear as small or large on two grounds. First, ex-ante, that is depending on the real or technical or natural generation capacities $\overline{K}_a$ and $\overline{K}_b$: Second, ex-post, that is depending on the alleged or declared or strategic capacities K_a and K_b. As shown in …gure 1 for given declared capacity, we will refer to regimes of

- low demand (D_L) if $D < \min(K_a; K_b)$
- medium demand (D_M) if $\min(K_a; K_b) < D < \max(K_a; K_b)$
- high demand (D_H) if $\max(K_a; K_b) < D < K_a + K_b$
- very high demand (D_V) if $(K_a + K_b) < D$

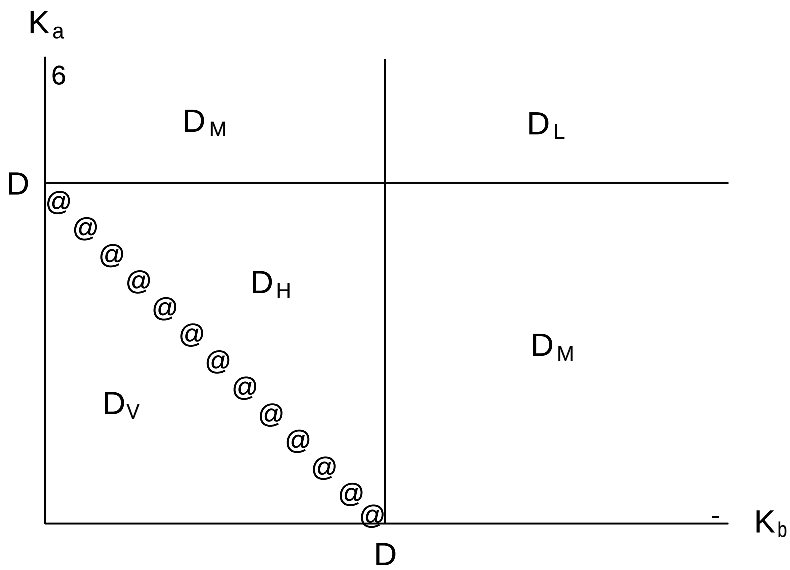

Figure 1: Alternative regimes of demand

Of course, because of the constraints $K_i \quad \overline{K}_i$ for $i = a; b$, the ex-post regime can only be a subset of the ex-ante regime. For instance, if the ISO announces an ex ante D_M regime, the generators cannot transform it in an ex post D_L regime. What they can do is to keep it in a D_M regime or to make it a D_H or even a D_V regime by withholding strategically a fraction of their natural capacity.

H_3) Price-cap and Rationing rules

Like in most papers (and like in the real world), we suppose there exists an upper limit to bids, denoted by $\hat{p}$. This can be interpreted as a regulated maximum price or as the reservation price of consumers. Beyond this threshold, demand is nil. Below this value, demand is totally inelastic.

When …rms bid di¤erent prices, they are dispatched according to the "merit order", that is the low bidder produces up to its declared capacity and the high bidder produces up to the residual demand, if any.

When the …rms bid the same price p, they are rationed according to their available capacity, which means that the gross revenue of …rm i is $pD\frac{K_i}{K_i + K_j}$. Although proportional rationing is commonly used in real markets, we also consider an alternative rationing mechanism, that is the e¢cient rationing. We will show that the rationing rule does not play any signi…cant role for the equilibria of the game.

H_4) Shortage penalty

What occurs when demand is very high? Under pure market mechanisms, the price should be $\hat{p}$, each …rm i receiving revenue $\hat{p}K_i$, and demand being rationed. Actually, one can observe that the political consequences of such shortage are dramatic and that market rules should be designed to avoid the situation where demand is larger than the available capacity. A simple rule can be to index the pro…t of the generators to make it a decreasing function of the di¤erence between demand and the total declared capacity; this would mean that pro…t increases with $K_a + K_b$. Said di¤erently, this simple rule prescribes that generators must be rewarded for declaring their capacity available. However, as this reward depends on the total capacity declared on the spot markets, it can create incentives for colluding behavior and free riding[4]. In our model we will assume that the market rules include a very simple penalty clause when a shortage occurs: firms are paid their marginal cost of generation[5].

2.1 Determination of the system marginal price

To describe the matching process and the determination of the "system marginal price" (henceforth, SMP), suppose that $K_a > K_b$ at the opening of the bidding stage. In figures 2a and 2b, we have represented four levels of (inelastic) demand: D_L, D_M; D_H and D_V.

When demand is low ($D_L \leq K_b$), each firm is able, alone, to provide the whole market. "Medium" demand D_M is such that $K_b < D_M \leq K_a$ which means that firm a can provide all the market while firm b cannot. Demand D_H is such that $K_a < D_H < K_a + K_b$, which means that none can supply all the market without the help of the other.

Finally, when $D_V > K_a + K_b$, there is a shortage of power.

[4] The "regulated capacity payments" are a feature of some electricity systems including Spain, Argentina, Australia. However, recently they have been widely criticised (see Newbery, 1997 and OECD, 1999) and they have been abandoned in the newly designed England and Wales pool.

[5] For an analysis of shortage penalties as a function of the difference between demand and the total declared capacity, see Crampes and Creti (2001a).

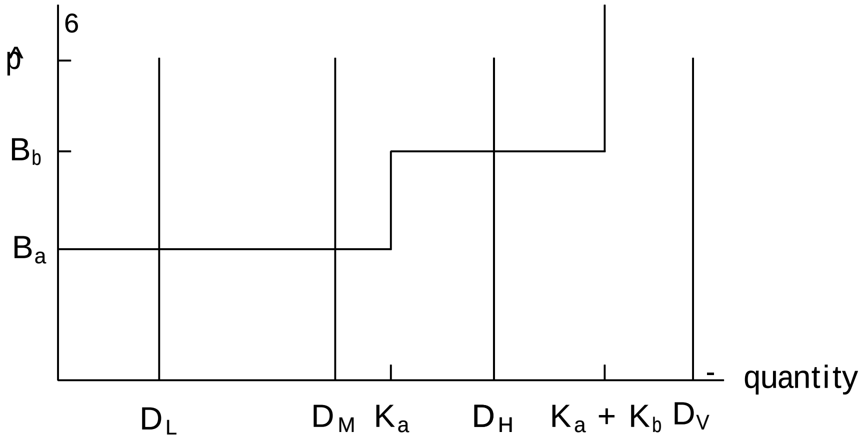

Figure 2a: …rm a is the low bidder

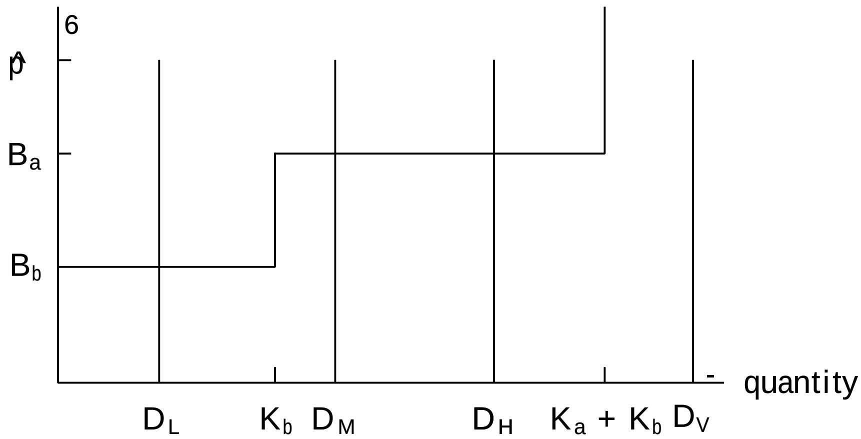

Figure 2b: …rm b is the low bidder

We observe that when demand is low, the clearing price is the bid B_i …xed by the low bidder. For a medium demand, the clearing price is …xed by the …rm with the higher generation capacity (…rm a in the case depicted here). With a high demand, the equilibrium price is the bid …xed by the high bidder. It results that, depending on the declared capacities K_a and K_b, and depending on the demand value, we have very di¤erent conditions of price competition. This also shows that it is essential to endogenise the capacity choice stage to understand the stylised facts observed in power markets.

2.2 Timing of the game and demand forecast

We consider different timings of the game, depending on whether demand is known before or after the capacity declarations by the operators. This corresponds to different institutional settings: for example, in Spain, the market operator communicates to the spot market participants the expected level of demand before the generators make their offer prices.

We first assume that, knowing the value of D, the firms play a standard two-stage game, as in Kreps-Sheinkman (1983):

i) firms a and b announce their available capacities K_a and K_b;

ii) knowing these capacities, the firms submit their bids B_a and B_b[6].

As a final step, the market operator matches demand D and the total supply function (i.e. the relation derived from the merit order) to compute the clearing price (or system marginal price) and pays the agents.

We also consider an alternative timing of the game, corresponding to the former English and actual Californian spot markets, among others. Demand forecast for the day ahead is announced to generators, and bids as well as capacity declarations are made before the true demand is known. Hence, when firms make their capacity choice, they consider demand as a stochastic variable $D \in \left[\underline{D}; \overline{D}\right]$ distributed according the distribution function $F(D)$. In this case, the timing is as follows:

i) the demand forecast is announced to the operators

ii) firms a and b announce their available capacities K_a and K_b.

iii) knowing the capacities, the firms submit their bids B_a and B_b:

In the final step, demand and supply are matched and generators are paid.

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[6] Note that this setting does not correspond totally to any real system. Actually, firms bid simultaneously on K_i and B_i in the Spanish framework. Nevertheless, when they have to announce in advance maintenance plans, it is true that $(K_a; K_b)$ are known before the choice of B_a and B_b.

3 Price competition

In this section, we determine the price equilibrium corresponding to each regime of demand, given the available capacity declared by the generators.

3.1 Low demand regime

In the low demand situation, there is no capacity constraint in generation. As the firms propose a perfectly homogeneous good, there is pure Bertrand competition that will necessarily be won by firm a since $c_a < c_b$ by hypothesis.

This simple reasoning drives the following result:

Proposition 1 When neither of the generators is capacity constrained, Bertrand competition leads to the following price equilibrium:

$$B_a^* = c_b - \epsilon \quad ; \quad B_b^* = c_b$$

and profits are

$$\pi_a = (c_b - c_a)D_L \quad ; \quad \pi_b = 0$$

Notice that ϵ is the smallest tick below c_b fixed by the rules of the market and that the system marginal price is c_b.[7].

3.2 Medium demand regime

When $K_a > D_M > K_b$, the system marginal price is the bid fixed by generator a since his capacity is the higher. Profits are respectively

$$\begin{array}{llll}
\pi_a = (B_a - c_a)(D_M - K_b) & ; & \pi_b = (B_a - c_b)K_b & \text{if } B_a > B_b \\
\pi_a = (B_a - c_a)D_M \dfrac{K_a}{K_a + K_b} & ; & \pi_b = (B_a - c_b)D_M \dfrac{K_b}{K_a + K_b} & \text{if } B_a = B_b \\
\text{and } \pi_a = (B_a - c_a)D_M & ; & \pi_b = 0 & \text{if } B_a < B_b
\end{array} \quad (1)$$

Comparing the profit functions in (1), we have the following result:

[7] To be more precise, $B_a^* = Max\{c_b - \epsilon; c_a\} = SMP$: In fact, the existence of the tick could sensitively lower generator a's profit and even make it negative, especially when the difference between the marginal costs is not very large. However, here we will not consider this problem of optimization in integer numbers.

Proposition 2 When demand is lower than the larger capacity declared available, there exists a continuum of equilibria of the price game, where the firm having the advantage in capacity bids the price cap and the competitor bids low enough avoiding the large firm to undercut. When $K_a > D > K_b$, price equilibria of the medium demand regime are as follows:

$$B_a^* = \hat{p} \quad ; \quad B_b^* \in [0; \gamma_b^M]$$

where $\gamma_b^M \stackrel{def}{=} c_a + (\hat{p} - c_a)\dfrac{D_M - K_b}{D_M}$
All the price equilibria give the same profits

$$\pi_a = (\hat{p} - c_a)(D_M - K_b) \quad ; \quad \pi_b = (\hat{p} - c_b)K_b$$

Proof. Notice that $\dfrac{D_M}{K_a + K_b} < 1$. Consequently, the best response of b is obviously to charge any price strictly below the bid of his competitor when some profit is feasible (i.e. as long as $B_a \geq c_b$) and anything that will result in no dispatching ($B_b > B_a$ is sufficient) when generation would mean losses, i.e. when $B_a < c_b$.
Concerning firm a, the choice is between the maximum price (when the highest bidder), which gives $(\hat{p} - c_a)(D_M - K_b)$ and a low bid ($B_a = B_b - \epsilon$) to provide all the market, which gives profit $(B_b - c_a)D_M$. By comparing these two profits, we determine the threshold $\gamma_b^M \stackrel{def}{=} c_a + (\hat{p} - c_a)\dfrac{D_M - K_b}{D_M}$. Therefore the best response of firm a is to bid $\hat{p}$ if $B_b < \gamma_b^M$ and $B_b - \epsilon$ otherwise. Reminding that the price is fixed by the generator with the highest capacity and that the generator bidding low is called into operation first concludes the proof. ■

In Figure 3, the best response of firm b is the shaded area and the best response of firm a is the discontinuous bold line. One can see that best responses intersect only along the bold segment in the upper part of the diagram.

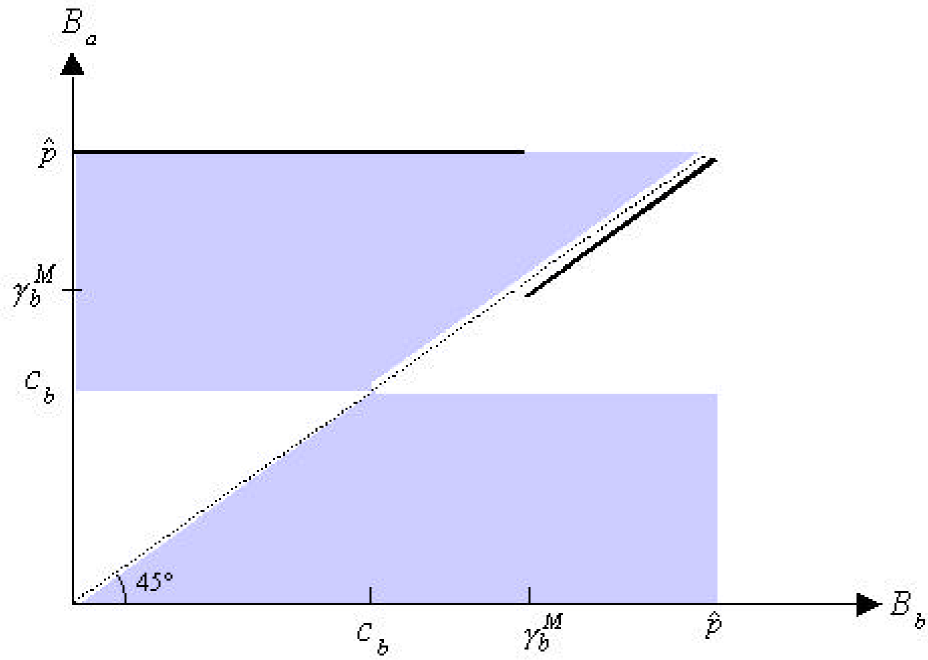

Figure 3: Price equilibria with a medium demand

Let us discuss the role of γ_b^M, that is the highest bid of firm B that induces A to be satisfied with selling the residual demand $D_M - K_b$ at the maximum price $\hat{p}$; rather than producing full capacity at the reduced price $\gamma_b^M - \varepsilon$. This threshold is the weighted average of c_a and $\hat{p}$. It is increasing with demand and decreasing with the capacity of b but independent of the capacity of a despite his role of price maker. Actually, when D_M is large and/or K_b is small, firm a faces a residual demand sufficiently high to justify not trying to compete and b benefits from the price fixed by a.

When $\gamma_b^M < c_b$, for example because b has a strong disadvantage in terms of generation cost or because the range $[K_b; K_a]$ is very narrow so that $(\hat{p} - c_a)(1 - \frac{K_b}{D_M}) < c_b - c_a$, there is an additional set of equilibria given by $B_b^* \in [\gamma_b^M; c_b]; B_a^* = B_b^* - \varepsilon$ where b bids below marginal cost and a undercuts that bid. But all these (Bertrand like) equilibria are dominated by those defined in Proposition 2: Consequently, we can discard them.

Of course, when the bidding game starts with $K_b > K_a$, firm b is the price maker. It results that the only possible equilibria[8] are

$$B_a^* \in [0; \gamma_a^M] \quad ; \quad B_b^* = \hat{p}$$

and the resulting profits are

$$\pi_a = (\hat{p} - c_a)K_a \quad ; \quad \pi_b = (\hat{p} - c_b)(D_M - K_a)$$

We see that when demand exceeds the available capacity of the smallest generator, but stays below the production possibilities of the largest operator, firms benefit from tacit collusion: the system marginal price goes up to $\hat{p}$ and at the equilibrium, both firms are called into operation, earning a large mark-up on their marginal costs. Of course, this is not ex-post efficient, since efficient dispatching would command generator a to supply the entire market.

3.3 High demand regime

Under this regime, as none of the generators is able to serve the whole demand, a and b will both be called into operation at equilibrium; therefore they are inclined toward bidding $\hat{p}$. However, the low-bidding generator will be called first and sell all of his declared capacity, which is an incentive to undecut the competitor's bid. Recall that now, the system marginal price is equal to the highest bid.

3.3.1 Equilibria in pure strategies

When demand exceeds the capacity of the larger generator, there exist equilibria in which either the larger or the smaller generator acts as the high-bidding firm, as Proposition 3 states (see figure 4).

Proposition 3 When demand is higher than the larger capacity declared available, there exists two sets of equilibria in pure strategies. Assuming $D > K_a > K_b$, one set of equilibria is:

$$B_a^* \in [0; \gamma_a^H]; \qquad B_b^* = \hat{p}$$

[8] Because $c_a < c_b$, it is always true that $\gamma_a^M \overset{def}{=} c_b + (\hat{p} - c_b)\frac{D_M - K_a}{D_M} > c_a$. Consequently, there is now only one set of equilibria.

where $\gamma_a^H \stackrel{def}{=} c_b + (\hat{p} - c_b)\dfrac{(D_H - K_a)}{K_b}$, giving profits $\pi_a = (\hat{p} - c_a)K_a$, $\pi_b = (\hat{p} - c_b)(D_H - K_a)$.
The second set is:

$$B_a^* = \hat{p}; \quad B_b^* \in [0; \gamma_b^H]$$

where $\gamma_b^H \stackrel{def}{=} c_a + (\hat{p} - c_a)\dfrac{D_H - K_b}{K_a}$, giving the profits $\pi_a = (\hat{p} - c_a)(D_H - K_b)$, $\pi_b = (\hat{p} - c_b)K_b$.

Proof. If b bids $B_b > \gamma_b^H \stackrel{def}{=} c_a + (\hat{p} - c_a)\dfrac{D_H - K_b}{K_a}$, generator a is indifferent among all the bids B_a strictly less than B_b and for $B_b \leq \gamma_b^H$; firm a is better off fixing $\hat{p}$ than fixing any other price.
Symmetrically, generator b is indifferent among all the bids B_b less than B_a when $B_a > \gamma_a^H \stackrel{def}{=} (\hat{p} - c_b)\dfrac{(D_H - K_a)}{K_b} + c_b$, and if $B_a \leq \gamma_a^H$; b prefers to fix $\hat{p}$: QED. ■

From $D_H < K_a + K_b$, it is easy to check that each firm prefers to be the low bidder so that we cannot eliminate one of these equilibria by an argument of Pareto dominance.

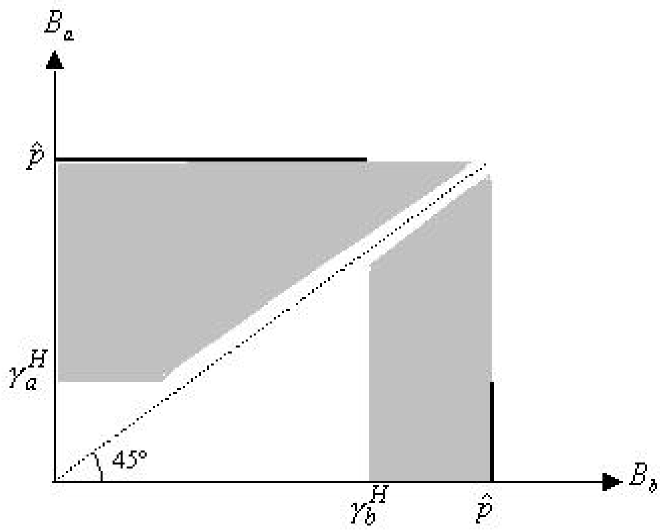

Figure 4: Price equilibria when demand is high

3.3.2 Equilibria in mixed strategies

The pure-strategy equilibria illustrate that when a firm chooses the offer price, her decision is based on two considerations: on one side, as both firms must be called into operation to serve all the demand, both of them will be despatched even bidding the highest admissible price; on the other side, bidding low enough to avoid undercutting ensures to be called into operation first in the merit order and to sell all the capacity. If its competitor plays a pure strategy, each firm knows exactly which of these considerations is relevant at a given offer price. By contrast when its competitor uses a mixed strategy, the firm cannot decide how far she has to lower her price to undercut the other firm in the market. In fact, in a mixed strategy equilibrium, the effects analyzed in the pure strategy just balance each other, so that, for a given strategy of the competitor, each firm is indifferent between all the prices over which it randomizes.

Assume that firm i's bids B_i $(i = a; b)$ are distributed on $[\underline{B}_i; \overline{B}_i]$according to some distribution function $G_i(B)$ with density $g_i(B)$[9]. The problem is to determine the distribution of probabilities $G_i(B)$ played by firm i such that player $j \neq i$ is indifferent between any bid belonging to its own support $[\underline{B}_j; \overline{B}_j]$.

Consider for example firm a. She wants to maximise her expected profit defined as follows:

$$\begin{aligned} E\pi_a(B_a; G_b(B_b)) &= \Pr[B_b < B_a](B_a - c_a)(D_H - K_b) + \int_{B_a}^{\overline{B}_b} [B - c_a]K_a dG_b(B) \\ &= G_b(B_a)(B_a - c_a)(D_H - K_b) + \int_{B_a}^{\overline{B}_b} [B - c_a]K_a dG_b(B) \quad (2) \end{aligned}$$

The first term is the expected pay-off when firm a is the price maker, while the second term is the expected revenue when b; submitting bids higher than firm a; becomes the marginal operator.

From the first order condition $\frac{\partial}{\partial B_a} E\pi_a(B_a; G_b(B_b)) = 0$, we obtain

$$g_b(B_a)[(B_a - c_a)(D_H - K_a - K_b)] + G_b(B_a)(D_H - K_b) = 0 \qquad (3)$$

[9] For alternative assumptions on firms' bids with mixed-strategy equilibria on a discrete support, see Crampes and Creti (2001b).

As the above condition must be true for any bid of generator b on the interval $[\underline{B}_b, \overline{B}_b]$, $\frac{g_b(B)}{G_b(B)} \equiv \frac{(D_H - K_b)}{(K_a + K_b - D_H)} \frac{1}{(B_a - c_a)}$ must be viewed as a differential equation. This equation must be solved to obtain the distribution function $G_b(B_b)$ such that, if b bids on the interval $[\underline{B}_b, \overline{B}_b]$ following that function, a is indifferent between all her bids, as her expected profit will be constant all along the support of her bids.

The solution of the differential equation is given by:

$$G_b(B_b) = \left(\frac{B_b - c_a}{C_b}\right)^{\delta_b} \quad \text{where} \quad \delta_b = \frac{(D_H - K_b)}{(K_a + K_b - D_H)} \tag{4}$$

Note that $\delta_b > 0$, as $K_b < D_H < K_a + K_b$

The constant C_b can be determined as follows. As $G_b(B_b)$ must be equal to 0 at the lowest bound of the interval, the lowest admissible bid has to be $\underline{B}_b = c_a$; moreover, $G_b(B_b)$ must be equal to 1 when generator b bids the highest admissible bid. This implies $C_b = \overline{B}_b - c_a$.

We conclude that if firm b's offer price is distributed over the interval $\left[c_a, \bar{B}_b\right]$, replacing $G_b(B_b) = \left(\frac{B_b - c_a}{\bar{B}_b - c_a}\right)^{\delta_b}$ in the expression for $E\pi_a(B_a, G_b(B_b))$, we obtain the expected profit of firm a:

$E\pi_a(B_a, G_b(B_b)) = (\bar{B}_b - c_a)(D_H - K_b)$ and, as required, it is independent of B_a.

We can follow the same argument to determine the distribution of probability of firm a. Finally, the mixed strategy equilibrium can be summarized as follows:

Proposition 4 When demand is higher than the larger capacity declared available, the equilibrium in mixed-strategies over a continuous support is as follows:

$$B_i \sim G_i(B_i) \text{ on } [c_j, \bar{B}_i]$$

$$\text{where} \quad G_i(B_i) = \left(\frac{B_i - c_j}{\bar{B}_i - c_j}\right)^{\delta_i} \quad ; \quad \delta_i = \frac{(D_H - K_i)}{(K_i + K_j - D_H)}$$

The expected profits are:

$$E\pi_i = (\bar{B}_j - c_i)(D_H - K_j)$$

with $i, j = a, b; \quad i \neq j$.

The fact that the supports of firms' bid do not coincide is not surprising, given marginal cost asymmetry. Indeed, Allen and Hellwig (1993) and Dencercke and Kovenock (1996) show that when firms have asymmetric capacities or production costs there is no reason why we should be able to fix a single parameter so as to satisfy the two boundary conditions of the mixed-strategy distributions.

One particular case that can be useful for the analysis of complex sequential games is when $\bar{B}_i = \bar{p}$ for both firms. In this case, expected profits are:

$$E\tilde{\pi}_a = (\bar{p} - c_a)(D_H - K_b) \quad ; \quad E\tilde{\pi}_b = (\bar{p} - c_b)(D_H - K_a) \qquad (5)$$

The choice of $\bar{p}$ as the upper bound of the support can be justified not only in terms of Pareto-dominance for generators but also considering game repetition and implicit collusion.

This mixed-strategy equilibrium resembles the one obtained by von der Fehr and Harbord (1993) in a context where firms have asymmetric variable production costs, equal exogenous capacities (normalized to one) and demand can be either equal to only one firm capacity, or to the sum of both firms' capacity.

However, with respect to the model by von der Fehr and Harbord (1993), our model places more emphasis on firms' capacities, that will be endogenised in the following stage of the game. Moreover, although the expected profit for firm i does not depend on K_i, capacities also play an important role in the characterization of the first-order stochastic dominance of the mixed strategies. On the one hand, straightforward calculations show that $K_a < (>) K_b \iff \delta_a > (<) \delta_b$. On the other hand, under the hypothesis $c_a < c_b$; it is always true that for $B < \bar{p}$:

$$\frac{B - c_a}{\bar{p} - c_a} > \frac{B - c_b}{\bar{p} - c_b} \qquad (6)$$

Therefore, $K_a < K_b$ is a sufficient condition for $G_a(B) \geq G_b(B)$: if the low-cost firm also declares the smallest capacity, her strategy profile first-order stochastically dominates that of the high-cost firm, which also exhibits the capacity advantage. Indeed, in expected terms, the low-cost/small-capacity firm submits bids higher than the high-cost/large capacity firm: hence we cannot exclude equilibria where the inefficient firm b bids lower than a and sells all of his capacity, being called into operation first. This is not ex-post efficient, as the social costs of generation are not minimized.

However, if the low cost firm announces the highest available capacity (and it is most likely), the first order stochastic dominance is not guaranteed.

Regarding the system marginal price, the following corollary applies:

Corollary 5 The system marginal price cannot be lower than the marginal cost of the least-efficient firm.

Proof. In the high demand regime, the SMP is fixed by the highest bidder. Even if, by Proposition 4, firm b (the inefficient generator) can bid on the interval $[c_a; c_b]$ with a positive probability, the SMP will be the bid of firm a (the highest bidder), which is less than c_b with zero probability. ■

As a consequence of the above corollary, a possible outcome of the game is the Bertrand equilibrium, but as both generators are needed to serve the demand, the least efficient firm is not excluded from the market.

In a model where firms have asymmetric exogenous capacities and production costs are sunk, Allen and Hellwigh (1993) also prove that, given the set of competitive prices at which market demand is equal to aggregate production capacity (which, depending on the properties of the market demand function, can also degenerate to a unique point), in equilibrium firms do not charge prices below the highest competitive price. This is similar to our finding that the system marginal price does not fall below c_b:

In some sense, one can say that the inefficient firm is protected by the efficient one against losses due to low bids. The possibility that a firm may bid below her marginal costs is a result that should be taken into account when econometricians try to evaluate the cost function using bid records.

3.4 Price game under efficient rationing

Under the assumption that the generators' marginal costs are known, an attracting alternative rule to break ties is efficient rationing: when firms bid the same price, the low cost firm is called into operation first, and the competitor is left serving the residual demand. This is the rule generally used in Bertrand-Edgeworth competition models, for example in Deneckere and Kovenock (1996).

In our model, the introduction of efficient rationing rule does not affect the equilibria of the price game, as Proposition 6 shows:

Proposition 6 When efficient rationing is used to break ties in offer prices, the price equilibrium and the profits for all the competition regimes are given by Proposition 1 to 5 and remain unchanged.

Proof. Under low demand regime, the efficient firm does not need to undercut her competitor's bids to obtain the whole demand. Equilibrium profits are the same as in Proposition 1

Under medium demand regime, for $K_a > D_M > K_b$, firms' profits are like in (1), except for $B_a = B_b$; where now we have:

$$\pi_a^{ER} = (B_a - c_a)D_M \quad ; \quad \pi_b^{ER} = 0 \quad \text{if} \quad B_a = B_b \tag{7}$$

where ER remind the efficient rationing rule assumption.

Consequently, the best response of b is to announce any bid below B_a as long as $B_a \geq c_b$: Concerning firm a, the best response is the same as in Proposition 2, except that now she does not necessarily undercut the competitor's bid to attract all the demand when $B_b \geq \gamma_b^M$: Therefore, the price equilibrium and the profits of firms are unchanged.

In the high demand regime, when firms play pure strategies, bidding the same price as the competitor is a strongly dominated strategy for generator b and a weakly dominated strategy for generator a (both under the hypothesis $K_a > K_b$ and $K_a < K_b$). Therefore, we can eliminate it. Note also that adopting the efficient rationing rule does not affect mixed strategy equilibria when calculated on a continuous support, as obviously the probability of ending up on a single point is of zero measure. ■

Efficient rationing is not a remedy against tacit collusion in the price game, as potential advantages from calling into operation first the low cost firm are totally offset by the uniform price mechanism.

4 Capacity choice

The multiplicity of equilibria in the second stage of the game, especially in the high demand regime, creates some difficulties for the analysis of capacity choice. Hereafter, we assume that when demand is high, firms play in mixed strategy and their expected (symmetric) profits are given by (5).

Remind that for the game in capacity, we assume that adding-up the installed capacity of each firm is sufficient to provide the whole demand, and that the low generation cost firm is also the one with the highest installed capacity. Also remember that there is no severe penalty for the unserved demand: firms are just paid their variable costs.

4.1 Deterministic demand

To analyse the capacity game when demand is known by the operators at the moment they announce their capacity availability, as in the Spanish spot market, we start describing the choice process when neither of the generator is capacity constrained, and then we extend the analysis to the case of medium and high demand forecast.

4.1.1 Low demand: none of the generators is capacity constrained

We first consider the case where the ISO announces that demand will be $D < \overline{K}_b$, which means that both firms have enough capacity to supply the whole demand individually.

- As we can see on Figure 1, if firm a thinks that b will declare $K_b > D$, for $K_a < D$ it will be in a D_M regime with profits $(\bar{p} - c_a)K_a$ and for $K_a \geq D$, it will be in a D_L regime with profits $(c_b - c_a)D$:
- if firm a thinks that b will declare $K_b \leq D$, with $K_a < D - K_b$, it will be in the D_V regime, earning 0. With $D - K_b \leq K_a < D$, it will be in the D_H regime, earning $E\hat{\pi}_a = (\bar{p} - c_a)(D - K_b)$. For $K_a > D$ it will be in the D_M regime, still earning $\pi_a = (\bar{p} - c_a)(D - K_b)$ since it will be dispatched for the residual demand $D - K_b$:

Consequently, the best response of firm a is $K_a = D - \varepsilon$ for $K_b > D$; and $\overline{K}_a \geq K_a \geq D - K_b$ for $K_b \leq D$ (Figure 5).

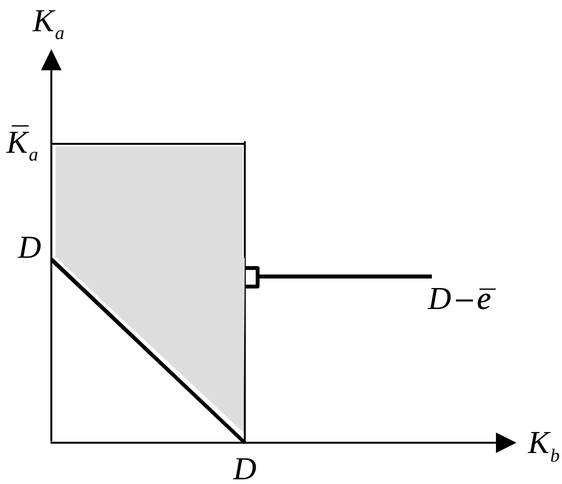

Figure 5: Firm a's best response function in the capacity game

Symmetrically, the best response function of firm b is to bid $K_b = D - \varepsilon$ for $K_a > D$ and $\overline{K}_b \geq K_b \geq D - K_a$ otherwise. Consequently, we can establish the following:

Proposition 7 If none of the generators is naturally capacity constrained ($D < \overline{K}_b < \overline{K}_a$), there are three families of equilibria for the capacity choice game:
i) $(K_a^*; K_b^*) = \{K_a; K_b | K_a \leq D; K_b \leq D; K_a + K_b \geq D\}$
ii) $\overline{K}_a \geq K_a^* \geq D; \quad K_b^* = D - \varepsilon$
iii) $K_a^* = D - \varepsilon; \quad \overline{K}_b \geq K_b^* \geq D$

Proof. The proof is directly obtained by intersecting the two best response functions. ■

In the type i) equilibria, we are in a high demand regime. Given that the expected profit of firm i defined in (5) is decreasing in the capacity of firm j, all the combinations such that $K_a^* + K_b^* = D$ are Pareto superior for generators.

The other two sets of equilibria give medium demand regime. In type ii) with firm a having an advantage in capacity, the expected profits are respectively $\pi_a^* = 0$ and $\pi_b^* = (\bar{p} - c_b)D$: Symmetrically, in the type iii) equilibria, the profits are $\pi_a^* = (\bar{p} - c_a)D$, $\pi_b^* = 0$:

We see that the elimination of equilibria by a Pareto-dominance argument is impossible: each of the generators is better-off when the other has the advantage in capacity and fixes the SMP at $\bar{p}$. Moreover, each generator prefers to sell all capacity in the D_M regime than earning the D_H profit level with only one fraction of the demand. These remarks imply i) that mixed strategies equilibria are very likely to exist and ii) that there is a strong incentive for the generators to agree on market sharing. Note that if they can agree to coordinate their capacity bids somewhere in the set $K_a + K_b = D$, generator a who has a cost advantage can use the credible threat to bid $K_a = D$ that guarantees at worst $\pi_a = (c_b - c_a)D$ in order to obtain a capacity advantage and the consequent profit advantage. He will deny any agreement such that $K_a \le \frac{c_b - c_a}{\bar{p} - c_a} D$. All these implicit or explicit agreements are obviously forbidden. But the model shows that the uniform price system gives strong incentives to transform a natural low demand regime into a medium or high demand regime by withholding capacities.

4.1.2 Medium and High demand forecast

The generators' reaction functions are symmetrical as long as capacity constraints are not binding. When $\overline{K}_b$ and $\overline{K}_a$ become binding, leading to natural medium and high demand regimes, some equilibria of the game where generators are unconstrained will be eliminated. If the ISO announces a medium demand regime ($\overline{K}_b < D < \overline{K}_a$), the capacity constraint of the least efficient generator puts downward pressure to firms' choice:

Proposition 8 If the smallest generator is naturally constrained, there are two families of equilibria:
i) $(K_a^*, K_b^*) = \{K_a, K_b \mid K_a \le D;\ K_b \le \overline{K}_b;\ K_a + K_b \ge D\}$
ii) $\overline{K}_a \ge K_a^* \ge D; \quad K_b^* = \overline{K}_b$.

Proof. The proof is similar to the proof of Proposition 7. ■

Type i) equilibria are similar to those in Proposition 7 but limited to a smaller set. By contrast, in type ii) equilibria, firm a now receives $\pi_a^* = (\bar{p} - c_a)(D - \overline{K}_b)$ instead of zero. There is no type iii) equilibrium.

This means that the set of conflicting Pareto superior equilibria is reduced to the segment $K_a^* + K_b^* = D$ with $K_b \in (0, \overline{K}_b)$. If generators can agree on capacity bids, and if $\overline{K}_b$ is low, (formally if $\overline{K}_b < \frac{(\bar{p} - c_b)D}{\bar{p} - c_a}$) firm a has a better position in the negotiation round since at worst she can obtain the medium regime profit.

Finally, when the ISO announces a natural D_H regime ($\overline{K}_a < D < \overline{K}_a + \overline{K}_b$), only high and very high strategic demand regime are feasible ex-post, but the common interest of the generators is to avoid the D_V area:

Proposition 9 If both generators are naturally constrained there exists one family of equilibria:
i) $(K_b^*; K_a^*) = \{K_a; K_b | K_a \le \overline{K}_a; K_b \le \overline{K}_b; K_a + K_b \ge D\}$.

Proof. The proof is similar to the proof of Proposition 7. ■

Our results point out that knowing demand at the time where capacity choice is made has a important announcement effect: when generators are unconstrained ex ante, the opportunity of withholding capacity, instead of playing as Bertrand competitors, is extremely appealing. In the worst case, firms create the conditions for the high demand regime to occur. At the equilibrium where generator i declares $K_i^* = D - \varepsilon$, which is only slightly below the announced demand, and the competitor chooses the capacity $K_j^* > D$, clearly only one operator withholds, but the SMP is the highest admissible price. Even if the ISO announces the medium demand regime, at equilibrium the D_H regime occurs, although the constrained generator may declare available his installed capacity; the only case where a possible outcome is $K_a^* = \overline{K}_a$ and $K_b^* = \overline{K}_b$ is when generators know from the beginning that each of them is constrained. In this case, absent capacity costs, withholding may not occur but it remains very likely because it is weakly Pareto superior for firms.

In all the cases, the set $K_a^* + K_b^* = D$ (with or without $K_i^* \le \overline{K}_i$) is very attractive for the generators but it can be sustained only through repetition arguments which are obvious in the real world but beyond the scope of the present paper.

4.2 Stochastic demand

In the timing of the game such that when generators choose their capacity, demand is a random variable (i.e. $D \in [\underline{D}; \overline{D}]$ is distributed according to the strictly increasing function $F(D)$), it is important to compare the support of the demand function and the installed capacity constraints, as they both have an impact on the competitive environment in which generators make their capacity bids.

4.2.1 None of the generators is capacity constrained

This case is extremely similar to the one detailed in Section 4.1. The smallest generator is able to serve the whole demand, even in peak hours: $\underline{D} < \overline{D} \leq \overline{K_b} < \overline{K_a}$; and generators will determine their capacities with respect to expected demand $E(D) = \int_{\underline{D}}^{\overline{D}} D dF(D)$: Absent any risk aversion, Proposition 7 remains true in terms of expected value of demand.

4.2.2 Peak hours capacity constraints

The case where none of the generators is constrained off-peak, but both of them must be called into operation to satisfy peak demand, that is $\underline{D} \leq \overline{K_b} < \overline{K_a} \leq \overline{D}$ represents the most realistic and interesting situation[10].

In order to calculate firms' reaction functions, we consider expected profits, assuming that generators are reimbursed their costs when joint declared capacity is insufficient to serve demand. Firm i's expected profit can be written as follows:

$$E(\pi_i) = \int_{\underline{D}}^{K_i+K_j} \pi_i(K_i, K_j, D)dF(D) + [1 - F(K_i + K_j)](\pi_i = 0)$$

$$K_i \leq \overline{K_i} \quad i, j = a, b \quad i \neq j \quad (8)$$

From the analysis of Section 3, we know that $\pi_i(K_i, K_j, D)$ is very dependent on the type of competition regime, and consequently incentives to release or to restrict capacity are quite different:

i) to avoid Bertrand competition (that occurs when each firm is at least able to serve off peak demand), generators have the incentive to declare less than the competitor and to restrict capacity;

ii) when the medium demand regime occurs, each agent prefers to leave the other with the capacity advantage: the biggest generator will make the SMP (equal to the price-cap) and the smallest will sell all his declared capacity at the maximum price;

[10] In the cases where one or both generators are constrained during off-peak hours, Bertrand competition never occurs. These games will not be detailed here, but, as in the case of deterministic demand, the results can be easily obtained by truncating the generators best reaction functions to take into account firms' capacity constraints.

iii) finally, if demand is high, the profit of generator i is independent from K_i.

The distribution function $F(D)$ weights all these different competition regimes. The choice of capacity results from a simple trade-off: if firm i increases her capacity, the probability of being "punished" for power shortage decreases, but at the same time the Bertrand competition regime becomes relatively more likely.

Lemma (10) suggests that candidates to the equilibrium of the capacity game must be found when both generators offer at least $\underline{D}$:

Lemma 10 Whatever K_j (with $i,j = a,b$ $i \neq j$), the best response function of firm i is $K_i(K_j) \geq \underline{D}$.

Proof. See the Appendix. ■

As a byproduct of Lemma (10), we can calculate firm i best response when j announces $K_j \leq \underline{D}$.

Corollary 11 The best response function of firm i for $K_j \leq \underline{D}$ (with $i,j = a,b$ $i \neq j$) is $K_i(K_j) = \overline{K_i}$ if $\overline{K_i} < \overline{D} - K_j$ and any value between $\overline{D} - K_j$ and $\overline{K_i}$ otherwise.

Proof. See the Appendix. ■

It results that the minimum total capacity available on the market is always higher or equal to $2\underline{D}$, so that all the competition regimes occur with positive probability. It remains to determine the best response of firm i to $K_j > \underline{D}$.

The relative distance between $\underline{D}$ and $\overline{D}$ has an impact on the intensity of competition: in fact, the "precision of demand forecast" determines the capacity combinations such that Bertrand competition, medium, high and very high demand may all have positive probability. If demand forecast is accurate, in the sense that the support where demand varies is not too large ($\overline{D} \leq 2\underline{D}$), the various competition regimes occur only if there is joint excess capacity with respect to peak demand ($K_a + K_b > \overline{D}$); by contrast, if the support of demand variability is large ($\overline{D} > 2\underline{D}$), there is a positive probability of having all the competition regimes both when $K_a + K_b < \overline{D}$ and $K_a + K_b > \overline{D}$. Consequently, the risks of voluntary power shortage can be different depending on the precision of demand forecast.

4.2.3 Peak hours capacity constraints: large demand support

In order to determine …rms' best reaction functions, it is useful to refer to the following …gure:

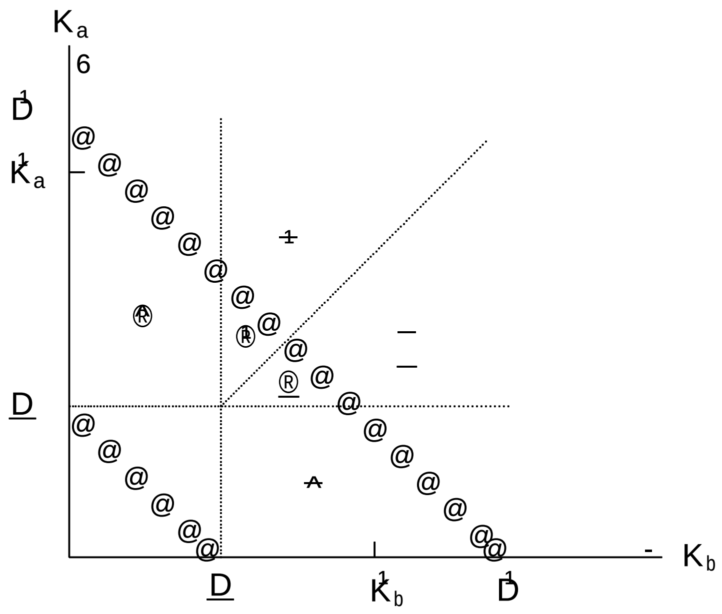

Figure 6. Pro…t zones when $\overline{D} > 2\underline{D}$

In the zones $\overline{®}$ and $\overline{¯}$ generator a has the capacity advantage, while the situation is reversed in the areas $\underline{®}$ and $\underline{¯}$. Also notice that in the subregions $\overline{®}$ and $\underline{®}$ total declared capacity does not cover peak demand (except on line $\overline{D}$), while in the subregions $\overline{¯}$ and $\underline{¯}$; the total declared is su¢cient to satisfy exceeds $\overline{D}$:

Firm a We calculate …rm a's expected pro…t in each of the subregions described by Figure (6).

Subregion $\overline{b}$
In zone $\overline{b}$; we know from Corollary (11) that …rm a's best response is $K_a^{b}(K_b) = \overline{K_a}$ if $\overline{K_a} < \overline{D}$ ¡ K_b and any value between $\overline{D}$ ¡ K_b and $\overline{K_a}$.

Subregion $\bar{\alpha}$

In the subregion $\bar{\alpha}$; a's expected profit is as follows.

$$E(\pi_a^{\bar{\alpha}}) = (c_b - c_a)\int_{\underline{D}}^{K_b} D dF(D) + (\bar{p} - c_a)\int_{K_b}^{K_a} (D - K_b) dF(D)$$
$$+(\bar{p} - c_a)\int_{K_a}^{K_a+K_b} (D - K_b) dF(D) \qquad (9)$$

$E(\pi_a^{\bar{\alpha}})$ is an increasing function of K_a, hence $K_a^{\bar{\alpha}}(K_b) = \bar{D} - K_b$ if $\bar{D} - K_b \le \overline{K_a}$ and $\overline{K_a}$ otherwise.

Subregion β

Lemma (10) ensures that when firm a thinks $\underline{D} < K_b \le \overline{K_b}$, firm a expected profit is an increasing function of her declared capacity, implying $K_a^{\beta}(K_b) = \underline{D}$:

Subregion $\underline{\alpha}$

In the subregion $\underline{\alpha}$; generator a's expected profit is:

$$E(\pi_a^{\underline{\alpha}}) = (c_b - c_a)\int_{\underline{D}}^{K_a} D dF(D) + (\bar{p} - c_a)\int_{K_a}^{K_b} K_a dF(D) \qquad (10)$$
$$+(\bar{p} - c_a)\int_{K_b}^{K_a+K_b} (D - K_b) dF(D)$$

The sign of $\frac{\partial E(\pi_a^{\underline{\alpha}})}{\partial K_a}$ is ambiguous. Firm a is now facing a trade-off: $K_a < K_b$ insures generator a that his profit in the medium demand regime is an increasing function of K_a; however increasing capacity also increases the probability to be in the low demand regime. When the incentive to increase capacity dominates, $\frac{\partial E(\pi_a^{\underline{\alpha}})}{\partial K_a}$ is positive for any K_a. The trade-off definitely depends on the shape of the function $F(.)$:

Lemma 12 F weakly convex is sufficient for $\frac{\partial E(\pi_a^{\underline{\alpha}})}{\partial K_a}$ to be positive $\forall K_a$ in the region $\underline{\alpha}$:

Proof. We calculate:

$$\begin{aligned} \frac{\partial E(\pi_a^{\alpha})}{\partial K_a} &= (c_b - c_a)K_a F'(K_a) + \\ &+ (p - c_a)[F(K_b) - F(K_a)] + (p - c_a)K_a[F'(K_a + K_b) - F'(K_a)] \end{aligned} \tag{11}$$

Because $c_b > c_a$, $K_b > K_a$ and $F'(.) > 0$, the two first terms of the derivative are positive. The sign of the third term of the RHS depends on the second derivative of $F(.)$. We see that $F''(.) \geq 0$ is sufficient for $\frac{\partial E(\pi_a^{\alpha})}{\partial K_a} > 0 \; \forall K_a$. ■

Consequently, under Lemma (12), the best response function of firm a in area $\underline{\alpha}$ is the upper frontier of the subregion.

Subregion γ

In this area, a earns:

$$E(\pi_a^{\gamma}) = (c_b - c_a)\int_{\underline{D}}^{b} D dF(D) + (p - c_a)\int_{K_b}^{\bar{D}} (D - K_b) dF(D) \tag{12}$$

$E(\pi_a^{\gamma})$ is clearly independent of K_a; consequently, a will choose any $K_a \geq \max(\bar{D} - K_b, \bar{K_a})$.

Subregion β

Expected profit of firm a is:

$$\begin{aligned} E(\pi_a^{\beta}) &= (c_b - c_a)\int_{\underline{D}}^{a} D dF(D) + (p - c_a)\int_{K_a}^{b} K_a dF(D) \\ &\quad + (p - c_a)\int_{K_b}^{\bar{D}} (D - K_b) dF(D) \end{aligned} \tag{13}$$

Since

$$\begin{aligned} \frac{E(\pi_a^{\beta})}{\partial K_a} &= (c_b - c_a)K_a F'(K_a) + \\ &\quad (p - c_a)[F(K_b) - F(K_a)] - (p - c_a)K_a F'(K_a) \end{aligned} \tag{14}$$

is negative at $K_a = K_b$; even if $F(.)$ is convex, equation (15) cannot be always positive. We have now the possibility of an interior solution to define the best response function of firm a, which is given below:

$$K_a^{\beta}(K_b) = \arg\{(\bar{p} - c_a)[F(K_b) - F(K_a)] = (\bar{p} - c_b)K_a F'(K_a)\} \qquad (15)$$

The qualitative study of the reaction function indicates that the best response function is positively sloped[11]. It passes through the frontier to subregion α at the point defined as follows:

$$\tilde{K}_b = \arg\left\{K_a^{\beta}(K_b) = \overline{D} - K_b\right\} \qquad (16)$$

To illustrate the interior solution given by (15), suppose that $F(D) = \frac{D - \underline{D}}{\overline{D} - \underline{D}}$; hence:

$$K_a^{\beta}(K_b) = \frac{\bar{p} - c_a}{2\bar{p} - c_a - c_b} K_b \qquad (17)$$

Equation (17) is solution only if $K_a^{\beta}(K_b) \geq \overline{D} - K_b$; that is for:

$$K_b \geq \tilde{K}_b = \frac{2\bar{p} - c_a - c_b}{3\bar{p} - 2c_a - c_b}\overline{D} \qquad (18)$$

The analysis of firm a's expected profit in the different subregions shows that if b is sufficiently low ($K_b \quad \overline{D}/2$) and $F(.)$ is convex[12], a has the incentive to declare to increase her capacity from the bottom frontier of region α (that is from capacity declarations equal to off peak demand) to the upper frontier of region β, which is given by installed capacity $\overline{K_a}$. For higher values of K_b, a's best response can be either negatively or positively sloped. In fact, convexity of the distribution function implies that the threat of incurring the shortage penalty overcomes the anti-competitive behavior of avoiding Bertrand competition by restricting capacity. Hence, as long as the shortage penalty has positive probability, firms' capacities are strategic substitutes. However, when the risk of ending-up with zero profit is excluded, the incentive to be the follower in the D_M regime may play a more important

[11] One can check that sign $K_a^{\beta\prime}(K_b) = $ sign $F'(K_b) > 0$.

[12] Apart from the uniform function $[0;1]$, an example of convex distribution function is the beta law $B(p;q)$ with $p > 1; q > 1$.

role, making firms' capacities strategic complements. Hence, for large enough capacity declarations of b, there is an endogenous switch from the strategic substitute to the strategic complements regime.

Firm a's best response is summarised by the following Lemma:

Lemma 13 If the demand distribution is uniform or skewed toward high realization of demand, a's best response function is as follows:

$$K_a(K_b) = \begin{cases} \overline{K_a} & \text{if } 0 \le K_b \le \overline{D} - \overline{K_a} \\ K_a \in \left[\overline{D} - K_b; \overline{K_a}\right] & \text{if } \overline{D} - \overline{K_a} < K_b \le \frac{\overline{D}}{2} \\ \overline{D} - K_b & \text{if } \frac{\overline{D}}{2} < K_b \le \hat{K}_b \\ K_a^{\beta}(K_b) \text{ defined by (15)} & \text{if } \hat{K}_b < K_b \le \overline{K_b} \end{cases}$$

Firm b Concerning firm b, the only difference is that when the low demand regime occurs, profits are zero as firm a provides all the energy demand at the positive margin $(c_b - c_a)$. Consequently, if $F(.)$ is convex, b's best response function is symmetric to the one of firm a as long as $K_a \le \frac{\overline{D}}{2}$ (that is, in subregions b, α, β).

In region β, the qualitative analysis of the best reaction function is similar to that of firm a, except that neither the marginal cost parameter nor the price-cap enter that function: the best choice of b will be to bid:

$$K_b^{\beta}(K_a) = \arg\{[F(K_a) - F(K_b)] = K_b F'(K_b)\} \quad (19)$$

provided that $K_a > \hat{K}_a$, where $\hat{K}_a$ is defined by:

$$\hat{K}_a = \arg\left\{K_b^{\beta}(K_a) = \overline{D} - K_a\right\} \quad (20)$$

The reaction function defined by (20) has a positive slope (sign $K_b^{\beta\prime}(K_b)$ = sign $F'(K_a) > 0$). When $F(D)$ is uniform, $K_b^{\beta}(K_a) = \frac{K_a}{2}$ and $\hat{K}_a = \frac{2}{3}\overline{D}$.

Equilibrium We can now characterise the equilibrium:

Proposition 14 If generators are capacity constrained during peak hours, the support of demand variability is large and $F(.)$ is uniform or skewed rightward, there exist three families of equilibria for the capacity game:

i) $(K_a^*; K_b^*) = \{K_a; K_b \mid K_a \geq \hat{K}_a;\ K_b \geq \hat{K}_b;\ K_a + K_b = \overline{D}\}$
ii) $(K_a^*; K_b^*) = \{K_a; K_b \mid K_a = K_a^{\beta}(K_b);\ K_b \in (\hat{K}_b; \overline{K_b}]\}$
iii) $(K_a^*; K_b^*) = \{K_a; K_b \mid K_a \in (\hat{K}_a; \overline{K_a}];\ K_b = K_b^{\beta}(K_b)\}$
where $\hat{K}_a$ and $\hat{K}_b$ are defined respectively by equation (20) and (16); $K_a^{\beta}(K_b)$ and $K_b^{\beta}(K_b)$ are given by (15) and (19).

Proof. The proof is directly obtained by intersecting the two best response functions. ■

The equilibrium of the game ensures that shortage never occurs, but withholding is not excluded. In particular, both generators restrict capacity in type i) equilibria[13], unless $\overline{D} = \overline{K_a} + \overline{K_b}$: in this case, knowing that values close to peak hours are very likely, generators can avoid the shortage penalty only declaring available all their installed capacity.

In the family of equilibria ii), firm b has the capacity advantage and her choice does not depend on the choice of a, this latter defined by (15). The most efficient firm always withholds, as K_a^* is at best equal to $\frac{\overline{D}}{2}$. If generators' installed capacities are significantly different (that is $\overline{K_b} \geq \frac{\overline{D}}{2}$); this family of equilibria does not exist at all.

Equilibria ii) and iii) are symmetric: in this latter set, a's choice does not depend on the choice of b : the most efficient firm might declare $\overline{K_a}$, as she is the largest generator, while the least efficient firm restricts capacity (see equation (19)).

4.2.4 Peak hours capacity constraints: large demand support and concavity of $F(.)$

As firms' reaction functions depend on $F(.)$ skewness, and Lemma (12) only provides a sufficient condition, we cannot exclude that if $F(.)$ is concave[14], that is if demand is unlikely to be in the upper part of the range $(\underline{D}; \overline{D})$; the last term of $\frac{\partial E(\pi_a^{\alpha})}{\partial K_a}$ (see equation (11)) might be high enough in absolute value to offset the two positive terms.

In the Appendix we show that for $F(.)$ strongly concave, firm a does not have incentive to increase her declared capacity (that is, equation (11) is

[13] Notice that when making capacity available is costly, equating marginal revenues to marginal costs, firms would declare less than peak demand. Therefore, shortage could not be excluded.

[14] An example of concave $F(.)$ is the Pareto distribution or the beta law with parameters $p > 1; q > 1$.

negative at $K_a = K_b$): The best response of firm a is the value of K_a such that equation (11) is zero or negative. Let denote this best response function as $K_a^{\underline{\alpha}}(K_b)$: The qualitative study of $K_a^{\underline{\alpha}}(K_b)$ also shows that in region $\underline{\alpha}$:

- if $\frac{\partial E(\pi_a^{\underline{\alpha}})}{\partial K_a} > 0$ at $K_a = \underline{D}$; then $K_b > K_a^{\underline{\alpha}}(K_b) > \underline{D}$;
- if $\frac{\partial E(\pi_a^{\underline{\alpha}})}{\partial K_a} \leq 0$ at $K_a = \underline{D}$; the best choice of a is $\underline{D}$;

$\hat{K}_b$ is the threshold value of K_b that separates the two solutions. Moreover, if equation (11) is negative in region $\underline{\alpha}$; then also $\frac{\partial E(\pi_a^{\bar{\alpha}})}{\partial K_a}$ defined by equation (14) is negative: hence firm a's reaction function is $K_a^{\underline{\alpha}}(K_b) = K_a^{\bar{\alpha}}(K_b) = \underline{D}$:

As in the case of convexity of $F(:)$; there is the possibility of an endogenous switch of different competition regimes: when $F(:)$ is strongly skewed leftward, either firm a's best response function is positively sloped or it sticks to off peak value of demand[15].

While in the case analysed in Section 4.2.3 this switch occurs at the upper frontier of region α; now this point is translated to the bottom frontier, at the border to region β. As in the case of deterministic demand, knowing that low demand realisations are very likely to occur, firms try to transform a natural low demand regime into high demand regime by withholding capacities. But by Lemma (10), in region β, firm a expected profit is always increasing in K_a; hence any incentive to restrict capacity is "transformed" into declaring $K_a = \underline{D}$; regardless the choice of b: This holds until K_b becomes large enough for the D_M regime to become attractive, hence generator a increases capacity as long as the competitor's capacity increases. Hence, if $F(:)$ is strongly concave, there is an endogenous switch from a competition regime where capacities are independent to the competition regime where they are strategic complements.

The best response function of b is symmetric to $K_a^{\underline{\alpha}}(K_b)$, except for the fact that b earns zero when Bertrand competition occurs:

We conclude that:

[15] Except for $K_b = \underline{D}$: Notice that by Lemma (10), when the competitor declares $K_b = \underline{D}$; firm a will declare $K_a > \underline{D}$:

Proposition 15 If $F(.)$ is strongly skewed leftward, support of demand variability is large and marginal generation costs are relatively close, there exists three families of equilibria for the capacity game:

i) if $\hat{K}_i^{*} \geq \overline{D} - \underline{D}$ and $\overline{K_i} \geq \overline{D} - \underline{D}$; $\hat{K}_j < \overline{K_j}$ $i,j = a,b$ $i \neq j$
$K_i^* \in [\overline{D} - \underline{D}; \overline{K_i}]$ $K_j^* = \underline{D}$.

ii) if $\hat{K}_{*} \geq \overline{D} - \underline{D}$ and $\overline{K_i} \geq \overline{D} - \underline{D}$ $\forall i = a,b$:
$K_i^* \in [\overline{D} - \underline{D}; \overline{K_i}]$ $K_j^* = \underline{D}$
$K_i^* = \underline{D}$ $K_j^* \in [\overline{D} - \underline{D}; \overline{K_j}]$: with $i,j = a,b$ $i \neq j$

iii) if $\overline{K_i} < \hat{K}_i < \overline{D} - \underline{D}$ and $\hat{K}_j < \overline{K_j}$ $i,j = a,b$ $i \neq j$:
$K_i^* = \overline{K_i}$ $K_j^* = \underline{D}$

If $\hat{K}_i < \overline{K_i}$ $\forall i = a,b$, there is no equilibrium.

Corollary 16 If $F(.)$ is weakly skewed leftward and firms' marginal costs are very different, Proposition 14 holds.

Proof. See the Appendix. ■

Notice that equilibria exist if for at least one firm the threshold value is either larger than $\overline{D} - \underline{D}$, if the installed capacity is beyond $\overline{D} - \underline{D}$; or larger than the installed capacity otherwise. Notice that type ii) equilibria are singletons and voluntary shortage might occur, as the total available capacity is lower than $\overline{D}$.

4.2.5 Peak hours capacity constraints: small demand support

If there is a small difference between the off-peak and peak demand levels ($\overline{D} \leq 2\underline{D}$), the equilibria will be either sets ii) and iii) of Proposition (14) or coincide with the ones of Proposition (15), depending on the skewness of the demand distribution fucntion.. However, as now there is only a small difference between peak and off peak demand, even when generators stick to very low calues of capacity declarations, shortage is excluded, as Proposition (17) points out:

Proposition 17 If generators are capacity constrained during peak hours, support of demand variability is small, shortage never occurs, regardless $F(.)$ skewness.

Proof. See the Proof of Proposition (15). ■

Interestingly, leaving generators with some uncertainty on demand realisations, but at the same time announcing that peak and off peak demand stay relatively close, provides a good remedy against voluntary shortages.

5 Conclusion

We have developed a two stage game to illustrate the competition in capacity and price between two generators. Although some of our results for the price-game are similar to those of the Bertrand-Edgeworth competition models, we must stress important differences. In Kreps and Scheinkman (1983), Allen and Hellwig (1986, 1993), for example, firms are symmetric, demand is price-elastic, consumers buy first from the low-price firms, while the residual demand for the high-price firm is rationed, often assuming that residual demand is proportional to market demand. In our model, each firm receives the same system marginal price, demand is inelastic and firms' are asymmetric. Rationing of demand only occurs if there are ties in offer prices; otherwise, the residual demand is sold at the system marginal price. This different specification turns out to be an advantage as it simplifies some computations, in particular when firms play mixed strategies. This occurs when demand exceeds the highest declared capacity, implying that both competitors will be called into operation, no matter their capacity. Therefore, the uniform price rule, together with demand inelasticity, make possible the calculation of the equilibrium, avoiding all the computational difficulties encountered by Deneckere and Kovenock (1996) in a Bertrand-Edgeworth type model with firms having asymmetric marginal production costs, as in our model.

Our results on the capacity choice game indicate that collusion enforced by strategic withholding is more likely when demand is deterministic and there is excess capacity. When demand is stochastic, the risk of voluntary shortage is very low if there is high probability of demand realisation close to peak time. By contrast, if low demand realisations are more likely to occur, generators are not concerned by the penalty rule: they may both align their capacity declarations to off peak demand, resulting in voluntary shortage when there is an important difference between the lowest and the highest demand realisations. Improving the accuracy of demand forecast would avoid the existence of such lock-in equilibrium.

Indeed, the topic of collusion when demand is stochastic has already been analyzed by several IO papers, as for example Haltiwanger and Harrington (1991), Staiger and Wolak, (1992), Fabra (2001). These papers mainly focus on the relationship between optimal collusive behavior of symmetric …rms in markets subject to cyclical ‡uctuations (the "business cycle"), considering in…nitely repeated games. In this literature, the results are highly dependent on the modelling assumptions, especially on the correlation between actual and future demand, and on capacity constraints. In particular, Staiger-Wolak, (1992) …nd support for the conventional view that periods of low demand lead to a breakdown of collusive pricing, through the emergence of excess capacity.

As Staiger-Wolak, (1992), we …nd that the nature of the price war depends on the degree of excess capacity. However, none of the papers considering the impact of capacity constraints on collusive behavior endogenise capacity choice. In our model, competition regimes become endogenous through the mechanism of capacity declarations. The negative relationship between excess capacity and ability to sustain collusion results reversed: the less …rms are capacity-constrained, the more they can play strategically with their capacity declarations, and this facilitates collusion. Moreover, the uniform pricing rule makes even more appealing the gains from tacit collusion by capacity withholding: when …rms restrict capacity, the SMP attains its maximum level, and so do scarcity rents.

Our model indicates that market design rules, such as uniform auctions and timing (and accuracy) of demand forecast can facilitate collusion and capacity withholding by generators. Hence, without excluding other possible sources of market power, the principal policy focus should be on targeting market rules that avoid gaming. However, the task of analysing withholding in electricity markets must include the interaction between spot trading and other markets: in particular, one has to consider whether capacity was economically withheld, providing ancillary services, or constrained down due to transmission congestion or environmental output restrictions.

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6 Appendix

6.1 Proof of Lemma10

Proof.

i) We analyse the decisions of firm i assuming $K_j < \underline{D}$ (with $i, j = a, b$ $i \neq j$):

1. with $K_i \leq \underline{D} - K_j$, generator i will be in the D_V regime, earning 0 with probability 1; therefore K_i is at least equal to $\underline{D} - K_j$:

2. with $\underline{D} - K_j < K_i \leq \underline{D}$; firm i will be in the D_V regime with probability $1 - F(K_i + K_j)$ and in the D_H regime with probability $F(K_i + K_j)$. The expected profit of i is:

$$E(\pi_{i1}) = (p - c_i) \int_{\underline{D}}^{K_i + K_j} (D - K_j) dF(D)$$

   Consequently, $K_i > \underline{D}$:

3. with $K_i > \underline{D}$; firm i has the capacity advantage and D_M, D_H or D_V regimes can occur, depending on demand realization; the expected profit is:

- with $K_i \leq \overline{D} - K_j$

$$E(\pi_{i2}) = (p - c_i) \left[\int_{\underline{D}}^{K_i} (D - K_j) dF(D) + \int_{K_i}^{K_i + K_j} (D - K_j) dF(D) \right] \quad (21)$$

- with $K_i > \overline{D} - K_j$

$$E(\pi_{i3}) = (p - c_i) \left[\int_{\underline{D}}^{K_i} (D - K_j) dF(D) + \int_{K_i}^{\overline{D}} (D - K_j) dF(D) \right] \quad (22)$$

which is independent of K_i:

For $K_j < \underline{D}$; we deduce that $K_i(K_j) \in \left[\overline{D} - K_j; \overline{K_i}\right]$ if $K_j \geq \overline{K_i} + \overline{D}$; and $K_i(K_j) = \overline{K_i}$ otherwise.

ii) Assume now that $\underline{D} < K_j \leq \overline{K_j}$ (with $i, j = a, b$ $i \neq j$): As long as $K_i < \underline{D}$, medium, high and very high demand regime may occur with positive probability. In the D_M regime generator j has the advantage in capacity and fixes the SMP. Hence expected profit of generator i is as follows:

$$E(\pi_{i4}) = (\bar{p} - c_i)\left[\int_{\underline{D}}^{K_j} K_i dF(D) + \int_{K_j}^{K_i+K_j} (D - K_j) dF(D)\right] \quad (23)$$

$E(\pi_{i4})$ is an increasing function of K_i.

As in all the above case analysed, firm i's profit increases when K_i increases, we conclude that for $\underline{D} < K_j \leq \overline{K_j}$ $K_i(K_j) \geq \underline{D}$.

iii) Assume now that $K_j = \underline{D}$. With $K_i = \underline{D}$, there exist two demand regimes: D_H and D_V. By increasing K_i above $\underline{D}$, firm i "transforms" some very high demands (that give zero profits) into high demands (that give positive profit)[16]. We conclude that for $K_j = \underline{D}$, $K_i \in \left[\overline{D} - \underline{D}, \overline{K_i}\right]$ if $\overline{K_i} > \overline{D} - \underline{D}$, and $K_i(K_j) = \overline{K_i}$ otherwise. ■

6.2 Proof of Corollary 11

Proof. Included in parts i) and iii) of Lemma (10) ■.

6.3 Proof of Proposition 15

Qualitative analysis of firm a's best response function Remember that region $\underline{\alpha}$ includes values such that $K_a \leq K_b$, the maximum installed capacity of b can be interior to such area or not (i.e. we consider $\underline{D} \leq K_b \leq \min\left(\overline{D} - \underline{D}, \overline{K_b}\right)$). In region $\underline{\alpha}$ the sum of declared capacities is at most equal to $\overline{D}$ (see Figure 6).

Firm a's best response is determined by analysing the followinf FOC:

$$\begin{aligned} \frac{\partial E(\pi_a^{\alpha})}{\partial K_a} &= (c_b - c_a)K_a F'(K_a) + \\ &\quad + (\bar{p} - c_a)[F(K_b) - F(K_a)] + (\bar{p} - c_a)K_a[F'(K_a + K_b) - F'(K_a)] \end{aligned} \quad (24)$$

[16] It also transforms some D_H elements into D_M demands but, being the leader in capacity, firm i expected profit is the same both in D_H and in D_M regime.

At $K_a = K_b$; equation (24) reduces to:

$$\frac{\partial E(\pi_b^{\underline{\alpha}})}{\partial K_b} = K_b\left[(\theta - c_a)F'(2K_b) - (\theta - c_b)F'(K_b)K_b\right] \qquad (25)$$

If $F(.)$ is not too concave, or if there is an important difference between c_b and c_a, equation (25) is positive and the analysis corresponds to Section 4.2.3. In particular, Proposition 14 holds.

If $F(.)$ strongly concave and c_a is not too different from c_b; equation (25) is negative. Firm a does not have any incentive to increase her capacity and to enter the subregion α (where her expected profit would be constant and equal to equation (12)).

This suggests that firm a profit's maximum should be found in subregion $\underline{\alpha}$; hence her reaction function is implicitly defined when equation (24) is equal to zero, or is negative. Denote this function by $K_a^{\underline{\alpha}}(K_b)$. One can check that:

$$\operatorname{sign} K_a^{\underline{\alpha}\prime}(K_b) = \operatorname{sign}\{F'(K_b) + K_aF''(K_a + K_b)\}:$$

Given that $F''(.)$ is negative, the sign of $K_a^{\underline{\alpha}\prime}(K_b)$ is positive when $F'(K_b)$ is large[17] and negative when K_a is large and/or $F(.)$ is strongly skewed leftward.

If the sign of $K_a^{\underline{\alpha}\prime}(K_b)$ is positive, then $K_a^{\underline{\alpha}}(K_b) = 0$ is interior to region $\underline{\alpha}$ and passes through the frontier at the point $\tilde{K}_b = \arg\{K_a^{\underline{\alpha}}(K_b) = \overline{D} - K_b\}$.

If the sign of $K_a^{\underline{\alpha}\prime}(K_b)$ is negative, then a moves to capacity choices lower than $\underline{D}$; that is in subregion $\underline{\gamma}$[18]. Let $\hat{K}_b < \min\left(\overline{D} - \underline{D}; \overline{K_b}\right)$ denote the value of K_b such that for $K_b \geq \hat{K}_b$, the slope of firm a's reaction fucntion is positive. $\hat{K}_b$ is defined as follows:

$$\hat{K}_b = \arg\{(c_b - c_a)\underline{D}F'(\underline{D}) + (\theta - c_a)\underline{D}\left[F(K_b) + F'(\underline{D} + K_b) - F'(\underline{D})\right] = 0\}$$

If $\hat{K}_b$ does not belong to region $\underline{\alpha}$ (that is $\hat{K}_b > \overline{D} - \underline{D}$), the FOC (24) continue to be negative for $K_b < \min\left(\overline{D} - \underline{D}; \overline{K_b}\right)$: In this case, $K_a^{\underline{\alpha}}(K_b) = \underline{D}$

Moreover, it can be easily shown that:

$$\frac{\partial E(\pi_a^{\overline{\beta}})}{\partial K_a} < \frac{\partial E(\pi_a^{\underline{\alpha}})}{\partial K_a} \qquad (26)$$

---

[17] The slope $K_a^{\underline{\alpha}\prime}(K_b)$ can be positive for values of K_a close to zero, which does not corresponds to the case we analyse.

[18] By Lemma (10), in subregion $\underline{\gamma}$, firm a expected profit is increasing in K_a; hence $K_a^{\underline{\gamma}}(K_b) = \underline{D}$.

where $\frac{\partial E(\pi_a^\beta)}{\partial K_a}$ is defined by equation (14) in Section 4.2.3. By transitivity, inequality (26) implies the following:

$$\frac{\partial E(\pi_a^\alpha)}{\partial K_a} < 0 \Rightarrow \frac{\partial E(\pi_a^\beta)}{\partial K_a} < 0 \qquad (27)$$

Then firm a's reaction function is $K_a^\alpha(K_b) = K_a^\beta(K_b) = \underline{D}$.

Qualitative analysis of firm b's best response function Region α includes values such that $\overline{D} - \underline{D} \geq K_a \geq K_b$; the sum of declared capacities is at most equal to $\overline{D}$ (see Figure 6). We study the following FOC:

$$\frac{\partial E(\pi_b^\alpha)}{\partial K_b} = [F(K_a) - F(K_b)] + K_b[F'(K_a + K_b) - F'(K_b)] \qquad (28)$$

The analysis is symmetric to the one of firm a and results simpler as marginal costs do not counterbalance the effect of concavity of $F(.)$. For instance, even when demand distribution function is "gently" skewed leftward, the function defined by (28) is negative at $K_b = K_a$. Also notice that $\hat{K}_a < \overline{D} - \underline{D}$ is defined as follows

$$\hat{K}_a = \arg\{[F(K_a) + F'(\underline{D} + K_a) - F'(\underline{D})] = 0\} \qquad (29)$$

If $\hat{K}_a$ belongs to region α, the slope of the reaction function $K_b^\alpha(K_a)$ defined by equating (28) to zero can be positive or negative, and the analysis is symmetric to the case of firm a. If $\hat{K}_a$ does not belong to region α, that is if equation (28) is negative, firm b's best reaction function will be to bid $\underline{D}$ not only in region α, but also in region β.