Competition between on line retailers and traditional shops

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Abstract

This article studies the competition between an electronic commerce firm and a traditional shop, that set respectively a uniform delivered price and a mill price. Furthermore, they face different geographical constraints: the proximity of consumers in the case of the traditional shop, and logistic costs in the case of the on line retailer that takes in charge home delivery. The aim of this paper is to assess the impact of the introduction of an electronic commerce firm on the level of competition and to understand the different advantages of these two kinds of firms.

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1 Introduction

The competition between electronic commerce and traditional market firms presents a new aspect in comparison to classical analysis of competition, because these two types of firms compete together but face very different constraints, especially the geographical constraint. In the case of a physical shop, the proximity of consumers is essential because they support transportation costs. On the contrary, in the electronic commerce this constraint is not acting anymore, as any consumer can accede to any web site without restriction. It has even been said that the development of the Internet would give a global dimension to competition. The accuracy of this argument is doubtful and depends on the sort of good we consider. For information or digital goods it remains valid, but for physical goods on line sellers have to support logistic costs that restrict the dimension of competition.

The aim of this paper is to assess the impact of the introduction of an electronic commerce firm on the level of competition: will it be higher even in the case of a "physical good"? The second issue of this paper is to understand the comparative advantages of these two kinds of firms that do not face the same geographical constraints.

Obviously, there may be many more features to take into account to study the competition between a web site and a physical shop as brand loyalty, consumer's confidence in online payment, etc. But we choose to focus on the geographical constraint aspect for two reasons. Firstly, logistic costs are very relevant to determine the prices charged by on line firms and secondly, from consumers point of view, home delivery is one of the main elements that distinguish on line shopping from traditional shopping.

We model a Hotelling competition between a traditional shop and an on line seller. A consumer who chooses to buy a good at the traditional shop has to support transportation costs whereas on line seller's price includes home delivery. Thus, from the consumers' point of view, there is no geographical constraint when they buy on line. But actually, this constraint exists for the electronic commerce firm that bears logistic costs and take them into account to determine its price. Furthermore, we consider that the on line seller also has a location, which could be the site of a warehouse.

As Hotelling's model, there are two different interpretations of our model: the geographical and the products differentiation interpretations. The former is also valid for mail order selling, but the latter is peculiar to electronic commerce.

The description we have made of the two types of pricing are known in the economic literature as "uniform delivery prices" for the web site (wich is an extreme case of price discrimination) and "mill prices" for the traditional shop. Some prior works study the competition between firms that set different pricing, discriminatory or mill pricing, but they focus mainly on the choice of the price policy (Thisse and Vives, 1988). Among these works, only a few of them deals with the special case of uniform delivered pricing and once again focus on the choice of pricing policy for a monopolistic firm (Beckmann, 1976, Cheung and Wang, 1996), a monopsonistic firm (Lofgren, 1997) or firms in duopoly (Katz and Thisse, 1993). In our model, the main issue is more to understand the competitive and location behaviors. The study made by Foncel, Guyot and Jouneau-Sion (2001) is more similar to ours. They look into the issue of the competition between an on line bank and a "localized" one. Their results may be applied to many other areas as digital or information goods but not to physical goods. Indeed, in their model the on line bank does not support any transportation costs. Our model considers the case of physical goods, where the on line firm bears logistic costs.

We use Hotelling's framework to study the competition between an on line seller and a traditional shop. In section 2, we assume that firms' location are fixed. In section 3, we study the case of endogenous locations. Section 4 proposes another interpretation of the model and section 5 concludes.

2 Competition with fixed locations

In this section, we study the competition between a traditional shop and an on line seller in the case of fixed locations.

2.1 The basic model

There are two sellers of a homogenous good, the on line seller denoted as i (for Internet) and the physical shop denoted as s. Their marginal production cost is zero, a common assumption in the industrial organization literature. They are located at the ends of a line of length 1, respectively at point 0 and 1. We suppose that customers are evenly distributed along this line and that each customer consumes exactly a single unit of the product. Since the product is homogenous, a customer will buy from the seller who quotes the least delivered price. Namely, the uniform delivered price p_i set by firm i

and the mill price p_s set by firm s plus the transportation cost $t(x)$ which is assumed linear with respect to the distance x as $t(x) = t.x$. Thus consumer $\overline{x}$ who is indifferent between buying to either firms is defined by:

$$p_s - t(1 - \overline{x}) = p_i$$

As the on line seller sets the same delivered price for each customer whatever his location may be (as long as he belongs to the area the firm decided to serve), *from consumers' point of view, there is no geographical constraint to buy on line.* But actually this constraint exists for firm i, whose location can be interpreted as the site of its warehouse. Firm i bears a logistic cost $l(x)$ of delivering a total distance x:

$$l(x) = \int_0^x T(z)\ dz$$

where $T(z)$ is the cost of delivering one customer at a distance z of the firm[1]. As function $T(z)$ is also assumed to be linear $T(z) = T.z$, then:

$$l(x) = \frac{T}{2}\ x^2$$

We choose an integral of linear costs to modelize the logistic costs for two reasons. Firstly, each z must be interpreted as a "group" of customers located in the same region. So it is unlikely that the same truck delivers all the consumers. The idea of new truck departure for each new area to deliver is more relevant. As each of these trips is similar to a linear cost, an integral of linear costs is quite appropriate. Secondly, the logistic costs are then more comparable to the transportation costs that consumers bear. We also assume that the electronic commerce firm enjoys economy of scale and a know-how such that it has a better technology than consumers, and then $T < t$.

Lastly, we assume that the utility of the good is sufficiently high for every customers to buy it.

The market boundary is determined by the location $\overline{x}$ of the consumer who is indifferent between buying to either firms. Then demand functions

[1] Here the logistic costs are supposed to be variable costs. Obviously, most of electronic commerce firms bear also important fixed costs. The aim of this paper is first to isolate the effects that occurs in such a model. Then, one of the main extensions of the model is to add fixed costs $F > 0$ to $l(x)$.

are given by:

$$D_i(p_i, p_s) = \begin{cases} 1 & \text{if} \quad p_i < p_s \\ \overline{x} = 1 - \dfrac{p_i - p_s}{t} & \text{if} \quad p_s < p_i < p_s + t \\ 0 & \text{if} \quad p_s + t < p_i \end{cases}$$

$$D_s(p_i, p_s) = \begin{cases} 1 & \text{if} \quad p_s < p_i - t \\ 1 - \overline{x} = \dfrac{p_i - p_s}{t} & \text{if} \quad p_i - t < p_s < p_i \\ 0 & \text{if} \quad p_i < p_s \end{cases}$$

Both firms compete in price schedules and choose their prices in order to maximize their profits:

$$\underset{p_s}{Max} \pi_s(p_s, p_i) = p_s.\ D_s(p_s, p_i)$$

$$\underset{pi}{Max} \pi_i(p_s, p_i) = p_i.D_i(p_s, p_i) - l\left(D_i(p_s, p_i)\right)$$

The first order conditions are given by[2]:

$$p_s = \begin{cases} p_i - t & \text{if} \quad p_i > 2t \\ \dfrac{p_i}{2} & \text{if} \quad \dfrac{tT}{2t+T} \leq p_i \leq 2t \end{cases}$$

$$p_i = \begin{cases} p_s & \text{if} \quad p_s > t + T \\ \dfrac{1}{2 + \frac{T}{t}} \left[(p_s + t)\left(1 + \frac{T}{t}\right)\right] & \text{if} \quad 0 \leq p_s \leq t + T \end{cases}$$

We note that the optimal response of firm s does not depend on consumers' transportation costs, but is fully determined by the price set by its competitor. In Hotelling's classical model, the optimal response function of a firm k to price p_j set by its competitor j (if the marginal cost is zero) is given by $p_k = (p_j + t)/2$. Firm i is the only one that takes into account consumers' transportation costs in its optimal response.

[2] As firms are supposed to have a profit at least equal to zero, firm s will always set a price such that $p_s \geq 0$. On the contrary, as $p_s \geq 0$, firm i would never set a price under $\frac{tT}{2t+T}$.

2.2 The equilibrium of the basic model

The resolution of the model leads to the following equilibrium values and results.

Equilibrium of the simultaneous price game

The unique pair of equilibrium prices is obtained from the first order conditions for any values of t and T:

$$p_s^* = \frac{t+T}{3+\frac{T}{t}} \qquad\qquad p_i^* = 2.\frac{t+T}{3+\frac{T}{t}}$$

The equilibrium prices p_s^* and p_i^* are increasing with t and T.

$$\begin{array}{ll} \frac{\partial p_s^*}{\partial t} = \frac{3t^2+2tT+T^2}{(3t+T)^2} & \frac{\partial p_i^*}{\partial t} = \frac{2(3t^2+2tT+T^2)}{(3t+T)^2} \\ \frac{\partial p_s^*}{\partial T} = \frac{2t^2}{(3t+T)^2} & \frac{\partial p_i^*}{\partial T} = \frac{4t^2}{(3t+T)^2} \end{array}$$

Proposition 1.a: *The mark up of both firms increases when t rises.*

Two effects can explain this result: one that modify consumer's trade off, and a strategic effect. On the one hand consumers, especially if they are far from the traditional shop, are ready to pay more to buy on line. Then, the electronic commerce firm takes advantage of a better position in consumers' trade off to rise its price. On the other hand the strategic response of the physical shop is to increase its price too.

On the contrary, when T rises, consumer's trade off is not modified. Firm i bears higher logistic costs and raises its price. Then, both firms increase their price, but only the strategic effect comes into play.

The asymmetry between the two firms is also illustrated by the following proposition:

Proposition 1.b: *When t is zero, prices reach their competitive level. On the contrary, when T is zero, both firms set a positive mark up.*

The firms take advantage of the existence of consumers' transportation costs to set a price above its competitive level. When t is 0, all consumers face the same situation and then enjoy a perfect competition situation. As t rises, consumers are all the more ready to pay a higher price to the on line seller since they are far from the traditional shop. When T is zero, the on line seller still benefits from a captive demand and then both firms set a positive mark up ($2t/3$ for firm i and $t/3$ for firm s).

In conclusion, despite of the existence of an on line seller whose offer gives the same utility to all consumers whatever their location may be, competition is limited as long as t is strictly positive.

Market shares

The equilibrium market shares are given by

$$D_s(p_s^*, p_i^*) = \frac{t+T}{3t+T} \qquad D_i(p_s^*, p_i^*) = 1 - \frac{t+T}{3t+T}$$

Proposition 2.a *:*

$$\begin{array}{ll} (i) & D_s(p_s^*, p_i^*) > D_i(p_s^*, p_i^*) \Leftrightarrow T < t \\ (ii) & \frac{\partial D_i(p_s^*, p_i^*)}{\partial t} > 0 \end{array}$$

As long as $T < t$, the electronic commerce firm gets a bigger market share than the physical shop. Moreover, the market share of the electronic commerce firm and the physical shop respectively increases and decreases with t. This is quite coherent with the previous results: when consumers' transportation costs rise, buying on line becomes more attractive to consumers.

Proposition 2.b : *When t is zero, the traditional shop serves the whole market. But when T is zero, it keeps one third of the market. Furthermore, the distribution of market shares when T is 0 is $(2/3; 1/3)$ whatever the value of t.*

When t is 0, there is a perfect competition and the physical shop is always able to set a lower price than its competitor that supports logistic costs. When T is 0, the price that would give the whole market the electronic commerce firm leads to zero profits.

Profits

The equilibrium profits are:

$$\pi_s(p_s^*, p_i^*) = t\left(\frac{t+T}{3t+T}\right)^2 \qquad \pi_i(p_s^*, p_i^*) = \frac{2t^2(2t+T)}{(3t+T)^2}$$

Lemma 3.a :

(i) $\quad \pi_s(p_s^*, p_i^*) < \pi_i(p_s^*, p_i^*)$ if $T < t$ or if $t < T < \sqrt{3}t$

(ii) $\quad \pi_s(p_s^*, p_i^*) \geq \pi_i(p_s^*, p_i^*)$ otherwise

When firms are located at the ends of the line, the on line seller has a better equilibrium profit than the traditional shop. This result is not only due to the hypothesis $T < t$. In the case $T > t$, there also exists some values of $(t; T)$ for which firm i have a better profit than firm s.

Remark: When $t = T = \tau$,

$$\pi_s(p_s^*, p_i^*) = \tau/4 \qquad \pi_i(p_s^*, p_i^*) = 3\tau/8$$

Proposition 3.b: *Setting a fixed price for home delivery gives a strategic advantage to the electronic commerce firm.*

Proof: If firm i sets a non discriminatory delivered price $p_i^{'}(x) + T.x$ to consumer x, then its market share is determined by $\overline{u}$ such that:

$$p_i^{'} + T\overline{u} = p_s + t(1 - \overline{u})$$

Then, if it sets an uniform delivered price p_i, it keeps the same market share, with p_i is such that:

$$p_i = p_s + t(1 - \overline{u})$$

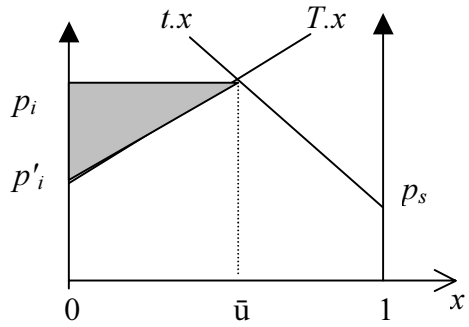
By setting an uniform delivered price, firm i increases its profit of:

$$\Delta\pi_i = p_i.\overline{u} - \int_0^{\overline{u}} T.xdx - p^{'}i.\overline{u}$$

Simple calculus leads to $\Delta\pi_i = \frac{T}{2}(\overline{u})^2$, the amount of logistic costs.

■

This result may also be illustrated by the following graph.



Benefit from uniform delivered pricing

The idea is quite intuitive. Firm i can set to all consumers a home delivery price equal to the cost of delivering consumer $\overline{u}$ (the furthest consumer from the warehouse) and keep the same market share.

Simple calculus also shows that the on line retailer profit increases with t. The intuition of this result has been previously explained.

On the contrary, we find a quite amazing result about the traditional shop profit: it increases with t although the consumers' trade off is less favorable to the shop when t rises.

$$\frac{\partial \pi_s}{\partial t} > 0$$

Proposition 3.c : *When the consumers' transportation cost rises, although consumers' trade off is less favorable to the shop, its profit increases.*

When t rises, not only the on line retailer's profit increases but also the traditional shop's profit does. Actually, there are two contrary effects on the shop's profit. On the one hand consumers' trade off is less favorable to the shop when t rises. But on the other hand, as its competitor increase its mark up (consumers become more captive to on line retailer), the shop can set a higher price too.

In conclusion, we found that although the on line seller proposes a fixed price including home delivery whatever the consumer's location is, the existence of consumer transportation costs prevents perfect competition to prevail.

2.3 First best optimum with fixed locations

In this section, we compare the former equilibrium with the first best situation in the case of fixed locations at (0,1).

The welfare W is the sum of consumers' surplus and firms' profit, then:

$$W = v - \int_0^{\overline{x}} Txdx - \int_{\overline{x}}^1 t(1-x)dx$$

At the equilibrium, the welfare is given by:

$$W^* = v - \frac{t\left(t^2 + 6tT + T^2\right)}{2\left(3t + T\right)^2}$$

The first best optimum maximizes W, i.e. minimizes the whole transportation costs that consumers and firm i bear. For fixed locations at (0,1), first best optimum market shares are given by:

$$D_i^{1st} = \frac{t}{t+T} \qquad D_s^{1st} = \frac{T}{t+T}$$

Then the first best welfare is given by:

$$W^{1st} = v - \frac{tT}{2(t+T)}$$

The comparison between the first best optimum and the equilibrium leads to the following proposition:

Proposition 4 : *For fixed locations at (0,1), the shop's equilibrium market share is higher than its first best value.*

This result comes from $T < t$ (we find the opposite result in the case $T > t$).

In conclusion, if the electronic commerce firm enjoys a know-how and a technology such that $T < t$, its equilibrium market share will always be lower than its first best value.

3 Competition with endogenous locations

In this section we consider that the firms are allowed to choose their location: l_s is the location of the traditional shop, and l_i is the location chosen by the electronic commerce firm for its warehouse. The hypothesis of the model are the same as in the fixed locations model apart from the choice of locations that takes place before the price competition. We also assume that the traditional shop chooses its location first. This assumption is quite relevant since the establishment of a new shop implies important fixed costs. We first give the intuition of the game before presenting it in a more formal way (section 3.1).

We take into account two different cases. In the first case, the different steps of the game are:

- *step 1*: firm s chooses l_s
- *step 2*: firm i chooses simultaneously l_i and the demand it will serve (we will explain why this issue is relevant with endogenous locations)
- *step 3*: firms compete in price

Implicitly, the traditional shop anticipates the reaction of its competitor when it determines its location. This is relevant for a recent establishment firm.

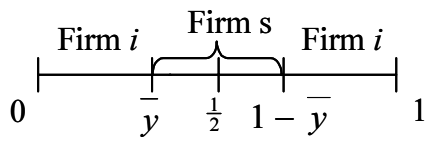
In the second case, the traditional firm did not expect the competition of the electronic commerce firm and, as a monopoly, chose to establish its shop in the middle of the line: $l_s = 1/2$. This illustrates the case of an older firm that settled in several decades ago. So l_s is assumed to be fixed at $1/2$ and the different steps of the game are:

- *step 1*: firm i chooses simultaneously l_i and the demand it will serve
- *step 2*: firms compete in price

We will first develop the second case. The case of a recent firm, more complex, is treated in appendix 7.2: we show that the equilibrium location is also $l_s^* = 1/2$ when the firm anticipates the behavior of the on line seller. Then the results we find for an older firm remain valid in this case.

3.1 The model when the shop is in the middle of the line

As we just explained it, the shop is assumed to be located at $l_s = 1/2$. So consumers who prefer to go to the shop will be around its location in $1/2$. This implies a particular form of demand functions.



There is not one, but two indifferent consumers: one on the left side noted $\overline{y}$ and the other on the right side located at point $1-\overline{y}$ where $\overline{y}$ is determined by $p_i = p_s + t(1/2 - \overline{y})$. Thus,

$$\overline{y} = \frac{1}{2} - \frac{pi - p_s}{t}$$

The consumers who go to the shop are between the two indifferent consumers. The others who would rather buy on line are divided in *two parts*, but the on line retailer may prefer not to serve all of them. If he decides to do so, then he would have to cross the area of the consumers that go to the shop. On the contrary, *if he decides not to serve some consumers, they will go to the shop* as we assumed that they prefer to buy the good anyway. Let h be the decision of firm i about the demand it decides to serve, h equals gd if it serves all of it, and h equals g (resp. d) if it chooses to serve only the left side of it (resp. the right side). The symmetry of the problem allows us to restrict the study to the case $h \in \{g, gd\}$.

In the first step of the game, a strategy of firm i is defined by (h, l_i) where $h \in \{g, dg\}$ and $l_i \in [0, 1]$. In the second step of the game, firms compete in price.

The resolution of the game is made by backward induction. We first resolve the last step of the game: price competition. One should note that it is necessary to make the distinction between two different cases $h = g$ and $h = gd$ because it induces particular specifications for demand and logistic costs functions. For each case h, we determine the subgame equilibrium profits $\pi_i^h(l_i)$ and $\pi_s^h(l_i)$ from the price competition game.

Then we resolve the first step of the game where firm i takes its decision (h^*, l_i^*) in the following way:

$$l_i^* = \arg\max_{l_i} \pi_i(l_i)$$

where $\pi_i(l_i)$ is defined by:

$$\pi_i(l_i) = \max_{\{g,gd\}} \left\{ \pi_i^g(l_i), \pi_i^{gd}(l_i) \right\}$$

Proposition 4.a: *When the traditional shop is located in the middle of the line, it dissuades the electronic commerce firm from serving some of the customers that would like to buy on line at the equilibrium prices ($h^* = g$).*

Proof: Appendix 7.1.

Proposition 4.b: *The electronic commerce firm always chooses not to serve all its demand to reduce competition rather to minimize logistic costs.*

Firm i chooses $h = g$ for every values of t and T, even when T is 0. So this choice does not depend on logistic costs. When firm i decides to give up the right side of its demand, these consumers go to the traditional shop. Then, firm s can set a higher price than the one it would have set if $h = gd$. Thus, there are two positive effects on firm i's profit. Firstly, $\overline{y}$ increases and secondly firm i may also rise its price. These two effects compensate for the loss of market share due to the decision $h = g$.

As $h = g$, demand functions are given by:

$$D_i(p_i, p_s) = \overline{y} = \begin{cases} 1/2 & \text{if} \quad p_i \leq p_s \\ \dfrac{1}{2} - \dfrac{pi - p_s}{t} & \text{if} \quad p_s \leq p_i \leq p_s + \dfrac{t}{2} \\ 0 & \text{if} \quad p_i \geq p_s + \dfrac{t}{2} \end{cases}$$

$$D_s(p_i, p_s) = 1 - \overline{y} = \begin{cases} 1 & \text{if} \quad p_s \leq p_i - \dfrac{t}{2} \\ \dfrac{1}{2} + \dfrac{pi - p_s}{t} & \text{if} \quad p_i - \dfrac{t}{2} \leq p_s \leq p_i \\ 1/2 & \text{if} \quad p_s \geq p_i \end{cases}$$

The logistic function is given by:

$$L(\overline{y}, l_i) = \int_0^{l_i} T.udu + \int_0^{\overline{y}-l_i} T.udu = \frac{T}{2}\left[(l_i)^2 + (\overline{y} - l_i)^2\right]$$

The other cases are developed in appendix 7.1.

3.2 The equilibrium when the shop is in the middle of the line

The resolution of the model leads to $h^* = g$ (proposition 4.a) and the following equilibrium values:

$$l_i^* = \frac{3t(2t+T)}{2\left(18t^2+10tT+T^2\right)}$$

Simple calculus shows that l_i^* is decreasing with T and increasing with t. Because of the logistic costs function specification, firm i always locates near the middle of the demand it serves (but not exactly, as explained at the time of the calculus of the equilibrium market shares). We will also see that firm i's market share is decreasing with T. So the warehouse mooves to the left when T rises. The opposite occures when t rises.

Equilibrium prices

Equilibrium prices are given by:

$$p_i^*(l_i^*) = \frac{3t(6t^2+6tT+T^2)}{2\left(18t^2+10tT+T^2\right)} \qquad p_s^*(l_i^*) = \frac{t\left(9t^2+7tT+T^2\right)}{18t^2+10tT+T^2}$$

The study of the equilibrium values confirms the intuitions and results of the fixed locations model. Furthermore, the comparison between the equilibrium prices for fixed and endogenous locations leads to the following result:

> **Proposition 5:** *When the shop is located at the middle of the line, it faces a new trade off and finally sets a higher price compared to the case of fixed locations at (0,1). On the contrary, the on line retailer lowers its price.*

Firm i lowers its price for two reasons. Firstly, it can reduce its costs by choosing its location and secondly, it has to be more competitive when the shop is in the middle of the line.

Firm s faces the following trade off. On the one hand, it may raise its price without loosing the consumers on its right (as firm i refuses to serve them). One the oter hand, if it does, it will reduce its market share on its

left. Obviously, firm s will make more profit by extracting the surplus of all the right side consumers than staying competitive for the left side consumers, and finally raises its price.

Markets shares:

The equilibrium market shares are given by:

$$D_i^*(l_i^*) = \overline{y}^*(l_i^*) = \frac{3t(3t+T)}{18t^2+10tT+T^2} \qquad D_s^*(l_i^*) = \frac{9t^2+7tT+T^2}{18t^2+10tT+T^2}$$

As for fixed locations, firm i's market share is decreasing with T, and increasing with t. Furthermore, firm i is not exactly located at the middle of the demand it serves. Actually, we have $l_i^* < \frac{\overline{y}^*(l_i^*)}{2}$. When firm i chooses its location, it takes into account not only the logistic costs, but also the consequences on prices, demand, and then on its location, etc.

Profits:

The equilibrium profits are:

$$\pi_i^*(l_i^*) = \frac{9t^2\,(2t+T)}{4\,(18t^2+10tT+T^2)} \qquad \pi_s^*(l_i^*) = t.\left[\frac{9t^2+7tT+T^2}{18t^2+10tT+T^2}\right]^2$$

The study of the profits confirms the intuitions and results of the fixed locations model. Furthermore, simple calculus leads to the following proposition.

Proposition 6: *When the shop is located at the middle of the line, it has a better profit compared to the fixed locations at (0,1) situation. Furthermore, it has a better profit than its competitor.*

Although firm i is the only one that can take advantage of the existence of consumers' transportation costs, firm s has a better equilibrium profit than its competitor because it can extract much more surplus to the right side consumers that firm i refuses to serve.

3.3 The endogenous choice of l_s

We also explored the case where the shop anticipates the competition of the electronic commerce firm and then determines its location. This may be the case for a recent establishment firm. The resolution of this game is more complex but quite similar to the case $l_s = 1/2$. We found the following result:

Proposition 7: *When the traditional shop anticipates the reaction of the electronic commerce firm, it chooses to locate in the middle of the line.*

Proof: appendix 7.2

So the previous results remain valid.

3.4 First best optimum with endogenous locations

The welfare W^E is the sum of consumers' surplus and firms' profit. For endogenous locations and $h = g$, the welfare is given by:

$$W^E = v - \int_0^{\overline{y}} T\left|l_i - x\right| dx - \int_{\overline{y}}^1 t\left|\frac{1}{2} - x\right| dx$$

At the equilibrium, the welfare is given by:

$$W^{E*} = v - \frac{t\left(t+T\right)\left(162t^3 + 108t^2T + 22tT^2 + T^3\right)}{4\left(18t^2 + 10tT + T^2\right)^2}$$

Lemma 9.a : *At the first best optimum, each firm's demand is on either side of the indifferent consumer.*

Proof : appendix 7.3

The first best optimum rules out the case where there may be some consumers who would rather buy on line on both sides of the shop. So the expression of the welfare is W^E. The first best values that maximise the welfare are given by:

$$l_i^{1st} = \frac{t}{2(t+T)} \qquad l_s^{1st} = 1 - \frac{T}{2(t+T)}$$

$$\overline{y}^{1st} = \frac{t}{t+T}$$

$$W^{1st} = v - \frac{tT}{4(t+T)}$$

Proposition 9.b : *For endogenous locations, the shop's equilibrium market share is higher than its first best value.*

This result is valid for every values of t and T such that $t > T$.

4 Products differentiation interpretation

The classical Hotelling model has got two possible interpretations: the geographical one and the products differentiation one. This model also offers a products differentiation interpretation: it could illustrate the competition between a firm that sells personalized products and another that proposes a standardized product. In the first case, the good perfectly meets the consumer's preferences and the latter may pay a higher price for that service (like the uniform delivered price in the previous model). In the second case, the good may be less expensive, but do not exactly correspond to the consumer's preferences and so he bears a cost with respect to the distance from his ideal good.

This new issue is also relevant in electronic commerce framework because there are many sellers on the Internet that propose personalized products.

In relation to the previous model, firm s supplies one single product whose characteristics are defined by l_s. Then t can be interpreted as consumer's

attachment to its ideal product. Firm i makes personalized product from a "basic product" defined by l_i. Thus $T(x)$ is the cost of making a good which is at a distance x of the basic product.

We can transpose all the results of the previous model that do not stem from the comparison between t and T (which is not relevant here). Thus we can state the following propositions.

- The more consumers are attached to their ideal product, the more firms make profits. This is relevant not only for firm i that supply personalized products, but also for firm s that sells one single product although its market share decreases (propositions 1, 2.a and 3.c).
- Setting the same price for a personalized good to all consumers gives a strategic advantage to firm i (proposition 3.b).
- When firm s chooses its product first, it stands in the middle of the line (proposition 7) and then gets a better profit than the firm who makes personalized products (proposition 6).
- In this case, firm i refuses to supply some consumers in order to reduce competition, even if it can personalize products for free (proposition 4.b).

5 Conclusion

We showed that although the on line seller proposes a fixed price including home delivery whatever the consumer's location is, the existence of consumers' transportation costs prevents a perfect competition. We also showed that uniform delivered pricing gives a strategic advantage to the electronic commerce firm. But even so, by choosing an appropriate location, the shop can dissuade the electronic commerce firm from serving some of its customers and thus makes a higher profit than the latter. When the shop is in the middle of the line, the electronic commerce firm always chooses not to serve all of its demand in order to reduce competition rather to minimize logistic costs.

The model also has a products differentiation interpretation. It may illustrate the competition between a firm that sells some personalized products and another that proposes a single product.

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7 Appendix

7.1 Choice of h when l_i is endogenous and $l_s = 1/2$

The resolution of the problem presented in section 3.2 is quite intuitive. Actually we will resolve the problem in the following way, which is technically simpler and perfectly equivalent to the former. Decision $h^* \in \{g, dg\}$ is determined by:

$$h^* = \arg\max_{\{g,gd\}} \left\{ \pi_i^g(l_i^g), \pi_i^{gd}(l_i^{gd}) \right\}$$

where $\pi_i^h(l_i^h)$ is the maximum profit that firm i can get by choosing the best location l_i^h when h, then:

$$\pi_i^h(l_i^h) = \max_{l_i} \pi_i^h(l_i)$$

7.1.1 Case $h = g$

Demand and logistic costs functions in the case $h = g$ have been defined in section 3.2. The optimal response functions are given by:

$$p_i(p_s) = \begin{cases} p_s & \text{if} \quad p_s > \frac{t}{2} + T\left(\frac{1}{2} - l_i\right) \\ \frac{1}{2t+T}\left\{t\left[\frac{t}{2} + T\left(\frac{1}{2} - l_i\right)\right] + (t+T)p_s\right\} & \\ & \text{if} \quad 0 \leq p_s \leq T\left(\frac{1}{2} - l_i\right) + \frac{t}{2} \end{cases}$$

$$p_s(p_i) = \begin{cases} p_i - \frac{t}{2} & \text{if} \quad p_i > \frac{3}{2}t \\ \frac{1}{2}\left(\frac{t}{2} + p_i\right) & \text{if} \quad \frac{t}{2} \leq p_i \leq \frac{3}{2}t \\ +\infty & \text{if} \quad 0 \leq p_i < \frac{t}{2} \end{cases}$$

Then we determine the subgame equilibrium:

$$p_i^g(l_i) = \frac{t}{2}\left[1 + \frac{2T\,(1-2l_i)}{3t+T}\right] \qquad p_s^g(l_i) = \frac{t}{2}\left[1 + \frac{T\,(1-2l_i)}{3t+T}\right]$$

and the subgame equilibrium values:

$$D_i^g(l_i) = \frac{3t + 2Tl_i}{2\,(3t+T)} \qquad D_s^g(l_i) = \frac{3t + 2T(1-l_i)}{2\,(3t+T)}$$

$$\pi_s^g(l_i) = t.\left[\frac{3t + 2T(1-l_i)}{2\,(3t+T)}\right]^2$$

$$\pi_i^g(l_i) = \frac{-4T\left[18t^2 + 10tT + T^2\right](l_i)^2 + 12tT(2t+T)l_i + 9t^2(2t+T)}{8\,(3t+T)^2}$$

Thus the optimal location and the equilibrium values are:

$$l_i^g = \frac{3t(2t+T)}{2\,(18t^2 + 10tT + T^2)}$$

$$p_i^g(l_i^g) = \frac{3t(6t^2+6tT+T^2)}{2\,(18t^2+10tT+T^2)} \qquad p_s^g(l_i^g) = \frac{t\,(9t^2+7tT+T^2)}{18t^2+10tT+T^2}$$

$$D_i^g(l_i^g) = \overline{y} = \frac{3t(3t+T)}{18t^2+10tT+T^2} \qquad D_s^g(l_i^g) = \frac{9t^2+7tT+T^2}{18t^2+10tT+T^2}$$

$$\pi_i^g(l_i^g) = \frac{9t^2\,(2t+T)}{4\,(18t^2+10tT+T^2)} \qquad \pi_s^g(l_i^g) = t.\left[\frac{9t^2+7tT+T^2}{18t^2+10tT+T^2}\right]^2$$

Then the value of $\pi_i^g(l_i^g)$ is given by:

$$\pi_i^g(l_i^g) = \frac{9t^2\,(2t+T)}{4\,(18t^2+10tT+T^2)}$$

7.1.2 Case $h = gd$:

Demand functions are given by:

$$D_i(p_i,p_s) = \begin{cases} 1 & \text{if} \quad p_i \leq p_s \\ 2\overline{y} = 1 - 2\left(\dfrac{pi - p_s}{t}\right) & \text{if} \quad p_s \leq p_i \leq p_s + \dfrac{t}{2} \\ 0 & \text{if} \quad p_i \geq p_s + \dfrac{t}{2} \end{cases}$$

$$D_s(p_i,p_s) = \begin{cases} 1 & \text{if} \quad p_s \leq p_i - \dfrac{t}{2} \\ 1 - 2\overline{y} = 2\left(\dfrac{pi - p_s}{t}\right) & \text{if} \quad p_i - \dfrac{t}{2} \leq p_s \leq p_i \\ 0 & \text{if} \quad p_s \geq p_i \end{cases}$$

N.B.: When $h = gd$, we must consider two cases: $l_i \leq \overline{y}$ and $\overline{y} \leq l_i \leq 1 - \overline{y}$.

If l_i is such as $l_i \leq \overline{y}$

When $l_i \leq \overline{y}$, the logistic costs function is given by:

$$L(l_i,\overline{y}) = \int_0^{l_i} T(l_i - u)du + \int_{l_i}^{\overline{y}} T(u - l_i)du + \int_{1-\overline{y}}^{1} T(u - l_i)du$$

Price competition leads to the following optimal response functions:

$$p_i(p_s) = \begin{cases} p_s & \text{if } p_s > \dfrac{t}{2} + T\left(\dfrac{1}{2} - l_i\right) \\ \dfrac{1}{4}\left(2p_s + t + T - 2Tl_i\right) & \text{if } 0 \leq p_s \leq \dfrac{t}{2} + T\left(\dfrac{1}{2} - l_i\right) \end{cases}$$

$$p_s(p_i) = \begin{cases} p_i - \dfrac{t}{2} & \text{if } p_i > t \\ \dfrac{p_i}{2} & \text{if } 0 \leq p_i \leq t \end{cases}$$

There is one single equilibrium:

$$p_i^{gd}(l_i) = \frac{1}{3}(t + T - 2Tl_i) \qquad p_s^{gd}(l_i) = \frac{1}{6}(t + T - 2Tl_i)$$

Then the subgame values are:

$$D_i^{gd}(l_i) = \frac{2t - T + 2Tl_i}{3t} \qquad D_s^{gd}(l_i) = \frac{t + T - 2Tl_i}{3t}$$

$$\pi_s^{gd}(l_i) = \frac{(t + T - 2Tl_i)^2}{18t}$$

$$\pi_i^{gd}(l_i) = \frac{2T\,(2T - 9t)\,(l_i)^2 + 4T(2t - T)l_i + (2t - T)^2}{18t}$$

The optimal location and the equilibrium values are:

$$l_i^{gd} = \frac{2t - T}{9t - 2T}$$

$$p_i^{gd}(l_i^{gd}) = \frac{t\,(3t + T)}{9t - 2T} \qquad p_s^{gd}(l_i^{gd}) = \frac{t\,(3t + T)}{2\,(9t - 2T)}$$

$$D_i^{gd}(l_i^{gd}) = \frac{3\,(2t - T)}{9t - 2T} \qquad D_s^{gd}(l_i^{gd}) = \frac{3t + T}{9t - 2T}$$

$$\pi_i^{gd}(l_i^{gd}) = \frac{(2t - T)^2}{2\,(9t - 2T)} \qquad \pi_s^{gd}(l_i^{gd}) = \frac{t}{2}\left(\frac{3t + T}{9t - 2T}\right)^2$$

If l_i is such as: $\overline{y} \leq l_i \leq 1-\overline{y}$

When $\overline{y} \leq l_i \leq 1-\overline{y}$, the logistic costs function is given by:

$$L(l_i, \overline{y}) = \int_0^{\overline{y}} T(l_i - u)du + \int_{1-\overline{y}}^{1} T(u - l_i)du = T.\overline{y}\,(1-\overline{y})$$

N.B.: When $\overline{y} \leq l_i \leq 1-\overline{y}$, the logistic costs are the same for all l_i. So firm i is indifferent about l_i.

Price competition leads to the following optimal response functions:

$$p_i(p_s) = \begin{cases} p_s & \text{if} \quad p_s > \dfrac{t}{2} \\ \dfrac{t^2 + 2(t-T)p_s}{2\,(2t-T)} & \text{if} \quad 0 \leq p_s \leq \dfrac{t}{2} \end{cases}$$

$$p_s(p_i) = \begin{cases} p_i - \dfrac{t}{2} & \text{if} \quad p_i > t \\ \dfrac{p_i}{2} & \text{if} \quad 0 \leq p_i \leq t \end{cases}$$

As firm i is indifferent about l_i, we determine the equilibrium values for $\overline{y} \leq l_i$:

$$p_i^{gd}(\overline{y} \leq l_i) = \frac{t^2}{3t-T} \qquad p_s^{gd}(\overline{y} \leq l_i) = \frac{t^2}{2\,(3t-T)}$$

$$D_i^{gd}(\overline{y} \leq l_i) = \frac{2t-T}{3t-T} \qquad D_s^{gd}(\overline{y} \leq l_i) = \frac{t}{3t-T}$$

$$\pi_i^{gd}(\overline{y} \leq l_i) = \frac{(2t-T)^3}{4\,(3t-T)^2} \qquad \pi_s^{gd}(\overline{y} \leq l_i) = \frac{t^3}{2\,(3t-T)^2}$$

Optimal choice of l_i when $h = gd$

When $l_i \leq \overline{y}$, firm i gets a profit $(2t-T)^2/2\,(9t-2T)$. When $\overline{y} \leq l_i$, it gets a profit $(2t-T)^3/4\,(3t-T)^2$. Simple calculus shows that for every values of (t, T) the optimal decision when $h = gd$ is such that $l_i \leq \overline{y}$. Then

$$\pi_i^{gd}(l_i^{gd}) = \frac{(2t-T)^2}{2\,(9t-2T)}$$

Firm i's decision about h

The choice of h is determined by:

$$h^* = \arg\max_{\{g,gd\}} \left\{ \pi_i^g(l_i^g), \pi_i^{gd}(l_i^{gd}) \right\}$$

Simple calculus shows that $\pi_i^g(l_i^g) > \pi_i^{gd}(l_i^{gd})$, $\forall(t,T)$. Then,

$$h^* = g$$

7.2 Case where l_s is endogenous

The traditional shop is now able to choose its location l_s and anticipates the reaction of the on line retailer. In this case, the shop faces new strategic decisions.

Either the shop is far enough from the ends of the line, then firm i's demand is divided in two parts (as for $l_s = 1/2$) and the latter has to choose $h \in \{g, d, gd\}$ and l_i. The symmetry of the problem allows us to restrict the study to the case $l_s \geq 1/2$, so we eliminate the case $h = d$.

Or the shop is close enough to the right end of the line, then each demand is on either side of the indifferent consumer and firm i's choice is limited to l_i.

The endogenous choice l_s implies a new strategy for the shop because *a priori*, l_s determines if firm i's demand is divided in two parts and, if it is, what firm i may decide about h (g or gd). Indeed, the greater l_s is, the smaller the right side of firm i's demand is, and the smaller the incentive to serve it will be. *A priori*, there should exist a value $\overline{l_s}$ above which $h = g$.

As the shop supplies the consumers left by the on line retailer, its trade off is the following. Setting on the right to make firm i choose $h = g$, but setting as left as possible to make the right side of firm i's demand (that the shop will finally serve) bigger. Actually, we show in section 3 that even for $l_s = 1/2$, firm i decides $h = g$ for every values of t and T, for other reasons than logistic costs (proposition 4.b).

In the formal study of the case where l_s is endogenous, simulations on t and T show that $\overline{l_s}$ is always under 0.41, but we assumed $l_s \geq 1/2$. So there is no value of l_s for which firm i decides to serve all its demand.

We also show that the shop always sets l_s such that firm i's demand is divided in two parts, as this latter always chooses $h = g$. Then the shop is setting as left as possible and chooses $l_s^* = 1/2$.

7.3 First best optimum with endogenous locations

We determine the case that prevails in the first best optimum.

Case $2l_s - \overline{y} \geq 1$:

see section 3.4.

N.B.: In this case, the maximum surplus is reached when $2l_s - \overline{y} = 1$.

Case $2l_s - \overline{y} < 1$ and $h = gd$.

As the consumers located between $\overline{y}$ and $2l_s - \overline{y}$ go to shop, the welfare is given by:

$$W = v - \int_0^{\overline{y}} T\left|l_i - x\right| dx - \int_{\overline{y}}^{2l_s-\overline{y}} t\left|l_s - x\right| dx - \int_{2l_s-\overline{y}}^{1} T\left(l_i - x\right) dx$$

In this case, the first best values that minimize the transportation costs would be $l_i = l_s = \overline{y} = 1/2$. Then, the welfare would be $W = v - T/4$.

The first best optimum

In the case $2l_s - \overline{y} < 1$ and $h = g$, the welfare function is the same as for the case $2l_s - \overline{y} \geq 1$. The maximum of this function is reached when $2l_s - \overline{y} = 1$. So, the case $2l_s - \overline{y} < 1$ and $h = g$ do not prevail at the first best optimum.

When we compare the maximum welfare in the case $2l_s - \overline{y} < 1$, $h = gd$ and the case $2l_s - \overline{y} = 1$, we easily find that for every $T > 0$:

$$\frac{tT}{4\left(t + T\right)} < 4T$$

Then the first best optimum is such that $2l_s - \overline{y} = 1$. The first best values are given in section 3.4.