Markov-Switching Models of Business Cycle: Can the Econometric Model detect the growth regime?

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Abstract

This article proposes first a univariate Markov-Switching Model of US GNP, decomposed into unobservable permanent and transitory components. Household consumption is then introduced into the model. The addition of this variable is interesting for two reasons :

- test of the permanent income theory through a direct approach.

- better detection of some recessions, notably those related to a deficit in global demand.

We will see that this type of modeling also enables one to find , in real time, the different recessions which affected the last thirty years. Following this, an RBC model with indivisible labour, in which occur two types of shocks (permanent and transitory governed by a first order Markov process), will be directly confronted with its natural empirical counterpart : the MS-UC bivariate model. Does the estimation carried out on simulated pseudo-observations allow us to find the reversal periods of the theoretical data generating process with greater accuracy?

keywords: Markov Switching; Unobservable Components; Business cycle; Consumption

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1 INTRODUCTION:

In the past, researchers endeavoured to assess the business cycle and to identify its turning points. The US Department of Commerce drew up economic indexes using the works of Burns and Mitchell (1946). But as Chauvet (1998) points out, this analysis doesn't provide a formal mathematical process assessment of business cycles. Chauvet fully estimates a dynamic factor model with switching regimes where the factor is common to four macro-economic variables and picks up accurately recession and expansion periods.

We are going to demonstrate in this article, that by using a stochastic factor, common to only two cointegrated variables, consumption and income, and that because of the MS model, it is possible to identify the majority of economy crises which have punctuated the last quarter of the twentieth Century. The article will demonstrate too that the general characteristics relating to the business cycle dynamic are in line with those accepted in literature. Finally, two models will be carried out. The first one will involve cycle/trend decomposition of the output. In the second, we will add an equation explaining the level of consumption. We will then ascertain to what extent (compared with the univariate case) the additional information gleaned from the individual's consumption can be of significant interest.

The second purpose of this article is to determine whether the econometric model can detect the different regimes which affect the simulated observations of an RBC model. We have chosen the theoretical model put forward by Danthine and al (1998).

From a general point of view, the sign of consumption cross correlations, labour productivity, investment with output in the pioneer RBC models, seem to match common observation. However, the levels of variance and autocorrelation of consumption remain too low when compared with those of the output. Moreover, some characteristics of the labour market do not reappear. The models generated a greater variability of productivity than that of working hours, something which is contrary to reality. Disappointed by these results, the authors wanted to extend the models in order to get closer to economic reality. In this way, Hansen (1985), assuming that labour was indivisible, reproduced the characteristics of the labour market, which up until then hadn't been properly identified. Danthine and al (1998), taking up again a more elaborate Hansen-type model, note that, under some conditions,the serie of consumption is similar to its empirical counterpart. This is one of the reasons why we'd rather implement this model, which seems to reproduce, under certain conditions, the time features of the series under study. Two types of shocks affect their economy : one is permanent and has an influence on the growth rate of labour productivity, the other alters the level of input and has only a transitory impact. These shocks, as we shall see later, can be assimilated to those which affect the transitory and permanent components of output in the econometric model. It seems relevant to place an RBC model with Marchov shocks against its empirical counterpart : a Marchov-switching econometric model. We will then examine whether the UC-MS model estimation on the pseudo observations, resulting from decision rules of the Danthine model, succeeds in detecting states of growth.

The paper is divided as follows. In the next section, the two empirical univariate and bivariate models will be developed along with the result of their estimation. Section three considers the development of the theoretical model. Following this, we examine calculations of the model's moments and the comparison between simulated and estimated chosen statistical characteristics. The final part summarises and offers suggestions for future research.

2 ECONOMETRIC MODELS:

An important advantage of the MS model is the fact of revealing expansion or recession phases and of locating turning points during a given period. With this objective in mind, we are going to develop two model types. The first will relate to an univariate context, where only the decomposition of output in trend/cycle will be examined. In the second, we will add an equation which links consumption to two income components. The question is whether the introduction of this second equation will identify periods of recession formerly unrecognised in the univariate context. The intuitive reaction is that consumption adds a new element and its slowdown could be the cause of a non-detected reversal in the course of the study on output behaviour alone.

In addition, the second model helps distinguish explicitly the influences of the two independent output components on consumption and so to put forward a view on the relevance of the permanent output theory. Are the real-life facts compatible with this way of thinking ?

2.1 Univariate Model with switching regime:

The model is the following:

$$\begin{aligned} Y_t &= Y_{0t} + Y_{1t} \\ \psi(L) Y_{0t} &= \varepsilon_{0t} \\ \varphi(L) \Delta Y_{1t} &= \mu_{S_t} + \varepsilon_{1t} \\ \mu_{S_t} &= \mu_0 + \mu_1 S_t \end{aligned}$$

$\varepsilon_{0t} \longrightarrow N(0, \sigma_0)$
$\varepsilon_{1t} \longrightarrow N(0, \sigma_1)$
$p = P(S_t = 1/S_{t-1} = 1)$
$q = P(S_t = 0/S_{t-1} = 0)$
Y_t represent the log of output.

Y_t is composed of a transitory and a permanent component, two elements which are not directly observable, but which will be assessed with time through a particular econometric process. This model clearly helps identify the transitory shock, affecting the cyclical component, Y_{0t}, and the permanent shock, altering the permanent component Y_{1t}.

Y_{0t} is a stationary autoregressive process.

Y_{1t} is random walk with drift.

Given that economies went through periods of crises followed by periods of expansion, it seems natural that the growth rate of the permanent component varies according to the regime. In this model, the change is inherent in the process. The regime $S_t(= 0 \text{ or } 1)$ plays a decisive role and affects the drift of the permanent component. In this way we can expect, that in an expansion period, the growth rate of the output trend is superior to that which applies in the case of a recession. The fact of considering the income in a cycle dynamic current brings the model closer to the data generating process.

p is the probability that the regime 1 will survive. q that of regime 0.

The regime follows first order Markov process : knowing the value of the regime at $t - 1$, is enough to determine the probability of it arrival at t.

$P(S_t = i/S_{t-1} = k, S_{t-2} = j, ...) = P(S_t = i/S_{t-1} = k)$

The state space form of the model is:

Measurement equation:

$$Y_t = \begin{bmatrix} 1 & 0 & \cdots & 0 & 1 & 1 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} \Delta Y_{1t} \\ \Delta Y_{1t-1} \\ \vdots \\ \Delta Y_{1t-q} \\ Y_{1t-1} \\ Y_{0t} \\ Y_{0t-1} \\ \vdots \\ Y_{0t-p} \end{bmatrix}$$

$Y_t = H\xi_t$

State Equation:

$$\begin{bmatrix} \Delta Y_{1t} \\ \Delta Y_{1t-1} \\ \vdots \\ \Delta Y_{1t-q+1} \\ Y_{1t-1} \\ Y_{0t} \\ Y_{0t-1} \\ \vdots \\ Y_{0t-p+1} \end{bmatrix} = \begin{bmatrix} \mu_{S_t} \\ 0 \\ 0 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ 0 \end{bmatrix} + \begin{bmatrix} \varphi_1 & \varphi_2 & \cdots & \varphi_q & 0 & 0 & 0 & \cdots & 0 \\ 1 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & \ddots & \cdots & \cdots & 0 & \vdots & \vdots & \ddots & \vdots \\ \vdots & 0 & \ddots & 0 & \vdots & 0 & 0 & \cdots & 0 \\ 1 & 0 & \cdots & 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 0 & \psi_1 & \psi_2 & \cdots & \psi_p \\ 0 & 0 & \cdots & 0 & 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 & 0 & 0 & \ddots & 0 & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \ddots & 0 \end{bmatrix} \begin{bmatrix} \Delta Y_{1t-1} \\ \Delta Y_{1t-2} \\ \vdots \\ \Delta Y_{1t-q} \\ Y_{1t-2} \\ Y_{0t-1} \\ Y_{0t-2} \\ \vdots \\ Y_{0t-p} \end{bmatrix} +$$

$$\begin{bmatrix} \varepsilon_{1t} \\ 0 \\ \vdots \\ \vdots \\ 0 \\ \varepsilon_{0t} \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$\xi_t = \tilde{\mu}_{S_t} + F\xi_{t-1} + \varepsilon_t$

$Q = E(\varepsilon_t \varepsilon_t')$

The state equation is the expression of unobservable variables in a function of the unobservable vector, delayed for a period and a vector depending on the state of the regime at any time t. The measurement equation expresses the relation between the observable variable and the unobservable variable.This writing will make Kim's estimation method (1994), which is a clever mixing of Hamilton's and Kalman's filters, possible.

2.2 The Bivariate Model:

The univariate context seems to be insufficient, insofar as the macroeconomic variables interact in an environment disrupted by shocks. In this article we will also consider the relation between consumption and output through the switching regime model.

The model is:

$$Y_t = Y_{1t} + Y_{0t}$$

in difference:

$$\Delta Y_t = \Delta Y_{1t} + \Delta Y_{0t}$$

The transitory component is stationnary and follow an AR(1):

$$Y_{0t} = \varphi_1 Y_{0t-1} + \varepsilon_{0t}$$

$\varepsilon_{0t} \rightarrow iid\ N(0, \sigma_0^2)$

Y_{1t} est marche aléatoire avec dérive:

$$\Delta Y_{1t} = \mu_{S_t} + \varepsilon_{1t}$$

où $\mu_{S_t} = \mu_0 + \mu_1 S_t$

The regime follows a first order markov-process:

$\Pr(S_t = i/S_{t-1} = j, S_{t-2} = k, S_{t-3} = l, ...) = \Pr(S_t = i/S_{t-1} = j)$ $i, j = 1, 0$

$p10 = \Pr(S_t = 0/S_{t-1} = 1)$

$p01 = \Pr(S_t = 1/S_{t-1} = 0)$

$\varepsilon_{1t} \rightarrow iid\ N(0, \sigma_1^2)$

In previous studies the existence of a cointegrated relation between consumption and output was widely recognised. This incurred a stochastic component common to both variables. As well as this, in order to assess the influence of the output's cyclical component on an individual's consumption, we put forward the following relation :

$$C_t - \gamma Y_{1t} = \gamma_0 + \beta Y_{0t} + \varepsilon_t$$

$\varepsilon_t \rightarrow iid\ N(0, \sigma^2)$

This time Y_{1t} and Y_{0t} are the components common to output and consumption. Y_{1t} being I(1) with drift acts as the common stochastic trend.

γ measures the influence of the permanent component of output on consumption.

β measures that of the cyclical component.

As a result, the theory of permanent income would be validated in part if we found a coefficient γ close to the unit and a value for β non significatly different from zero.

ε_t represents autonomous consumption as it is the part of consumption which isn't explained by output.

The state space form is as following:

Measurement Equations:

$\Delta Y_t = \Delta Y_{1t} + \Delta Y_{0t}$

$C_t - \gamma Y_{1t} = \gamma_0 + \beta Y_{0t} + \varepsilon_t$

$$\begin{bmatrix} \Delta Y_t \\ C_t - \gamma Y_{1t} \end{bmatrix} = \begin{bmatrix} 0 \\ \gamma_0 \end{bmatrix} + \begin{bmatrix} 1 & 1 & -1 \\ 0 & \beta & 0 \end{bmatrix} \begin{bmatrix} \Delta Y_{1t} \\ Y_{0t} \\ Y_{0t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ \varepsilon_t \end{bmatrix}$$

$Z_t = c + H\xi_t + \tilde{\varepsilon}_t$

$R = E(\tilde{\varepsilon}_t \tilde{\varepsilon}_t') = \begin{bmatrix} 0 & 0 \\ 0 & \sigma^2 \end{bmatrix}$

State Equations:

$\Delta Y_{1t} = \mu_{S_t} + \varepsilon_{1t}$

$Y_{0t} = \varphi_1 Y_{0t-1} + \varepsilon_{0t}$

$$\begin{bmatrix} \Delta Y_{1t} \\ Y_{0t} \\ Y_{0t-1} \end{bmatrix} = \begin{bmatrix} \mu_{S_t} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & \varphi_1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta Y_{1t-1} \\ Y_{0t-1} \\ Y_{0t-2} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{0t} \\ 0 \end{bmatrix}$$

$\xi_t = \tilde{\mu}_{S_t} + F\xi_{t-1} + \eta_t$

$$Q = E(\eta_t \eta_t') = \begin{bmatrix} \sigma_1^2 & 0 & 0 \\ 0 & \sigma_0^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The same estimation process as this of the previous case, provides us maximum likelihood estimators of interest parameters as well as transition probabilities. Unobservable components are also extracted.

2.3 EMPIRICAL RESULTS:

The empirical analysis focuses on the log of quarterly US GNP and Household Consumption multiplied by 100, for the period (1970:01-2000:02). Whether it be for the bivariate or univariate model, different autoregressive structures were tested, with the help of the maximum likelihood test. They were all put aside in favour of those that appear in this paper.

2.3.1 The Univariate case:

Tests lead us to choose AR(1) and AR(0) representation for respectively, cyclical and permanent components.

parameters

ψ_1	φ_1	μ_0	μ_1	$\sigma^*_{\varepsilon_0}$	$\sigma^*_{\varepsilon_1}$	q	p	l
0.98	0.07	0.95	-1.27	0.38	0.68	0.95	0.74	-39.75

*:Without constraining the parameter, σ_{ε_0} and σ_{ε_1}tend towards zero, so it is difficult to extract the true behaviour of the components of the output. To get round this difficulty we fix a priori these two variances to estimated values in the context of the same model without Markov switching regime.

The transitory component (fig. 4 in annex) is stationnary, but the autoregressive coefficient is next to one. The growth rate in a high period ($S_t = 0$) is about 4% per year, whereas it is negative in a recession (-0.32% per quater). The estimation, then puts forward two distincts regimes (high growth and negative growth) of the US economy during last thirty years.

The survival probability of a high regime is greater than that of a low regime (0.95 against 0.74).Consequently, the high state of the economy is more persistent than the low, since its average duration is 20 quaters>4 quaters.

Fig 1. represents the probability of expansion at t using information up to t.

Fig 2. represents the probability of expansion at t using full sample information.

Using this, we can identify the crises of American economy. It's to be noticed that the 1990' s crise is revealed contrary to the case of Hamilton's univariate MS GNP model (1989) as Chauvet noticed. It's interesting to compare this estimation with that of the bivariate model in which we examine the joint influence of consumption and output.

2.3.2 The bivariate case:

The results are :

parameters		*parameters*	
γ_0	77.87	σ_{ε_0}	0.55
γ	1.04	σ_ν	0.19
β	-0.07	σ_{ε_1}	0.5
ψ_1	0.81	q	0.95
μ_0	1.03	p	0.86
μ_1	-0.96	ℓ	-14.75

We observe that the influence of the transitory component on consumption is insignificant.($\beta = -0.07$). Individuals seem to determine their consumption exclusively according to their so-called permanent income ($\gamma = 1.04$). In this way the theory appear to be confirmed.

Moreover, this estimaton verifies the cointegrated relation between income and consumption for the period studied here. Stochastics and linear trends are represented by Y_{1t}, and the linear combination $C_t - \gamma Y_{1t}$ is stationnary since Y_{0t} is. As C_t and Y_{1t} are both I(1), the cointegrated relation linking output and consumption seems obvious. With cointegrating tests for this period, we draw similar results.

The quaterly growth rate in a period of expansion is about 1% ($\simeq$ 4% per year) whereas it's only 0.07% in a time of recession. Contrary to the former estimation, we notice a positive growth rate even in a low state. These figures appear closer to those usually met in literature. The estimation, then puts forward two distincts regimes (high growth and low growth) of the US economy during last thirty years.

Moreover the survival probability of a high growth state is greater than that of a recession regime.

The average duration is about the same that found previously.

So, if we compare this with the first model, we get more accurate, and informative results, relating to figures of growth rates in the different states.

We can interpret graphs in annex in the following way:

The marked drop of the income in 1982 seems to be closely linked to the sharp decline of the transitory component (fig. 8). At the same time, consumption and permanent income (fig. 7) didn't reveal such a big drop. Graphs of probability (fig 5 and 6) enables us to find principal recessions of the last thirty year. They are identical to those calculated by Chauvet. The 1990 recession is located contrary of the case of Stock and Watson's index (1993). So the study of the link between consumption and output in a Markov-Switching framework, is enough to identify with accuracy the different recessions.

These graphs must be compared to those of the previous study:

First, the transitory component is more stationnary, and the permanent smoother. In the univariate case , as in the bivariate, we observe distinct peaks before a strong recession, in anticipation of the imminent crisis.

The 1990's crise is more evident in the second model. Here,we can interpret that this drop particularly concerned the consumption.

To summarise, the bivariate model has the following advantages:

- the two components of income are well identified.

- it heps us to verify, using a direct approach, an implication of the permanent income hypothesis.

- it finds the well-known cointegrated relation.

- different crises are better located and values of growth rates are more in line with what is expected.

It stays to check if such a model can detect with accuracy turning points of an artificial RBC economy with switching regime.

3 THEORETICAL MODEL:

3.1 Basic Structure:

So as not to distance ourselves too much from the reality of the labour market, we prefer to develop a growth model with labour indivisibility, of the type done by Hansen (1985). This assumption is in fact driven by the observation that most people either work full time or not at all.

3.1.1 Preferences:

The utility function of (all identical) individuals depends on their level of consumption and leisure. People decide either to work a constant number of hours h_0 or not to work at all. As a result the fluctuation of working hours depends on the movement connected to entering and leaving a job, rather than the fluctuation in hours per employed worker, an idea often preferred in pioneer models and which corresponds to a perfect divisibility of labour.

Finally, so as to bypass the convexity problem engendered by such a modelisation, one which would entail difficulties in the resolution, each individual choose ex-ante a probability of working ξ_t.

Rogerson's and then Danthine's essential contributions state that, depending on their situation in the labour market, individuals do not have the same level of consumption. The utility functions are thus written as the following :

$$u(c_t, 1 - l_t) = \frac{(c_{1t}^{\gamma}(1 - l_t)^{1-\gamma})^{1-\phi}}{1 - \phi} \tag{1}$$

if the individual works.

$$u(c_t, 1 - l_t) = \frac{c_{2t}^{\gamma(1-\phi)}}{1 - \phi} \tag{2}$$

if the individual doesn't work:

$1 - l_t = h_t$ represents the number of hours worked in such a way as the total time spent working and given to leisure is equal to the dotation normalised at 1.

We expect $c_{1t} > c_{2t}$.

3.1.2 Productive activity.

The production function has the following form :

$$y_t = f(k_t, h_t P_t) = k_t^{\alpha}(h_t P_t)^{1-\alpha}\lambda_t \tag{3}$$

k_t is the capital stock at t.

P_t corresponds to work productivity with a growth rate which varies according to the nature of the economy regime..

$$P_{t+1} = x_{t+1} P_t \tag{4}$$

where $x_{t+1} = 1 + r_{t+1}$. Whith r_{t+1} equal to the growth rate.

The process followed by x_t is a first order markov process with a transition matrix :

$$\begin{array}{cc} & x_{t+1} \\ & \begin{array}{cc} x_1 & x_2 \end{array} \\ x_t \begin{array}{c} x_1 \\ x_2 \end{array} & \left[\begin{array}{cc} \pi_1 & 1-\pi_1 \\ 1-\pi_2 & \pi_2 \end{array} \right] \end{array} \tag{5}$$

It takes then two possible states x_1 et x_2.

An exogeneous shock λ_t has a direct effect on the technology of production. It is level stationary, first markov process, with a transition matrix :

$$\begin{array}{cc} & \lambda_{t+1} \\ & \begin{array}{cc} \lambda_1 & \lambda_2 \end{array} \\ \lambda_t \begin{array}{c} \lambda_1 \\ \lambda_2 \end{array} & \left[\begin{array}{cc} \pi & 1-\pi \\ 1-\pi & \pi \end{array} \right] \end{array} \tag{6}$$

The growth rate of labour productivity, with two possible states, of the theoritical model can be assimilated to the growth rate of the permanent component affected by possible change. In addition to this, the transitory component of the bivariate model is subject to transitory shocks comparable to those which have an influence on the output level in this model RBC. As a result, from this point of view, these two models automatically lend themselves to comparison.

The equilibrium of this economy is described by a range of functions which are solutions of :

$$\max E \sum_{t=0}^{+\infty} \beta^t \left\{ \xi_t \frac{(c_{1t}^{\gamma}(1-l_t)^{1-\gamma})^{1-\phi}}{1-\phi} + (1-\xi_t) \frac{c_{2t}^{\gamma(1-\phi)}}{1-\phi} \right\} \tag{7}$$

subject to:

$$\xi_t c_{1t} + (1-\xi_t) c_{2t} + i_t \leq k_t^{\alpha} (h_t P_t)^{1-\alpha} \lambda_t \tag{8}$$

the various uses of commodities mustn't exceed the output.

$$h_t = \xi_t h_0 \tag{9}$$

the aggregate hours supplied equals the number of hours worked multiplied by the probability of working.

Lastly:

$$k_t = (1 - \Omega)k_{t-1} + i_t \tag{10}$$

describes the evolution of the stock of capital through time. Ω is its period depreciation rate. i_t denotes aggregate investment in period t.

3.2 Model analysis and solution:

3.2.1 Analysis:

As King and al (1988) points out , the model's output can be made stationary under the appropriate change of variables. Each variable incorporates the same trend corresponding to the labour productivity which grows through time. Consequently, by dividing them by P_t , this trend is extracted and the variables are considered stationary.

Let us make the following definition: $\hat{c}_{1t} = c_{1t}/P_t$, $\hat{c}_{2t} = c_{2t}/P_t$, $\hat{k}_t = k_t/P_t$, etc....

Then the solution to the problem (7) must satisfy :

$$\begin{aligned} v(\hat{k}, x, \lambda) &= \max_{\{\xi, \hat{c}_1, \hat{c}_2\}} \{\xi u(\hat{c}_1, 1 - h_0) + (1 - \xi)u(\hat{c}_1, 1 - h_0) \\ &\quad + \beta \int \int (x')^{\gamma(1-\phi)} v(\hat{k}', x', \lambda') dG(x', x) dF(\lambda', \lambda)\} \end{aligned} \tag{11}$$

subject to:

$$\xi \hat{c}_1 + (1 - \xi)\hat{c}_2 + \hat{i} \leq \hat{k}^\alpha (\xi h_0)^{1-\alpha} \lambda \tag{12}$$

dG and dF being respectively the conditional distribution function of x and λ. A prime variable indicates next period's value of that variable . The first order conditions defining the optimal policy rules bring the following solutions:

$$\hat{c}_1 = \sum_s \frac{1}{A} \left\{ (1 - \Omega)k - x_s k' + \hat{c}_1^{1-\frac{1}{\alpha}} [AM_1 - M_1 + k^\alpha M_1^{1-\alpha} h_0^{1-\alpha} \lambda] \right\} P(x' = x_s / x) \tag{13}$$

$$\hat{c}_2 = A\hat{c}_1 \quad (14)$$

$$\xi = (\frac{M}{D\hat{c}_1})^{\frac{1}{\alpha}} \quad (15)$$

où:
$A = (1-h_0)^{\frac{a}{b-1}}$
$a = (1-\gamma)(1-\phi)$
$b = \gamma(1-\phi)$
$D = A^b - (1-h_0)^a - c(A-1)(1-\phi)$
$M = (1-\phi)c(1-\alpha)h_0^{(1-\alpha)}\lambda k^\alpha$
$M_1 = (\frac{M}{D})^{\frac{1}{\alpha}}$

These solutions depend on the states $\hat{k}, x, \lambda$. A method of value function iteration enables us to discover the fixed point of this function. By creating a grid of different values of capital and shocks, this method provides optimal functions $c_1^*(\hat{k}, x, \lambda), c_2^*(\hat{k}, x, \lambda), \xi^*(\hat{k}, x, \lambda)$ of interest variables for each combination $(\hat{k}, x, \lambda)$. See Christiano (1990) for a detailed review of the discrete state space procedures. Obviously, if there is a high number of values $\hat{k}$ composing the grid , the process of numerical maximisation can be longer. An anonymous researcher suggests a way of reducing the time of calculation of these optimal functions using the property of continuity of consumption.

3.2.2 Solution:

Since we intend to simulate the above model, we first need to calibrate it. In order to check the validiy of our resolving method, we retain the same choice of calibration as Danthine and al (1998).

In this way:
$\pi_1 = \pi_2 = 0.95; \pi = 0.95.$
$x_1 = 1.007; x_2 = 1;$
$\lambda_1 = 1.0115; \lambda_2 = 0.985;$

The growth rate of labour productivity in a high state is 0.7% per quarter whereas it's zero when the state is low. The other basic parameters are calibrated in the tradition of dynamic equilibrium analysis:
$\beta = 0.99, \Omega = 0.025(\simeq 1\%$ per year$), \gamma = 0.33, \alpha = 0.36.$

When optimal functions are optained, we simulate ,according to the transition probability, 1000 shock and growth rate of labour productivity series of length, the sample length of data used in the former estimations. And then create 1000 series of interest variables.

The calculation of the moments are the following:

	mean	standard deviation
$\hat{k}$	9.677	0.468
ξh_0	0.304	0.027
$\hat{TC}$	0.784	0.020
$\hat{y}$	1.057	0.043

TC is total consumption.

The standard deviation of the capital stock is too great compared with that found by Danthine and al. So it seems necessary to reflect on the choice of the capital grid or the maximisation method. The results of this in-depth investigation is forthcoming.

3.3 Comparison of simulated and estimated recession periods:

The UC-MS bivariate model is estimated for each of the 1000 simulated series. Each series is associated with a particular vector of shocks and growth rate of labour productivity. If $x_t = 1.007$, then the state of the economy is said to be in expansion, otherwise the economy is in recession.

These vectors must be compared with the estimated recession probability. By definition, when this probability is above 0.5, the regime is low. An insignificant difference between these two vectors would be synonymous with an accurate detection of reversal periods.

At this stage of the research we only carry out three simulations.

Results are the following.

length=200;

simulations	γ_0	γ	β	ψ_1	μ_0	μ_1
1	0.29	0.99	-0.4	0.39	0.002	0.029
2	-0.20	0.09	-0.47	1	0.09	-0.08
3	0.33	1	-0.33	0.53	0.005	-0.004

simulations	σ_{ε_0}	σ_ν	σ_{ε_1}	q	p
1	0.02	1^e-12	0.005	0.99	8^e-12
2	0.03	1^e-12	0.01	3.1^e-8	0.99
3	0.03	0.004	0.005	0.98	0.98

	simulation 3
Probability of detecting the right state:	66.5%
Probability of detecting the low state:	70%
Probability of detecting the high state:	59.4%

In two of three cases, regimes emerging from the estimation are of a different type to those coming from the danthide model.

One of the states corresponds to a very high quaterly growth rate (9% for case 2, 3% for case 1).

The other one is equivalent to a quaterly growth rate equal to zero for case 1 and 2. In this cases, very high state of economy is not persistent at all contrary to the low state. So this results are completly different in comparison of regimes simulated.

For the third simulation, all parameters are similar to those emerging from the estimation of the section 2. So, series which come from theoretical reflexion and real series seem to have a close behaviour.

The two states detected are the following:

- high growth rate: 0.5% per quarter
- low growth rate: 0.001% per quarter

These figures are close to the simulated growth rates.

Finaly, the probability of detecting the low regime is 70 %. Then, in this case, the econometric model succeed quite well in locating reversal periods of the theorical DGP.

However these conclusions have to be moderated since the number of simulations is small. And a further estensive study requiring a long calculation time is currently underway.

4 CONCLUSION:

With the aim of locating periods of different growth regime of US economy, we've developed two econometric models.

The first one includes trend/cycle decomposition of output in an environment where the regime follows a first order markov process with two states, and influences the drift of the permanent component. The recession probabilities of this estimated model represent a precise index of major reversals.

The second one includes an additional consumption equation: this addition brings two major points of interest:

- the ability to find with more exactitude reversal periods identified expost by the NBER

- the ability to test directly the theory of permanent income.

Thus if the study of income in a cyclical dynamic alone seems insufficient for economic interpretation, the joint study of consumption/output using a UC-MS model seems satisfactory. It has been shown that the true behaviour of agents can be identified as being compatible with the theory of permanent income.

Then, as far as I known, this article puts forward for the first time, an RBC model with switching regimes and an Unobservable-Component Markov-Switching Model. It is particularly compared simulated and estimated recession periods. If the simulated DGP were close to the real behaviour of individuals, an if the difference in prediction of recession was marginal, the UC-MS Model would be a useful tool for detecting reversal periods of the real economy. In our short study, the econometric model doesn't perform well in two cases. However a more exhaustive study must be undertake.

Finally, for future research, we could add the labour to the econometric model. In this way, the model would be more complete, more faithful and then more comparable with Danthine and al's model.

References

[1] Burns, A., Mitchell, W., 1946. Measuring Business Cycle (New York: National Bureau of Economic Research).

[2] Chauvet, M., 1998. An econometric characterization of business cycle dynamics with factor structure and regime swithching. International Economic Review 39, 969-996.

[3] Christiano, L., 1990. Solving a particular growth model by linear quadratic approximation and by value-function iteration. Journal of Business and Economic Statistics 8, 23-26.

[4] Danthine, J.P., Donaldson, J.B., Johnsen, T., 1998. Productivity growth, consumer confidence and business cycle. European Economic Review 42, 1113-1140.

[5] Danthine, J.P., Donaldson, J.B., Mehra, R., 1989. On some computational aspects of equilibrium business cycle theory. Journal of Economic Dynamics and Control 13, 449-470.

[6] Hamilton, J., 1989. A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle. Econometrica 57, 357-384.

[7] Hansen, G., 1985. Indivisible labor and the business cycle. Journal of Monetary Economics 16, 309-327.

[8] Kim, C.J., 1994. Dynamic Linear Models with Markov-Switching. Journal of Econometrics 60 1-22.

[9] King, R., Plosser, C., Rebelo, S., 1988a. Production growth and business cycles I: The basic neoclassical model. Journal of Monetary Economics 21, 195-232.

[10] King, R., Plosser, C., Rebelo, S., 1988a. Production growth and business cycles II: New directions. Journal of Monetary Economics 21, 309-341.

[11] Rogerson, R., 1988. Indivisible labor, lotteries, and equilibrium. Journal of Monetary Economics 21, 3-16.

[12] Stock, J.H, Watson, M.W., 1993. A Procedure for Predicting Recessions with Leading indicators: Econometric Issues and Recent experience, in J.H. Stock and M.W. Watson, eds., Business Cycles, Indicators and Forecasting (Chicago: University of Chicago Press for NBER, pp. 255-284).

ANNEX:

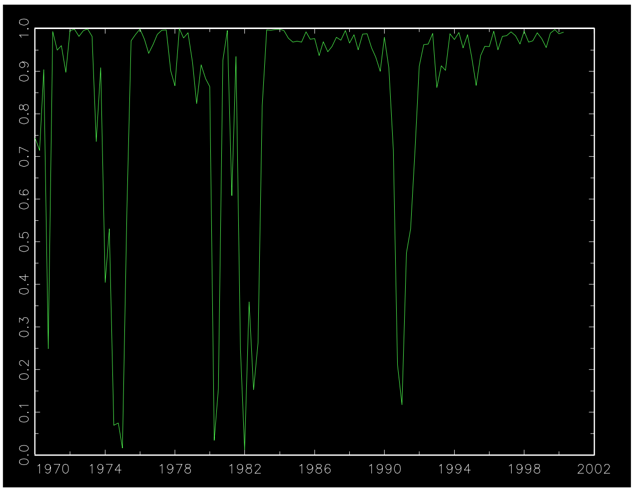

fig. 1: filtered expansion probability

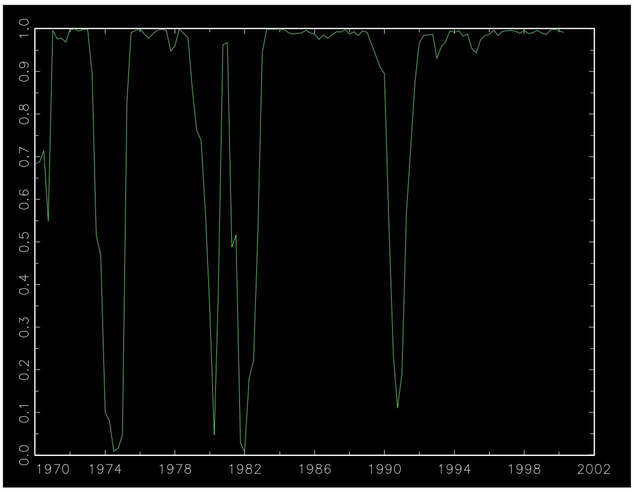

fig. 2: smoothed expansion probability

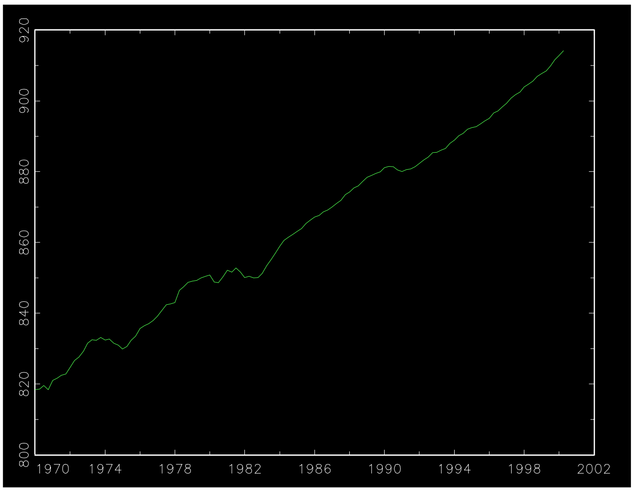

fig. 3:permanent component

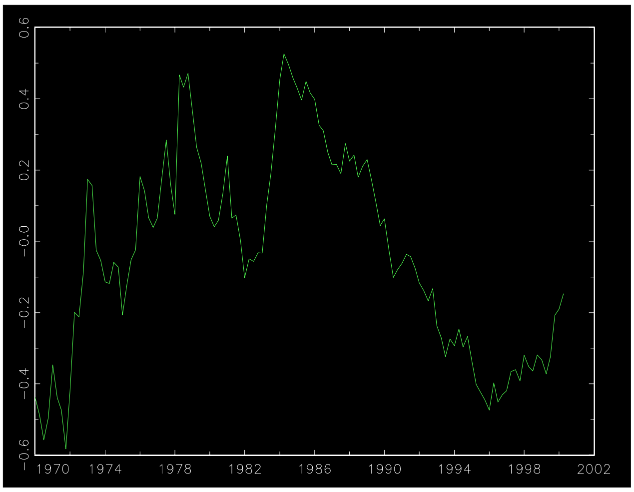

fig. 4: cyclical component

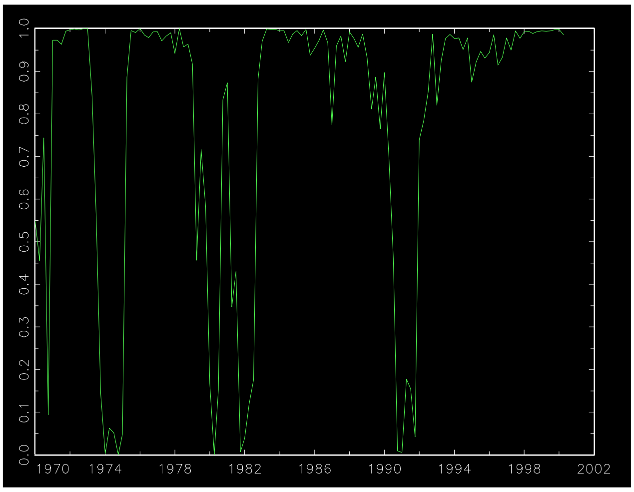

fig. 5: filtered expansion probability

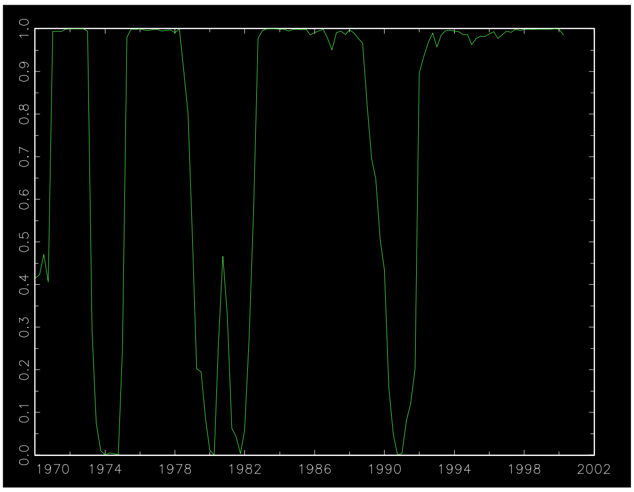

fig. 6: smoothed expansion probability

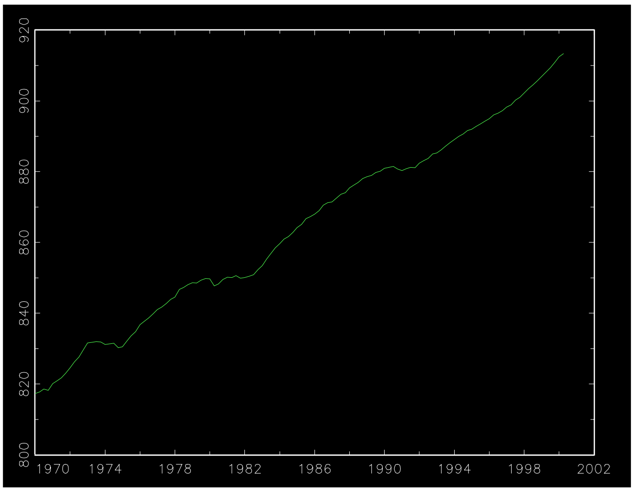

fig. 7: permanent component

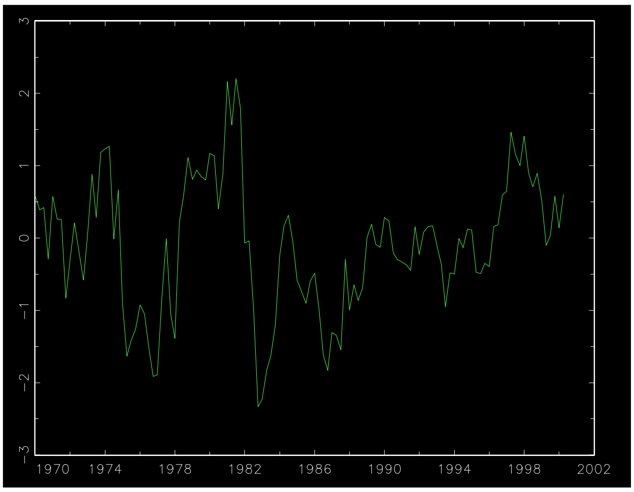

fig. 8: cyclical component