A model of employer–employee effects industry correlations[*]

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PRELIMINARY
Do not quote
February 2002

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[*] The authors gratefully acknowledge the financial support of the National Science Foundation and the Centre de Recherche en Économie et Statistique (CREST), Paris. This research is a part of the U.S. Census Bureau's Longitudinal Employer-Household Dynamics Program (LEHD), which is partially supported by the National Science Foundation Grant SES-9978093 to Cornell University (Cornell Institute for Social and Economic Research), the National Institute on Aging, the U.S. Department of Labor (ETA) and the Alfred P. Sloan Foundation. Any opinions, findings, and conclusions or recommendations expressed in this publication are those of the authors and do not necessarily reflect the views of the U.S. Census Bureau, the National Science Foundation, or the Institut National de la Statistique et des Études Économiques. Confidential data from the LEHD Program were used in this paper. The U.S. Census Bureau is preparing to support external researchers use of the U.S. data under a protocol to be released in the near future; please contact Ron Prevost, `Ronald.C.Prevost@census.gov`. For further information concerning the French data, contact INSEE/CREST, 15, bd Gabriel Péri, 92245 Malakoff Cedex, France.

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Abstract

Recent work on linked employer–employee data has shown, among other results, that person effects and firm heterogeneity in wages explain about half of the inter-industry wage differentials each. One puzzling feature of this decomposition is the pattern of correlations: intra-industry correlation between firm and worker effects is negative, while it is positive on the inter-industry level. As no ready theory can explain this, we investigate a tractable parametric model of unobserved heterogeneity among workers and firms and in an ongoing project estimate it on aggregate moments.

Keywords: Inter-industry wage differentials, firm-size wage differentials, linked employer–employee data.
JEL classification: J31

1 Motivation

The persistence of inter-industry and firm-size wage differences has been widely documented in the empirical literature.[1] Using linked employer–employee data (as documented by Abowd and Kramarz 1999a,b) researchers have been able to study these differences at the firm and employee levels. Following Abowd, Kramarz, and Margolis (1999), wages can be decomposed as a worker effect (in turn, with both explained and unexplained components) and a firm effect. Abowd et al. (2001) and Abowd and Kramarz (2000) have show the relative importance of each of those effects on wages: person effects and firm heterogeneity in wages explain about half of the inter-industry wage differentials each. A puzzle has surfaced from these studies: worker effects and firm effects are negatively and significantly correlated on the intra-industry level, while they are positively correlated on the inter-industry level. This holds for both France and the U.S., and is shown in table 1 for the U.S. In figure 1 we show at the inter-industry level the positive correlation.

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[1] Krueger and Summers (1987, 1988) for example.

It would therefore appear that workers with a large personal component are attracted or selected to industries with high average firm effects, but that inside each industry they are negatively selected to firms with low firm effects. The difficulty lies in the abstract nature of the firm effect, since it hides the necessary heterogeneity of the workforce. These correlations might therefore proceed from a statistical artifact in the estimation.

To shed some light on this problem, we will consider the simplest model that will allow the estimation of the underlying parameters, based on the population moments observed. It will consist of unexplained worker heterogeneity, as Gibbons and Katz (1992) have argued, as well as firm heterogeneity. Differences in production functions will lead to differences in hiring patterns and systematic wage differences. Those differences, as argued in section 3, will be warranted by differences in incentives.

We proceed as follows: in the next section we develop exact formulas for the estimation of worker and firm effects in a multi-sector economy. In section 3 a model of wage setting is presented, incorporating both incentive wage and efficiency wage characteristics, fitting into the framework of section 2. In section 4 we settle the parametric setup. Finally we present the estimation procedure and some future developments.

2 General formulas of worker and firm effects

Let the economy consist of two types of workers. This is not as limiting as it sounds, for it may only represent an heterogeneity inside a particular class of workers, i.e., once we consider effects net of observables.

The workers are hired by a continuum of firms, indexed by the share x

Table 1: Variances and correlations in four U.S. states

	$\ln w$	θ	ψ	$x\beta$	resid	θ_j	ψ_j	θ_k	ψ_k
std	0.842	0.772	0.345	0.638	0.335	0.294	0.288	0.125	0.188
var	0.709	0.595	0.119	0.407	0.112	0.087	0.083	0.016	0.035
	$\ln w$	θ	ψ	$x\beta$	resid	θ_j	ψ_j	θ_k	ψ_k
$\ln w$	1.000	0.501	0.487	0.288	0.387	0.216	0.346	0.142	0.358
θ		1.000	**0.000**	-0.509	0.000	0.392	-0.020	0.166	0.029
ψ			1.000	0.108	0.000	-0.036	0.837	0.092	0.547
$x\beta$				1.000	0.000	-0.152	0.032	-0.056	0.152
resid					1.000	0.000	0.000	0.000	0.000
θ_j						1.000	**-0.042**	-0.002	-0.001
ψ_j							1.000	0.000	0.000
θ_k								1.000	**0.163**
ψ_k									1.000

Notes: θ is worker effect, ψ is firm effect; θ_j and ψ_j are firm-level values (what we call *intra-industry* variables) and θ_k and ψ_k are industry-level values (*inter-industry*). Source: authors' computations.

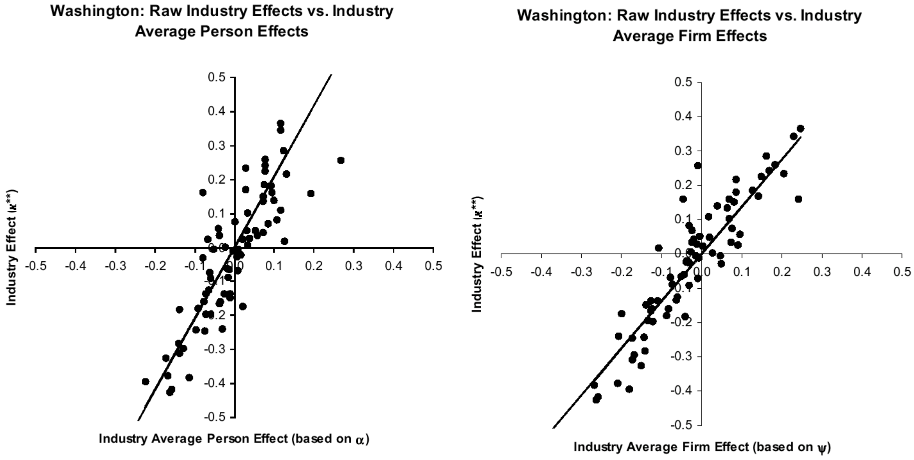

Figure 1: Raw industry effects vs. average person and firm effects, from Abowd and Kramarz (2000).

of type 1 workers each firm hires, the size t of the firm, and the sector k. The economy is divided in a finite number $\bar{k}$ of sectors. In sector k, there is a mass (density) $\mu_k(x, t)$ of firms of type (k, x, t).

We omit from what follows the k index except when it is necessary. The economy as a whole is represented by the space $\Omega = \{\Omega_1, \dots, \Omega_{\bar{k}}\}$ with $\Omega_k = [0, 1] \times \mathbb{R}^+$, the space defining each sector. There is a quantity (or number) $J = \int_\Omega d\mu$ of firms, $J_k = \int_{\Omega_k} d\mu_k$ in each sector. Firm (x, t) hires xt of type 1 workers, and $(1 - x)t$ of type 2. We write $\int_\Omega xt\, d\mu(x, t) = M$ and $\int_\Omega (1 - x)t\, d\mu(x, t) = N$ so that there are $M + N$ workers in the economy. Type 1 workers earn $w(x, t, k)$ in firm (x, t) in sector k while type 2 earn $u(x, t, k)$; the wage only depends on the sector, the share of type 1 workers, and the size of the firm.

As in Abowd et al. (1999) (henceforth AKM99) we write the individual wage in each period of time as the sum of a person effect and a firm effect.

$$\ln w_{i,j(i,t),t} = \theta_i + \psi_{j(i,t)} + \varepsilon_{i,j(i),t}$$

However we consider here only the limiting (and purely theoretical) case where the number of time periods goes to infinity and the workers visit ergodically all firms and all sectors. This frees us from the identification problem discussed in detail in the above paper. Moreover, the effects estimated in AKM99 would converge in probability to the theoretical values we show below.

In this case, even though the type of the worker is not directly known by the econometrician, observing the workers jump from job to job provides enough information to be able to reduce the problem to the following:

we look for W and U (worker effects), and $\psi(k, x, t)$ (firm effect in industry k for a firm of size t hiring a proportion x of type 1 workers) which minimize

$$\min_{W,U,\psi} \int_{\Omega} \Big\{ xt\big(w(x,t) - W - \psi(x,t)\big)^2 \\ + (1-x)t\big(u(x,t) - U - \psi(x,t)\big)^2 \Big\} d\mu(x,t).$$

This is the equivalent of the sum of squared residuals in the ordinary least squares setting. The solutions to this problem are not unique (they are equal up to a constant), and we can show that:

Theorem 1. *Under integrability assumptions on w and u, the solutions are of the type*

$$W = \frac{\int x(1-x)tw(x,t)}{\int x(1-x)t} + \Lambda, \tag{1}$$

$$U = \frac{\int x(1-x)tu(x,t)}{\int x(1-x)t} + \Lambda, \tag{2}$$

$$\psi(x,t) = x(w(x,t) - W) + (1-x)(u(x,t) - U) - \Lambda \tag{3}$$

where the integrals are all taken on Ω with respect to the measure μ, and Λ is an arbitrary constant (we will set $\Lambda = 0$ in the rest of the paper).

Proof. Set $U = 0$ for the time being, as the solutions are only identified up to a constant. Let us write $J(W, \psi) = \int_{\Omega} \rho(W, \psi)(x, t)\, d\mu(x, t)$. J is defined on $\mathbb{R} \times L^2(\Omega, \mu)$, and may take the value $+\infty$. It is clear that ρ is convex in its two arguments (as sum of two squares); hence J is also convex, and can be show to be strictly convex if $w(x,t) - u(x,t)$ is not a constant. Hence it has a unique minimum under this assumption, characterized by the first order constraints

$$\int_{\Omega} xt(w(x,t) - W - \psi(x,t))\, d\mu(x,t) = 0 \tag{4}$$

$$xt(w(x,t) - W - \psi(x,t)) + (1-x)t(u(x,t) - \psi(x,t)) = 0 \tag{5}$$

Hence $W = \int xt(w-\psi)\, d\mu / \int xt\, dB$ and $\psi = x(w-W) + (1-x)u$. Replacing ψ by this last value in equation (4) and simplifying with the fact that $\int x^2 t = M - \int x(1-x)t$ and $\int x^2 tw = \int xtw - \int x(1-x)w$ proves the theorem. $\square$

Remark 1 (Some special cases).
The case where μ_k for all k is $\delta_{1/2,2}$, a Dirac distribution of firms of equal size and same number of type 1 workers, leads to $W = \sum_k w_k/\bar{k}$, $U = \sum_k u_k/\bar{k}$, $\psi_k = \frac{1}{2}(w_k - \bar{w} + u_k - \bar{u})$, where w_k is the wage in sector k.
The case where the distribution β has all mass in t at $t = 2$ and $w(x,t) = u(x,t) + \delta$ leads to $W = U + \bar{\delta}$ and $\psi(x,t) = u(x) - U$.

We define the firm-employee effects covariances $\mathrm{cov}(\hat{\theta}, \hat{\psi})$ as

$$\frac{\int (xtW + (1-x)tU)\psi(x,t)\, d\mu}{\int t\, d\mu} - \frac{\int (xtW + (1-x)tU)\, d\mu}{\int t\, d\mu} \frac{\int \psi(x,t)\, d\mu}{\int t\, d\mu} \tag{6}$$

For total covariance the integrals are taken over Ω, for within-sector covariance over Ω_k for all k, and the between-sector covariance is the discrete covariance between the means (i.e. integrals) on the Ω_k. Total covariance is as usual the sum of between-sector and within-sector covariances.

3 An economic model

3.1 Setup and equilibrium

We consider an economy with a continuum of firms indexed by j in an arbitrary space. Each firm must fill two types of jobs (1 and 2) to be able to produce a unique good of price p. Firms differ by their production function, which exogenously fixes the size of the firm and the amount of a fixed production factor. Thus they will hire different quantities of workers in each type of jobs.

$$F_j(K_j, N_{1j}, N_{2j}) = \min\left(aK_jN_{1j}, b_k(K_j)\,N_{2j}\right),$$

where K_j is the exogenously fixed production factor specific to each firm (we could think of it as some sort of capital, or as some history-dependent production structure), $b_k(K_j)$ is the (possibly sector dependent) productivity of jobs of type 2. The size of each firm is fixed to N_j.

The production in each job is separable from the production of all other jobs. The firm offers two types of contracts corresponding to the two techniques of production: incentive and fixed wages.

In the first type of contract workers are paid an incentive wage w_1. In the second type of contract workers are paid a fixed wage w_2. The first type of contract—incentive pay—is used to compensate workers in type 1 jobs; the second type of contract—fixed pay—is used to compensate workers in type 2 jobs.

In the economy, there are two types of workers as in the previous section, endowed with a cost of effort κ_1 and $\kappa_2 > \kappa_1$. This type is known to the worker but not to the firm. Firms post a number of contracts for each type of job; workers apply to one job in one firm. We will look for the equilibria of the underlying game that have the following properties:

- type 1 workers apply to type 1 jobs;
- type 2 workers apply to type 2 jobs;
- firms maximize profits given workers' behavior.

The utility of a worker of type i in a firm j with wage w depends on the amount of extra effort e she exerts above the minimum amount of effort $\bar{e}$ enforceable by the firm:

$$U_i(w,e) = w - \bar{\kappa}\bar{e} - \frac{1}{2}\kappa_i e^2$$

where $\bar{\kappa}$ is the utility cost of work for minimum effort.[2] Workers are risk neutral, and have an outside opportunity worth $\overline{u}$ in utility units. Workers in a fixed wage job will only produce the amount corresponding to the minimum effort level $\bar{e}$, and thus will be willing to work for any wage greater than $\overline{u} + \bar{\kappa}\bar{e}$.

Workers in an incentive pay job optimally choose their effort level according to the piece rate α. Productivity in the job is a random variable π with mean μ_j and second moment σ_j^2, observed after the contracts have been signed but before production occurs. The optimal effort is therefore $e = \alpha\pi/\kappa_i$, expected output is $\mu_j\bar{e} + \alpha\sigma_j^2/\kappa_i$ and expected utility of a type i worker in type 1 job is

$$U_{i,1} = \alpha\mu\bar{e} + \frac{\alpha^2\sigma_j^2}{2\kappa_i} - \bar{\kappa}\bar{e}$$

Workers in a fixed pay job only work the minimum effort level $\bar{e}$ and

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[2] We follow Paarsch and Shearer (2000) for the description of the utility function.

are paid $\bar{u}+\bar{\kappa}\bar{e}$. It is possible for $\bar{e}$ to depend on both the sector and the capital of the firm.

Assumption 1. *The minimum effort $\bar{e}$ in type 1 jobs does not depend on the firm's capital K_j (but it may be industry dependent), while in type 2 jobs $\bar{e}_k(K_j)$ is increasing in the firm's capital for type 2 jobs, inside each sector k.*

Assumption 2. *The profit that the firm extracts from a type 1 job is decreasing in the piece rate α, for all values of α such that the employee receives at least her reservation utility.*

This ensures the fact that α will be chosen so that the utility of type 1 workers in type 1 jobs is the reservation utility, and hence is constant in each industry. This is not a strong constraint since the profit is a concave parabola attaining its maximum at $p/2-\mu\bar{e}_k(K_j)\kappa_i/\sigma_j^2$.

Theorem 2. *Under assumptions 1 and 2, type 2 workers will choose the fixed wage job and will be indifferent between all firms; type 1 workers will choose incentive pay jobs and will also be indifferent between all firms. Firms maximize profit for each type job.*

Proof. By assumption 2, the firm sets α so that $U_{1,1}=\overline{u}$. But $U_{2,1}<U_{1,1}$ for a given α, since $\kappa_1<\kappa_2$. Thus type 2 workers will not choose a type 1 job because they would receive a lower utility, while type 1 workers are indifferent between the two types of jobs (we could always set $U_{1,1}=\overline{u}+\varepsilon>\overline{u}>U_{2,1}$ so that type 1 workers strictly prefer type 1 jobs). Notice that, since $U_{1,1}$ is increasing in α, this is the minimal value accepted by the workers. Firms' profit on type 2 jobs is decreasing in the wage w_2, so profit is maximized for the minimal value of w_2 accepted by the workers, i.e. $\overline{u}+\bar{\kappa}\bar{e}_k(K_j)$. ☐

Assumption 3. *The elasticity of productivity in the second type of job is less than 1, i.e.* $b'_k(K)K/b_k(K) < 1$*, for all* K.

Theorem 3. *Under assumptions 1, 2 and 3, under the constraint that* K_j *and* N_j *are exogenously fixed, the firm's choice of* N_{1j} *is decreasing in* K_j *while firm's choice of* N_{2j} *is increasing in* K_j*.*

Proof. Due to the Leontieff specification of the output function, maximizing output per worker leads to $N_{1j} = \frac{b_k(K_j)N_j}{aK_j+b_k(K_j)}$ and $N_{2j} = \frac{aK_jN_j}{aK_j+b_k(K_j)}$. Differentiating with respect to K_j gives

$$\frac{\partial N_{2j}}{\partial K_j} = \frac{N_j a b_k(K_j)}{(aK_j + b_k(K_j))^2}\left(1 - \frac{b'_k(K_j)K_j}{b_k(K_j)}\right),$$

which is negative by assumption 3. The derivative of N_{1j} with respect to K_j is the opposite of the derivative of N_{2j} above, which concludes our proof.

□

Notice that $\frac{N_{1j}}{N_j}$ in each industry is independent of N_j, and is decreasing in K_j. Therefore we can invert this relationship and write K_j as a function of $x \equiv \frac{N_{1j}}{N_j}$ (as in section 2), and use $t \equiv N_j$

Main results

- Wage for type 1 workers is constant in each sector, potentially different across sectors.
- Wage for type 2 workers is decreasing in the share of good workers, by assumption 3.
- Firms can be indexed by the pair consisting of the share of type 1 workers and the size of the firm; if we assume these to be drawn from a bivariate distribution, we may use the results of section 2.

3.2 Covariances induced by a simpler model

We interpret the previous results in the terms used in section 2, with a simplifying assumption on the distribution of (x, t).

We assume in this subsection that given x there is only one size of firm in each industry. Thus we can use x as the index of firms in this economy, and write $t = \tau_k(x)$. If τ is increasing, then inside each industry N_{1x} (type 1 workers in firm x) increases with x, whereas N_{2x} decreases.

Lemma 1. *Inside each industry, the total person effect for each firm ($xtW + (1-x)tU$ in the notations of section 2) will be increasing in x.*

The shape of the firm effect is ambiguous. Under the assumptions 1 through 3, the firm effect will be decreasing inside each sector if $(1-x)\tau(x)(u(x)-U)$ is decreasing (in the notations of the first section). This holds if $(1-x)\tau(x)$ is not too decreasing.

Lemma 2. *Inside each sector, the firm effect can be decreasing in x.*

These lemmas show that the covariance between firm and person effects inside each sector can be negative (the firm effect is decreasing in x while the person effect is increasing in x). Thus within-industry correlations can be negative in this model, provided some assumptions on capital, minimum effort and size of the firm hold.

The between-industry correlation is governed by the mean in each sector of the person and the firm effect. If sectors are ranked by mean size of firms, then it is possible for the mean person effect to increase with the mean size of the firm, while the mean firm effect is increasing, for example if $b_k(\bar{K}_k)$ is increasing with k.

4 Estimating the model

We draw on the implied structure of wages from section 3 and on the firm and worker effects from section 2.

4.1 Parametric setup

In the notations of section 1, we assume the following specification:

Wage type 1 $w(x,t,k) = w_k$, the wage is constant in each industry (K)

Wage type 2 $u(x,t,k) = u_{0k} + u_{1k}x + u_{2k}t + u_{3k}xt + \epsilon_u$ $(4K)$

We expect $u_1 < 0$ from the incentive argument and $u_2 > 0$ from the results on firm-size wage differentials.

Distribution of firm sizes is assumed to be log-normal in each sector, $f(t|k) = \text{log-normal}(\mu_k, \sigma_k^2)$ $(2K)$

Distribution of share of type 1 workers conditional on firm size is a Beta distribution with parameters depending on the size of the firm, sector varying; $f(x|t,k) = \text{Beta}(p_k(t), q_k(t))$ with $p_k(t) = p_{0k} + p_{1k}t$, $q_k(t) = q_{0k} + q_{1k}t$ $(4K)$

Sector sizes in number of firms, J_k for all k The size of each sector in terms of workers is given by $J_k \iint tf(t)f(x|t)\, dxdt$. (K)

The total number of unknown parameters is therefore $12K$. Identifiable parameters: $12K - 1$, since only $J_k / \sum_{k'} J_{k'}$ is identified.

The measure μ used in section 2 would therefore be $\mu_k(x,t) = J_k f(t|k) f(x|t,k)$. Formulas for the log-normal and the beta densities is given in appendix A.1.

We will use the following moments to fit the parameters:

Intra-industry variances and covariances of firm and person effects $(3K)$

Intra-industry variances and covariances by size of firm $(3KT)$

if we partition firms in T categories according to size.

Total moments available: $3(T+1)K$. Therefore we must have $T \geq 3$ to be able to estimate the parameters.

4.2 Correlation formulas

Given this parametric structure, we can construct the function $\Psi : \mathbb{R}^{12K} \to \mathbb{R}^{3(T+1)K}$ that, given the parameters $(w_k, u_{jk}, J_k, \mu_k, \sigma_k, p_j, q_j) \in \mathbb{R}^{12K}$, calculates the covariances and variances of the underlying model. By asymptotic least squares we will therefore be able to find the parameters that best fit the observed moments.

We will show first that all the double integrals appearing in section 2 can be simplified to simple integrals, and that all covariances can be written as simple functions of a finite number of simple integrals.

Definition 1. *We write* $\mathbb{S}(g(X,T)|k)$ *the integral* $\iint_{\Omega_k} g(x,t)f(t)f(x|t)\,dxdt$ *for arbitrary integrable* g.

Theorem 4. *Any integral of the type* $\mathbb{S}(X^i T^j|k)$ *can be calculated as a simple integral; we have*

$$\mathbb{S}(X^i T^j|k) \equiv \iint_{\Omega_k} x^i t^j f(t) f(x|t)\,dxdt = \int_{\mathbb{R}^+} t^j \frac{B(p(t)+i, q(t))}{B(p(t), q(t))} f(t|k)\,dt. \tag{7}$$

Proof.

$$\begin{aligned}\iint_{\Omega_k} x^i t^j f(t) f(x|t)\, dx dt &= \int_{\mathbb{R}^+} t^j f(t|k) \int_0^1 \frac{x^{p_k(t)+i-1}(1-x)^{q_k(t)-1}}{B(p_k(t), q_k(t))}\, dx\, dt \\ &= \int_{\mathbb{R}^+} t^j f(t|k) \frac{B(p_k(t)+i, q_k(t))}{B(p_k(t), q_k(t))}\, dt \\ &= \int_{\mathbb{R}^+} t^j \frac{B(p_k(t)+i, q_k(t))}{B(p_k(t), q_k(t))} f(t|k)\, dt,\end{aligned}$$

□

Since $B(p+1, q) = \frac{p}{p+q} B(p, q)$ and $B(p, q) = B(q, p)$ the ratios of the beta functions has an explicit expression $\frac{B(p+i,q)}{B(p,q)} = \frac{p(p+1)\cdots(p+i-1)}{(p+q)\cdots(p+q+i-1)}$.

Setting $y = \log t$ the integral is fairly easy to calculate numerically, since $e^y f(e^y)$ is gaussian, $p(e^y) = p_0 + p_1 e^y$ and $q(e^y) = q_0 + q_1 e^y$.

$$\mathbb{S}(X^i T^j | k) = \int_{\mathbb{R}} e^{jy} R(y, i) \phi\left(\frac{y - \mu_k}{\sigma_k}\right) dy \tag{8}$$

with $R(y, i)$ a rational fraction in e^y, $R(y, i) \equiv \frac{p(p+1)\cdots(p+i-1)}{(p+q)\cdots(p+q+i-1)}$.

4.2.1 The person effects

We show that $\iint x(1-x) t(\cdot) f(x, t)\, dx dt$, with $(\cdot)$ equal to $w(x, t)$, $u(x, t)$ and 1, are written as sums of $\mathbb{S}(X^i T^j | k)$ terms.

We have

$$\begin{aligned}
\iint x(1-x)\,tw(x,t)f(x,t)\,dxdt &= \sum_k J_k \iint x(1-x)\,tw(x,t,k)f(x|t)f(t)\,dxdt \\
&= \sum_k w_k J_k \left[\int_{\mathbb{R}^+} tf(t,k)\int_0^1 x(1-x)f(x|t)\,dx\,dt\right] \qquad (9)\\
&= \sum_k w_k J_k \mathbb{S}[X(1-X)T \mid k] \qquad (10)\\
&= \sum_k w_k J_k [\mathbb{S}(XT|k) - \mathbb{S}(X^2T|k)] \qquad (11)
\end{aligned}$$

and

$$\begin{aligned}
\iint x(1-x)\,tf(x,t)\,dxdt &= \sum_k J_k \mathbb{S}[X(1-X)T \mid k] \qquad (12)\\
&= \sum_k J_k [\mathbb{S}(XT|k) - \mathbb{S}(X^2T|k)] \qquad (13)
\end{aligned}$$

Thus W is the between-sector mean of the wages of type 1 workers, weighed across sectors by $J_k[\mathbb{S}(XT|k) - \mathbb{S}(X^2T|k)]$.

For the workers of type 2 we have

$$\begin{aligned}
&\iint x(1-x)\,tu(x,t)f(x,t)\,dxdt \\
&\quad = \sum_k J_k \iint x(1-x)\,t(u_{0k} + u_{1k}x + u_{2k}t + u_{3k}xt)f(x|t)f(t)\,dxdt \\
&\quad = \sum_k J_k [u_0\mathbb{S}(XT) + u_2\mathbb{S}(XT^2) - u_1\mathbb{S}(X^3T) + u_3\mathbb{S}(X^3T^2) \\
&\qquad\qquad + (u_1 - u_0)\mathbb{S}(X^2T) + (u_3 - u_2)\mathbb{S}(X^2T^2)]
\end{aligned}$$

where we have dropped the $_k$ indices for simplicity.

4.2.2 Firm effects

Firm effects are given by the formula

$$
\begin{aligned}
\psi(x,t,k) &= x(w(x,t,k) - W) + (1-x)(u(x,t,k) - U) \qquad (14) \\
&= x(w_k - W) + (1-x)(u_{0k} - U + u_{1k}x + u_{2k}t + u_{3k}xt) \\
&= u_{0k} - U + (w_k - W + u_{1k} - u_{0k})x - u_{1k}x^2 \\
&\quad + u_{2k}t + (u_{3k} - u_{2k})xt - u_{3k}x^2 t \qquad (15)
\end{aligned}
$$

This is a polynomial in x and t, easy to calculate.

4.2.3 Variances and covariances

The total worker effect in a firm (x, t) is equal to

$$
\theta(x,t) = t\theta_m(x,t) = xtW + (1-x)tU \qquad (16)
$$

Since $\iint xtf(x,t)\,dxdt = M$ and $\iint (1-x)tf(x,t)\,dxdt = N$, we have

$$
\begin{aligned}
\bar{\theta} = \frac{\mathbb{S}(\theta)}{\mathbb{S}(T)} &= \frac{1}{M+N}\iint \theta(x,t)f(x,t)\,dxdt \\
&= \frac{1}{M+N}(MW+NU) \qquad (17) \\
\operatorname{var}\theta &= \frac{1}{M+N}\iint (xtW^2 + (1-x)tU^2)f(x,t)\,dxdt - (\bar{\theta})^2 \\
&= \frac{MN}{(M+N)^2}(W-U)^2 \qquad (18) \\
\operatorname{var}(\theta|k) &= \frac{S(XT|k)}{S(T|k)}\left(1 - \frac{S(XT|k)}{S(T|k)}\right)(W-U)^2 \qquad (19)
\end{aligned}
$$

The mean firm effect, as weighed by the size of each firm, is

$$\begin{aligned}\bar{\psi} &= \frac{\mathbb{S}(T\psi)}{\mathbb{S}(T)}\\ &= \Big(\sum_k J_k \mathbb{S}(T|k)\Big)^{-1} \cdot\\ &\quad \sum_k J_k \mathbb{S}\Big((u_{0k} - U)T + (w_k + U - W + u_{1k} - u_{0k})XT\\ &\qquad - u_{1k}X^2T + u_{2k}T^2 + (u_{3k} - u_{2k})XT^2 - u_{3k}X^2T^2 \;\Big|\; k\Big).\end{aligned}$$

The variance is equal to

$$\operatorname{var}\psi = \Big(\sum_k J_k \mathbb{S}(T|k)\Big)^{-1} \sum_k J_k \mathbb{S}(T\psi^2|k) - (\bar{\psi})^2 \tag{20}$$

and its full formula is given in the appendix A.2.

The covariances are defined as

$$\operatorname{cov}(\theta,\psi|k) = \frac{1}{\mathbb{S}(T|k)} \int_{\Omega_k} (\theta(x,t) - \mathbb{S}(\theta|k))(\psi(x,t) - \mathbb{S}(\psi|k)) f(x|t) f(t)\, dxdt$$

but again the formula for the cross product is in the appendix.

5 Estimation and further directions

We are thus left with the problem of estimating $12K - 1$ parameters by fitting a function that needs the evaluation of 19 numerical integrals, not infeasible but computationally intensive. The function $x \mapsto (\psi(x) - \hat{m})'(\psi(x) - \hat{m})$ of squared distance to the estimated moments $\hat{m}$ is non-convex, and possibly has a large number of local minima. We implement the minimization both by an Adaptive Simulated Annealing method (Ingber, 1989, 1996) or by standard non-linear least square routines. Initial results are interesting though much remains to be done in this new way of looking at industry wage differentials.

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A Appendix

A.1 Distributions

Distributions: log-normal (μ,σ^2)

$$f(t) = \frac{1}{\sqrt{2\pi\sigma^2}} \frac{1}{t} \exp\left(-\frac{1}{2\sigma^2}(\log t - \mu)^2\right) \tag{21}$$

Beta (p,q)

$$f(x|t) = \frac{x^{p-1}(1-x)^{q-1}}{B(p,q)} = \frac{\Gamma(p+q)}{\Gamma(p)\Gamma(q)} x^{p-1}(1-x)^{q-1} \tag{22}$$

where $B(p,q)$ is the beta function and $f(x,t,k) = J_k f(x|t,k) f(t,k)$; J_k is the size (in firms) of sector k.

A.2 Variance of the firm effect

$$\begin{aligned}
\mathbb{S}(T\psi^2|k) = \mathbb{S}\Big\{ &(U-u_0)^2 T + X^4 T {u_1}^2 + 2T^2(-U+u_0)u_2 + T^3 {u_2}^2 \\
&+ 2X^4T^2u_1u_3 + 2X^3T^3(u_2-u_3)u_3 \\
&+ X^4T^3{u_3}^2 + 2XT^3u_2(-u_2+u_3) + X^2T^3({u_2}^2 - 4u_2u_3 + {u_3}^2) \\
&+ 2X^3T^2(u_1(u_2-2u_3) + u_3(-U+W+u_0-w_0)) \\
&- 2XT(U-u_0)(U-W-u_0+u_1+w_0) \\
&- 2X^3Tu_1(U-W-u_0+u_1+w_0) \\
&+ 2XT^2((-U+u_0)u_3 + u_2(2U-W-2u_0+u_1+w_0)) \\
&+ 2X^2T^2(u_2(-U+W+u_0-2u_1-w_0) + u_3(2U-W-2u_0+u_1+w_0)) \\
&+ X^2T^2({u_0}^2 + {u_1}^2 + (U-W+w_0)^2 + 2u_1(2U-W+w_0) - 2u_0(U-W+2u_1+w_0)) \;\Big|\; k\Big\}
\end{aligned}$$

$$\begin{aligned}
E(\theta\psi|k) = E\Big(&TU(-U+u_0) + TX^3(U-W)u_1 + T^2Uu_2 \\
&+ T^2X^3(U-W)u_3 + T^2X((W-2U)u_2 + Uu_3) \\
&+ T^2X^2((U-W)u_2 + (W-2U)u_3) \\
&+ TX^2((U-W)u_0 + (W-2U)u_1 - (U-W)(U-W+w_0)) \\
&+ TX((W-2U)u_0 + U(2U-2W+u_1+w_0)) \;\Big|\; k\Big)
\end{aligned}$$