Stackelberg Bargaining and Strategic Inefficiencies[*]

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Abstract

In Rubinstein - Ståhl bargaining between two sellers and a buyer on prices and quantities, strategic inefficiencies arise, similar to the Stackelberg outcome. In equilibrium when goods are substitutes, agreement will be with the larger seller first. Efficiency is decreasing in the relative size of the sellers.

Keywords : Bargaining, Labor demand.

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1 Introduction

Many agreements in society are determined through bargaining. Wages are often described as being determined through a perhaps implicit negotiation. Sales contracts are often bilateral and long term. Although much can be learned by assuming the outcomes to be the result of independent negotiations, it is important to analyze what types of effects can arise if they are not.

In this paper we will study simultaneous negotiations with perfect information. Although negotiations are simultaneous, the bids are not. In each period one bid and response will be made in one negotiation, and this will be observed by all players. The model will be a simple generalization of Rubinstein-Ståhl bargaining in prices and quantities. We will consider the case where two sellers bargain with one buyer, and in some cases generalize to an arbitrary number of sellers.

When two sellers negotiate with a single buyer, total sales will be higher than the optimum when sellers are substitutes. The first agreement will have higher quantities than optimum, and the second agreement will be for an amount less than optimum. The outcome is similar in spirit to the Stackelberg oligopoly model. In the standard Stackelberg model, the possibility of one seller to commit to a higher quantity results in total quantities larger than the Cournot outcome. With full information bargaining it is the buyer who by strategically committing to high quantities in the first agreement, obtains a higher profit than with the efficient quantities.

In this market, we get a prediction of who trades with who. Similarly to the results of van Damme & Hurkens with endogenous timing in Stackelberg, the more efficient order of agreement will prevail in our setting. In the context of labor theory, the model predicts that if a firm bargains with two unions, it will come to agreement with the high productivity workers first. Another implication of the model is that small asymmetries lead to large differences in treatment. For example, if there is a small difference in willingness to work, the firm always hires the worker with the largest willingness to work first, leading to long work hours for the first worker and short for the last. For example, a small difference in preferences between men and women results in fairly large differences in outcome. In particular, the differences in work hours are larger than those motivated by efficiency considerations. The advantage of being concentrated described here is additional to those found in Horn & Wolinsky (1988).

The strategic inefficiencies of Stackelberg bargaining depend on a concentration asymmetry between sellers and buyers. More specifically, the more asymmetric the workers are, the less inefficient is the outcome. We find a justification for the idea of *countervailing power* – high concentration on one side of the market can justify increased concentration on the other.

Beyond potential applications of this mechanism, there are two reasons why the results are important. Firstly, it shows that the relationship between bargaining and efficiency can be more problematic than is commonly assumed. In the labor literature for instance, it is commonly assumed that bargaining in prices and quantities results in efficient contracts. Bargaining in wage and employment in this literature is often called "efficient bargaining." We describe a mechanism whereby inefficiencies arise for strategic reasons.

Secondly, there is a literature that implicitly assumes that this strategic possibility is not used. In Björnerstedt & Stennek (2001), the outcome is efficient because firms are assumed to have representatives negotiating. If for some reason such as lack of information we do not believe that the mechanism of our paper is empirically relevant, a result of this paper is to say that we then can use the model of Björnerstedt & Stennek. There are no *other* strategic effects that we miss if we do so.

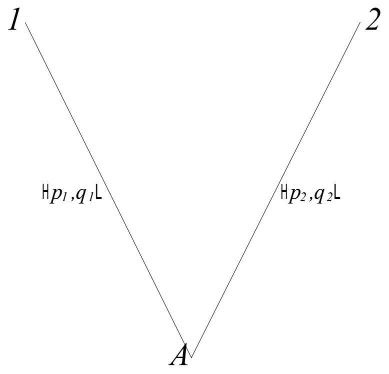

Figure 1: Link structures, with two sellers and one buyer.

2 The Model

We will consider asynchronous simultaneous bargaining between a firm A and two workers 1 and 2 (see figure 1).

The firm will bargain with both workers simultaneously over prices and quantities (p_i, q_i). In each period however, only one bid and response will be made, observed by all. Agreement is immediate and binding.

We assume that bids alternate between $(1, A)$ and $(2, A)$, with equal probability for the seller and buyer making the bid. Alternatively we could let the negotiation also be random or let bids alternate in a deterministic order.[1]

We will begin by considering the general case where the bargaining is over four pies, the size depending on who is negotiating with A and whether it is the first or last agreement. Later we will impose more structure on the analysis by assuming specific profit and utility functions for the players.

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[1] We could also let A select whether to bid to 1 or 2, when given an opportunity to bid.

2.1 Last Period

The analysis will be divided in to several steps. In characterizing the subgame perfect equilibrium, we begin by looking at the last period.

Once agreement has been reached in one negotiation, the remaining play in the negotiation remaining is standard Rubinstein–Ståhl bargaining. The agreement will depend on the quantity agreed upon in the other negotiation, as it affects A's payoffs.

Given agreement in negotiation $(1, A)$ on q_1, the equilibrium agreement will maximize total profits conditional on this agreement. Let π_2 denote the maximal profit that 2 and A can share. Similarly, let π_1 denote this profit when $(1, A)$ agree last. Given the strict concavity of the profit and utility functions, for any q_1 there exists a unique $(p_2(q_1), q_2(q_1))$ maximizing last period total profits and splitting this profit equally (by standard R-S reasoning). Note that quantities depend on δ as optimal production in the first period with $\delta < 1$ is slightly higher. π_1 does not depend on δ however.

$$\begin{aligned}
w_1(q_2) &= \delta\, v_1(q_2) \\
v_1(q_2) &= \frac{\delta}{2}\frac{\pi_1}{1+\delta} \\
w_2(q_1) &= \delta\, v_2(q_1) \\
v_2(q_1) &= \frac{1}{2}\frac{\pi_2}{1+\delta}
\end{aligned}$$

In expectation if last, worker 1 obtains

$$\frac{w_1(q_2)}{2} + \frac{v_1(q_2)}{2} = \frac{1}{2}\frac{\delta\,\pi_1}{1+\delta} + \frac{1}{2}\frac{\pi_1}{1+\delta} = \frac{\pi_1}{2}$$

Let Π_1 denote the total surplus that A and 1 can negotiate over, including the fact that the agreed upon quantity includes the effect this will have on A's profits in the subsequent negotiation with 2. Let Π_2 be similarly defined.

2.2 First period

The equilibrium will depend on which outcome is more efficient. Let ε denote the relative efficiency of agreeing with 1 first instead of 2.

$$\varepsilon = \Pi_1 + \frac{1}{2}\pi_2 - \left(\Pi_2 + \frac{1}{2}\pi_1\right). \tag{1}$$

The term Π_1 is the sum of what A and 1 obtain when agreeing first and $\frac{1}{2}\pi_2$ is what 2 gets in the last agreement. The variable ε thus indicates how much bigger the total surplus is when agreement is with 1 first than when 2 is first.

The relative size of the gains of the first and second agreement also determine the equilibrium structure. Let γ_i be defined by

$$\gamma_i = \Pi_i - \pi_i$$

Let V_i and W_i denote the value to A of bidding and receiving a bid from worker i, and v_i and w_i denote the value to worker i of bidding and receiving a bid. Let P_i be the probability that A gives a bid to i when bidding and p_i the probability that i gives an acceptable bid. Defining $q_i = (P_i + p_i)/2$, the value functions are given by:

$$
\begin{aligned}
V_1 &= (1 - P_1)\ W_1 + P_1\ (\Pi_1 - w_1)\,, \\
W_1 &= \delta\ \left(\frac{V_2}{2} + \frac{W_2}{2} \right), \\
v_1 &= (1 - p_1)\ w_1 + p_1\ (\Pi_1 - W_1)\,, \\
w_1 &= \delta^2\ \left(\frac{\pi_1}{2} q_2 + (1 - q_2)\ \left(\frac{v_1}{2} + \frac{w_1}{2} \right) \right), \\
V_2 &= (1 - P_2)\ W_2 + P_2\ (\Pi_2 - w_2)\,, \\
W_2 &= \delta\ \left(\frac{V_1}{2} + \frac{W_1}{2} \right), \\
v_2 &= (1 - p_2)\ w_2 + p_2\ (\Pi_2 - W_2)\,, \\
w_2 &= \delta^2\ \left(\frac{\pi_2}{2} q_1 + (1 - q_1)\ \left(\frac{v_2}{2} + \frac{w_2}{2} \right) \right).
\end{aligned}
$$

where

$$q_i = \frac{P_i}{2} + \frac{p_i}{2}.$$

2.2.1 Immediate agreement equilibrium

First we will consider under what conditions always agreeing with 1 or 2 can be an equilibrium. Letting $P_i = p_i = q_i = 1$ in the system of value equations, we get

$$
\begin{aligned}
V_1 &= \Pi_1 - \delta^2\, \frac{\pi_1}{2}, \\
V_2 &= \Pi_2 - \delta^2\, \frac{\pi_2}{2}, \\
W_1 &= \frac{2\delta}{4 - \delta^2} \left(\left(\Pi_2 - \delta^2\, \frac{\pi_2}{2} \right) + \frac{\delta}{2} \left(\Pi_1 - \delta^2\, \frac{\pi_1}{2} \right) \right), \\
W_2 &= \frac{2\delta}{4 - \delta^2} \left(\left(\Pi_1 - \delta^2\, \frac{\pi_1}{2} \right) + \frac{\delta}{2} \left(\Pi_2 - \delta^2\, \frac{\pi_2}{2} \right) \right), \\
v_i &= \Pi_i - W_i, \\
w_i &= \delta^2\, \frac{\pi_i}{2},
\end{aligned}
$$

In order for it to be an equilibrium, no player should have an incentive to set the probability P_i or p_i less than 1. This is true iff

$$W_1 \leq \Pi_1 - \delta^2 \frac{\pi_1}{2}$$
$$W_2 \leq \Pi_2 - \delta^2 \frac{\pi_2}{2}$$

With $a = \Pi_1 - \frac{\delta^2}{2}\pi_1$ and $b = \Pi_2 - \frac{\delta^2}{2}\pi_2$, this can be written as:

$$\delta \frac{\delta a + b}{2} \leq a$$
$$\delta \frac{a + \delta b}{2} \leq b$$

In order for this to be true for all δ we must have $a = b$. Now, we know that

$$a = b \iff W_1 = W_2 \iff V_1 = V_2 \iff \varepsilon(\delta) = 0$$

where $\varepsilon(\delta)$ is defined by

$$\varepsilon(\delta) = \left(\Pi_1 - \delta^2 \frac{\pi_1}{2}\right) - \left(\Pi_2 - \delta^2 \frac{\pi_2}{2}\right)$$

To characterize the equilibrium, first note that

$$\varepsilon - \gamma_1 + \gamma_2 = \frac{\pi_1}{2} - \frac{\pi_2}{2}.$$

If $\varepsilon = 0$ and $\gamma_1 > \gamma_2$ then $\frac{\pi_1}{2} < \frac{\pi_2}{2}$. We can thus write $\varepsilon(\delta)$ as

$$\varepsilon(\delta) = \varepsilon + \left(1 - \delta^2\right)\left(\frac{\pi_1}{2} - \frac{\pi_2}{2}\right) = \left(2 - \delta^2\right)\varepsilon + \left(1 - \delta^2\right)(\gamma_2 - \gamma_1).$$

Thus $\varepsilon(\delta) = 0$ for all δ iff $\varepsilon = 0$ and $\gamma_1 = \gamma_2$. Conversely we see that if $\gamma_1 = \gamma_2$ the sign of ε and $\varepsilon(\delta)$ are always the same. The immediate agreement equilibrium only exists in the symmetric case where $\Pi_1 = \Pi_2$ and $\pi_1 = \pi_2$. Denote these equilibrium strategies by σ_i.

2.2.2 Pure strategy equilibrium with agreement with worker 1

Assume that bidding is only with 1. We then have equal split in the first negotiation.

$$\begin{aligned} V_1 = v_1 &= \left(1 - \frac{\delta^2}{2}\right) \Pi_1, \\ W_1 = w_1 &= \frac{\delta^2 \, \Pi_1}{2}, \\ V_2 = W_2 &= \frac{\delta}{2} \Pi_1, \\ v_2 = w_2 &= \delta^2 \, \frac{\pi_2}{2}. \end{aligned}$$

In order for A not to want to bid to 2, the value to A has to be less than waiting, i.e.

$$\Pi_2 - \delta^2 \, \frac{\pi_2}{2} < \delta \left(\frac{V_1}{2} + \frac{W_1}{2}\right) = \delta \frac{\Pi_1}{2}$$

If both

$$\begin{aligned} \Pi_2 - \delta^2 \, \frac{\pi_2}{2} &< \delta \frac{\Pi_1}{2} \\ \Pi_1 - \delta^2 \, \frac{\pi_1}{2} &< \delta \frac{\Pi_2}{2} \end{aligned}$$

hold, there are two pure stationary equilibria. Denote these equilibrium strategies by σ_{1p}. Similarly, strategies implying immediate agreement with 2 is denoted σ_{2p}.

2.2.3 Mixed equilibrium

To randomize (i.e. for example $0 < p_i < 1$), the players have to be indifferent between the alternatives in the randomization. We thus have

$$\begin{aligned} w_1 &= \Pi_1 - W_1 \quad \text{if} \quad 0 < p_1 < 1 \quad \text{or} \quad 0 < P_1 < 1 \\ w_2 &= \Pi_2 - W_2 \quad \text{if} \quad 0 < p_2 < 1 \quad \text{or} \quad 0 < P_2 < 1 \end{aligned}$$

Regardless of whether these equations hold or not, the system depends only on the sums q_i.

Assume that both these conditions hold,

$$\begin{aligned} V_1 = W_1 &= \delta \left(\frac{V_2}{2} + \frac{W_2}{2} \right), \\ V_2 = W_2 &= \delta \left(\frac{V_1}{2} + \frac{W_1}{2} \right), \end{aligned}$$

which implies that $V_1 = \delta^2 \, V_1$, which cannot hold.

Assuming that the condition holds for 2, and that $P_1 = 1$ and $p_1 = 0$ or vice versa. As A has to prefer to put all probability weight on bidding, the value of bidding has to be greater than waiting. From the first equation in the system we thus have:

$$\Pi_1 - w_1 \geq W_1$$

Similarly 1 has to weakly prefer to wait in order to set $p_1 = 0$, thus by the second equation

$$w_1 \geq \Pi_1 - W_1$$

The equality of these two imply the same contradiction as with completely mixed strategies.

In the case of $P_1 = p_1 = 1$ we get $q_1 = 1$ and

$$\begin{aligned} V_1 &= \frac{\left(2 - \delta^2\right) \, \left(\Pi_2 - \delta^2 \, \frac{\pi_2}{2}\right)}{\delta}, \\ W_1 &= \delta V_2 = \delta \, \Pi_2 - \delta^3 \, \frac{\pi_2}{2}, \\ v_1 &= \delta^3 \, \frac{\pi_2}{2} + \Pi_1 - \delta \, \Pi_2, \\ w_1 &= \Pi_1 - \frac{\left(2 - \delta^2\right) \, \left(\Pi_2 - \delta^2 \, \frac{\pi_2}{2}\right)}{\delta}, \\ v_2 &= w_2 = \delta^2 \, \frac{\pi_2}{2}, \\ q_2 &= \frac{2 \, \left(1 - \delta^2\right)}{\delta^2} \frac{\Pi_2 - \delta^2 \, \frac{\pi_2}{2} - \delta \, \frac{\Pi_1}{2}}{\delta \, \Pi_1 - \delta \frac{\pi_1}{2} - \Pi_2 + \delta^2 \frac{\pi_2}{2}}. \end{aligned}$$

In order for $q_2 = 1$ to be an equilibrium, we must have:

$$\delta \Pi_1 - \delta^3 \frac{\pi_1}{2} = \left(2 - \delta^2\right) \left(\Pi_2 - \delta^2 \frac{\pi_2}{2}\right) \tag{2}$$

Assuming now that $q_2 < 1$, as $\delta \to 1$ this expression simplifies to

$$\Pi_2 - \frac{\pi_2}{2} < \Pi_1 - \frac{\pi_1}{2}$$

Given that we also have $q_2 > 0$ this means that

$$\Pi_2 - \delta^2 \frac{\pi_2}{2} > \delta \frac{\Pi_1}{2} \tag{3}$$

For $\delta = 1$, this simplifies to

$$\begin{aligned} V_1 &= W_1 = V_2 = \Pi_2 - \frac{\pi_2}{2}, \\ v_1 &= w_1 = \Pi_1 - \left(\Pi_2 - \frac{\pi_2}{2}\right), \\ v_2 &= w_2 = \frac{\pi_2}{2}, \\ q_2 &= 0. \end{aligned}$$

Denote these equilibrium strategies by σ_{1m}.

In the mixed strategy equilibrium, bids are given with certainty to the more efficient worker, i.e. 1. Bids are given with a probability converging to zero to the other worker. The possibility of agreement with worker 2 puts a lower limit on what A can get in negotiating with 1 in a similar manner as standard bargaining with an outside option. In the limit as $\delta \to 1$, player A gets the outside option, i.e. the whole cake Π_2 minus what 2 would get on the equilibrium path $\frac{\pi_2}{2}$. Given that both A and 2 are indifferent between bidding and not, when bidding to each other and they are randomly selected only the average probability q_2 matters.

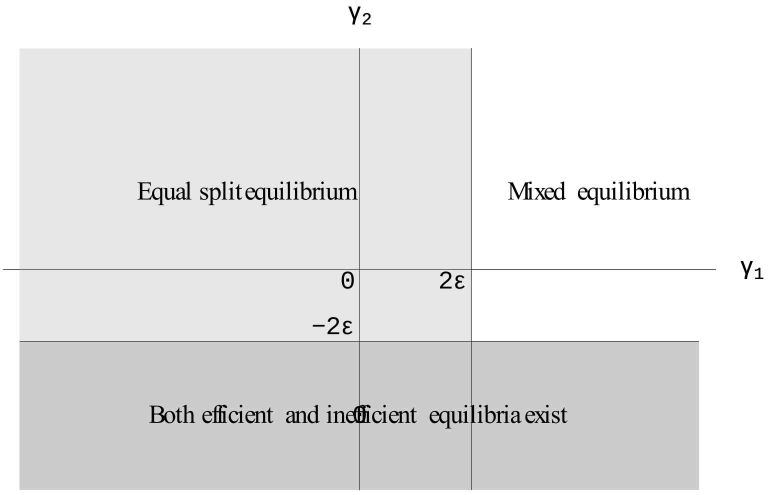

Figure 2: Equilibrium regions depending on the relative size of first to second period agreements γ_i compared to 2ε.

2.3 Equilibrium Characterization

Using γ_i and ε (4) can be rewritten as

$$2\varepsilon < \gamma_1$$

We also have agreement with the inefficient worker if γ_2 is sufficiently small.

$$-2\varepsilon > \gamma_2$$

Note that if

$$\Pi_2 - \frac{\pi_2}{2} > \frac{\Pi_1}{2} \tag{4}$$

holds, then so does (3). Summarizing, we get the following proposition

Proposition 1 *For all Π_i, and π_i with $\varepsilon > 0$, the set of stationary SPE is characterized as follows:*

1. *If* $\gamma_1 < 2\varepsilon$ *there is an equilibrium* σ_{1p} *where* A *and 1 agree first.*

2. *If* $\gamma_1 \geq 2\varepsilon$ *there is a mixed equilibrium* σ_{1m} *where* A *and 1 always agree and* A *and 2 agree first with a probability* q_2*.*

3. *If* $\gamma_2 \leq -2\varepsilon$ *there is an equilibrium* σ_{2p} *where* A *and 2 agree first. .*

4. *If* $\gamma_1 = \gamma_2$ *and* $\varepsilon = 0$ *there is an equilibrium* σ_i *where* A *agrees with* 1 *or* 2 *immediately.*

   As $\delta \to 1$ *the probability of* A *and 2 agreeing in* σ_{1m} *converges to zero and equilibrium payoffs are given by*

$$\begin{array}{ll} \sigma_{1p}: & \left(\frac{\Pi_1}{2}, \frac{\Pi_1}{2}, \frac{\pi_2}{2}\right) \\ \sigma_{1m}: & \left(\Pi_2 - \frac{\pi_2}{2}, \Pi_1 - \left(\Pi_2 - \frac{\pi_2}{2}\right), \frac{\pi_2}{2}\right) \\ \sigma_{2p}: & \left(\frac{\Pi_2}{2}, \frac{\pi_1}{2}, \frac{\Pi_2}{2}\right) \end{array}$$

Although the proposition only characterizes stationary equilibria, the next proposition shows that in general the outcome in terms of payoff can not be that different. Nonstationary equilibria result in the same payoffs when there is a unique stationary equilibrium. When two stationary equilibria exists, the payoffs of all nonstationary equilibria are within the bounds of the payoffs of the stationary equilibria.

Proposition 2 *For any sequence of subgame perfect equilibria as* $\delta \to 1$*, the minimal and maximal payoffs for all players are given by the minimal and maximal payoffs of the stationary equilibria.*

Proof: See Björnerstedt & Westermark (2002).

3 Applications

In this section the firm has the following profit function

$$r\ (q_1+q_2)-p_1\,q_1-p_2\,q_2-c\,\left(\frac{{q_1}^2}{2}+s\,q_1\,q_2+\frac{{q_2}^2}{2}\right).$$

The parameter $s\in[-1,1]$ is the level of substitute/complement of the inputs in the production. With $s=1$, the inputs are perfect substitutes, with c being the slope of the aggregate demand function.

The utility of worker i is given by:

$$p_i\,q_i-\frac{1}{2\,u_i}\,(q_i)^{\,2}$$

where u_i parameterizes i's willingness to work. To simplify analysis, we sometimes normalize u_1 and u_2 as follows:

$$u_1=1-\lambda$$
$$u_2=\lambda.$$

Then, if $s=1$, λ is the share of output of worker 2. In the figures, we also use $r=1+c$ in addition the above normalization. Then efficient production is normalized to one.

3.1 Stage 2

Assume that the firm and worker 1 have come to agreement on q_1. Conditional on this agreement, the firm and worker 2 will agree upon q_2 that maximizes joint profit. In other words, we want to maximize

$$r\,(q_1+q_2)-p_1q_1-\frac{c}{2}\left(q_1^2+2sq_1q_2+q_2^2\right)-\frac{1}{2u_2}q_2^2-\left(rq_1-p_1q_1-\frac{c}{2}q_1^2-\frac{1}{2u_1}q_1^2\right)$$

with respect to q_2. The solution for q_2 is $q_2 = \frac{r-csq_1}{1+cu_2}u_2$. Now, let us determine the price paid to worker 2. Note that the price p_2 is set to get an equal split of the bilateral surplus;

$$rq_2 - p_2q_2 - \frac{c}{2}\left(q_1^2 + 2sq_1q_2 + q_2^2\right) + \frac{1}{2}q_1^2 = p_2q_2 - \frac{1}{2u_2}q_2^2.$$

The solution is given by $p_2q_2 = \frac{1}{4}\frac{cu_2+3}{u_2}q_2^2$.

3.2 Stage 1

Now, let us analyze bargaining between the firm and worker 1. The total surplus for the firm and worker 1 is given by

$$\max_{q_1} r\left(q_1 + q_2\right) - p_2q_2 - \frac{c}{2}\left(q_1^2 + 2sq_1q_2 + q_2^2\right) - \frac{1}{2u_1}q_1^2 \tag{5}$$

where $p_2q_2 = \frac{1}{4}\frac{cu_2+3}{u_2}q_2^2$ and $q_2 = \frac{r-csq_1}{1+cu_2}u_2$. Solving the problem gives

$$q_1 = r\frac{2c + \frac{2}{u_2} - cs}{2c^2 + \frac{2c}{u_1} + \frac{2c}{u_2} + \frac{2}{u_1u_2} - c^2s^2} \text{ and } q_2 = r\frac{2c + \frac{2}{u_1} - 2cs}{2c^2 + \frac{2c}{u_1} + \frac{2c}{u_2} + \frac{2}{u_1u_2} - c^2s^2}.$$

Note that, when the firm agrees first with worker 2, the quantities are almost identical to above. The numerators change to $2c+\frac{2}{u_2}-2cs$ for worker 1 and to $2c + \frac{2}{u_1} - cs$ for worker 2.

Solving for optimal quantities, we get:

$$\begin{aligned}
\Pi_1 &= \frac{(1+c)^2\,\left(2 + \left(-1 + c\,(-3+2\,s)\,(-1+\lambda)\right)\,\lambda\right)}{4 + 2\,c\,\left(2 + c\,\left(-2+s^2\right)\,(-1+\lambda)\,\lambda\right)} \\
\Pi_2 &= \frac{(1+c)^2\,\left(1 + \left(1 + c\,(-3+2\,s)\,(-1+\lambda)\right)\,\lambda\right)}{4 + 2\,c\,\left(2 + c\,\left(-2+s^2\right)\,(-1+\lambda)\,\lambda\right)}, \\
\pi_1 &= \frac{2\,(1+c)^2\,\left(-1 + c\,(-1+\lambda)\right)\,(-1+\lambda)\,\left(-1 + c\,(-1+s)\,\lambda\right)^2}{\left(2 + c\,\left(2 + c\,\left(-2+s^2\right)\,(-1+\lambda)\,\lambda\right)\right)^2}, \\
\pi_2 &= \frac{2\,(1+c)^2\,\left(1 + c\,(-1+s)\,(-1+\lambda)\right)^2\,\lambda\,(1+c\,\lambda)}{\left(2 + c\,\left(2 + c\,\left(-2+s^2\right)\,(-1+\lambda)\,\lambda\right)\right)^2},
\end{aligned}$$

Using these expressions, we can see that

$$\varepsilon = (1-2\lambda)\,\frac{c^2\,(1+c)^2\,s^2\,\lambda\,(1-\lambda)}{2\,\left(2+2\,c+\,(2-s^2)\,(1-\lambda)\,c^2\,\lambda\right)^2}$$

The sign of this expression is determined by the numerator. More precisely $\varepsilon \geq 0$ iff

$$(1-2\lambda) \geq 0$$

i.e., iff $\lambda \leq 1/2$. The mixed equilibrium will thus specify the larger worker agreeing first in equilibrium.

3.3 Multiple equilibria with complementarities

This section analyzes under what conditions we can have multiple equilibria. We first describe a sufficient condition on s that rules out multiplicity. Then, some cases when multiplicities arise are described. The following Lemma gives a sufficient condition on s such that the complementarity conditions never hold.

Lemma 3 *For any $s \geq -0.3$, the complementarity conditions does not hold.*

Proof: (See appendix)

With strong complementarities, all equilibria can exist. For $s = -.45$ immediate agreement with 1 is an equilibrium for all $\lambda \leq 1/2$. For some parameter values immediate agreement with 1 and 2 are both equilibria. Plotting γ_1, γ_2 and 2ε in figure 3 we get the inequalities $\gamma_1 > 2\varepsilon$ and $-\gamma_2 < 2\varepsilon$ satisfied for all λ in the shaded region.

3.4 Over- and Underproduction

Now, let us compare equilibrium production with efficient production. Let the efficient quantities be denoted q_1^e and q_2^e. Efficient production is found by solving the following problem

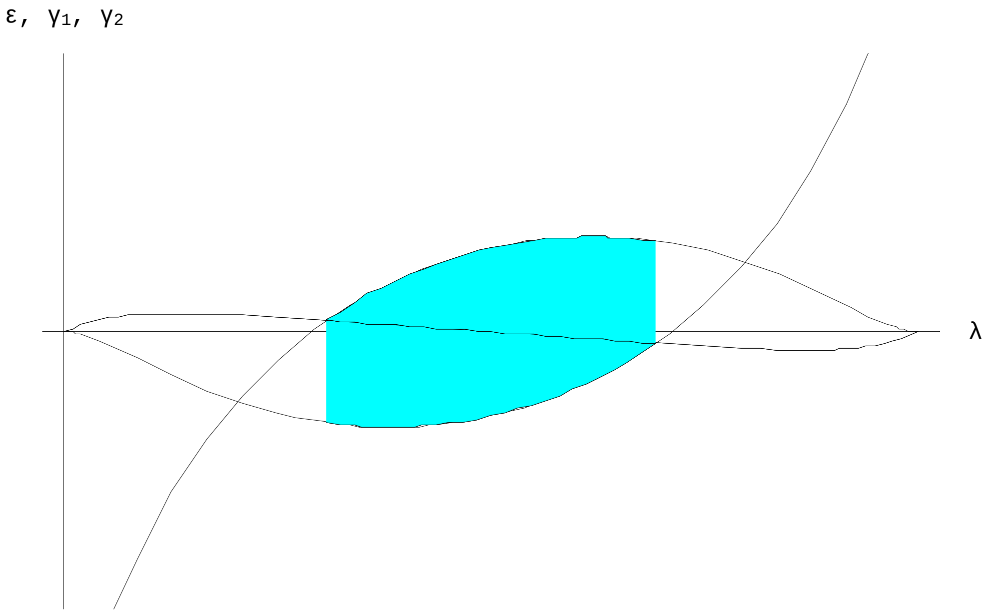

Figure 3: Plot of γ_1, γ_2 and 2ε as functions of λ with $s=-\frac{1}{2}$. In the shaded region both equilibria with immediate agreement with 1 and 2 exist.

$$\max_{q_1,q_2}\left(r\left(q_1+q_2\right)-\frac{c}{2}\left(q_1^2+2sq_1q_2+q_2^2\right)-\frac{1}{2u_1}q_1^2-\frac{1}{2u_2}q_2^2\right).$$

The solution to this problem is

$$q_1^e=r\frac{c+\frac{1}{u_2}-cs}{c^2+\frac{c}{u_1}+\frac{c}{u_2}+\frac{1}{u_1u_2}-c^2s^2}\text{ and }q_2^e=r\frac{c+\frac{1}{u_1}-cs}{c^2+\frac{c}{u_1}+\frac{c}{u_2}+\frac{1}{u_1u_2}-c^2s^2}.$$

The difference between equilibrium production and efficient production is

$$q_1+q_2-(q_1^e+q_2^e)=rs\frac{c\left(c^2\left(1-s\right)^2+\frac{1}{u_1u_2}+\frac{c}{u_1}\left(1-s\right)+\frac{c}{u_2}\left(1-s\right)\right)}{\left(2c^2+\frac{2c}{u_1}+\frac{2c}{u_2}+\frac{2}{u_1u_2}-c^2s^2\right)\left(c^2+\frac{c}{u_1}+\frac{c}{u_2}+\frac{1}{u_1u_2}-c^2s^2\right)}.$$

Hence, since the ratio is positive, the sign is determined by the sign of s. If

$s > 0$ then we have overproduction and if $s < 0$ we have underproduction. Obviously, when $s \neq 0$, the equilibrium sum of utilities is smaller than the social optimum, since equilibrium quantities are not the same as the efficient. Equilibrium production is efficient only when $s = 0$.

The difference between equilibrium and efficient production for worker 1 is

$$q_1 - q_1^e = rcs \frac{\frac{1}{u_1 u_2} + \frac{c}{u_1} + c^2 (1-s) + \frac{c}{u_2}(1-s)}{\left(2c^2 + \frac{2c}{u_1} + \frac{2c}{u_2} + \frac{2}{u_1 u_2} - c^2 s^2\right)\left(c^2 + \frac{c}{u_1} + \frac{c}{u_2} + \frac{1}{u_1 u_2} - c^2 s^2\right)}.$$

Hence q_1 is too high when $s > 0$ and too low when $s < 0$. For worker 2, we have

$$q_2 - q_2^e = -\frac{r\left(\frac{1}{u_1} + c(1-s)\right) s^2 c^2}{\left(2c^2 + \frac{2c}{u_1} + \frac{2c}{u_2} + \frac{2}{u_1 u_2} - c^2 s^2\right)\left(c^2 + \frac{c}{u_1} + \frac{c}{u_2} + \frac{1}{u_1 u_2} - c^2 s^2\right)}$$

Thus, q_2 is always too low.

Note that when $s > 0$, the results above have similarities with a duopoly model where one of the firms is a Stackelberg leader. This follows, since in this model and the duopoly model the first agreement on quantities is too high and the second agreement is too low.

The strategic inefficiency of the Stackelberg bargaining equilibrium depends on the relative size of the workers. The more "asymmetric" the workers are, the lower the loss (see figure 4). Note that, as asymmetries grows large, the bargaining model converges to a situation with a firm bargaining with one worker. With only one worker, the bargaining outcome is efficient.

Some limited results are also available when the workforce is large. In particular, for $s = 1$ and symmetric workers, we have the following result.

Proposition 4 *Overproduction increases as the number of workers increases.*

Proof: See appendix.

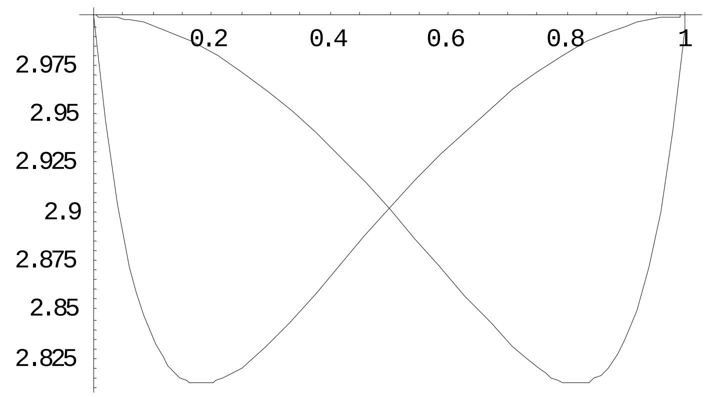

Figure 4: Inefficiency of equilibrium agreement with 1, and 2 as functions of λ compared to efficient production of 3.

3.5 Labor Applications

Note that an implication of the model when the workers are substitutes, i.e., $s = 1$, is that, *given* the hourly wages p_1 and p_2, one worker works too much. The other might work too much or too little, depending on parameter values. To see this, note that the effect on worker utility of a change in working time, taking wages p_i as given, is $p_i - \frac{1}{u_i}q_i$ where $u_1 = 1 - \lambda$ and $u_2 = \lambda$. Consider a situation where worker 1 likes work at least as much as worker 2, i.e., we have $\lambda \leq \frac{1}{2}$. The results above shows that, if $\lambda < \frac{1}{2}$, the firm must agree first with worker 1 in equilibrium. Consider the effects on the utility of worker 2 of a small change in q_2. From above, we have, $p_2 = \frac{1}{4}\frac{c\lambda+3}{\lambda}q_2$. The effect on utility of a change in working time is then $\frac{1}{4}\frac{c\lambda-1}{\lambda}q_2$. The sign depends on $c\lambda - 1$. If c is large, then the utility of worker 2 increases and hence worker 2 works too little. Otherwise worker 2 works too much. Now, consider worker 1. For worker one we have $p_1 = \frac{\frac{1}{2(1-\lambda)}q_1^2+\frac{1}{4}\frac{c\lambda+1}{\lambda}q_2^2}{q_1}$. Hence, the effect on utility is

$$\frac{\frac{1}{4}\frac{c\lambda+1}{\lambda}q_2^2 - \frac{1}{2\lambda}q_1^2}{q_1}.$$

Some algebra shows that the sign is determined by

$$2c\lambda^2 - 4\lambda\left(1-\lambda\right)c + 2\lambda - 4\left(1-\lambda\right) - \left(1-\lambda\right)c^2\lambda^2. \tag{6}$$

This expression is negative for any $\lambda \in [0, \frac{1}{2}]$ and $c > 0$. Too see this, note that expression 6 has at most one local minimum with respect to λ in the interval $(0, \frac{1}{2})$. The minimum is $\lambda = \frac{-2+\frac{1}{3}c+\frac{1}{3}\sqrt{(18+c^2)}}{c}$. Also, expression 6 is negative $\lambda = 0$ and $\lambda = \frac{1}{2}$. Hence, expression 6 must be negative for any $\lambda \in [0, \frac{1}{2}]$. Hence, worker 1 works too much. Since we do not know whether worker 2 works too much or too little, it seems reasonable to expect that, in a large economy, a share of the type 2 workers perceive that they work too much. Then, in the economy as a whole, more workers perceive that they work too much rather than too little.

This is in line with empirical evidence in Bell & Freeman. Evidence from German GSOEP data indicates that workers feel that the actual working time is larger that the desired working time, taking the effect of reduced pay into account. The workers were asked the following question : "If you could choose the extent of your hours at work, taking into account that your earnings would change corresponding to the time, how many hours would you work?" The difference between actual and desired hours is significant. The average difference for all workers is approximately four to five hours per week, with a slightly larger difference for men and smaller for women.

The model has a potential to explain some of the differences in work hours and pay between men and women. To see this, note that introducing small asymmetries in a symmetric setup leads to potentially large asymmetries in treatment of workers. In a symmetric economy with many firms as above with two types of workers all workers are - on average - symmetrically treated. However, if a small asymmetry in preferences is introduced we know that the equilibrium prescribes agreement with the largest worker first. An example could be that men prefer to work a little more than women.

From above, we know that $q_1 > q_1^e$ and $q_2 < q_2^e$. Hence, the worker that is more/less inclined to work increases/decreases work hours more than what is motivated by efficiency considerations. Thus, the large difference in work hours between men and women could at least partially be caused by strategic effects. The price difference is, when $\lambda = \frac{1}{2}$, $p_1 - p_2 = \frac{2c}{(c+4)(c^2+8c+8)} > 0$. Since the price difference is continuous in λ, we have $p_1 > p_2$ for λ close to $\frac{1}{2}$. Thus, men have higher wages than women, at least if men and women are fairly similar. From the previous section, we also know that the men perceive that they work too much. Also, note that, in equilibrium, men and women are indifferent between switching occupations.

The model in this paper is related to the model in Horn & Wolinsky. Horn & Wolinsky analyzes a model where the firm bargains with two workers that supply a fixed amount of labor. When the workers are substitutes, given that one worker is hired, the addition of the second worker only leads to a small increase in production. The firm can use this when bargaining and play out the workers against each other and threaten each worker to be last. Since the surplus is small when being last, wages are low. In the model presented in this paper, the firm has two methods of pressuring down wages. First, the same mechanism as in Horn and Wolinsky can be used. Second, the firm also has the opportunity vary the quantities in order to decrease wage payments. In particular, the firm decreases the quantity for the last worker in order to decrease the surplus even more when bargaining with this worker. This leads to a further wage cut for workers.

4 Concluding remarks

Given that there are simultaneous negotiations, and contracts are observable, this paper shows that in theory there is a possibility for the firm to obtain higher profits by offering the Stackelberg bargaining quantities. Whether firms actually do so is an open question. As shown in the previous section however, the model has a number of predictions that might be empirically testable.

Several factors might limit the scope of using Stackelberg strategies. The outcome implies treating symmetric workers differently. The possibility of arbitrage might also affect the results. In the model it is assumed that 1 and 2 cannot agree upon letting 2 do some of the work of 1. The possibility of arbitrage between workers in bargaining after agreement might reintroduce efficiency. If the workers are perfect substitutes, restrictions on such arrangements might be difficult. However, arbitrage between workers seems to be uncommon. In applying the theory to questions of industrial organization this might be of greater concern.

It should be noted that the firm will not have an incentive to renegotiate the contract with 1 to reduce the quantity agreed upon. Although quantities would be more efficient, in bargaining with worker 1 again, the firm has to split surplus equally. This will not be beneficial for the firm. The equilibrium is thus renegotiation proof in some sense.

Although the focus of the application of the model of this paper is on bargaining in the labor market, it can also be used to discuss topics in industrial organization such as merger control. The model provides support of the concept of *countervailing power*: high market concentration on one side can motivate an increase on the other.

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A Proofs

Lemma 5 *3 For any s such that $2+2s^3-7s^2+4s \geq 0$, the complementarity conditions does not hold.*

Proof: Consider the complementary conditions. The first complementarity condition is

$$\frac{\Pi_2}{2} - \Pi_1 + \frac{\pi_1}{2} = -\frac{(1-\lambda)\left(2 + ca_1 + \lambda c^2 a_2 + \lambda^2 (1-\lambda) c^3 a_3\right)}{\left(2c^2\lambda - 2c^2\lambda^2 + 2c + 2 - c^2 s^2 \lambda + c^2 s^2 \lambda^2\right)^2}$$

where $a_1 = 2(\lambda+1) + 4s\lambda$, $a_2 = 2(2-\lambda) + 4s - (\lambda+3)s^2$ and $a_3 = 2 + 4s - 7s^2 + 2s^3$.

The effect of a change in c on the numerator is

$$(1-\lambda)\left(a_1 + 2c\lambda a_2 + 3c^2\lambda^2 (1-\lambda) a_3\right).$$

The numerator at $c = 0$ is $2(1-\lambda)$. A sufficient condition on s for the complementarity condition to be negative is found by choosing s such that the coefficients in front of c, c^2 and c^3 are positive. Since the numerator then is increasing in c the numerator is positive for any $c > 0$, implying that $\frac{\Pi_2}{2} < \Pi_1 + \frac{\pi_1}{2}$.

Consider a_1. Note that $a_1 = 2\lambda(1+s) + 2(1+s\lambda) \geq 0$.

Consider a_2. For $s = 0$ we have $a_2 = 2(2-\lambda) > 0$ for $\lambda < 1$. Also, the effect of a change in s on a_2 is $4 - 2s\lambda - 6s$. When $s < 0$ we have $4 - 2s\lambda - 6s > 0$. At $s = 1$ we have $a_2 = 5 - 3\lambda > 0$. When $s > 0$ the coefficient attains a maximum at $s = \frac{2}{\lambda+3}$. Since this is a local maximum and a_2 is positive when $s = 0$ and $s = 1$, the coefficient is positive for any $s \in (0,1)$. The value of s when the coefficient is zero is

$$4(1+s) - 2\lambda - s^2\lambda - 3s^2 = 0.$$

Note that, by differentiating the expression above, the solution for s is an increasing function of λ. Then, the maximum value of s when a_2 is zero is when $\lambda = 1$. This gives $s = \frac{1}{2} - \frac{1}{2}\sqrt{3}$. Since the coefficient is increasing in s when $s < 0$, the coefficient is positive for any $s > \frac{1}{2} - \frac{1}{2}\sqrt{3}$.

Consider a_3. The constant attains a local maximum at $s = \frac{1}{3}$ and has the second derivative $12s - 14 < 0$. Since $a_3 > 0$ at $s = 0$ and at $s = 1$, it is positive for any $s \geq 0$. Also, a_3 is increasing in s for $s < 0$. The point where the coefficient is zero is

$$2 + 2s^3 - 7s^2 + 4s = 0. \tag{7}$$

Note that, at $s = \frac{1}{2} - \frac{1}{2}\sqrt{3}$, we have $2 + 2s^3 - 7s^2 + 4s = -0.5$. Thus, the value of s that solves the expression above is larger than $\frac{1}{2} - \frac{1}{2}\sqrt{3}$.

Since a_1, a_2 and a_3 are positive for s larger than the value of s that solves 7, we have $\frac{\Pi_2}{2} < \Pi_1 + \frac{\pi_1}{2}$.

A similar argument as below establishes the result for the other complementarity condition. ■

A.0.1 Proposition 3

In this section, we analyze the equilibrium employment level in the firm and compare it with the efficient employment level. We restrict attention to a model where the workers have the payoff function $p_i q_i - \frac{u}{2} q_i^2$ and the firm has the payoff function

$$r \sum_{i=1}^{n} q_i - \sum_{i=1}^{n} p_i q_i - \frac{c}{2} \left(\sum_{i=1}^{n} q_i \right)^2 .$$

Also, we focus on the limit case when δ converges to one. The main result is that the total equilibrium quantity is larger than the total efficient quantity. To illustrate the main result in a simple framework, let us analyze an example with two workers.

Example 6 *Let us first analyze bargaining between the firm and worker* 2*, conditional on that the firm and worker* 1 *has agreed on* (p_1, q_1)*. Given this agreement, the firm and worker* 2 *will agree upon* q_2 *that maximizes joint profit;*

$$r(q_1+q_2) - p_1 q_1 - \frac{c}{2}(q_1+q_2)^2 - \frac{u}{2}q_2^2.$$

It is easily verified that the solution to this problem is given by $q_2 = \frac{r-cq_1}{c+u}$ *and* $p_2 = \frac{1}{4}\frac{3u+c}{c+u}(r-cq_1)$*. Now, consider bargaining between the firm and worker* 1 *when no agreement has been signed with any of the workers. When bargaining with worker* 1*, the firm knows that the subsequent agreement with worker* 2 *is as above. The firm and worker* 1 *will agree upon* q_1 *that maximizes joint profit subject to* $q_2 = \frac{r-cq_1}{c+u}$ *and* $p_2 = \frac{1}{4}\frac{3u+c}{c+u}(r-cq_1)$*. The solution to this problem is given by* $q_1 = r\frac{2u+c}{2u^2+4cu+c^2}$*. This gives* $q_2 = r\frac{2u}{2u^2+4cu+c^2}$ *and total production is then*

$$q_1 + q_2 = r\frac{4u+c}{2u^2+4cu+c^2}.$$

Now, consider the socially optimal production. It is found maximizing the social surplus;

$$r(q_1+q_2) - \frac{c}{2}(q_1+q_2)^2 - \frac{u}{2}q_1^2 - \frac{u}{2}q_2^2.$$

The socially optimal production (q_1^e, q_2^e) *is* $q_1^e = q_2^e = \frac{r}{2c+u}$*. Since*

$$q_1 + q_2 - (q_1^e + q_2^e) = rc\frac{u}{(2u^2+4cu+c^2)(2c+u)} > 0,$$

we have overproduction.

Note from the example that the firm agrees on a larger quantity with worker 1 than with worker 2. The reason for this is the following. If the quantity when being last is small, the surplus that the firm and worker 2 bargain over is small, leading to a low wage for worker 2. The firm can use this to push down the wage for worker 1, by threatening to agree first with the other worker instead. By doing this, the firm can push the utility of worker 1 to the same level as for worker 2, which leads to low wages for both workers. Note that the quantities q_1 and q_2 must be on opposite sides of the efficient quantity. This follows since the response function $q_2 = \frac{r-cq_1}{c+u}$ is decreasing in q_1. Because the argument above implies that q_2 is low, the quantity agreed upon first must be larger than the efficient quantity. Since q_1 is larger than the efficient quantity and the slope of the reaction function is $-\frac{c}{c+u}$ it follows that q_1 increases more than q_2 decreases. Hence, the total equilibrium quantity is larger than the efficient quantity. Now, let us analyze whether the overproduction result remains as the workforce grows large. This is modelled as, given some fixed demand or firm size, workers become smaller and smaller. Let n denote the number of workers and assume that the coefficient u is equal to nu^1, where u^1 denotes the value of u when the firm bargains with only one worker. Thus construction leaves aggregate supply unchanged as n is increased. As is shown below, it turns out that overproduction increases as we let the number of workers grow.

First, consider efficient demand. The social optimum per worker is given by $q_1 = \frac{r}{n(c+u^1)}$. Total demand is then

$$nq_1 = \frac{r}{c+u^1}.$$

Second, consider equilibrium demand. First, note that if we have n workers, a slight modification of the two worker example above describes the agreements of the last two workers, given that an agreement has been reached on $q_1, \ldots, q_{n-2}$. For simplicity, let $Q_i = \sum_{j=1}^{i} q_j$. We get $q_{n-1} = \frac{2u+c}{2u^2+4cu+c^2}(r - Q_{n-2})$ and $q_n = \frac{2u}{2u^2+4cu+c^2}(r - Q_{n-2})$. Thus, the hiring decisions are ratios of $(r - Q_{n-2})$. Similarly, if an agreement has been reached upon $q_1, \ldots, q_{n-1}$, we get $q_n = \frac{1}{c+u}(r - Q_{n-1})$. In general, given that an agreement has been reached with i workers, let ρ_j^i denote the ratio of q_j to $(r - Q_{n-2})$. Then, given that an agreement has been reached on $q_1, \dots, q_{n-2}$ we have $q_{n-1} = \rho_{n-1}^{n-2}(r - Q_{n-2})$ and $q_n = \rho_n^{n-2}(r - Q_{n-2})$. Furthermore, if an agreement has been reached upon $q_1, \dots, q_{n-1}$, we get $q_n = \rho_n^{n-1}(r - Q_{n-1})$. Note that there is a relationship between these ratios. It is easy to verify that we have $\rho_n^{n-2} = \rho_n^{n-1}\left(1 - c\rho_{n-1}^{n-2}\right)$. This is a result that turns out to hold in general and that is useful in the remaining analysis.

Lemma 7 *For all $i = 1, \dots n-1$, we have, for $j = i+1, \dots, n$ that $q_j = \rho_j^{i-1}(r - Q_{i-1})$ and, for $j = i+1, \dots, n$, that $\rho_j^{i-1} = \rho_j^i\left(1 - c\rho_i^{i-1}\right)$. Also, these ratios are independent of Q_{i-1}.*

Proof: We prove this by induction. Suppose we have $q_j = \rho_j^i(r - Q_i)$ for $j = i+2, \dots, n$ and that ρ_j^i, for $j = i+1, \dots, n$, are independent of Q_i.

First, let us find $p_j q_j$ for $j \geq i$. We have

$$p_j q_j - \frac{u}{2} q_j^2 = \frac{u+c}{4} q_n^2 \Rightarrow p_j q_j = \frac{u+c}{4} q_n^2 + \frac{u}{2} q_j^2 \tag{8}$$

Note that

$$q_j = \rho_j^{j-1}\left(r - c\sum_{k=1}^{j-1} q_k\right) = \rho_j^{j-1}\left(r - c\sum_{k=1}^{j-2} q_k - c\rho_{j-1}^{j-2}\left(r - c\sum_{k=1}^{j-2} q_k\right)\right)$$

$$= \rho_j^{j-1}\left(r - c\sum_{k=1}^{i} q_k\right)\prod_{l=1}^{j-i-1}\left(1 - c\rho_{n-l}^{n-(l+1)}\right).$$

Then

$$p_j q_j = \left[\frac{u+c}{4}\left(\rho_n^{n-1}\prod_{l=1}^{n-i-1}\left(1 - c\rho_{n-l}^{n-(l+1)}\right)\right)^2 .\right.$$

$$\left. + \frac{u}{2}\left(\rho_j^{j-1}\prod_{l=1}^{j-i-1}\left(1 - c\rho_{j-l}^{j-(l+1)}\right)\right)^2\right]\left(r - c\sum_{k=1}^{i} q_k\right)^2 .$$

Also, note that

$$\sum_{j=i+1}^{n} q_k = \left(r - c\sum_{k=1}^{i} q_k\right) \sum_{j=i+1}^{n} \rho_j^i \tag{9}$$

Now, let us solve for q_i. Net surplus when bargaining with worker i is

$$r\left(\sum_{k=1}^{i-1} q_k + q_i + \sum_{k=i+1}^{n} q_k\right) - \sum_{k=1}^{i-1} p_k q_k - \sum_{k=i+1}^{n} p_k q_k - \frac{c}{2}\left(\sum_{k=1}^{i-1} q_k + q_i + \sum_{k=i+1}^{n} q_k\right)^2 - \frac{u}{2} q_i^2$$

Let $R_{i+1} = \sum_{j=i+1}^{n} \rho_j^i$ and let

$$S_{i+1} = \sum_{j=i+1}^{n} \left[\frac{u+c}{4}\left(\rho_n^{n-1} \prod_{l=1}^{n-i-1}\left(1 - c\rho_{n-l}^{n-(l+1)}\right)\right)^2 + \frac{u}{2}\left(\rho_j^{j-1} \prod_{l=1}^{j-i-1}\left(1 - c\rho_{j-l}^{j-(l+1)}\right)\right)^2\right].$$

The first-order condition is, using 8 and 9,

$$r\left(1 - cR_{i+1}\right) + 2c\left(r - cQ_{i-1} - cq_i\right) S_{i+1}$$

$$-c\left(1 - cR_{i+1}\right)\left(rR_{i+1} + \left(1 - cR_{i+1}\right)\left(Q_{i-1} + q_i\right)\right) - uq_i = 0$$

Then we get

$$q_i = \frac{\left(1 - cR_{i+1}\right)^2 + 2cS_{i+1}}{c\left(1 - cR_{i+1}\right)^2 + 2c^2 S_{i+1} + u}\left(r - cQ_{i-1}\right) = \rho_i^{i-1}\left(r - cQ_{i-1}\right).$$

Now, for $j > i$ we use that

$$q_j = \rho_j^i\left(r - cQ_i\right) = \rho_j^i\left(r - cQ_{i-1} - cq_i\right) = \rho_j^i\left(1 - c\rho_i^{i-1}\right)\left(r - cQ_{i-1}\right).$$

Then

$$\rho_i^{i-1} = \tfrac{(1-cR_{i+1})^2+2cS_{i+1}}{u+c(1-cR_{i+1})^2+2c^2S_{i+1}},$$
$$\vdots$$
$$\rho_{i+1}^{i-1} = \rho_{i+1}^{i}\left(1-c\rho_i^{i-1}\right),$$
$$\vdots$$
$$\rho_n^{i-1} = \rho_n^{i}\left(1-c\rho_i^{i-1}\right).$$

Also, since ρ_j^i are independent of Q_i and ρ_i^{i-1} is independent of Q_{i-1} we have ρ_j^{i-1} independent of Q_{i-1}. ■

Let us use the result in the Lemma above to compute the equilibrium quantities. We have the following result.

Proposition 8 *Overproduction increases as the number of workers increases.*

Proof: Suppose there are n workers. If no agreement has been reached, we have $\sum_{j=1}^{n} q_j = \sum_{j=i}^{n} \rho_j^0 r$ by definition of ρ_j^0. Using the Lemma above gives

$$\sum_{j=1}^{n} q_j = \left(\rho_1^0 + \left(1-c\rho_1^0\right)\sum_{j=2}^{n}\rho_j^1\right) r = \left(\rho_1^0\left(1-c\sum_{j=2}^{n}\rho_j^1\right)+\sum_{j=2}^{n}\rho_j^1\right) r. \tag{10}$$

Note that, since $\sum_{j=2}^{n}\rho_j^1 r$ is the total equilibrium quantity when we have $n-1$ workers, it follows that total equilibrium quantity increases if we have $1-c\sum_{j=2}^{n}\rho_j^1>0$. We show that this must hold by induction. First note that

$$c\sum_{j=1}^{n} q_j = c\sum_{j=i}^{n}\rho_j^0 r = \left(c\rho_1^0 + \left(1-c\rho_1^0\right)\left(c\sum_{j=2}^{n}\rho_j^1\right)\right) r. \tag{11}$$

Also, when $n=2$, we have, $0 < cq_{n-1}+cq_n = r\frac{4uc+c^2}{2u^2+4cu+c^2} < 1$ implying that $c\left(\rho_{n-1}^{n-2}+\rho_n^{n-2}\right)<1$. Induction and 11 implies that

$$1 > c\sum_{j=1}^{n}\rho_j^0 > c\sum_{j=2}^{n}\rho_j^1. \tag{12}$$

Note that from 10, $\sum_{j=i}^{n}\rho_j^0 > \sum_{j=2}^{n}\rho_j^1$ as long as $\sum_{j=2}^{n}\rho_j^1 < \frac{1}{c}$. From 12 above, this holds. Thus, demand increases monotonically. ■

As the theorem shows, overproduction increases as the number of workers increase. Since the sequence $\{\sum_{j=1}^{n} q_j\}_{n=1}^{\infty}$ of equilibrium production is monotonic, it has a limit. It is possible to establish an upper bound for this limit. Since we know that the limit of $\sum_{j=1}^{n}\rho_j^1$ is at most $\frac{1}{c}$ it follows that $\frac{r}{c}$ is an upper bound of the sequence.

Another fact from the two-worker example is that the first worker supplies more labor than the last. This also turns out to be a general result. The following Lemma shows that workers that agrees with the firm early sells more labor to the firm.

Lemma 9 *We have $\rho_j^{i-1} > \rho_k^{i-1}$ when $k > j$.*

Proof: Let us prove this by induction. Suppose it holds when agreement has been reached with i workers. By the two worker example above, we know that it holds when $i = n-2$. If it holds when agreement has been reached with i workers, we have $\rho_{i+1}^i > \rho_{i+2}^i > \ldots > \rho_n^i$. We have, using the previous Lemma, $\rho_{i+1}^{i-1} > \ldots > \rho_n^{i-1}$. It remains to show that $\rho_i^{i-1} > \rho_{i+1}^{i-1}$ (or $\frac{\rho_i^{i-1}}{\rho_{i+1}^{i-1}} > 1$). To do this, consider

$$\frac{\rho_i^{i-1}}{\rho_{i+1}^{i-1}} = \frac{\rho_i^{i-1}}{\rho_{i+1}^{i}\left(1-\rho_i^{i-1}\right)} = \frac{\left(1-cR_{i+1}\right)^2 + 2cS_{i+1}}{u\rho_{i+1}^{i}}$$

Note that $\frac{\rho_i^{i-1}}{\rho_{i+1}^{i-1}} > 1$ is equivalent to

$$\left(1-cR_{i+1}\right)^2 + 2cS_{i+1} - u\rho_{i+1}^{i} > 0. \tag{13}$$

The remaining part of the proof shows that 13 is positive.

Let us establish some facts about the terms in 13. Noting that $1 - c\sum_{j=i+1}^{n}\rho_j^i = \left(1-c\rho_{i+1}^i\right) - c\sum_{j=i+2}^{n}\rho_j^i$ and using that $\rho_j^i = \rho_j^{i+1}\left(1-c\rho_{i+1}^i\right)$

gives

$$\left(1-cR_{i+1}\right)^2=\left(1-c\rho_{i+1}^{i}\right)^2\left(1-cR_{i+2}\right)^2. \tag{14}$$

Also, we have

$$S_{i+1}=\left(1-c\rho_{i+1}^{i}\right)^2 S_{i+2}+\frac{u+c}{4}\left(\rho_n^{n-1}\prod_{l=1}^{n-i-1}\left(1-c\rho_{n-l}^{n-(l+1)}\right)\right)^2+\frac{u}{2}\left(\rho_{i+1}^{i-1}\right)^2 \tag{15}$$

Using 14 and 15, 13 can be rewritten as

$$\left(1-cR_{i+1}\right)^2+2cS_{i+1}-u\rho_{i+1}^{i}=\left(1-c\rho_{i+1}^{i}\right)^2\left(1-cR_{i+2}\right)^2$$

$$+2c\left(1-c\rho_{i+1}^{i}\right)^2 S_{i+2}+2c\frac{u+c}{4}\left(\rho_n^{n-1}\prod_{l=1}^{n-i-1}\left(1-c\rho_{n-l}^{n-(l+1)}\right)\right)^2+cu\left(\rho_{i+1}^{i-1}\right)^2-u\rho_{i+1}^{i}$$

$$>\left(1-c\rho_{i+1}^{i}\right)^2\left[\left(1-cR_{i+2}\right)^2+2cS_{i+2}\right]+u\left(c\rho_{i+1}^{i}-1\right)\rho_{i+1}^{i}$$

The inequality follows since the middle term in 15 is positive.

We claim that the right hand side is zero. To see this, note that, from the previous lemma, we have

$$1-c\rho_{i+1}^{i}=1-c\frac{\left(1-cR_{i+2}\right)^2+2cS_{i+2}}{u+c\left(1-cR_{i+2}\right)^2+2c^2S_{i+2}}=\frac{u}{u+c\left(1-cR_{i+2}\right)^2+2c^2S_{i+2}}$$

This implies

$$\left(1-c\rho_{i+1}^{i}\right)\left[\left(1-cR_{i+2}\right)^2+2cS_{i+2}\right]-u\rho_{i+1}^{i}=0$$

and the conclusion follows. ■