

Monetary Policy and Lexicographic Preference Ordering

John Driffill* and Zeno Rotondi†
(Preliminary and Incomplete)

20 January 2002‡

Abstract

In this paper we argue that the objectives given to the European Central Bank in the Maastricht Treaty are not well represented by the widely used weighted sum of squared deviations of inflation and output from target (plus possibly terms in squared changes in interest rates to pick up interest rate smoothing). Instead the stated lexicographic ordering should be taken at face value and its implications explored fully. We set out a number of models that do this, and comment on their implications.

JEL classification: E52; E58

Keywords: Monetary Policy; Time Inconsistency; Lexicographic Preferences, European Central Bank

*Birkbeck College; email: jdriffill@econ.bbk.ac.uk.

†Universita' di Roma "La Sapienza", Facolta' di Scienze Statistiche, and Banca di Roma; email: zeno.rotondi@bancaroma.it.

‡The authors are grateful to participants of seminars at the University of South Bank and the University of Manchester for helpful comments on a previous version of the paper. The financial support of the ESRC Programme "Understanding the evolving macroeconomy", grant no. L138251003, is gratefully acknowledged. The usual disclaimer applies.

1 Introduction

There have been a lot of analyses of the effects of monetary policy on prices and output fluctuations under EMU, assuming that the independent European Central Bank (ECB) operates monetary policy as specified in the Maastricht Treaty. The Treaty famously requires the ECB to pursue the single goal of price stability, with no trade-off permitted between that and the stabilization of real economic activity. The ECB is allowed to pursue real economic stability only insofar as it is consistent with the goal of price stability, where price stability is usually understood as zero or close to zero inflation. Analyses of this policy have nevertheless typically represented these instructions by an objective function which puts heavy weight on price stability, but includes a small weight on output stability. While this is analytically convenient, it arguably does not accurately represent the intention of the Treaty, which clearly implies a lexicographic preference ordering.

The main rationale for this explicit restriction, as with the adoption of monetary targeting, has been the attempt to ensure continuity with respect to the past, in order to help the ECB to inherit the anti-inflationary credibility earned by the Bundesbank. Indeed, the hierarchical formulation of goals is consistent with the well-known formulation of the Bundesbank's goals, where "safeguarding the currency" was interpreted as the primary goal and "support the general economic policy of the Federal Government, but only in so far as this is consistent with the aim of safeguarding the currency" was

interpreted as the secondary goal.¹

The introduction of a hierarchical formulation of goals, with medium- or long-run price stability as primary goal has increased the accountability of European central bankers in pursuing low inflation and has eliminated the uncertainty about the relative weights attached by the policy maker to the achievement of the different goals.

However in order to evaluate the price stability mandate we need a fuller understanding of the implications of a lexicographic preference ordering for the implementation of monetary policy, the control of the money market and the interest rate. In particular, if the ECB's monetary instrument has to be used to achieve price stability, then a first important question is whether there are any degrees of freedom left for moderating real fluctuations. One possible answer to this question might be related to the definition of price stability and the issue of how price stability can be maintained. As explained by Svensson (1999): *'defining price stability involves deciding between price-level stability and low (including zero) inflation, choosing the appropriate price index, and selecting the appropriate level for a quantitative target. It also involves deciding on the role of real variables, like output, in the objectives for monetary policy. Thus defining price stability boils down to defining the monetary-policy loss function'*.

In this paper we explore a number of models in an attempt to formalize policy with lexicographic objectives. Section 2 discusses at greater length

¹See Svensson (1995) and von Hagen (1995).

the motivation for this exploration and the weaknesses that we find in the bulk of the extant literature. Section 3 sets out a simple formalization of lexicographic preferences, under alternative monetary targeting regimes, that embodies the idea that forecast future inflation is the primary objective but that policy can respond to short-term shocks to output. Section 4 looks at interest rate rules in the context of a model with expected future inflation in the aggregate supply function. Section 5 formalizes the target as a zone for inflation rather than as a point and explores the possibilities of multiple equilibria in this setting. Section 6 offers some concluding thoughts.

2 Modeling Monetary Policy

In this section of the paper we review a number of issues raised by the current literature on the modelling of monetary policy, in the context of the “new paradigm”, that is to say the use of short term interest rates by independent central banks to achieve an inflation target. Despite the primacy given to the stabilization of inflation, most academic analyses assume that the central bank’s objectives include, in addition to deviations of inflation from target, deviations of output from a target value, and also very often a term involving changes in interest rates. This is justified in the face of the stated objectives of central banks by noting that central banks typically do not attempt to achieve inflation targets continuously or immediately, regardless of the cost in terms of fluctuations of output, exchange rates, and interest rates. Central banks instead typically aim to get inflation on target within a period of

a year or two from the date of the policy action. They may also specify the target as a range of acceptable rates of inflation rather than a point. They seem reluctant to move interest rates up and down too much, being particularly reluctant to have short term reversals in the direction of interest rate changes. Thus they appear to smooth interest rates. The one-year horizon within which inflation is brought into line also reflects the lags in the effects of interest rates on inflation and maybe also uncertainty about what the effects on interest rate changes are. Attempting to bring inflation into line too rapidly might, it is believed, lead to instrument instability. Central banks themselves declare, as the Bank of England has done, that they are not “inflation nutters”.

For all these reasons it is argued that an objective function such as

$$L = E_0 \sum_{t=0}^{\infty} \left[(\pi_t - \bar{\pi})^2 + \beta (y_t - \bar{y})^2 + \zeta (i_t - \bar{i})^2 \right], \quad (1)$$

is appropriate to represent central banks’ objectives. However, when combined with a supply function that has expectational elements, such as the Lucas price-surprise supply function

$$y_t = \alpha (\pi_t - \pi_t^e) - \varepsilon_t, \quad (2)$$

it inevitably introduces the phenomenon of time-inconsistency, and much of the academic literature continues to be preoccupied with it. Taking this model as its starting point, much analysis goes on to assume that policy-making is discretionary, i.e., based on period-by-period optimization of the objective function, rather than adherence to a rule that has good long-run

properties. It is argued that the predictions of this model accord well with observed behavior of central banks.

However, this kind of analysis does not take the stated objectives of, say, the European Central Bank at face value, and in a number of ways seems inappropriate for modelling central banks. The assumed objectives are arguably inappropriate. Many central banks seem to have accepted with alacrity that they should have a responsibility for stabilizing inflation, and appear very happy to have, only as subsidiary objective, a responsibility for output and employment. As part of a wider-ranging comment on modelling monetary policy, Eric Rasmussen (1998) has argued that central banks might strongly prefer to have responsibility only for matters that are under their control. Hence they accept willingly the inflation objective and reject responsibility for output. The advantage for the CBs is that their performance is more easily measured if they have the sole objective: that is, it enhances transparency of policy. Rasmussen argues that CBs want to establish a reputation for carrying out their duties competently. For this reason they prefer to have targets that they can achieve. The output target does not meet this criterion. The natural level of output, around which they might stabilize actual output is measured with wide margins of error, and errors in setting the output target would lead to persistent inflation or deflation. For example, the natural rates of unemployment in the United States and in the United Kingdom in the 1990s appear to have fallen to levels much lower than anyone would have predicted in advance. Monetary policy

that had targeted output at the estimated natural level for this period would have been unnecessarily restrictive. Blinder, in discussing central banks, has argued that time-inconsistency is not an issue. They do not want to spring surprises on the public in order to stimulate short-term increases in output. Mervyn King of the Bank of England has famously remarked that they (the Bank of England) want to make monetary policy boring. Even though one might not wish to take central bankers' statements about themselves entirely at face value, all these observations are consistent with the view that central bank objectives are not such as to lead to the dilemmas for policy engendered by the one given above.

The assumption of discretionary policy that is carried through in much of the literature may also be inappropriate. Independent central banks have frequently been granted freedom to conduct monetary policy as they see fit to meet governmentally determined objectives, and generally have institutional features designed to reinforce their independence of political pressure. Their senior officials – the governor and so on – often have long tenures of office and in some cases may be allowed only one term so that the desire to be re-appointed cannot lay them open to pressure. The institutional culture of a central bank is likely to produce continuity across the terms of individual governors. In these circumstances, central banks are likely to behave so as to establish reputations for doing their job well, and this suggests that their behavior is likely to be better represented by a pre-committed policy rule rather than short-term optimization.

The above model also effectively assumes that central banks can influence inflation immediately by setting interest rates appropriately, since current inflation is taken as the control variable. If in fact central banks can only influence inflation rates with a lag of six months or longer, then the time-inconsistency problem is less important. If the policy lag is in fact longer than the life of currently existing contracts, then there may be no time-inconsistency issue at all, as Goodhart and Huang have argued.

3 A simple framework with monetary targeting

In order to build a useful framework for examining monetary policy under the case of lexicographic preferences we consider a discretionary regime, i.e. a regime where monetary policy is time consistent and the policy maker is unable to pre-commit ex ante to a rule for setting the instrument. Let's assume that the supply function takes the form of a standard expectations augmented Phillips curve

$$y_t = \alpha (\pi_t - \pi_t^e) - \varepsilon_t, \tag{3}$$

where $\alpha > 0$ and ε_t is a random shock with mean zero and variance σ_ε^2 . Private sector's inflation expectations are rational, i.e. $\pi_t^e = E_{t-1}\pi_t$. The instrument is money growth m_t and is related to inflation by the a simple equation of the form

$$\pi_t = m_t. \tag{4}$$

Given the present aim of describing the conduct of monetary policy under lexicographic preferences, adding a velocity shock or a control error to equation (4) complicates the algebra without yielding important additional insights.

Society and government have the following loss function

$$L_t^{s,g} = [(\pi_t - \bar{\pi})^2 + \lambda(y_t - \bar{y})^2]. \tag{5}$$

The central bank has lexicographic preferences. As primary goal the central bank has price stability, expressed as

$$L_t^1 = (E_{t-1}\pi_t - \bar{\pi})^2. \tag{6}$$

As secondary goal the central bank has output stability, expressed as

$$L_t^2 = (y_t - \bar{y})^2. \tag{7}$$

In the present framework the optimization process is divided into two steps: first the primary objective is minimized; second as long as the first order condition for minimizing the primary objective remains satisfied it is possible to use the residual degrees of freedom for minimizing the secondary objective. In other words the optimization of the secondary objective is conditional on the optimization of the primary objective. Moreover, solutions which imply a lower value for L_t^1 are strictly preferred by the central bank

and similarly solutions which imply the same value of L_t^1 but a lower value of L_t^2 are strictly preferred as well.

The expression (6) is one possible definition of price stability. An alternative definition of price stability, sometimes used in the literature, is the following²

$$E_{t-1}\pi_t = \bar{\pi}. \tag{8}$$

The problem with this last condition is that it is too general and, as price stability is not expressed in terms of a loss function, it does not allow to order the multiple solutions that satisfy the above condition.

Price stability can also be defined in terms of price level stabilization, but even if this is an interesting theoretical case it is not adopted in practice.

In the next sections we will examine lexicographic preferences in the context of a monetary targeting regime with an announced long-run money growth target. In all the cases considered announcements made by the central bank will play a key role in the optimization process. The reason is the following. If the period loss function L_t^1 is minimized by choosing ex post - after expectations are formed and shocks are realized - the actual value of money growth there will be no degrees of freedom left for achieving other objectives. Indeed in this case the optimal value of m_t is $\bar{\pi}$, which implies a strictly preferred value for L_t^1 . However this last case is not interesting as lexicographic preferences coincide with the case of a single objective. Thus

²See for example Smets (2000).

only if the optimization of L_t^1 is made ex ante it is possible to have some degrees of freedom left for optimizing other objectives. This explains why the primary objective is minimized by choosing an optimal announcement on the level of instrument ex ante.

Moreover we will consider also alternative monetary regimes: interest rate targeting - with an operative target expressed in terms of a rule - and inflation zone targeting. In this second part of the analysis announcements will play a less important role.

3.1 Disciplined discretion

Let's start with the case when deviations from the announcement made are not costly. Suppose that in each period before expectations are taken by the private sector the central banker announces a reference target for money growth, \bar{m}^a , consistent with the achievement of the primary objective of price stability. We can express deviations from the announcement made in the following way

$$\pi_t = m_t = \bar{m}^a + \Delta_t, \tag{9}$$

where \bar{m}^a is the announcement made ex ante and Δ_t is the adjustment made ex post for stabilizing the secondary target, given of course that it is consistent with the achievement of the primary objective. Minimization of L_t^1 with respect to \bar{m}^a , with the constraint that $\pi_t = \bar{m}^a + \Delta_t$, yields the following first order condition

$$E_{t-1}\pi_t = \bar{\pi}. \quad (10)$$

This condition implies that

$$\bar{m}^a = \bar{\pi} - E_{t-1}\Delta_t. \quad (11)$$

Substituting this last expression back in the expression for inflation we get

$$\pi_t = \bar{\pi} + \Delta_t - E_{t-1}\Delta_t. \quad (12)$$

The term $(\Delta_t - E_{t-1}\Delta_t)$ in (12) represents the concept of disciplined discretion within the present framework. In other words it expresses the margin available for stabilizing output fluctuations, given that the first order condition required for price stability is satisfied.

Laubach and Posen (1997) discuss at length the idea of disciplined discretion for the case of the Bundesbank and the Swiss central bank without, however, providing an analytical framework. In particular they question the highly stylized framework used in the ‘rules versus discretion’ debate. What emerges from their study is: *‘an interpretation of German and Swiss monetary practice that we call “disciplined discretion”. The practice followed by the central banks of the two countries should not be constructed simply as a more complicated rule; it should be seen, instead, as a system of commitments meant to clarify publicly and continuously the intent and stance of monetary policy. [...] It is not necessary to bind a central bank’s hands extremely tightly*

in order to sustain low inflation. It is, however, crucial that a central bank achieve transparency and provide structured accountability over the medium term.

In order to find the value of $(\Delta_t - E_{t-1}\Delta_t)$ we minimize the secondary objective with respect to Δ_t , by taken the announcement and private sector's expectations as given. This yields the following first order condition

$$\Delta_t = -\bar{m}^a + \pi_t^e + \frac{1}{\alpha}\bar{y} + \frac{1}{\alpha}\varepsilon_t. \quad (13)$$

Taking conditional expectation of the above expression we get

$$E_{t-1}\Delta_t = -\bar{m}^a + \pi_t^e + \frac{1}{\alpha}\bar{y}. \quad (14)$$

Now by subtracting this last expression from the first order condition (13)we obtain

$$\Delta_t - E_{t-1}\Delta_t = \frac{1}{\alpha}\varepsilon_t. \quad (15)$$

Hence we can completely eliminate the variability of output without violating the first order condition for the minimization of L_t^1 . The fact that Δ_t can be chosen only for stabilizing the shock ε_t implies that the announcement made by the central bank will be

$$\bar{m}^a = \bar{\pi}, \quad (16)$$

where it is assumed that, if there is no advantage in announcing a long-run target different from the average value of money growth in equilibrium,

the target announced is equal to trend money growth.

So in this case we have the following equilibrium values:

$$\begin{aligned}\pi_t^e &= \bar{\pi}; \\ \pi_t &= \bar{\pi} + \frac{1}{\alpha}\varepsilon_t; \\ y_t &= 0.\end{aligned}\tag{17}$$

This equilibrium implies excessively high inflation volatility, from the point of view of both the society and the central bank. Hence the question that we will try to answer in the subsequent sections is whether there exist better equilibria.

3.2 A mixed strategy based on the announced monetary target

Now let's assume instead that the central bank considers the following mixed strategy for conducting monetary policy. With a given probability γ it deviates from the announced target for money growth, while with probability $(1 - \gamma)$ it sets actual money growth equal to the announced target, or

$$\pi_t = \bar{m}^a.\tag{18}$$

The idea is to see whether it might be optimal for the society and the central bank to introduce some degree of uncertainty about its commitment to the announced target for money growth. Notice that this is a different game with respect to the one considered before with pure strategy.

In the case under examination inflation expectations are given by

$$\pi_t^e = \gamma(\bar{m}^a + \Delta_t^e) + (1 - \gamma)\bar{m}^a, \quad (19)$$

which can be rewritten as

$$\pi_t^e = \bar{m}^a + \gamma\Delta_t^e. \quad (20)$$

Minimization of L_t^2 with respect to Δ_t yields the following first order condition

$$\Delta_t = -\bar{m}^a + \pi_t^e + \frac{1}{\alpha}\bar{y} + \frac{1}{\alpha}\varepsilon_t. \quad (21)$$

By taking conditional expectations we get

$$\Delta_t^e = -\bar{m}^a + \pi_t^e + \frac{1}{\alpha}\bar{y}. \quad (22)$$

This implies that inflation expectations become

$$\pi_t^e = \bar{m}^a + \frac{\gamma}{(1 - \gamma)\alpha}\bar{y}. \quad (23)$$

So after some algebraic passages we have that

$$\pi_t = \bar{m}^a + \frac{1}{(1 - \gamma)\alpha}\bar{y} + \frac{1}{\alpha}\varepsilon_t, \quad (24)$$

with

$$\Delta_t = \frac{1}{(1 - \gamma)\alpha}\bar{y} + \frac{1}{\alpha}\varepsilon_t. \quad (25)$$

The first order condition for minimizing L_t^2 with respect to \bar{m}^a again is

$$E_{t-1}\pi_t = \bar{\pi}, \quad (26)$$

and we can write this condition as

$$\gamma \left(\bar{m}^a + \frac{1}{(1-\gamma)\alpha} \bar{y} \right) + (1-\gamma) \bar{m}^a = \bar{\pi}. \quad (27)$$

Now we can choose \bar{m}^a in order to satisfy the first order condition for L_t^2 .

We have

$$\bar{m}^a = \bar{\pi} - \frac{\gamma}{(1-\gamma)\alpha} \bar{y}. \quad (28)$$

Thus in the case of a deviation from the escape clause rule we will have the following equilibrium values:

$$\begin{aligned} \pi_t^e &= \bar{\pi}, \\ \pi_t &= \bar{\pi} + \frac{1}{\alpha} \bar{y} + \frac{1}{\alpha} \varepsilon_t, \\ y_t &= \bar{y}. \end{aligned} \quad (29)$$

While in the case of no deviation we will have the following values:

$$\begin{aligned} \pi_t^e &= \bar{\pi}, \\ \pi_t &= \bar{m}^a = \bar{\pi} - \frac{\gamma}{(1-\gamma)\alpha} \bar{y}, \\ y_t &= -\frac{\gamma}{(1-\gamma)} \bar{y} - \varepsilon_t. \end{aligned} \quad (30)$$

Notice that these equilibria imply that $E\pi_t = Em_t = \pi_t^e = \bar{\pi}$, i.e. there is perfect credibility, but, contrary to the case with pure strategy examined in the previous section, average money growth is higher than the long-run target \bar{m}^a . It is interesting to observe that this result reflects an important stylized fact of the historical evidence on the Bundesbank, which as we observed before is a typical example of a central bank thought to have lexicographic preferences. Fratianni and Huang (1995) and Fratianni (1995) show that in the 1984-1994 period the Bundesbank had at the same time a stronger reputation for low inflation and a poorer tracking record in achieving the monetary target than the bank of Italy.

Now it is possible to derive threshold values for σ_ε^2 which ensure that the equilibrium with the considered mixed strategy is strictly preferred to the pure strategy case examined in the previous section by both the society and the central bank. For the society we must have

$$\sigma_\varepsilon^2 > \bar{\sigma} \equiv \frac{(1 + \lambda\alpha^2)\gamma}{(1 - \gamma)^2(1 - \lambda\alpha^2)}\bar{y}^2, \quad (31)$$

with

$$1 - \lambda\alpha^2 > 0. \quad (32)$$

While for the central bank we must have

$$\sigma_\varepsilon^2 > \tilde{\sigma} \equiv \frac{\gamma}{(1 - \gamma)^2}\bar{y}^2, \quad (33)$$

with $\bar{\sigma} > \tilde{\sigma}$. See the appendix for details on the derivation.

3.3 Costly deviations from the announced monetary target

Now, let's return to pure strategy, but consider the case when there is a cost for deviating from the announced reference target for money growth. Suppose that, along the line of Rogoff's (1985), the government introduces an incentive scheme for achieving the announced money growth target. In this case L_t^2 is given by

$$L_t^2 = (y_t - \bar{y})^2 + \omega (m_t - \bar{m}^a)^2, \quad (34)$$

where the parameter $\omega > 0$ is chosen by the government.

Minimization of L_t^2 with respect to m_t yields

$$\pi_t = \frac{\alpha \bar{y} + \alpha^2 \pi_t^e + \omega \bar{m}^a + \alpha \varepsilon_t}{\alpha^2 + \omega}. \quad (35)$$

After taking expectations at time $t - 1$, the above expression becomes

$$\pi_t^e = \frac{\alpha \bar{y}}{\omega} + \bar{m}^a. \quad (36)$$

Inserting the value obtained for expectations back into first order condition we get

$$\pi_t = \frac{\alpha \bar{y}}{\omega} + \bar{m}^a + \frac{\alpha}{\alpha^2 + \omega} \varepsilon_t. \quad (37)$$

Now we can substitute the expressions obtained for π_t and π_t^e in the loss function relative to the primary objective and minimize it with respect to the announcement \bar{m}^a . This yields the following optimal announcement

$$\bar{m}^a = \bar{\pi} - \frac{\alpha \bar{y}}{\omega}. \quad (38)$$

Sustituting the optimal announcement in the expression for inflation we get

$$\pi_t = \bar{\pi} + \frac{\alpha}{\alpha^2 + \omega} \varepsilon_t, \quad (39)$$

Again this equilibrium implies that $E\pi_t = Em_t = \pi_t^e = \bar{\pi}$, i.e. there is perfect credibility, but average money growth is higher than the long-run target \bar{m}^a .

Now it is interesting to observe that if the government sets

$$\omega = \frac{1}{\lambda}, \quad (40)$$

then for the society and the government it is possible to achieve the same value of the loss function obtained under a regime with commitment.³

4 Interest rate rules

In this section we develop a framework for analyzing the case of interest rate rules under lexicographic preferences. The model considered here is a stylized New Keynesian model, which is a simplified version of Clarida, Gali and Gertler (1999). The supply function is given by a Phillips curve that relates inflation positively to the output gap

³Under a regime with commitment the optimal level of inflation for the given society's loss function is $\pi_t = \bar{\pi} + [\lambda\alpha / (1 + \lambda\alpha^2)] \varepsilon_t$.

$$\pi_t = \delta E_t \pi_{t+1} + \eta y_t + v_t. \quad (41)$$

We have also an IS equation which relates inversely the output gap to the real interest rate

$$y_t = -\beta (r_t - E_t \pi_{t+1}) + u_t. \quad (42)$$

The central bank has again lexicographic preferences. As primary goal the central bank has price stability, expressed as

$$L^1 = E_0 \sum_{t=0}^{\infty} \delta^t L_t^1, \quad (43)$$

with $\delta > 0$ the discount factor. The period loss function corresponding to the primary objective is

$$L_t^1 = (E_{t-1} \pi_t - \bar{\pi})^2. \quad (44)$$

As secondary goal the central bank has output stability, expressed as

$$L^2 = E_0 \sum_{t=0}^{\infty} \delta^t L_t^2. \quad (45)$$

The period loss function corresponding to the secondary objective is now

$$L_t^2 = (y_t - \bar{y})^2 + \varphi (r_t - \bar{r}_t)^2, \quad (46)$$

with $\varphi > 0$ and \bar{r}_t an operative target for the interest rate chosen according to an optimal rule that minimizes the period loss function corresponding to the primary objective.

In our framework the operative target can be state contingent. As observed by Svensson (1997) and Beetsma and Jensen (1999) state contingent targeting may not be feasible in general. However in our case, given the preferences of the central bank and the structure of the economy, it is possible for private agents to determine rationally the value of the operative target as it is chosen endogenously ex post (after expectations are formed and before r_t is chosen) by the central banker in order to achieve the primary objective.

Notice that usually the assumption made in the literature on interest rate rules is that φ is infinite, or alternatively there is no possibility of deviating from the interest rate rule. The only exception is when some degree of monetary inertia (usually due to the presence of a financial stability motive in the central bank's loss function) explicitly in the analysis. Hence the present framework is more flexible of the standard one used in the literature and probably closer to the real world too.

The first order condition for minimizing L_t^2 with respect to r_t is given by

$$-\beta(y_t - \bar{y}) + \varphi(r_t - \bar{r}_t) = 0. \quad (47)$$

Inserting (42) in (47) and collecting for r_t we get

$$r_t = \frac{1}{\varphi + \beta^2} \left(\beta^2 E_t \pi_{t+1} + \varphi \bar{r}_t + \beta u_t \right). \quad (48)$$

Inserting expression (48) back in the expressions (41) and (42) we obtain output and inflation as a function of the operative target \bar{r}_t . In particular for inflation we have

$$\pi_t = \frac{\delta(\varphi + \beta^2) + \eta\beta\varphi}{\varphi + \beta^2} E_t \pi_{t+1} - \frac{\eta\beta\varphi}{\varphi + \beta^2} \bar{r}_t + \frac{\eta\varphi}{\varphi + \beta^2} u_t + v_t \quad (49)$$

Using this new expression for inflation we can show that the first order condition for minimizing L_t^1 with respect to \bar{r}_t is given by

$$E_{t-1} \pi_t = \bar{\pi}, \quad (50)$$

It is possible to show that the condition (50) implies two rules for \bar{r}_t . The first rule consists in setting the target equal to a constant value given by

$$\bar{r}_t = -\frac{(\varphi + \beta^2)k}{\eta\varphi\beta} \bar{\pi} + \frac{\beta}{\varphi} \bar{y} \equiv \bar{r}, \quad (51)$$

with

$$k = 1 - \frac{\delta(\varphi + \beta^2) + \eta\beta\varphi}{\varphi + \beta^2}. \quad (52)$$

In this case the expression for inflation becomes

$$\pi_t = \frac{\delta(\varphi + \beta^2) + \eta\varphi\beta}{\varphi + \beta^2} E_t \pi_{t+1} + k\bar{\pi} + \frac{\eta\varphi}{\varphi + \beta^2} u_t + v_t, \quad (53)$$

and the first order condition for the primary objective is satisfied only if

$$\frac{\delta(\varphi + \beta^2) + \eta\beta\varphi}{\varphi + \beta^2} < 1. \quad (54)$$

The condition (54) is fulfilled for

$$\begin{aligned}
0 < \varphi < \frac{\beta^2(1-\delta)}{\delta + \eta\beta - 1} \equiv \bar{\varphi} \text{ if } \delta + \eta\beta - 1 > 0; \\
\varphi > 0 \text{ if } \delta + \eta\beta - 1 < 0.
\end{aligned} \tag{55}$$

Hence, with a constant interest rate target not necessarily the weight on the achievement of the operative target for the interest rate should be high in order to reach the primary objective.

Let's see the second case when there is also a forward-looking component in the interest rate rule for the operative target. In this case the rule for \bar{r}_t is given by

$$\bar{r}_t = \frac{\delta(\varphi + \beta^2) + \eta\varphi\beta}{\eta\varphi\beta} E_t\pi_{t+1} - \frac{(\varphi + \beta^2)}{\eta\varphi\beta} \bar{\pi}. \tag{56}$$

In this last case the first order condition for the primary objective is satisfied for $\varphi > 0$.

In both cases equilibrium inflation will be equal to

$$\pi_t = \bar{\pi} + \frac{\eta\varphi}{\varphi + \beta^2} u_t + v_t. \tag{57}$$

A problem with this equilibrium is that it is not clear how private agent may co-ordinate on one of the two possible rules for setting the operative target for the interest rate. One possibility available for the government for solving this problem of multiple equilibria would be to delegate monetary policy to a central banker with $\varphi > \bar{\varphi}$. But this solution works only if $(\delta + \eta\beta - 1) > 0$.

5 Inflation zone targeting

Let's examine an alternative framework. Instead of having a point target for inflation we consider now a target range. The case of inflation zone targeting is discussed by Orphanides and Wieland (1999) and Terlizzese (1999).⁴ Here we focus on escape clause regimes and we ask whether there might be the risk of having multiple equilibria, as shown for example by Obstfeld (1991) and Obstfeld and Rogoff (1996) in the case of fixed exchange rates.⁵

Here for simplicity we eliminate time subscript. Moreover, opposite to the previous analysis we assume that the shock ε is uniformly distributed with support $[-\bar{\varepsilon}, \bar{\varepsilon}]$, that the monetary instrument is the actual inflation rate and the parameter α in the supply function is equal to one.

If inflation is within the target range $[0, \bar{\pi}]$, central bank's preferences are given by the following period loss function

$$L = (y - \bar{y})^2. \tag{58}$$

⁴Terlizzese (1999) does an analysis similar to the present one. However he does not take into account the possibility of deviations from the target range and, hence, he neither considers the implications of the presence of fixed versus flexible costs for deviating in the policy maker's loss function.

⁵As in Obstfeld and Rogoff (1996), also Alexius (1999) extends the Obstfeld (1991) model to the case of a uniform distribution for the supply shocks, instead of a triangular distribution. But contrary to them he does not realize that multiple equilibria exist also under this extension.

On the contrary, if inflation is greater than $\bar{\pi}$ its preferences are given by

$$L = (y - \bar{y})^2 + \chi_0 + \chi_1 (\pi - \bar{\pi})^2. \quad (59)$$

Finally, if inflation is negative its preferences are given by

$$L = (y - \bar{y})^2 + \psi_0 + \psi_1 \pi_t^2. \quad (60)$$

In the above expressions it is assumed that any deviation from the target range leads to both a fixed and a variable (quadratic) extra cost to the central banker. The parameters χ_0, χ_1, ψ_0 and ψ_1 are all positive.

5.1 Equilibria

Let us consider first the case when inflation is within the target range. In this case equilibrium inflation is derived by minimizing (58) with respect to π . From this first order condition we can obtain the threshold values for the shock ε in this case. We have

$$\begin{aligned} \varepsilon &\leq \bar{\varepsilon}_u \equiv \bar{\pi} - \pi_t^e - \bar{y}; \\ \varepsilon &\geq \bar{\varepsilon}_l \equiv -\pi_t^e - \bar{y}. \end{aligned} \quad (61)$$

When these conditions are satisfied with the inequality sign, equilibrium output is equal to \bar{y} and equilibrium inflation is

$$\pi = \pi^e + \bar{y} + \varepsilon. \quad (62)$$

While when the supply shock is equal to one of the above threshold values inflation is equal to one of the two extreme values of the target range.

Let's consider the case when the supply shock is greater than $\bar{\varepsilon}_u$ or lower than $\bar{\varepsilon}_l$. In this case the central banker must decide whether to deviate from the target range or stick to one of the extreme values of the target range. He takes this decision by comparing the two losses corresponding to the two possibilities.

When he deviates from the upper bound of the target range the equilibrium inflation rate is found by minimizing (59) with respect to π . We have in this case

$$\pi = \frac{\pi^e + \varepsilon + \bar{y} + \chi_1 \bar{\pi}}{1 + \chi_1}. \quad (63)$$

Similarly, by minimizing (60) we find that when he deviates from the lower bound of the target range equilibrium inflation is given by

$$\pi = \frac{\pi^e + \varepsilon + \bar{y}}{1 + \psi_1}. \quad (64)$$

Substituting these values for inflation back in their corresponding loss functions and comparing them with the case when inflation is equal to one of the two extreme values of the target range we obtain the following requirements for not deviating and sticking to one of the two extreme values of the target range. We have

$$\varepsilon \leq \varepsilon_u \equiv \bar{\pi} - \bar{y} - \pi^e + \sqrt{\chi_0(1 + \chi_1)}; \quad (65)$$

$$\varepsilon \geq \varepsilon_l \equiv -\bar{y} - \pi_t^e - \sqrt{\psi_0(1 + \psi_1)}.$$

It easy to see that

$$\bar{\varepsilon}_l < \varepsilon_l < 0 < \bar{\varepsilon}_u < \varepsilon_u. \quad (66)$$

Private sector's inflation expectations are given by

$$\begin{aligned} E\pi &= \quad (67) \\ &E[\pi \mid \bar{\varepsilon}_l < \varepsilon < \bar{\varepsilon}_u] \Pr(\bar{\varepsilon}_l < \varepsilon < \bar{\varepsilon}_u) \\ &+ E[\pi \mid \bar{\varepsilon}_u \leq \varepsilon \leq \varepsilon_u] \Pr(\bar{\varepsilon}_u \leq \varepsilon \leq \varepsilon_u) \\ &+ E[\pi \mid \varepsilon < \varepsilon_l] \Pr(\varepsilon < \varepsilon_l) \\ &+ E[\pi \mid \varepsilon > \varepsilon_u] \Pr(\varepsilon > \varepsilon_u); \end{aligned}$$

where we have used the fact that $E[\pi \mid \varepsilon_l \leq \varepsilon \leq \bar{\varepsilon}_l] \Pr(\varepsilon_l < \varepsilon < \bar{\varepsilon}_l)$ is equal to zero.

In equilibrium expectations must be rational and hence we must have that

$$\pi^e = E\pi. \quad (68)$$

After substituting the expressions from (61) to (65) in (67) we get

$$E\pi = \frac{\varepsilon_u - \bar{\varepsilon}_u}{2\bar{\varepsilon}} \bar{\pi} + \frac{\bar{\varepsilon}_u - \bar{\varepsilon}_l}{2\bar{\varepsilon}} (\pi^e + \bar{y}) + \frac{\varepsilon_l + \bar{\varepsilon}}{2\bar{\varepsilon}} \left(\frac{\pi^e + \bar{y}}{1 + \psi} \right) \quad (69)$$

$$\begin{aligned}
& + \frac{\bar{\varepsilon} - \varepsilon_u}{2\bar{\varepsilon}} \left(\frac{\pi^e + \bar{y} + \chi\bar{\pi}}{1 + \chi} \right) + \frac{\bar{\varepsilon}_u^2 - \bar{\varepsilon}_l^2}{4\bar{\varepsilon}} \\
& + \frac{\varepsilon_l^2 - \varepsilon^2}{4\bar{\varepsilon}(1 + \psi)} + \frac{\bar{\varepsilon}^2 - \varepsilon_u^2}{4\bar{\varepsilon}(1 + \chi)}.
\end{aligned}$$

Using (68) we can solve the expression (69) for π^e .

Consider first the case when $\chi_1 = \psi_1 = \theta_1$. In this case we have a unique solution

$$\pi^e = \frac{2(2\bar{\varepsilon} + \theta_1\bar{\pi})\bar{y} + \theta_1(2\bar{\varepsilon} - \bar{\pi})\bar{\pi} - (1 + \theta_1)(\chi_0 - \psi_0)}{2\theta_1(2\bar{\varepsilon} - \bar{\pi})}. \quad (70)$$

Which is positive if $\chi_0 = \psi_0$, $\bar{\varepsilon} > \bar{\pi}$.

Now consider the case when $\chi_1 > \psi_1$ and assume, in order to simplify algebra, that $\chi_0 = \psi_0 = \theta_0$. In this case we get two solutions:

$$\pi_1^e = \frac{\chi_1\bar{\pi} - \bar{\varepsilon}(\chi_1 + \psi_1) + \bar{y}(\psi_1 - \chi_1) + \psi_1\chi_1(\bar{\pi} - 2\bar{\varepsilon}) - \sqrt{\Phi}}{\chi_1 - \psi_1}, \quad (71)$$

and

$$\pi_2^e = \frac{\chi_1\bar{\pi} - \bar{\varepsilon}(\chi_1 + \psi_1) + \bar{y}(\psi_1 - \chi_1) + \psi_1\chi_1(\bar{\pi} - 2\bar{\varepsilon}) + \sqrt{\Phi}}{\chi_1 - \psi_1}, \quad (72)$$

with

$$\Phi \equiv (1 + \psi_1)(1 + \chi_1) \left(4\bar{\varepsilon}\bar{y}(\chi_1 - \psi_1) + \psi_1\chi_1(2\bar{\varepsilon} - \bar{\pi})^2 \right). \quad (73)$$

For $\bar{\varepsilon} > \bar{\pi}$ the expression (73) in the square root is positive. In order to rule out the possibility of having multiple equilibria we can observe that

$$\begin{aligned}\lim_{\chi \rightarrow \psi} \pi_1^e &= \text{undefined}, \\ \lim_{\chi \rightarrow \psi} \pi_2^e &= \pi^e,\end{aligned}$$

with

$$\begin{aligned}\chi_0 &= \psi_0; \\ \chi_1 &= \theta_1.\end{aligned}$$

Hence we can rule out the solution (71) using a continuity argument.

6 Conclusions

In this paper we have taken issue with the preferences or objectives conventionally held to underlie central banks' behavior, and have instead proposed that the lexicographic ordering set out in the Treaty of Maastricht for the European Central Bank should be taken at face value. We have explored a number of formulations of the policy problem with these objectives. The exercises that we have carried out here are preliminary ones, which are merely suggestive. In further work on this topic we hope to find more satisfactory models. The lexicographic ordering, which puts inflation stabilization first and other objectives second, may be seen as way of enshrining commitment to inflation stabilization. The lexicographic ordering may nevertheless permit a considerable degree of output smoothing, as the models of section 3,

and indeed the models of section 5, show. Modelling the inflation target as a range, as in section 5, with additional penalties for letting inflation stay outside the range, does not seem to lead to multiple equilibria. However, this result needs further investigation and it may reveal more pitfalls in monetary policy making.

Appendix

Here we derive the threshold values for σ_ε^2 given in expressions (31) and (33). The unconditional expectation of society's loss function under the solution (17) is equal to

$$EL^{s,g} = \frac{1}{\alpha^2} \sigma_\varepsilon^2 + \lambda \bar{y}^2. \quad (\text{A1})$$

The unconditional expectation of society's loss function under the solutions (29) and (30) is equal to

$$\begin{aligned} EL_{ms}^{s,g} &= \gamma \left[\frac{1}{\alpha^2} (\sigma_\varepsilon^2 + \bar{y}^2) \right] \\ &+ (1 - \gamma) \left[\left(\frac{\gamma}{(1 - \gamma) \alpha} \right)^2 (1 + \lambda \alpha^2) \bar{y}^2 + \lambda \sigma_\varepsilon^2 \right]. \end{aligned} \quad (\text{A2})$$

Now comparing (A1) with (A2) allows us to find the threshold value $\bar{\sigma}$ for the society given in the text, which ensures that $EL^{s,g} > EL_{ms}^{s,g}$.

For the central bank we have under the solution (17)

$$EL^1 = \frac{1}{\alpha^2} \sigma_\varepsilon^2, \quad (\text{A3})$$

and

$$EL^2 = \bar{y}^2. \quad (\text{A4})$$

While in the case of the considered mixed strategy we have

$$EL_{ms}^1 = \frac{\gamma}{(1-\gamma)\alpha^2} \bar{y}^2 + \frac{\gamma}{\alpha^2} \sigma_\varepsilon^2, \quad (\text{A5})$$

and

$$EL_{ms}^2 = \frac{1}{1-\gamma} \bar{y}^2 + (1-\gamma) \sigma_\varepsilon^2. \quad (\text{A6})$$

Comparing (A3) with (A5) allows us to find the threshold value $\tilde{\sigma}$ for the society given in the text, which ensures that $EL^{s,g} > EL_{ms}^{s,g}$.

References

Alexius, A. (1999), Inflation rules with consistent escape clauses, *European Economic Review*, 509-523.

Beetsma, R., and Jensen H. (1999), "Optimal inflation targets, "conservative" central banks, and linear inflation contracts: Comment", *American Economic Review*, 342-347.

Clarida, R., J. Galí and M. Gertler (1999), The science of monetary policy: a New Keynesian perspective, *Journal of Economic Literature*, 1661-1707.

Fратиани, M. and H. Huang (1995), Central bank reputation and conservativeness, Financial Market Group LSE, Discussion Paper no. 216.

Fратиани, M. (1995), "La politica monetaria necessita di un'ancora anti-inflazionistica", *Economia Italiana*, 349-365.

Laubach, L. and A.S. Posen (1997), *Disciplined discretion: monetary targeting in Germany and Switzerland*, Essays in International Finance, No. 206, Princeton.

Orphanides, A. and V. Wieland (1999), Inflation zone targeting, ECB working paper, no. 8.

Obstfeld, M. (1991), Destabilizing effects of exchange rate escape clauses, Working paper no. 3603, NBER.

Obstfeld, M. and K. Rogoff (1996), *Foundations of international macroeconomics*, MIT.

Rasmussen, E. (1998), A theory of trustees and other thoughts, in: Tihire Akder (ed.), *Public debt and its finance in a model of a macroeconomic policy*

game, Central Bank of the Republic of Turkey, April, 35-60.

Rogoff, K. (1985), The optimal degree of commitment to an intermediate monetary target, *Quarterly Journal of Economics*, 1169-1190.

Smets, F. (2000), What horizon for price stability, ECB Working Paper Series, No 24.

Svensson, L.E.O. (1995), Optimal inflation targets, 'conservative' central banks, and linear inflation contracts, IIES working paper.

Svensson, L.E.O., (1997), "Optimal inflation targets, "conservative" central banks, and linear inflation contracts", *American Economic Review*, 98-114.

Svensson, L.E.O., (1999), "Price Stability as a Target for Monetary Policy: Defining and Maintaining Price Stability", IIES working paper.

Terlizzese, D. (1999), A note on lexicographic ordering and monetary policy, Mimeo, Bank of Italy.

Von Hagen, J. (1995), "Inflation and monetary targeting in Germany", in: L. Leiderman and L.E.O. Svensson (eds), *Inflation targets*, CEPR.