Social Interactions, Human Capital and Mobility (Very Preliminary and Incomplete)*

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1 Introduction

The interplay between social arrangements, norms of behavior and economic variables has long been studied within economics. Codes of behavior, expectations about the behavior of others and beliefs regarding one's future opportunities may guide various decisions and thus have an effect on market behavior. Observed differences in wealth and welfare between countries, neighborhoods or ethnic groups are often explained as multiple equilibria resulting from such differences in the underlying norms of behavior, in particular in situations which require some form of coordination or involve some degree of complementarities.

In this paper we examine how the underlying social arrangements, such as the matching arrangements within a social group, can interact with decisions the agents make regarding investments in human capital. In particular, we assume that acquiring human capital affects both the agent's prospects of future income directly, but also the social group the agent is part of and hence his\her prospects for future mates. Consequently, the prevailing norms, e.g., who elects to invest in human capital, have an indirect effect on the individual's incentives to invest. While differences in human capital investments across groups are probably also due in part to differences in opportunities, we believe some of these differences can be attributed to differences in norms of behavior. For example, can differences in social norms

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explain differences in performance and attendance in schools beyond observed characteristics such as parental income, parental education, spending per pupil etc.? In particular, can the residual be attributed to some choice of "effort" level which builds on what is socially acceptable, e.g., whether going to school and being a successful student is considered "cool" or not? If so, these differences in non-market behavior, coupled with the resulting differences in the levels of human capital acquired, have a direct effect on the welfare of the group. Moreover, we seek to address the question, can social arrangements leading to inferior economic outcomes co-exists with more efficient arrangements when agents can choose their social affiliation? If so, under what conditions can one observe persistent income inequality, attributed to such differences, across social groups which offer ex-ante equal occupational opportunities?

The multiple-equilibria approach mentioned above usually presumes that different groups are isolated entities, i.e., agents only interact with members of their own group, and they can not change their social affiliation. In this paper we want to focus on the role of mobility, as a form of social insurance across generations, in the formation of norms of behavior across different social groups. Following Benabou (1993) and Durlauf (1992), a group will be identified with a residential location, e.g., a neighborhood. Individuals interact locally within their social group. In particular, they meet their partner for marriage from among the group, where their desirability depends on the prevailing matching hierarchy within that group.[1] Couples, once formed, can elect to move to a different neighborhood. However, in order to do so they must be willing to pay for a housing unit in their desired neighborhood more than a couple who was born there. Therefore, financial constraints will play a role in shaping the composition of the social groups. In most models, as done in this paper as well, residential segregation is primarily achieved through differentials in real estate prices across communities. While in the two papers mentioned above, this social polarization is deplored on grounds of creating inequality in educational opportunities through public financing, in this model, we assume equal educational opportunities across communities. However, mobility provides a mechanism for intergenerational transfers which serves as a form of isurance. As a result, differences in the underlying matching arrangements and the prospects for mobility will create differences in human capital investments, and consequently wealth, across the groups.

[1] There are no issues of search frictions or asymmetric information involved in the process.

1.1 Related literature

1.1.1 Theoretical Literature

Early models by Becker and Tomes (1987) and Loury (1981) attribute differences in human capital investments to (perpetuating) differences in parental income. Parents affect their children's earnings-potential through investment in human capital. Human capital markets are assumed incomplete in the sense that human capital formation cannot be financed by issuing claims against a child's future earnings due to the lack of enforceability of such contracts. Hence these investments are constrained by the level of parental income. As a result, high income families are better able than poor families to make such investments and income disparities are passed on across generations.

Durlauf (1992) constructs a dynamic model of income inequality which persists through the impact of human capital formation on earnings, while focusing on the role of community income. In particular, education (and hence the level of human capital) is a public good financed by local tax, allowing differences in opportunities to arise between communities. Residential segregation can be achieved by neighborhoods setting a minimum income requirement. Related to this is Benabou's (1993) model which links between residential choice, education, and productivity in a city composed of several communities. Education is modeled as a public good with local complementarities in the costs of investment. Moreover, the spillover from high-skill workers affects the two types of investment asymmetrically. This sorting condition induces residential segregation as the high-skilled workers are willing to bid more for the company of others like them.[2]

The foundations of the model investigated in this project are based on the one proposed in Mailath and Postlewaite (2002). They investigate the effects of different matching structures on individuals' welfare in a heterogenous population where some agents possess a heritable attribute. They identify conditions under which a "mixed" matching arrangement, where some couples are composed of a high-income no-attribute agent and a low-income agent with the attribute, is stable. This mixing provides a form of social insurance against individual income shocks. In particular, when

[2] Many empirical studies has made an attempt to document peer effects. However, it is a very delicate matter since it requires controlling for many unobservables and it is subject to selection effects. The second order assumption of greater resuction in the cost of acquiring education for the more able is therefore even harder to support empirically. For a recent study on peer effects see Hoaxby (2000). The model suggested in this paper avoids making these assumptions.

stable, this arrangement leads to higher welfare for all agents than welfare under matching which is assortative on income and the attribute.[3]

1.1.2 Casual empirical evidence

First, let me mention some casual empirical evidence related to differences in human capital investments across groups. Early work by Summers and Wolfe (1977) shows that average level of education (school level, district etc.) explains income controlling for parental education which serves as a proxy for ability. Datcher (1982) and Corcoran, Gordon, Laren and Solon (1989) show that neighborhoods characteristics are an important determinant of individual income levels. Wilson, in "The Truly Disadvantaged" (1987), has documented the growth and persistence of the chronically poor in a number of studies. He emphasized the idea that as middle- and upper-class blacks moved outside of historically segregated neighborhoods, hence changing the composition of the population, the remaining residents have found themselves confronted by a breakdown of social and economic institutions which has rendered poverty in these neighborhoods self-perpetuating.

More recently, Cook and Evans (2000) show that the substantial narrowing of the Black-White test gap over time is primarily due to the narrowing of the within-school gap, i.e., the gap between white-black with the same level of parental education who attend the same school. It explains roughly 75% of the documented convergence. This is consistent with a story which emphasizes differences in effort levels across groups, due to differences in norms of behavior, rather than differences in ability or educational opportunities. The remaining 25% are attributed to changing family and school characteristics. This piece of empirical evidence is in the spirit of the model pursued in this paper. It suggest that an underlying component, e.g., effort, is influenced by a person's social environment and has direct impact on his human capital investment.[4]

2 Model

There is an infinite sequence of two-period lived generations. Each generation contains a continuum, of measure one, of men and women. Each agent can be of two types, either high ability or low ability. In the first period of

[3] There is still a possibility of multiple stable social arrangments, i.e., a range of the parameters such that both assortative matching and mixed matching are stable.

[4] Many studies document the effect of human capital investments, such as years of schooling, on earnings and other economic variables.

their lives agents make investments in human capital, where the effect on future income is different for the different types. In the second period of their life, income is realized and is observed by all, men and women match, and each couple consumes their joint income and have two offspring. Utility of each agent comes from consumption resulting from joint income and the average utility of their two children. The discount factor for inter-generational utility is β.

Ability is a heritable trait. If both parents are high (low) ability, their two offspring are high (low) ability. If one parent is high ability and the other is low ability, each of there children has a probability of a half to be either high or low ability. Without much loss of generality we assume that they have exactly one child of each type.[5] Ability as inherited from one's parents is not a productive trait, i.e., it does not affect distribution of income as adult. However, a high-ability agent can choose to develop the trait in the first period in life into a productive trait. If he elects to do so his future distribution of consumption is the lottery $(H, 1/2 + k; L, 1/2 - k)$, i.e., his realized income will be high with probability $1/2 + k$ and otherwise low. A high-ability agent who does not invest in human capital has an identical future income distribution as a low-ability agent which is $(H, 1/2-k; L, 1/2+k)$. However, those high-ability agents who choose to invest in human capital are forced to match with each other.[6] Let L_{y} (H_{y}) denote high-ability agent with a low (high) realized income and correspondingly L_{n} (H_{n}) a low-ability agent with low (high) income.

For the analysis of the equilibrium when there is one population we can, without loss of generality, normalize utility so that $U(2H) = 1$, $U(2L) = 0$ and , $U(L + H) = u$ where $u \in [1/2, 1)$.

A matching structure is **stable**[7] if no pair (a man and a woman) can increase their utility by marrying each other relative to following the matching structure, taking into account the effect on their children's' utility assuming that they follow the existing structure, and a high-ability agent's decision in the first stage is maximizing utility given the matching structure. An **equilibrium** (μ, m) is a steady-state composition, where μ is the share of

[5] Reference to extentions of the law of large numbers for a continuum.

[6] One can show, at least for the case of one-population that it is not necessary to force exclusion on the high-ability agents that choose to invest in human capital but rather that if there are costs involved with schooling then there is a range such that they deter the low-ability agents from going to school just in order to be able to match with the high-ability agents. This implies that there is no equilibrium with investment and mixed matching.

[7] The pair-wise stability concept is borrowed from Mailath and Postelwaite (2002).

high-ability agents, and a stable matching structure m.

In Section 3 we will analyze the case of a population located in one "city". In Sections 4 we will extend the model to include two locations, where we assume a fixed housing stock in each location, with a total of one housing unit per couple. Children are born where their parents are located. They spend their first period of life in that neighborhood, realize their income and match locally. Then, as a couple, they get to make a locational decision. If they prefer to migrate to the other neighborhood, and they are able to "bid" out a couple in the other neighborhood the two couples swap locations. For simplicity, an equilibrium will be (μ, p, m, d) where μ, m are as before, p is the percentage of high-ability who live in neighborhood 1 and d is the mobility structure, i.e., it prescribes which couples get to move from one neighborhood to another in the steady state, i.e., while keeping the proportions of high- to low-ability agents fixed, according to a demand and supply analysis. Note that the value of a neighborhood to a certain agent depends on all aspects of the equilibrium and hence all four components must be determined simultaneously.

3 No-investment equilibrium in a one-population model

First, we want to illustrate in a simple set-up how the matching structure can affect human-capital investment decisions. One stable arrangement is where the high-ability agents choose to invest in the early stage and so define their future social group to be one where they can only match with others a like. Once income is realizes, they match assortatively on income. The low-ability agents do the same. The resulting matching structure is:

$$\begin{aligned} &H_{\mathsf{y}} - H_{\mathsf{y}} \\ &H_{\mathsf{n}} - H_{\mathsf{N}} \\ &L_{\mathsf{y}} - L_{\mathsf{y}} \\ &L_{\mathsf{n}} - L_{\mathsf{n}} \end{aligned}$$

This gives rise to the following value equations:

$$\begin{aligned} V_{\mathsf{y}} &= (1/2 + k)((1-\beta) + \beta V_{\mathsf{y}}) + (1/2 - k)\beta V_{\mathsf{y}} \\ V_{\mathsf{n}} &= (1/2 - k)((1-\beta) + \beta V_{\mathsf{n}}) + (1/2 + k)\beta V_{\mathsf{n}} \end{aligned}$$

and so $V_y = (1/2 + k)$ and $V_n = (1/2 - k)$. For this matching to be stable it must be that no pair can increase their utility by deviating from the matching structure. The candidates for deviating are high-ability agents with low income and low-ability agents with high income. These adult types can choose to marry each other rather than follow the assortative structure if a high-ability agent chooses not to invest in human capital in the first stage and remains part of the collective social group.[8] Therefore, the following two constraints must hold:

$$\begin{aligned} y \text{ (ex-ante)} \quad &: \quad V_y \geq (1/2 - k)((1-\beta) + \beta V_y) \\ & \qquad + (1/2 + k)((1-\beta)u + \beta/2(V_y + V_n)) \\ & \quad \text{or} \\ H_n \quad &: \quad (1-\beta) + \beta V_n \geq (1-\beta)u + \beta/2(V_y + V_n), \end{aligned}$$

The second constraint implies that the low-ability high-income agents prefer assortative matching to mixing, in which case it is clear that the high-ability agents prefer to invest (ex-ante) and match assortatively over not investing since it gives them a better lottery over future income. Or, it is the case that the high ability agents prefer to invest and commit to match assortatively in the second period, over not to invest and match, if happen to be poor, with a low-ability high-income mate. Note that if high-ability low-income agents prefer to match assortatively ex-post, i.e., in the second stage after not investing, it must be that they prefer to invest in the first period.[9] Also, if the second constraint holds, and so the high-ability agents do not have the option to mix, they prefer to invest ex-ante. And so, the matching structure if one of the two constraints holds. This simplifies so that:

$$\begin{aligned} \beta k &\leq (1-\beta)(1-u) \\ & \text{or} \\ \beta k &\geq (1-\beta)((u - 4k/(1+2k)) \end{aligned}$$

The second possible social arrangement is one where high-ability agents elect not to invest in human capital in the first period in order to avoid

[8] The couples at the top, i.e., H_y, are matched with each other, enjoying the best possible outcome. Therefore, they have no incentive to deviate from this arrangement. The couples at the bottom, L_n, would prefer to marry anyone else, however, they are not desireable mates.

[9] In particular, this implies that a sufficient but not necessary condition is one where they prefer to match assortatively ex-post.

seclusion from the entire (heterogeneous) group, which includes low-ability agents as well as their own type. Matching in the second period is such that it provides the high-ability agents "social insurance" in the event that they get a low realization of income, in which case they match with a high-income low-ability partner. Note, however, that by choosing not to develop their ability in the early stage they increase the probability of low income in the second period.

Formally, the matching structure is as follows;

$$\begin{aligned} &H_{\mathsf{y}} - H_{\mathsf{y}} \\ &H_{\mathsf{n}} - L_{\mathsf{y}} \\ &L_{\mathsf{y}} - H_{\mathsf{n}} \\ &L_{\mathsf{n}} - L_{\mathsf{n}} \end{aligned}$$

We refer to this as mixed matching since while at the top and at the bottom matching is assortative on both income and ability, the middle is mixed. High-ability agents with low income marry low-ability agents with high income. Each party is trading off value by matching this way rather than assortatively (on income and ability); while the low-income high-ability agents are enjoying a higher joint consumption today, one of their offspring will have low ability as a result of the match. Consequently, facing inferior future prospects. On the other hand, the high-income low-ability agents are foregoing current consumption for better prospects for their children, since by matching this way one of them will be high ability. For this matching structure to be stable one needs to verify that for these mixed couples this prescription is better than marrying assortatively.

In an equilibrium that involves no investment by high-ability agents, the shares of the four types of agents are $(H_{\mathsf{y}}, \mu(1/2-k); L_{\mathsf{y}}, \mu(1/2+k); H_{\mathsf{n}}, (1-\mu)(1/2-k); L_{\mathsf{n}}, (1-\mu)(1/2+k)$. For simplicity, assume first that $\mu = 1/2-k$. Then, a mixed matching structure as described above has $(1/2-k)$ couples of high-income high-ability agents, $2(1/2-k)(1/2+k)$ mixed couples and $(1/2+k)$ low-income low-ability couples. This induces the following values:

$$\begin{aligned} V_{\mathsf{y}} &= (1/2-k)((1-\beta)+\beta V_{\mathsf{y}}) + (1/2+k)((1-\beta)u + \beta/2(V_{\mathsf{y}}+V_{\mathsf{n}})) \\ V_{\mathsf{n}} &= (1/2-k)((1-\beta)u + \beta/2(V_{\mathsf{y}}+V_{\mathsf{n}})) + (1/2+k)\beta V_{\mathsf{n}} \end{aligned}$$

For this matching to be stable the following three constraints must hold:

$$\begin{aligned} L_{\mathsf{y}} \text{ (implied)} &: \quad (1-\beta)u + \beta/2(V_{\mathsf{y}}+V_{\mathsf{n}}) \geq \beta V_{\mathsf{y}} \\ y_{\text{ex-ante}} &: \quad V_{\mathsf{y}} \geq (1/2+k)((1-\beta)+\beta V_{\mathsf{y}}) + (1/2-k)\beta V_{\mathsf{y}} \\ H_{\mathsf{n}} &: \quad (1-\beta)u + \beta/2(V_{\mathsf{y}}+V_{\mathsf{n}}) \geq (1-\beta) + \beta V_{\mathsf{n}} \end{aligned}$$

The first constraint ensures that a low-income high-ability agent prefers to marry a high-income low-ability agent rather than his own type. This constraint is implied by the second one, which requires that the ex-ante decision of the high-ability agent is not to invest, i.e., that a young high-ability agent in this group, anticipating the mixed-matching structure, chooses not to invest in human capital. The third constraint requires that mixing is preferred by a high-income low-ability agent as well. Simplifying the stability conditions and using the value equations we get that:

$$(1-\beta)((1-2k)/(1+2k)-(1-u)) \geq \beta/2(V_{\mathsf{y}}-V_{\mathsf{n}}) \geq (1-\beta)(1-u) \quad (1)$$

where

$$V_{\mathsf{y}}-V_{\mathsf{n}} = 2(1-\beta)/(2-\beta)((1/2+k)u+(1/2-k)(1-u)).$$

When $\mu < 1/2 - k$, there are more high-income low-ability agents than low-income high-ability ones. As a result, some of the latter type match with each other. When $\mu > 1/2 - k$, the low-income high-ability agents are the long side of the market. In the former case, when mixed matching is stable, while the value of the high-ability agents does not change directly as μ increases in the range it increases through the second order effect on the value of the low-ability agents, which increases as the two sides of the market become of equal size. In the latter case, while the value to the low-ability agents does not change directly, since they are the short-side of the market, it changes as a result of the change to the value of the high-ability agents. This value decreases as μ increases since the amount of social insurance that they get decreases. In particular, the values to both types co-move in μ and achieve their maximal value at $\mu = 1/2 - k$. Since the values to both types under assortative matching do not depend on μ, one can establish that under the stability condition above, mixed matching is stable for a range of μ around $\mu = 1/2 - k$ for which it was derived. Since the option to invest and match assortatively always exists, when mixed matching is stable it pareto dominates assortative matching, i.e., $V_{\mathsf{y}}^{\mathsf{mix}} \geq V_{\mathsf{y}}^{\mathsf{ass}}$ and $V_{\mathsf{n}}^{\mathsf{mix}} \geq V_{\mathsf{n}}^{\mathsf{ass}}$, though it induces an inferior distribution of income in the population.

MP discuss the welfare implication of mixed matching in their model.[10] In particular, they show that mixed matching, when stable, pareto dominates assortative matching, i.e., each of the four types of (adult) agents is at least as well off in terms of utility, and some are strictly better, under

[10] Recall, that they do not have the investment feature but rather high ability is automatically a productive trait, i.e., it provides the agents with the $(\mathsf{H}, 1/2+\mathsf{k}; \mathsf{L}, 2/1-\mathsf{k})$ distribution over future income.

the mixed imposes an expected loss of size $k(1-2k)(H-L)$ in total income in the economy relative to an equilibrium with assortative matching and investment. Therefore, while agents are benefiting from mutual insurance it has negative implications on the total wealth of the group.

Figure 1 illustrates ranges of the parameters where both equilibria are stable (level 2), where only the mixed equilibrium with no investment is stable (level 1) and where only the assortative equilibrium with investment is stable (level 0).

4 Two social groups

We have seen in the previous section that mixed matching can be stable at the same time as assortative matching, in which case the former pareto dominates the latter, though it leads to lower total wealth. Suppose now that there are two neighborhoods with a fixed housing stock, each with a number of units enough to accommodate half of the total population. Neighborhood affiliation determinism the agent's social group. Specifically, it determined the agent's pool for mates and the appropriate matching arrangements. Each son inherits his parents' house. At their adult phase, after marrying, couples can choose where they want to reside subject to what they can afford in the market. That is, in order for a couple to move from the neighborhood they grew in to another neighborhood, they must be willing to pay the market price for a house in their desired neighborhood, where prices are determined by supply and demand. In the first step we will assume there is an "efficient swap" in each generation — we find a price such that all couple who are willing to pay this price swap with couples from the neighborhood who are willing to sell at that price and the population composition remains unchanged. However, we will assume this mobility is imposed on the poor couples, i.e., they are not compensated for the move, leaving all the rent from the move to the couples who move to the good neighborhood. In the second step, we will assume that rents are split between the two sides leaving both sides better off than staying at their current location.

The question becomes what social arrangements can co-exist? Could there be persistent differences in investments in human capital, leading to differences in income distribution and wealth between the two groups? In the language of this paper, can we want identify conditions under which an asymmetric equilibrium exists, where one group is characterized by assortative matching and investment in human capital while the other is governed by mixed matching and no investment. Moreover, it is the neighborhood where high-ability agents invest in human capital which is the desired one, i.e., although both social arrangements are stable the mixed one with no investment is inferior to the assortative one with investment, unlike the result with isolated populations.

In the two populations' model $\mu = 1/2$ while differences in compositions are captured by p, which denotes the percentage of high-ability agents in neighborhood 1. In terms of social arrangements there are three possible combinations for an equilibrium; two symmetric equilibria and two asymmetric. In the symmetric equilibria, either both neighborhoods are characterized by assortative matching and investment in human capital, or both are characterized by mixed matching and no investment. In the asymmetric equilibrium, one neighborhood is governed by assortative matching while the other by mixed, and each can be the more desired location.

As for a symmetric equilibrium with assortative matching. Since the (expected) values to both types in assortative matching do not depend on the composition of the population, when assortative matching is stable in one population, as characterized in Section 3, it is also stable for both populations to be governed by assortative matching with any $p \in [0, 1]$ and no mobility between neighborhood.

There is no equilibrium with both neighborhoods governed by mixed matching and $p \neq 1/2$. Suppose that $p \neq 1/2$, the neighborhood where the share of high-ability agents is lower than a half is the one where the values of both types of agents are higher, making this neighborhood a desired location.[11] Therefore, mobility must imply that high-income couples from the less desired location must swap with low-ability low-income couples from the desired location. As a result, there are high-ability couples moving in but not moving out, implying the share of high-ability agents increases. Therefore, it cannot stay constant as required in a steady state.[12]

Next, we show that there could not exist an asymmetric equilibrium with one neighborhood governed by assortative matching with investment and the other by mixed matching with no investment where the former neighborhood being a preferred location. With this matching structure the

[11] This is implied by the monotonicity established in Section 3.

[12] If we follow the population over the generations, assuming each generation believes the social arrangements, i.e., the mixed matching, is going to persist, then either the share of high-ability converges to a half or the stability of the mixed matching breaks down.

following is the composition of the population:

Neighborhood 1:		Neighborhood 2:	
$H_y - H_y$:	$p(1/2 + k)$	$H_y - H_y$:	$(1 - p)(1/2 - k)$
$H_n - H_n$:	$(1 - p)(1/2 - k)$	$H_n - L_y$:	$(1 - p)(1/2 + k)$
		$L_y - H_n$:	$(1 - p)(1/2 + k)$
$L_y - L_y$:	$p(1/2 - k)$	$H_n - H_n$:	$p(1/2 - k) - (1 - p)(1/2 + k)$
$L_n - L_n$:	$(1 - p)(1/2 + k)$	$L_n - L_n$:	$p(1/2 + k)$

where we assume that the majority of the high-ability agents are in neighborhood one, in particular, $p(1/2 - k) > (1 - p)(1/2 + k)$, i.e., $p > 1/2 + k$. The mobility structure, m, is such that rich couples (both low and high ability) and some mixed couples from neighborhood 2 move to neighborhood 1 pushing all the low-income couples out. For the shares of high- and low-ability agents to remain the same, it has to be that:

$$
\begin{aligned}
p(1/2 - k) &= (1 - p)(1/2 - k) + w2(1 - p)(1/2 + k) \\
&\text{and} \\
(1 - p)(1/2 + k) &= (1 - p)(1/2 + k) + w2(1 - p)(1/2 + k),
\end{aligned}
$$

where w is the share of mixed couples that moves out of neighborhood 2. Solving the two equations gives $p = (3/2 + k)/(5/2 - k)$. This implies the following value equations for the four types:

$$
\begin{aligned}
V_y^1 &= (1/2 + k)\big((1 - \beta) + \beta V_y^1\big) + (1/2 - k)\beta V_y^2 \\
V_n^1 &= (1/2 - k)((1 - \beta) + \beta V_n^1) + (1/2 + k)\beta V_n^2 \\
V_y^2 &= (1/2 - k)((1 - \beta) + \beta V_y^1) \\
&\quad + (1/2 + k)\big((1 - \beta)u + \beta/2(p_y(V_y^1 + V_n^1) + (1 - p_y)(V_y^2 + V_n^2))\big) \\
V_n^2 &= (1/2 - k)(p_m((1 - \beta)u + \beta/2(p_n(V_y^1 + V_n^1) + (1 - p_n)(V_y^2 + V_n^2)) \\
&\qquad + (1 - p_m)((1 - \beta) + \beta V_n^1)) + (1/2 + k)\beta V_n^2
\end{aligned}
$$

where $p_m = (1-p)(1/2+k)/(p(1/2-k))$, $p_y = p_n = w/((1-p)(1/2+k))$. The constraints for the stability of the matching arrangement in neighborhood 1 are:

$$V_y^1 \geq (1/2-k)((1-\beta) + \beta V_y^1) + (1/2+k)((1-\beta)u + \beta/2(V_y^1 + V_n^1)) \quad \text{(ex-ante)}$$

or

$$(1 - \beta) + \beta V_n^1 \geq (1 - \beta)u + \beta/2(V_y^1 + V_n^1).$$

In neighborhood 2:

$$V_{\mathsf{y}}^{2} \geq (1/2 + k)((1-\beta) + \beta V_{\mathsf{y}}^{1}) + (1/2 - k)\beta V_{\mathsf{y}}^{2} \text{ (ex-ante)}$$

and

$$(1-\beta)u + \beta/2(p_{\mathsf{n}}(V_{\mathsf{y}}^{1} + V_{\mathsf{n}}^{1}) + (1-p_{\mathsf{n}})(V_{\mathsf{y}}^{2} + V_{\mathsf{n}}^{2})) \geq (1-\beta) + \beta V_{\mathsf{n}}^{1}.$$

The underlying assumption for constructing this equilibrium (which the mobility structure builds on) was that $V_{\mathsf{y}}^{1} \geq V_{\mathsf{y}}^{2}$ (and $V_{\mathsf{n}}^{1} \geq V_{\mathsf{n}}^{2}$). However, under this mobility structure, the value for a high-ability agent from investing and matching assortatively does not depend on where he was born, since he gets to live in neighborhood 1 if he gets a high-income draw and in neighborhood 2 otherwise. For mixing to be stable, it must be that, ex-ante, the high-ability agents prefer not to invest and hence commit to match assortatively, but rather to not invest and mix. Therefore implying that $V_{\mathsf{y}}^{2} \geq V_{\mathsf{y}}^{1}$. For generic parameter values, there is no solution to the four value equations with this additional constraint that $V_{\mathsf{y}}^{1} = V_{\mathsf{y}}^{2}$.

Therefore, to be able to construct an equilibrium where assortative matching with investment in human capital dominates mixed matching with no investment due to mobility, where the comparison would have otherwise been reversed, requires a positive transfer between those who move into the good neighborhood and those who are pushed out. In other words, such an equilibrium can only exist due to a positive price differential in the value of the two houses. To illustrate the role of a price differential in creating differences between (ex-ante) identical neighborhoods we start with a simple example.

4.1 Example:

Suppose there are two locations of equal size and one type of risk averse agents who face the a lottery over future income of the form $(H, q; L, 1-q)$, i.e., high income with probability q and otherwise low income. Each parent has one child. Each parent cares about the utility stream to all future offspring. There exists an equilibrium where there is a price differential between the two neighborhoods although there is no material difference between them (except for this price differential). Movement between the neighborhoods serves as a transfer mechanism between a parent and his child, where a rich parent pays the differential and move to the good neighborhood and in the event that his child happens to be poor, he "cashes in" on this transfer by moving to the bad neighborhood and consuming the price differential. When $q > 1/2$, there is an equilibrium where rich people compete and move to one neighborhood, which is considered good, paying a price differential for housing, while pushing out the low-income agents from this group. Formally, this gives rise to the following value equations:

$$\begin{aligned} V^1 &= q((1-\beta)u(H) + \beta V^1) + (1-q)((1-\beta)u(L+r) + \beta V^2) \\ V^2 &= q\left[(1-q)/q((1-\beta)u(H-r) + \beta V^1) + (2q-1)/q((1-\beta)u(H) + \beta V^2)\right] \\ &\quad +(1-q)((1-\beta)u(L) + \beta V^2) \end{aligned}$$

For mobility to be voluntary it must be that:

$$\begin{aligned} (1-\beta)u(L+r) + \beta V^2 &\geq (1-\beta)u(L) + \beta V^1 \\ &\quad and \\ (1-\beta)u(H-r) + \beta V^1 &\geq (1-\beta)u(H) + \beta V^2 \end{aligned}$$

Finally, since there are more rich agents who compete for houses in the good neighborhoods, we assume that the price is bid up to the point that they are indifferent between moving to neighborhood 1 and staying in neighborhood 2, i.e., where the second constraint binds.

In order to solve numerically for the equilibrium we assume that $u(x) = x^{(1-g)}/(1-g)$. Refer to Figure 2 for a graph of the equilibrium price, which solves the system of two linear equations in the values and one non-linear in the price, as a function of the coefficient of risk-aversion g and discount factor β for $q = 0.6$ and $q = 0.7$.

Going back to the analysis of an asymmetric equilibrium which incorporates a price differential, we rewrite the value equations and the stability constraints to account for the transaction price as done in the example. Next we assume the same utility function as in the example and search numerically for the values and an equilibrium price, leaving both parties to the transaction with some rent, for different parameters values of k, β, g. See Figure 3 for ranges of existence of such an equilibrium with $V_y^1 > V_y^2$ and $V_n^1 > V_n^2$.

5 Bibliography

References

[1] Becker, G. S., and Tomes, N. (1979), "An Equilibrium Theory of the Distribution of Income and Intergenerational Mobility," The Journal of Political Economy, Vol. 87, No. 6, 1153-1189.

[2] Benabou, R. (1993), "Working of a City: Location, Education and Production," The Quarterly Journal of Economics, 108, 619-652.

[3] Cook, M. D., and Evans, W. N. (2000), "Families or Schools? Explaining the Convergence in White and Black Academic Performance," Journal of Labor Economics, October 2000.

[4] Corcoran, M., Gordon, R., Laren, D., and Solon, G. (1989), "Effects of Family and Community Background on Men's Economic Status," NBER Working Paper no. 2896.

[5] Datcher, L. (1982), "Effects of Community and Family Background on Achievement," Review of Economics and Statistics, 64, 32-41.

[6] Durlauf, N. S. (1992), "A Theory of Persistent Income Inequality," Journal of Economic Growth, 1, 75-93.

[7] Fernandez, R. and Rogerson, R. (1996), "Income Distribution, Communities, and the Quality of Public Education," The Quarterly Journal of Economics, 111, 135-164.

[8] Fernandez, R.(2001), "Education, Segregation and Marital Sorting: Theory and an Application to UK Data," NBER Working Paper no. 8377.

[9] Kremer, M. (1997), "How Much Does Sorting Increase Inequality?," The Quarterly Journal of Economics, 112, 115-139.

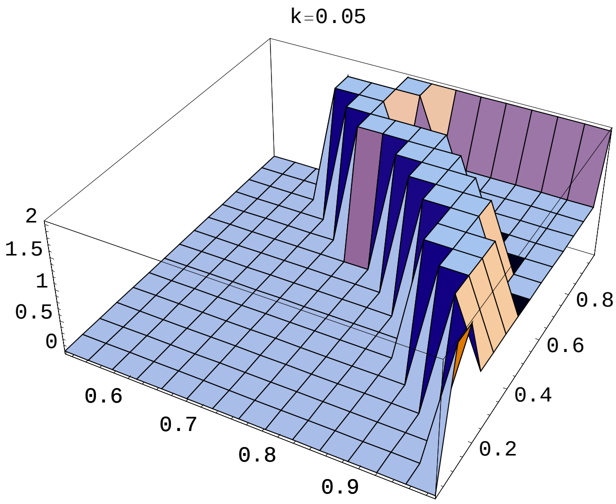
[10] Loury, G. (1981), "Intergenerational Transfers and the Distribution of Earnings," Econometrica, Vol. 49, No. 4, 843-867.

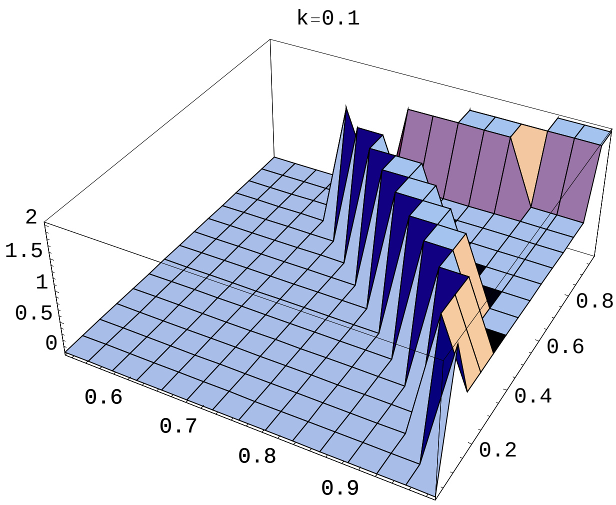
[11] Mailath, G. J., and Postlewaite A. (2002), "Social Assets," Working paper.

[12] Summers A., and Wolfe B. L. (1977), "Do Schools Makes a Difference?," American Economic Review.

[13] Wilson, W. J. (1987), The Truly Disadvantaged, University of Chicago Press.

Figure 1: Stability of equilibria in the one population model. Both equilibria are stable (level 2), only the mixed equilibrium is stable (level 1), only the assortative equilbrium is stable (level 0).





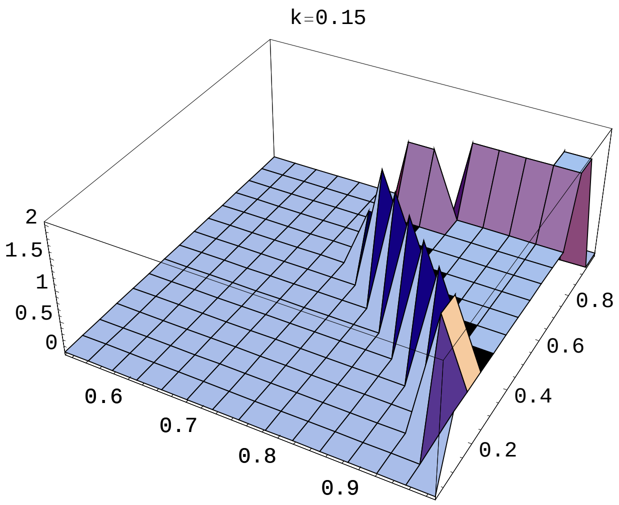
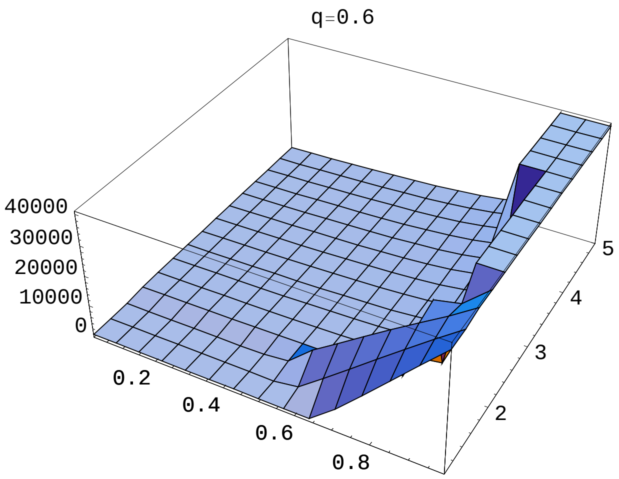

k=0.15
2
1.5
1
0.5
0
0.6
0.7
0.8
0.9
0.2
0.4
0.6
0.8

Figure 2: Price differentials for q=0.6,q=0.7 as a function of the discount factor and the risk-aversion coefficient



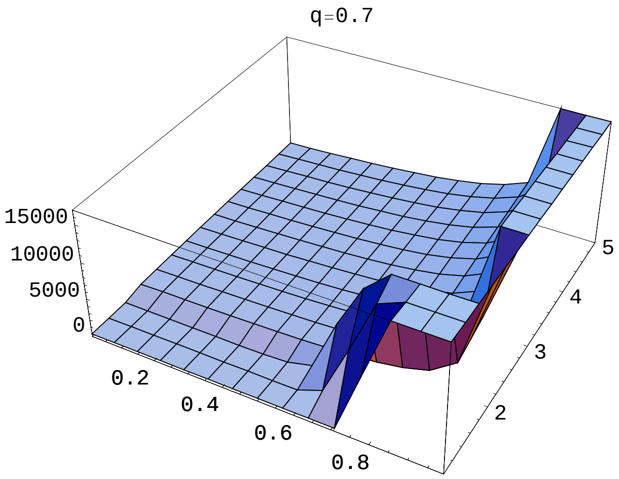
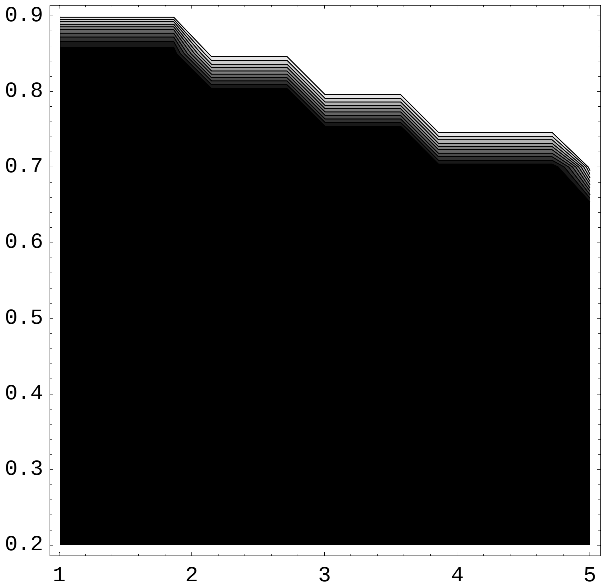


Figure 3 : Existence of an equilbrium for different levels of productivity k as a function of the risk aversion coefficient and the discount factor. Light areas indicate ranges of the parameters where the equibrium exists.

k=0.05



k=0.15