CHEAP TALK IN REPEATED MONETARY POLICY GAMES *

Nurlan Turdaliev †
Department of Economics
McGill University

Abstract

This paper gives a novel approach to modeling communication of central banks. A central bank and the public are engaged in an infinitely repeated monetary policy game with cheap talk. The central bank's type is private information. First, when the central bank is sufficiently impatient, there is only uninformative babbling equilibrium. This result suggests that impatient central banks can only be secretive. Second, if the central bank is sufficiently patient, there exists another equilibrium in which the central bank fully reveals its private information. Third, if the central bank is sufficiently patient then the fully revealing equilibrium is Pareto superior to the babbling equilibrium. This result supports the view that the recent move by some central banks to more open monetary policies should be encouraged.

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[†] Department of Economics, McGill University, 855 Sherbrooke Street West, Montreal, QC, H3A 2T7, Canada. Tel.: (514) 398-4827; fax: (514) 398-4938. E-mail: nurlan.turdaliev@mcgill.ca

1 Introduction

Central bankers around the world seem to spend much effort on cheap talk.[1] Let us give some examples. In the semiannual Humphrey-Hawkins testimony before Congress, the Fed chairman has to present the Fed's objective for the growth of money supply. Forecasts of inflation, unemployment, and economic growth are also reported. Since the Fed is not accountable for its forecasts, this may be viewed as cheap talk.

In December 1998 the FOMC also decided to announce the "bias" in its attitude to future actions on interest rates immediately after each meeting. This new procedure was first implemented in May 1999. However, as early as August 1999, the FOMC concluded that investors had been overreacting, often treating a bias as a guarantee of FOMC action at the next meeting. In December 1999 the FOMC changed its announcement procedure. First, the FOMC would make a public statement immediately after each meeting. (Previously, such statements were made only in case of a policy change or a major shift in its economic outlook.) Second, the Committee decided to use new language to describe the outlook. In its January 19, 2000 press release it is described in the following way: [2]

> The new language will provide the FOMC's assessment of the risks to satisfactory economic performance (one set of bracketed words will be chosen at each meeting to reflect the Committee's view about prospective developments):
>
> > "Against the background of its long-run goals of price stability and sustainable economic growth and of the information currently available, the Committee believes that the risks are [balanced with respect to prospects for both goals] [weighted mainly toward conditions that may generate heightened inflation pressures] [weighted mainly toward conditions that may generate economic weakness] in the foreseeable future."

The FOMC then comments on some phrases in the above language. This may be viewed as an example of the FOMC coaching the public as to what each of the several possible messages literally means. Of course, the public is free to interpret any message in the new language as it wishes, no matter what the FOMC wants it to think. This is again an example of cheap talk.

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[1] In cheap talk games (for an overview see Farrell and Rabin, 1996) players' payoffs are not affected by message sending. Talk is "cheap". This means of communication contrasts with costly signaling (e.g, the education game in Spence, 1974).

[2] See Board of Governors (2000).

Even minutes of the FOMC meetings can be interpreted as cheap talk. At each meeting the Committee votes on the text of the minutes of the previous meeting. Thus, one can view this text as a carefully censored version of the meeting, which is edited by taking into consideration all the events that have happened between the two meetings. This editing can hardly be a *random* noise added to the control error as some authors seem to suggest (e.g., see Faust and Svensson, 1999).

Let us raise several questions here. Why would a central bank spend its time and effort on cheap talk? Probably, it wants to form public opinion with regard to future monetary policy. Given that the central banker has a strategic incentive to engage in cheap talk (to possibly influence the public's opinion), is this cheap talk informative or uninformative? Does the central bank's strategic incentive regarding communication serve the interest of the public?

These questions are related to the phenomenon of secrecy of central banks. The bias towards secrecy of the Federal Reserve in the U.S., for example, is well documented (see Goodfriend, 1986; Mayer, 1987). This paper provides theoretical support to the recent move by some central banks to more open monetary policies. Using a repeated cheap talk game between a central bank and the public, we have obtained the following results.

- When the central bank is impatient, there is no informative equilibrium. In other words, the central bank talks uninformatively, and the public disregards this talk. We conclude that impatient central banks can only be secretive.
- When the central bank is patient enough, there is an informative equilibrium in which the central bank talks truthfully, and the public believes the central bank.
- Moreover, this informative equilibrium is proven to be Pareto superior to the uninformative equilibrium.

This research is motivated by the following three points. First, central banks have private information: they may have some information regarding the economy that is not available to the private sector; they may have their own forecasts of macroeconomic variables such as inflation or output; their views on what kind of monetary policy ought to be conducted may not be known to the private sector. Second, central banks make announcements: there are press releases, speeches given by the officials, testimonies before Congress, etc. This is one possible channel to reveal private information. Stein (1989) is the first to model this channel of communicating central bank private information through a game of cheap talk. However, his model is static. Third, monetary policy involves repeated interactions between central banks and the private sector. Kydland and Prescott (1977) have first shown that Nash equilibrium of a monetary policy game can be inefficient. Barro and Gordon (1983) demonstrated that the reputation effect modeled through repeated games mitigates this inefficiency. However, these models have not used communication, and the results do not shed much light on how this issue of reputation affects the central bank's ex-ante incentives to communicate.

The last two points indicate a gap between modeling monetary policy in a static environment with a communication channel and modeling monetary policy in a dynamic environment without a communication channel. This paper is an attempt to combine dynamic nature of repeated monetary policy games with explicit modeling of central bank communication through cheap talk.

In order to address questions about central banks' cheap talk, we use a model of asymmetric information where communication can be used to convey a central bank's private information. We think that any such model should have the following features: (i) the central bank must have several possible types; (ii) the information about its type must be private; (iii) talk is cheap, i.e. communication does not *directly* affect the agents' utilities; (iv) the public is rational. Then the question above can be formalized as questions about a Bayesian game with the above characteristics.

We use a version of a Sender-Receiver cheap-talk game introduced by Crawford and Sobel (1982). More precisely, we model the relationship between a central bank and the private economy as a repeated monetary policy game with cheap talk. There are two players, a central bank (CB) and the public (P). CB can be of two possible types, "strong" or "weak". CB's type is private information; the type is chosen once and for all. Before period zero, CB announces (not necessarily truthfully) its type. In every period P forms an inflation expectation, and then CB chooses actual inflation. The actions of both players are observable. First, we prove that when CB's discount factor is close to zero, there is no equilibrium in which CB truthfully reveals its type. The reason is that the weak type has an incentive to misreport its type. We strengthen this result by showing that there exists no partially revealing equilibrium, in which the two types talk imprecisely but choose different talking strategies. Thus, when CB is impatient, the only equilibrium is the "babbling" equilibrium in which both types send the same message so that P's belief does not change. This result suggests that impatient central banks can only be secretive.

The main results of the paper are:

1. when CB is sufficiently patient, there is a fully revealing equilibrium in which CB truthfully announces its type;
2. the best fully revealing equilibrium is Pareto superior to the best babbling equilibrium.

The intuition behind the first result is that when CB is patient enough, reputational considerations are important. If the weak type pretends to be strong and subsequently cheats, it can be punished by high inflation expectations in the future. This high inflation occurs because if CB ever cheats, P will believe forever that CB is weak. The second result is true because when the type is truthfully announced, P and CB can attain the Ramsey outcome, the best possible outcome with rational expectations. In contrast, when CB's type is not exactly known, it takes some time for P to learn it, and during this time any possible outcome is worse than the Ramsey outcome.

We extend our results to two different environments. In the first one, CB's type changes every period following an i.i.d. process, and CB makes announcements every period. In the second environment, CB does not have perfect control over inflation and its inflation target is not observable. In both cases the best fully revealing equilibrium is superior to the best babbling equilibrium.

Let us contrast our results with those in Stein (1989), which is the only paper known to us that addresses the phenomenon of central bank secrecy through games of cheap talk. Stein justifies the tendency of the Fed to be secretive by arguing that the Fed cannot precisely and credibly communicate its private information. In his static model, the Fed can talk informatively, but ex-ante incentives to cheat are great enough to prevent any precise communication. In contrast, in our dynamic model there exists an equilibrium in which the central bank can communicate precisely, and the public believes it. This difference arises because in the dynamic model the reputation effect plays a role.

The paper is organized as follows. Section 2 describes the environment. Section 3 contains the main results. Section 4 presents two extensions of the basic model. Section 5 reviews the related literature and concludes.

2 Model

In this section we describe the model. Following the literature (see, e.g., Persson and Tabellini, 1990), we model the relationship between a central bank and the private sector as a game of two players, the central bank and the public. The public is assumed to behave "myopically", i.e. in each period it maximizes that period's utility.[3]

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[3] Consider an infinite-horizon model with a central bank and a continuum of private agents. We assume that private agents are identical and anonymous. The central bank chooses an inflation target. Agents choose nominal wage inflation. It can be shown that in this environment where agents are atomistic and anonymous and have nothing to carry from one period to another, the infinite horizon optimization problem of a household

Since this paper studies several games with different time horizons, the following notational convention is used. A one-shot game is denoted by a letter without a superscript, say $\mathcal{G}$. An infinitely repeated game is denoted by a letter with the superscript ∞, say $\mathcal{G}^{\infty}$.

We first consider a one-shot monetary policy game Γ. There are two players, the central bank (CB) and the public (P). CB sets inflation (or an inflation target) x, whereas P chooses an action y. This action y can, for example, be the nominal wage which varies one for one with expected inflation (see Persson and Tabellini (1990) for more details). There are two types of CB, s (strong) and w (weak). The strong type is less tolerant to high inflation than the weak type.[4] The set of types is denoted by I, so that $I = \{s, w\}$. Probability of type i is $p^i \in (0,1), i \in I$, where $p^s + p^w = 1$. We break up the stages of Γ into numbered stages:

1. Nature chooses CB's type according to the distribution (p^s, p^w). CB's type is private information, i.e. P does not observe the realized type.
2. In the "announcement" stage, CB announces its type by sending a message, which does not have to be truthful. More precisely, CB sends one of two possible messages, s or w. Messages are assumed to be costless. [5]
3. P forms inflation expectation y. Having observed y, CB chooses actual inflation x. The payoffs to both players depend on x and y: P's payoff is $-(x-y)^2$ whereas payoff to CB of type i is $u^i(x, y)$.

Our benchmark game is the infinitely repeated game Γ^{∞}. In this game:

1. Nature chooses CB's type, s or w, according to the distribution (p^s, p^w). The type is chosen once and for all.
2. CB announces its type by sending one of the two messages, s or w.
3. Next come periods $0, 1, 2, \ldots$. In every period t, P moves first by choosing period t inflation expectation y_t. CB observes y_t, and then chooses actual inflation x_t. At the end of the period both players know (x_t, y_t).

Let $\delta \in (0,1)$ denote CB's discount factor. Type i CB maximizes the discounted utility $(1-\delta)\sum_{t=0}^{\infty} \delta^t u^i(x_t, y_t)$, whereas P maximizes $-(x_t - y_t)^2$ in every period t. We study this game in section 3.

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reduces to a sequence of one-period optimization problems. (See, for example, Atkeson and Kehoe, 2000). Then the game is equivalent to a two-player game between the central bank and the public in which P in every period maximizes that period's utility.

[4] One way to justify this setup in a richer model would be to assume that the population consists of two groups, and each type of CB is a representative of one of the groups (see Turdaliev, 1999).

[5] Technically, G is a game of cheap talk, not a Spencian signaling game.

In section 4 we analyze two modifications of Γ^∞. In the first modified game, $\Gamma^\infty_{i.i.d.}$, CB's type changes every period following an i.i.d. process. Timing in period t is as follows: Nature chooses CB's type; CB sends a message; P forms inflation expectation y_t; having observed y_t, CB chooses actual inflation x_t, which is immediately observed by P.

In the second modified game, G^∞, CB's type is chosen once and for all, and CB announces its type only once. There are two differences between Γ^∞ and G^∞: CB does not have perfect control over inflation, and P does not perfectly observe CB's action. Timing in every period is as follows. P forms inflation expectation y_t. Having observed y_t, CB chooses an inflation target x_t. After that both players observe actual inflation z_t, where $z_t = x_t + \epsilon_t$. The random variables ϵ_t are i.i.d. with a commonly known distribution function. Type i CB maximizes $(1-\delta)E\sum_{t=0}^{\infty}\delta^t u^i(z_t, y_t)$, whereas P maximizes $-E(z_t - y_t)^2$ in period t.

It is well known that in games of cheap talk, i.e. games with costless pre-play communication, there is always a "babbling" equilibrium in which announcements are uninformative and thus ignored. Sometimes all equilibria are of this type. In the next two sections we study the games described above to answer the following questions: Is there an equilibrium of the one-shot game in which CB's announcements are informative? How does the answer change as we go from the one-shot game to the infinitely repeated game? If an informative equilibrium exists, is it better than a "babbling" equilibrium?[6] We also consider how the results depend on the discount factor.

In extensions of the basic model, we also address the following questions: Do the results hold if CB's type changes over time? Do the results hold if CB does not have perfect control over inflation, and P does not perfectly observe CB's inflation target?

3 Benchmark Game

In this section we study our benchmark game. It is the case of perfect inflation control and perfect monitoring. In other words, CB can choose actual inflation, and each player's action is immediately observed by both players. We show that the one-shot game has only babbling equilibrium. In contrast, if CB is sufficiently patient, the infinitely repeated version of the game has a fully revealing equilibrium in which CB truthfully announces its type. We prove that the best fully revealing equilibrium is better than the best babbling equilibrium.

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[6] In making welfare comparisons among equilibria, the ex ante utility of CB is treated as a social welfare function.

3.1 One-shot Game

In this subsection we study the one-shot game Γ introduced in the previous section. We first study a game of complete information, i.e. a game in which CB's type is known. For this game we establish uniqueness of Nash equilibrium. A Ramsey outcome arises when CB can commit to its policy. We show that inflation in Nash equilibrium is higher than in the Ramsey outcome. Then we proceed to the one-shot game of incomplete information Γ (CB's type is unknown). The main result of this subsection is that Γ has only babbling equilibrium.

Let us remind the reader that P's utility function is $-(x-y)^2$, and the utility function of type i CB is $u^i(x,y)$. We assume that the action space of CB as well as the action space of P is a closed interval X on the real line $\mathbb{R}$. Throughout the paper the functions u^i are assumed to have the following functional form:

$$u^i(x,y) = U(x-y) + V^i(x), \qquad x, y \in X. \tag{3.1}$$

This is a generalization of the functional form used by Barro and Gordon (1983) and Canzoneri (1985). The functions V^s, V^w, and U are assumed to be twice differentiable.

We need the following assumption which assures, among other things, that U is concave, and V^s, V^w are "sufficiently concave".

Assumption 1. *The functions* U,V^s, *and* V^w *satisfy* [7]

$$\begin{array}{rcl} U'' & < & 0, \qquad (V^s)' < (V^w)', \\ (V^s)'' & < & -c_1, \qquad (V^w)'' < -c_1, \end{array}$$

where c_1 *is a positive constant.*

a. Complete Information Case

It is helpful to first examine the case of complete information, i.e., when CB's type is public information. Consider the game $\Gamma(i)$ in which CB's type $i \in I$ is public. Timing in $\Gamma(i)$ is as follows: (i) P forms inflation expectation y; (ii) CB observes y, and then chooses actual inflation x.

In the game $\Gamma(i)$, for any P's action y, an action x is said to be a best response to y if it maximizes $u^i(\cdot, y)$. Clearly, for any given inflation rate x, P's best response is $y = x$. Thus, a *Nash equilibrium* of $\Gamma(i)$ is a pair

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[7] As usual, $U'' < 0$ means $U''(x,y) < 0$ for all $x, y \in X$; similarly for other functions.

$(x, y) \in \mathbb{R}^2$ such that $y = x$ and x is a best response to y. Since $x = y$ in a Nash equilibrium, we will refer to just x as the Nash equilibrium.

The following claim will be often used. It says that there is a unique Nash equilibrium when CB's type is public. In addition, it establishes that the equilibrium inflation rate with the weak CB is higher than that with the strong CB.

Claim 1. *Under Assumption 1, the game* $\Gamma(i)$ *has a unique Nash equilibrium* x^i*. Furthermore,* $x^s < x^w$*.*

(All proofs are relegated to Appendices B and C.)

b. Complete Information Case: Efficient outcome vs. Nash equilibrium

We will use the notion of *Ramsey outcome* in our analysis of repeated games in the next section. A Ramsey outcome arises when CB can commit to a particular inflation rate. Thus, we can think of a Ramsey outcome of the game $\Gamma(i)$ as a Nash equilibrium of the game that can be obtained from $\Gamma(i)$ by "flipping" the sequence of moves: CB moves first, and P moves after observing CB's action. Clearly, a Ramsey outcome of $\Gamma(i)$ is such an x that maximizes $u^i(x, x)$. This implies that a utility payoff to CB in a Nash equilibrium is no higher than that in a Ramsey outcome. Another implication of this fact is the following claim that establishes Pareto efficiency of the Ramsey outcome.

Claim 2. *Let* x *be a Ramsey outcome of* $\Gamma(i)$*. Then the pair of actions* (x, x) *for P and CB is Pareto efficient.*

In our analysis of repeated games below we use uniqueness of a Ramsey outcome established in the following claim.

Claim 3. *Under Assumption 1, each game* $\Gamma(i)$*,* $i \in I$*, has a unique Ramsey outcome* x_r^i*. Furthermore,* $x_r^w > x_r^s$*.*

The above inequality mirrors Claim 1: the Ramsey outcome with the weak CB is higher than that with the strong CB. We need the following technical assumption.

Assumption 2. *The function* U *satisfies:* $U'(0) > 0,\ \ U(x^w - x^s) > U(0)$*.*

Next, we establish inefficiency of Nash equilibrium. The following claim states that inflation in the Nash equilibrium of $\Gamma(i)$ is higher than that in the Ramsey outcome.

Claim 4. *If Assumptions 1 and 2 are satisfied, then the Nash equilibrium inflation level is larger than the Ramsey level in* $\Gamma(i): \; x^i > x^i_r, \; i \in I.$

Thus, the Nash equilibrium is not efficient: since $u^i(x,x)$ is strictly concave as a function of x and attains its maximum at x^i_r, we conclude that $u^i(x^i,x^i) < u^i(x^i_r,x^i_r)$.

c. Incomplete Information Case

Now we examine the game Γ where CB's type is private. Let us define the notion of strategy in Γ. A strategy σ for CB is a pair of contingent plans for types s and w. The contingent plan for type i specifies two things. First, it says which message to send (mixing is allowed). Second, given P's action, it specifies CB's action. A strategy π for P is a pair of actions, one for each possible message received. The solution concept we use for the game Γ is perfect Bayesian equilibrium.[8] Since P does not know CB's type, in order to choose an action after receiving CB's message, it has to form a belief (probability distribution) about the type. A pair of strategies (σ,π) is called a *perfect Bayesian equilibrium* (or just equilibrium), if (i) CB's strategy σ is a best response to π given CB's type; (ii) P's strategy π is a best response to σ given P's belief, where the belief is formed by Bayes' rule whenever possible.[9]

A perfect Bayesian equilibrium of Γ is said to be a *fully revealing cheap talk equilibrium (FRCTE)* if in the announcement stage type $i \in I$ sends message i with probability one.[10] Thus, in a FRCTE the two types separate from each other. An equilibrium is said to be a *partially revealing cheap talk equilibrium (PRCTE)* if in the announcement stage at least one type uses a totally mixed strategy, and the announcement strategies of the two types are different. Note that in a PRCTE, P's posteriors differ from its priors, and therefore the two types partially separate. An equilibrium is said to be a *babbling equilibrium* if both types of CB have the same announcement stage strategy. Thus, in a babbling equilibrium, P's posteriors coincide with the priors, i.e., the two types pool.

The set of equilibria of the game Γ has the following features. By definition, any perfect Bayesian equilibrium is either a FRCTE, a PRCTE, or a babbling equilibrium. There always exists a babbling equilibrium: if P ignores CB's messages then CB has no incentive to reveal its type; as a result,

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[8] See Fudenberg and Tirole (1991).

[9] See Appendix A for a more rigorous definition.

[10] We restrict ourselves to truthful equilibria in which type i sends message i. One other possible fully revealing cheap talk equilibrium is when messages are flipped: type s sends message w, whereas type w sends s.

CB babbles, i.e. the two types pool their announcements, and P justifiably ignores the announcements.

Now we can show that *if* announcements are believed, then type s does not want to pretend to be w, whereas type w wants to pretend to be s. To make the statement of the following claim easier to understand, let us make the following comment. If announcements are believed, and if a message $m \in I$ is received, P plays x^m, the one- shot Nash equilibrium of $\Gamma(m)$. If the actual type of CB is i, in response to x^m it will play its best response $B^i(x^m)$. Thus, if, say, type w sends message s and is believed, then P plays x^s, whereas type w CB plays $B^w(x^s)$. The payoff to type w CB is $u^w(B^w(x^s), x^s)$.

Claim 5. *Suppose Assumptions 1 and 2 are satisfied. Then type w has incentives to misrepresent its type, whereas type s has no incentives to do so. More precisely, the following holds:*

$$\begin{aligned} u^s(B^s(x^s), x^s) &> u^s(B^s(x^w), x^w), \\ u^w(B^w(x^w), x^w) &< u^w(B^w(x^s), x^s), \end{aligned}$$

where B^i is type i's best response function.[11]

It is this incentive of type w to misreport its type that results in non-existence of an equilibrium with truthful announcements of types. Now we turn to the main result of this subsection.

Proposition 1. *If Assumptions 1 and 2 are satisfied, then Γ has only babbling equilibria.*

To prove the proposition one has to show nonexistence of both FRCTE and PRCTE. Nonexistence of the former follows from Claim 5. Nonexistence of the latter uses some properties of the utility functions u^i.

Let us discuss here how this result relates to a result of Stein (1989) who showed that in a similar model there could not be a fully revealing equilibrium but did not rule out a partially revealing equilibrium. In the proof of Proposition 1 we make sure that there exists no partially revealing equilibrium. In Stein's model CB has a continuum of types which allows him to show that in spite of nonexistence of fully revealing equilibrium there exists partition equilibrium first found by Crawford and Sobel (1982). Stein uses this result to justify the ambiguity of central bank announcements. We do not have this type of equilibrium since the set of types is finite.

The point of this result is not that we should believe it but – to the contrary – that its lack of credibility should persuade us that the static model is inappropriate to study the question.

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[11] Lemma 3 in Appendix A states that for both types the best response correspondence is actually a function.

3.2 Repeated Game

Now we turn to the infinitely repeated version of Γ denoted Γ^∞ which is the serious, credible model in this paper. The game is defined in the previous section. Let us describe strategy spaces in Γ^∞. A strategy for CB is a contingent plan. First, it specifies what to announce for a given realization of type; at the announcement stage mixing of actions is allowed. Second, in every period t, it specifies which action to choose for a given type of CB, a message, a history of actions taken by both players in the previous periods, and P's action in period t (recall that in any period, CB observes P's action before it takes its own action). A strategy for P specifies which action to take in every period t for a given message and a history of both players' actions in the previous periods. To solve its maximization problem, P has to have a *belief* about CB's type. A belief is Bayesian if it is formed by using Bayes's rule whenever possible; a Bayesian belief may be arbitrary when this rule is not applicable. A *perfect Bayesian equilibrium*, the solution concept we use for Γ^∞, is a triple $\langle$strategy for CB, strategy for P, belief$\rangle$ such that: (i) after any history of actions, CB's continuation strategy maximizes its expected discounted utility given P's continuation strategy; (ii) P's action equals the expected value of actions taken by the two types of CB, where the expectation is taken with respect to the belief after the given history; (iii) the belief is Bayesian.[12]

The definition of a *fully revealing cheap talk equilibrium (FRCTE)* is the same as in the one-shot case: a FRCTE is a perfect Bayesian equilibrium in which CB truthfully reveals its type: each type i sends message i.[13] We say that a perfect Bayesian equilibrium *supports the Ramsey outcome* if, given that CB's type is $i \in I$, in every period both CB and P play the one-shot Ramsey outcome x_r^i with probability one.

Main Results

We will study the case when CB is sufficiently patient as well as the case when it is sufficiently impatient. We start with the former. The following proposition makes two points. First, unlike the one-shot game, the repeated game has a FRCTE. Second, such an equilibrium can support the Ramsey outcome.

Proposition 2. *Under Assumptions 1 and 2, if the discount factor δ is sufficiently close to one, then there exists a FRCTE in the game Γ^∞ that supports the Ramsey outcome.*

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[12] See Appendix A for a rigorous definition.

[13] See footnote 10.

The Ramsey outcome is supported by the following pair of strategies. If the realized message is i, P plays the one-shot Ramsey outcome x_r^i as long as CB has played this action in *all* previous periods, and plays x^w, the one-shot Nash equilibrium for type w, otherwise. The belief is defined accordingly: P believes that CB is type s with probability 1 if CB has played x_r^s in all previous periods, and believes that CB is type w with probability 1, otherwise. Type i CB plays the Ramsey outcome x_r^i as long as it has played this action in all previous periods, and plays the best response $B^i(x^w)$ otherwise. Thus, the Ramsey outcome is supported by the off-equilibrium threat that in case of deviation P will believe forever that CB is weak (type w). The Ramsey outcome x_r^w yields higher payoff to type w than the Nash equilibrium x^w. Thus, if CB is sufficiently patient, type w would not want to deviate. Type s would not deviate because of Claim 5: it has no incentive to have P believe that it is type w. The formal proof in Appendix B shows that the two types have no incentives to misrepresent themselves at the announcement stage.

It can be easily shown that both the set of FRCTE and the set of babbling equilibria in Γ^∞ are large. In this paper we compare the two sets by comparing the best equilibria in each set.

Proposition 3. *If Assumptions 1 and 2 hold, and the discount factor is sufficiently close to one, then the best FRCTE in* Γ^∞ *is Pareto superior to the best babbling equilibrium.*

It suffices to show that the FRCTE supporting the Ramsey outcome is Pareto superior to any babbling equilibrium. In any babbling equilibrium, there are two possibilities after realization of a message: either the two types never separate, i.e., always choose the same action, or they separate at some point. If they never separate, CB's expected payoff in any period is strictly less than the payoff in the FRCTE:

$$p^s u^s(x,x) + p^w u^w(x,x) \quad < \quad p^s u^s(x_r^s, x_r^s) + p^w u^w(x_r^w, x_r^w), \tag{3.2}$$

because $x_r^s \neq x_r^w$ by Claim 3. If the two types separate, then: (i) after separation CB's expected continuation payoff is no higher then that in the FRCTE because the types are now known; (ii) before separation CB's expected payoff in any period is strictly less than that in the FRCTE (see (3.2)); (iii) in the separation period, CB's expected payoff is strictly lower than that in the FRCTE because of concavity of the utility function.

Now we investigate the case of an impatient CB. Applying limiting arguments to Proposition 1, one can prove the following proposition.

Proposition 4. *Suppose Assumptions 1 and 2 are satisfied. Then for* δ *sufficiently close to zero the game* Γ^∞ *has only babbling equilibria.*

Thus, when CB is impatient it can only be secretive.

4 Extensions of the Basic Model

The benchmark game Γ^{∞} allows us to obtain interesting results, namely existence of FRCTE and its Pareto superiority to babbling equilibrium. Two features of this game, however, may not be "realistic". First, CB's type may change over time (due to changes in the composition of the FOMC, for example). Second, central banks do not directly control the inflation rate. Rather, they control some other variable, say, interest rate. And the realized inflation rate may differ from what the central banker had in mind. In order to address these issues, we study some modifications of the benchmark game and demonstrate that our main results are robust to these changes.

In this section we study two extensions of the benchmark game Γ^{∞}. In the first modified game, CB's type changes every period following an i.i.d. process. Accordingly, CB announces its type every period. In the second modified game, CB's type does not change over time, but CB has no perfect control over inflation, whereas P does not observe CB's inflation target. We show that our main results still hold.

4.1 CB's Type Changes Over Time

In Γ^{∞} CB's type was chosen once and for all, and there was only one announcement stage. Suppose now that CB's type changes every period, and it is i.i.d. with the probability of type i being $p^i, i \in I$. And suppose that CB announces its type in every period t, before any actions are taken. P observes the announcement and then forms inflation expectation y_t. Having observed y_t, CB chooses actual inflation rate x_t. P immediately observes x_t. We denote this infinitely repeated game $\Gamma^{\infty}_{i.i.d.}$. The solution concept we use for this game is again perfect Bayesian equilibrium. A FRCTE is such a perfect Bayesian equilibrium that along the equilibrium path CB truthfully announces its type.[14] A babbling equilibrium is such a perfect Bayesian equilibrium that along the equilibrium path CB babbles, i.e., after any finite history both types mix messages in the same manner.[15]

Proposition 5. *Suppose Assumptions 1-2 are satisfied. If δ is sufficiently close to one, then there exists a FRCTE in $\Gamma^{\infty}_{i.i.d}$. The best FRCTE in this game is Pareto superior to the best babbling equilibrium. If δ is sufficiently close to zero, then all equilibria are babbling ones.*

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[14] We allow any announcement off the equilibrium path.

[15] Babbling after different histories on the equilibrium path may differ.

In the proof of Proposition 2, P's off-equilibrium threat for the case of deviation was to believe forever that CB is weak. This threat cannot be used here because CB's type changes every period. However, it turns out that the following threat suffices to support the Ramsey outcome: if CB deviates from playing the Ramsey outcome, P will ignore CB's messages forever and play a one-shot babbling equilibrium. That the expected payoff in a babbling equilibrium is lower than the expected payoff in Ramsey outcomes is established by the technique used in the proof of Proposition 3, which uses concavity of the utility function. Then, if CB is sufficiently patient, deviations are not profitable.

4.2 Imperfect Control and Imperfect Monitoring

Now we slightly change the environment in the benchmark game Γ^∞ by supposing that CB does not have perfect control over inflation: it chooses an inflation target, but the realized inflation may differ from the target. P, on the other hand, never observes CB's inflation target. This new game is one of both incomplete and imperfect information.[16]

Let us describe the one-shot game G. First, Nature chooses CB's type $i \in I$ according to the probability distribution (p^s, p^w). Next, CB announces its type by sending a message $m \in I$. P receives the message and then forms inflation expectation $y \in \mathbb{R}$. CB observes it and chooses an inflation target $x \in \mathbb{R}$. After this both players observe the realized inflation rate z, where $z = x + \epsilon$. The random variable ϵ with the support $\mathbb{R}$ is assumed to have a continuous, strictly positive p.d.f. ϕ. The function ϕ does not depend on CB's type. The corresponding c.d.f. is denoted Φ. We assume that $E[\epsilon] = 0$. It is convenient to assume that ϕ belongs to a family $\{\phi_v\}_{v \in \mathbb{R}_{++}}$ of p.d.f.s with mean zero and different variances v.

P's utility payoff is $-(z-y)^2$; the payoff to CB of type i is $u^i(z, y)$. For the sake of notational simplicity we introduce the functions $\bar{u}^i \colon \mathbb{R}^2 \to \mathbb{R}$ by

$$\bar{u}^i(x, y) = E_\epsilon[u^i(x + \epsilon, y)], \qquad i \in I. \tag{4.1}$$

These are expected utility functions: $\bar{u}^i(x, y)$ is the expected payoff to type i CB conditional on actions x and y. It is easily seen that all the assumptions made above about the functions u^i are inherited by the functions $\bar{u}^i$. For example, if Assumption 1 holds then $\bar{u}^i_{11} < -c_1$, $i \in I$.

Similar to the previous section, let $G(i)$, $i \in I$, denote the game in which it is common knowledge that CB's type is i. It is straightforward to prove

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[16] Two early contributions into the monetary policy literature that deal with games of both incomplete and imperfect information are Canzoneri (1985) and Cukierman and Meltzer (1986).

that the counterparts of Claims 1-4 and Proposition 1 hold if u^i is replaced by $\bar{u}^i$. We will use the following notation: $\bar{x}^i$ is the Nash equilibrium of $G(i)$, and $\bar{x}_r^i$ is the Ramsey outcome of $G(i)$.

In the infinitely repeated game G^∞, CB's type follows an i.i.d. process and is drawn every period according to the probability distribution (p^s, p^w). Timing within each period t is as before: CB's type is chosen first, and it is private information; P forms inflation expectation y_t; CB observes y_t and then chooses x_t; the realization of variable $z_t = x_t + \epsilon_t$ is observed by both players.

A strategy for CB consists of two strategies: σ^s, a strategy for type s, and σ^w, a strategy for type w. For each type $i \in I$, after every appropriate history[17] σ^i specifies: (i) the message to be announced; (ii) CB's action in the current period. A strategy for P is defined in a similar manner. A belief specifies a probability distribution over CB's types whenever it is P's turn to move. A belief is called Bayesian if it is formed according to Bayes's rule. A perfect Bayesian equilibrium is a triple $\langle$strategy for CB, strategy for P, belief$\rangle$ that satisfies the following conditions: (i) after any history, CB's continuation strategy maximizes its expected discounted utility given P's continuation strategy; (ii) P's action equals the expected value of the two CB types' actions, where expectation is taken with respect to the belief after the given history; (iii) the belief is Bayesian.

A pair of strategies (σ^i, π) together with the distribution ϕ induce a distribution λ_i over the set of infinite histories. Type i CB maximizes $(1-\delta)E\sum_{t=0}^{\infty}\delta^t u^i(z_t, y_t)$, where the expectation is taken with respect to the distribution λ_i. As in Γ^∞, P is assumed to be a myopic optimizer: in any period t it maximizes $-E_\epsilon[(z_t - y_t)^2]$; in other words it tries to match CB's action.

The proof of non-existence of either FRCTE or PRCTE in the one-shot game G can be adapted from the previous section with almost no change. The issue of existence of a FRCTE in G^∞ proves to be a non-trivial issue. We prove its existence below. Furthermore, it turns out that the best FRCTE is better than the best babbling equilibrium.

Let us describe a fully revealing cheap talk equilibrium in G^∞ that is better than any babbling equilibrium. We use a Green-Porter type trigger equilibrium that has two possible types of periods, normal and reversionary.[18] CB truthfully announces its type every period. There are two thresholds for the inflation rate, $\bar{z}^s$ and $\bar{z}^w$. In period t, if announcement s is made, we say that the threshold has been breached if the actual inflation is *higher* than

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[17] Such a history would consist of CB's messages, actions taken by P, and realized inflation rates in the previous periods.

[18] See Green and Porter (1984).

the corresponding threshold, i.e., $z_t > \bar{z}^s$; similarly, if announcement w is made, we say that the threshold has been breached if the actual inflation is *lower* than the corresponding threshold, i.e., $z_t < \bar{z}^w$. After any history, the current period is normal if the previous period was normal (the first period is always normal), and the threshold was not breached. Otherwise the period is reversionary. In a normal period, both CB and P play x_t, which is "close" to $\bar{x}^i_r$, the Ramsey outcome of the one-shot game $G(i)$. In a reversionary period, CB and P play the Nash equilibrium for $G(i): \; x_t = y_t = \bar{x}^i$.

Proposition 6. *Suppose Assumptions 1 and 2 are satisfied. Then* G^∞ *has a FRCTE. Furthermore, there exists such a FRCTE that is Pareto superior to any babling equilibrium.*

5 Conclusions

In this paper we study the phenomenon of central bank secrecy. We analyze a monetary policy game of cheap talk. Our first result (Proposition 1) is that in the one-shot game a central bank cannot talk credibly. As a consequence (Proposition 4), if the central bank is impatient, then in the infinitely repeated game the same result holds: it cannot talk credibly, it will only "babble". In contrast, our second result (Proposition 2) shows that when the central bank is sufficiently patient, there is a fully revealing equilibrium, in which it can talk truthfully and credibly. Our third result (Proposition 3) is that the best fully revealing equilibrium is Pareto superior to the best babbling equilibrium, in which the central bank talks uninformatively, and the public ignores this talk.

This paper deals with cheap talk games introduced into the literature by Crawford and Sobel (1982). Typically, in a cheap talk game there are two players, the sender and the receiver. The sender sends costless messages trying to convey her private information. The receiver observes the message and then takes an action that affects both players' payoffs. The literature on repeated cheap talk games was initiated by Sobel (1985). In the equilibrium of this finite-horizon game a "bad" sender, who observes the realization of a random variable and should report it, mimics the behavior of a "good" sender to build a reputation, and ultimately cheats, after which her reputation is damaged unrecoverably. Morris (1999) extends Sobel's model to a game in which the sender observes a noisy signal of a binary random variable. The good sender is assumed to have the same preferences as the receiver. The bad sender's preferences are biased in a particular direction (say, linear in the receiver's action). Morris proves the following, which is "opposite" to our result: if the sender is concerned about her reputation, i.e., when she is sufficiently patient, there are no informative equilibria. The intuition is that the good sender may be concerned about adverse inferences about her type, which may affect the receiver's attitude to announcements in the future. The good sender, therefore, may have an incentive to lie. Our result differs from Morris's result for at least two reasons. First, the bad sender's, i.e. the weak CB's, preferences are not biased in a particular direction; in other words, the bad sender's payoff does not *monotonically* increase in the receiver's action. Second, in addition to messages, the sender in our game can influence the receiver's belief through her actions, whereas in Morris's model the receiver updates his belief by observing the true state in the previous period.

There are two papers on repeated cheap talk games with conclusions similar to ours. Kim (1996) models infinitely repeated interactions between a plaintiff and a defendant. Because of the reputation effect the plaintiff, who has private information about damage, talks informatively (but not always truthfully). What makes this model different from ours is that the defendant has access to costly verification of the plaintiff's claim. In equilibrium the defendant uses this verification with positive probability. Stocken (2000) proves that a one-shot game between a manager, who possesses private information about his firm, and an investor has no informative equilibrium. However, if the game is repeated infinitely many times, there may be an equilibrium in which the manager almost always truthfully reveals his information. Stocken does not provide any welfare comparison.

The literature on central bank secrecy is large. Both pros and cons of secrecy have been offered. Goodfriend (1986) first considered the Federal Reserve's arguments for secrecy, and by informal arguments rejected invalidity of all of them. Dotsey (1987) and Tabellini (1987) found that secrecy adversely affects the private sector's forecasting accuracy. A recent empirical investigation by Thornton (2000) shows that when the Fed's intentions were not known, the market's reaction to the Fed's actions was inconsistent with the Fed's intentions. Possibility of potential welfare gains from central bank policy announcements are demonstrated in Andersen (1989) and Walsh (1999). Faust and Svensson (1999), in an interesting extension of Cukierman and Meltzer (1986), argue that increased transparency is socially beneficial, but complete transparency leads to the worst of all outcomes.

Arguments for secrecy have also been provided. Garfinkel and Oh (1995) argue that imprecise announcements mitigates the credibility problem. Cosimano and Van Huyck (1993) demonstrate that secrecy can increase the value of commercial banks. Bhattacharya and Weller (1997) find that secrecy about the scale of an intervention operation in the foreign exchange market is desirable. Rudin (1988) demonstrates, in contrast to Dotsey (1987) and Tabellini (1987), that reducing secrecy can reduce forecasting accuracy of the private sector if one recognizes that some agents actually watch the Fed's activity closely.

We now comment on two important papers on which our work is built. Stein (1989) applies the Crawford and Sobel (1982) cheap talk model to rationalize ambiguity of the Fed's announcements. This is probably the only paper in the monetary policy literature that uses the language of cheap talk games. In this two-period model the Fed has private information regarding its target for the exchange rate. Because of quadratic preferences the Fed has an incentive to make the public believe that its target is larger than it actually is. This makes precise announcements on the part of the Fed impossible. Nevertheless, there is an equilibrium with imprecise announcements: the Fed can credibly announce the interval within which its actual target lies.

Cheap talk is not the only way to convey private information. Spence-type costly signaling is another possible means of communication. Most of the literature on central bank secrecy uses costly signaling. For example, Cukierman and Meltzer (1986) provide a rationalization for central bank ambiguity in a game of both incomplete and imperfect information. The main objective of their paper is to derive conditions under which central banks prefer ambiguity to full disclosure of objectives. This is achieved by allowing the central bank to choose the level of noise in money control. The higher the variance of the noise the higher is ambiguity; that is, ambiguity is measured by this variance. The key result is that under certain assumptions the optimal level of the variance is positive.

Stein's model is a cheap talk game which allows him to address strategic issues that arise as far as central bank announcements are concerned; but this model is static. The game in Cukierman and Meltzer (1986) is dynamic but does not explicitly model central bank announcements. Both papers justify ambiguity of central bank announcements. Our approach combines the strengths of these two papers: we study repeated cheap talk games. A key result in our paper is that the reputation issue arising in the repeated game may drastically change their results. Central banks can talk precisely, and this improves welfare.

Appendix A: Definitions

a. One-shot Game Γ

Let us give a precise definition of a strategy in Γ. A strategy σ for CB is a pair (σ^s, σ^w) with $\sigma^i = \big(\alpha^i(s), \alpha^i(w), x^i(s), x^i(w)\big) \in \mathbb{R}^4$, $i \in I$, such that $\alpha^i(s) + \alpha^i(w) = 1$. A strategy of P is a pair $\pi = (y(s), y(w)) \in \mathbb{R}^2$. The interpretation is that type i CB sends message $m \in I$ with probability $\alpha^i(m)$. If the realized message is $m' \in I$, then type i CB plays $x^i(m')$, whereas P plays $y(m')$.

Definition 5.1. *A perfect Bayesian Nash equilibrium of the game* Γ *is a pair of strategies* (σ, π) *such that:*

1. $x^i(m)$ *maximizes* $u^i(\cdot, y(m)), \quad i, m \in I;$
2. $\alpha^i(m) \in (0,1)$ *only if* $u^i(x^i(s), y(s)) = u^i(x^i(w), y(w)), \quad i, m \in I;$
3. $y(m) = \mu(s|m)\, x^s(m) + \mu(w|m)\, x^w(m)$, *where*

$$\mu(i|m) = \frac{p^i\, \alpha^i(m)}{p^s\, \alpha^s(m) + p^w\, \alpha^w(m)}, \qquad i, m \in I, \tag{5.1}$$

where the last equation is used whenever possible. If $p^s\, \alpha^s(m) + p^w \alpha^w(m) = 0$*, then* $\{\mu(s|m), \mu(w|m)\}$ *is an arbitrary distribution.*

Parts 1 and 2 of the definition are usual optimization requirements. Part 3 reflects two things. First, the feature of P's utility is such that its best response is the expected value of x. Second, for updating beliefs Bayes' rule is used whenever possible; however, beliefs are arbitrary after zero-probability events. In (5.1) $\mu(i|m)$ is the posterior probability of CB being type i conditioned on message m.

b. Infinitely Repeated Game Γ^∞

Let h^0 represent the null history. A *public history for P* is a sequence $h_p^t = (m, h_1, \ldots, h_{t-1})$ with $t \geq 1$, where $m \in I$ is CB's message, and $h_\tau = (x_\tau, y_\tau)$ is a pair of actions in period τ. Similarly, a *public history for CB* is a sequence $h_{cb}^t = (m, h_1, \ldots, h_{t-1}, y_t)$ with $t \geq 1$, where m and h_τ have the same meaning as above, and y_t is P's action in period t. Let H_p and H_{cb} denote the sets of public histories for P and CB respectively. The difference in histories for the two players is due to timing of moves: in every period, CB moves after observing P's action.

A strategy of CB of type i is a mapping σ^i from $h^0 \cup H_{cb}$ into $\mathbb{R}$ such that $\sigma^i(h^0) \in [0,1]$. Here $\sigma^i(h^0)$ is interpreted as probability of announcing s, whereas $\sigma^i(h_{cb}^t)$ is CB's action after h_{cb}^t. P's strategy is a mapping π from H_p into $\mathbb{R}$. Here $\pi(h_p^t)$ is interpreted as P's action after history h_p^t. A strategy of CB is a pair of strategies of the two types and is denoted σ, i.e., $\sigma = (\sigma^s, \sigma^w)$. A *belief function* is a mapping from $h^0 \cup H_p$ into $\Delta(I)$. The interpretation is that after any public history for P, a belief function provides posterior belief (probability distribution) about CB's type.

We will use the following convenient notation. If $\sigma^i(h^t) = x$, then $\sigma^i(z|h^t) = 1$ if $z = x$ and $\sigma^i(z|h^t) = 0$ otherwise; $\sigma^i(m|h^0) = \sigma(h^0)$ if $m = s$ and $\sigma^i(m|h^0) = 1 - \sigma(h^0)$ otherwise. A similar notation is used for π. Also, $\mu(i|h^t) = \mu(h^t)$ if $i = s$, and $\mu(i|h^t) = 1 - \mu(h^t)$ otherwise. We will need a special notation for expected discounted utility function for CB. Notice that for a fixed type $i \in I$ and a public history h^t_{cb}, the strategies σ^i, π, and the belief function μ induce a probability distribution over the set of $H_{cb}(h^t_{cb})$ of public histories for CB with the common beginning part h^t_{cb}. The expected discounted utility of type i CB conditioned on the history h^t_{cb} is denoted $\mathcal{U}^i(\sigma^i, \pi \mid h^t_{cb})$.

The solution concept we use for Γ^∞ is perfect Bayesian equilibrium.

Definition 5.2. *A perfect Bayesian equilibrium in Γ^∞ is a pair of strategies $(\sigma, \pi) = (\sigma^s, \sigma^w, \pi)$ and a belief function μ such that:*

1. *(Sequential rationality) For any type $i \in I$, any $h^t_{cb} \in H_{cb}$, and any alternative strategy $\hat{\sigma}^i$ for type i CB,*

$$\mathcal{U}^i(\sigma^i, \pi \mid h^t_{cb}) \geq \mathcal{U}^i(\hat{\sigma}^i, \pi \mid h^t_{cb}).$$

   For any $h^t_p \in H_p$,

$$\pi(h^t_p) = \mu(s|h^t_p)\sigma^s(h^t_p) + \mu(w|h^t_p)\sigma^w(h^t).$$

2. *(Correct initial belief) $\mu(s|h^0) = p^s$.*

3. *(Action-determined belief) For any $h^t \in H_p$, and any $x, y, y' \in \mathbb{R}$,*

$$\mu(\cdot|h^t, x, y) = \mu(\cdot|h^t, x, y').$$

4. *(Bayesian updating) For any $h^t_p \in H_p$ and $x, y \in \mathbb{R}$, if there is $j \in I$ with $\mu(j|h^t_p) > 0$ and $\sigma^i(h^t_p) = x$, then for any $i \in I$*

$$\mu(i|h^t_p, x, y) = \frac{\mu(i|h^t_p)\sigma^i(x|h^t_p)}{\sum_{k\in I} \mu(k|h^t_p)\sigma^k(x|h^t_p)}. \tag{5.2}$$

Appendix B: Proofs in the Basic Model

We start with the following implications of Assumption 1.

Lemma 1. *If Assumption 1 is satisfied, then for all $i \in I$,*

$$u^i_{11} < -c_1, \tag{5.3}$$
$$u^i_{11} + u^i_{12} < -c_1, \tag{5.4}$$
$$u^i_{12} + u^i_{22} = 0, \tag{5.5}$$
$$u^i_{11}u^i_{22} - (u^i_{12})^2 > 0, \tag{5.6}$$
$$u^i_{12} > 0. \tag{5.7}$$

Proof. We will provide the proof of (5.3) only. The remaining proofs are similar. Indeed, by Assumption 1:

$$u^i_{11} = (U(x-y))'' + (V^i(x))'' < (V^i(x))'' < -c_1.$$

The remaining proofs are similar. ∎

Lemma 2. *If Assumption 2 is satisfied then there is a positive constant c_2 such that $u^i_2(x,x) = -c_2$ for all $i \in I$, $x \in \mathbb{R}$.*

Proof. Note that $u^i_2(x,x) = -U'(x-x) = -U'(0)$. By Assumption 2, $U'(0) > 0$. Take $c_2 = U'(0)$. ∎

The following two facts are useful in the proofs below.

Fact 1. *Let $f : \mathbb{R} \to \mathbb{R}$ be twice differentiable with $f'' \leq -c < 0$, for some positive constant c. Then f has a unique maximum.*

Fact 2. *Let $f : \mathbb{R} \to \mathbb{R}$ be continuously differentiable with $|f'| \geq c > 0$, for some positive constant c. Then $f^{-1}(0)$ is unique.*

Lemma 3. *Suppose Assumption 1 is satisfied. In the game $\Gamma(i)$, $i \in I$, for any P's action $y \in \mathbb{R}$, CB has a unique best response $B^i(y)$.*

Proof. Fix $y \in \mathbb{R}$ and $i \in I$. By Assumption 1 and (5.3), the function $u^i(\cdot, y)$ is twice differentiable and its second derivative $u^i_{11}(\cdot, y) \leq -c_1$ for a positive constant c_1. By Fact 1, $u^i(\cdot, y)$ must have a maximum which is unique. ∎

Proof of Claim 1. Fix $i \in I$. Suppose $\Gamma(i)$ has an equilibrium $(\bar{x}, \bar{y})$. We will first show that $\bar{y} = \bar{x}$. When $\bar{x}$ is fixed, P maximizes $-(\bar{x} - y)^2$. The F.O.C. implies $\bar{y} = \bar{x}$. Then (5.3) and Lemma 3 imply that $(\bar{x}, \bar{x})$ is an equilibrium of $\Gamma(i)$ iff the following F.O.C. is satisfied:

$$u^i_1(\bar{x}, \bar{x}) = 0.$$

Define the function $f:\mathbb{R}\to\mathbb{R}$ by $f(x) = u_1^i(x,x)$. From above, $(\bar{x},\bar{x})$ is an equilibrium of $\Gamma(i)$ iff $\bar{x}$ is a zero of f. By Assumption 1 and (5.4),

$$f'(x) = u_{11}^i(x,x) + u_{12}^i(x,x) \le -c_1 < 0.$$

Thus, f has a unique zero by Fact 2.

Let us show that $x^s < x^w$. Indeed, x^i is the unique solution to the equation $u_1^i(x,x) = 0$, which is equavalent to: $U'(0)+(V^i(x))' = 0$. Therefore $(V^s(x^s))' = (V^w(x^w))'$. By Assumption 1, $(V^s(x))' < (V^w(x))'$. The claim follows from the last two equations and the fact that the functions $(V^i)'$ are decreasing. ∎

Proof of Claim 3. Fix $i \in I$. The function f given by $f(x) = u^i(x,x)$ has the following second derivative:

$$f'' = [u_{11}^i + u_{12}^i] + [u_{12}^i + u_{22}^i].$$

By (5.4) and (5.5), $f'' < -c_1 < 0$. The existence and uniqueness follow from Fact 1.

Let us show that the Ramsey outcome in $\Gamma(s)$ is smaller than that in $\Gamma(s)$. Denote the Ramsey outcomes by x_r^s and x_r^w. Recall that x_r^i is the unique solution to $f'(x) = 0$. Since $f(x) = u^i(x,x) = U(0) + V^i(x)$, we have: $f'(x) = (V^i(x))'$. Thus, x_r^i is the solution to $(V^i(x))' = 0$. By Assumption 1, $(V^w)' > (V^s)'$. Since V^s and V^w are strictly concave, $x_r^w > x_r^s$. ∎

Proof of Claim 4. Fix $i \in I$. Recall from the proofs of Claims 1 and 3 that the Nash equilibrium x^i satisfies the following F.O.C.:

$$u_1^i(x^i,x^i) = U'(0) + (V^i(x^i))' = 0,$$

whereas the Ramsey outcome x_r^i satisfies

$$u_1^i(x_r^i,x_r^i) + u_2^i(x_r^i,x_r^i) = (V^i(x_r^i))' = 0.$$

By Assumption 2, $U'(0) > 0$ which implies that $(V^i(x^i))' < 0$. Since V^i is strictly concave, $x^i > x_r^i$. ∎

Proof of Claim 5. We first show that type s does not want to pretend to be w. Introduce the function $f:\mathbb{R}\to\mathbb{R}$ by $f(x) = u^s(x,x^w)$. Then $f(x) = U(x - x^w) + V^s(x)$. It is easy to see that

$$f'(x^w) = U'(0) + (V^s(x^w))' < U'(0) + (V^s(x^s))' = 0,$$

where the inequality follows from $(V^s(x^s))' < 0$ and $x^s < x^w$, whereas the last equality is the F.O.C. $u^s(x^s,x^s) = 0$. By similar arguments $f'(x^s) > 0$.

Since $u^s_{11} < 0$, f is strictly concave. Therefore f attains maximum in (x^s, x^w). Since $(V^s(x))' < 0$ and $U'(x - x^w) > 0$ when $x \in (x^s, x^w)$, it is clear that

$$U(x - x^w) + V^s(x) < U(0) + V^s(x^s), \qquad x \in (x^s, x^w).$$

Therefore $u^s(B^s(x^w), x^w) < u^s(x^s, x^s)$.

Now we show that type w has incentives to misreport its type. Indeed, by Assumption 2,

$$u^w(x^w, x^s) = U(x^w - x^s) + V^w(x^w) > U(0) + V^w(x^w) = u^w(x^w, x^w).$$

Therefore,

$$u^w(B^w(x^s), x^s) \geq u^w(x^w, x^s) > u^w(x^w, x^w) = u^w(B^w(x^w), x^w).$$

∎

Lemma 4. *Under Assumption 1, the functions v^i given by $v^i(y) = u^i(B^i(y), y)$, $i \in I$, are strictly concave and*

$$(v^i)'' < -c_1 < 0.$$

Proof. Fix $i \in I$. The best response B^i satisfies the F.O.C.

$$u^i_1(B^i(y), y) = 0, \qquad y \in \mathbb{R}. \tag{5.8}$$

Then

$$u^i_1(B^i(y), y) = U'(B^i(y) - y) + (V^i(y))' = 0, \qquad y \in \mathbb{R}.$$

Thus,

$$U'(B^i(y) - y) = -(V^i(y))', \qquad y \in \mathbb{R}. \tag{5.9}$$

Since $v^i(y) = u^i(B^i(y), y)$, the derivative of v^i:

$$\begin{aligned} (v^i(y))' &= u^i_1(B^i(y), y)(B^i)' + u^i_2(B^i(y), y) \\ &= u^i_2(B^i(y), y) = -U'(B^i(y) - y) = -(V^i(y))', \quad y \in \mathbb{R}, \end{aligned}$$

where the second and last equalities are due to (5.8) and (5.9). Therefore,

$$(v^i)' = (V^i)', \tag{5.10}$$

and recalling Assumption 1,

$$(v^i)'' = (V^i)'' < -c_1.$$

∎

Lemma 5. *The derivative of B^i is bounded:* $0 < (B^i)' < 1$.

Proof. Fix $i \in I$. The best response B^i satisfies the following F.O.C.:

$$u_1^i(B^i(y), y) = 0, \qquad y \in \mathbb{R}. \tag{5.11}$$

Differentiating this equation we obtain:

$$u_{11}^i (B^i)' + u_{12}^i = 0.$$

Thus,

$$(B^i)' = -\frac{u_{12}^i}{u_{11}^i}. \tag{5.12}$$

The claim follows from (5.3), (5.4), (5.7), and (5.12). ∎

We first prove a lemma which is essential for the proof of Proposition 1. The following notation is used. If $\xi \in \mathbb{R}^2$, then $\xi \equiv (\xi_1, \xi_2)$. For any function $f\colon \mathbb{R}^2 \to \mathbb{R}^2$ let f^i be its i-th component, i.e., $f(\xi) = (f^1(\xi), f^2(\xi))\ \forall \xi \in \mathbb{R}^2$.

Lemma 6. *Suppose a function $f\colon \mathbb{R}^2 \to \mathbb{R}^2$ is such that*

1. *$f_1^1 \le -C_1$ for some positive constant C_1;*
2. *there is a positive constant C_2 such that*

$$\frac{f_1^1 f_2^2 - f_2^1 f_1^2}{f_1^1} \le -C_2 < 0. \tag{5.13}$$

Then the equation

$$f(\xi) = 0 \tag{5.14}$$

has a unique solution.

Proof. Fact 1 and (i) imply that for any ξ_2 there is a unique ξ_1 such that $f^1(\xi_1, \xi_2) = 0$. This defines an implicit function g such that

$$f^1(g(\xi), \xi) = 0 \qquad \forall \xi \in \mathbb{R}^2. \tag{5.15}$$

Then solving (5.14) is equivalent to solving the following equation:

$$f^2(g(\xi_2), \xi_2) = 0. \tag{5.16}$$

We need to show that (5.16) has a unique solution. It suffices to show that the function $F\colon \mathbb{R} \to \mathbb{R}$ defined by $F(\xi_2) = f^2(g(\xi_2), \xi_2)$ has the derivative bounded away from zero and apply Fact 2. Differentiating (5.15) yields:

$$f_1^1 g' + f_2^1 = 0. \tag{5.17}$$

Taking derivative of F and using (5.17) yields:

$$F' = f_1^2 g' + f_2^2 = \frac{f_1^1 f_2^2 - f_2^1 f_1^2}{f_1^1}.$$

Using further (5.13) yields $F' \le -C_2 < 0$. ∎

Proof of Proposition 1. The proof is broken into two steps. In Step 1 we prove that there exist neither FRCTE nor PRCTE. In Step 2 we prove that there exists a babbling equilibrium.

Step 1. Suppose there exists a FRCTE. From Claim 1 it follows that after message $m \in I$ the equilibrium x^m of the game $\Gamma(m)$ is played. Claim 5 implies that type w has an incentive to deviate from sending message w. Therefore, no FRCTE exists.

Suppose there exists a PRCTE. Let y^m denote P's action after message $m \in I$ has been received. We will first show that $y^s \neq y^w$. Suppose on the contrary, that they are equal and $y^s = y^w = \bar{y}$).Then the actions of the two types do not depend on the signal received by P. Each y^i is a weighted average of the two types' actions. Since in a PRCTE posterior beliefs are different upon reception of different messages, the actions taken by s and w must coincide (and be independent of the realization of the message) and equal $\bar{y}$. CB plays best response to P's action, therefore $(\bar{y}, \bar{y})$ must constitute a Nash equilibrium of both $\Gamma(s)$ and $\Gamma(w)$. This contradicts Claim 1. Thus, $y^s \neq y^w$.

Next, let us show that $y^j \in [x^s, x^w], j \in I$. Fix $j \in I$. Suppose to the contrary that, for example, $y^j < x^s$. By definition, $B^i(x^i) = x^i, i \in I$. This and Lemma 5 imply that $B^i(y^j) > y^j, i \in I$. But then P's action y^j cannot be a weighted average of the two types' actions $B^s(y^j)$ and $B^w(y^j)$. Thus, $y^j \in [x^s, x^w]$.

Let us demonstrate next that it cannot be the case that type s uses a mixed strategy at the announcement stage. Indeed, if he were using one, then he would have been indifferent between P's playing y^s and y^w. We'll show that it is impossible. It suffices to prove that the function $v^s(y) = u^s(B^s(y), y)$ is strictly monotone on $[x^s, x^w]$. As has been mentioned,

$$B^s(y) \le y, \qquad y \in [x^s, x^w].$$

Then by (5.10), Lemma 4 and Assumption 2,

$$(v^s(y))' = -U'(B^s(y) - y) < 0, \qquad y \in [x^s, x^w].$$

Therefore type s cannot be indifferent between P playing y^s and y^w, and thus cannot use a mixed announcement strategy.

By definition of PRCTE, type w must mix messages. Without loss of generality we can assume that type s sends message s. Then it is clear that if message w is received, P knows that CB's type is w. Thus,

$$y^w = x^w,$$

where x^j denotes the Nash equilibrium in $\Gamma(j)$.

Due to the preceding analysis, type w is indifferent between P's playing y^s and y^w. Thus the function $v^w(y) = u^w(B^w(y), y)$ attains the same value at y^s and y^w.

Let $\{q^s, q^w\}$ be P's posterior belief if message s was received. In this case P's action y^s is expectation of the actions $B^s(y^s)$ and $B^w(y^s)$ of the two types:

$$y^s == q^s B^s(y^s) + q^w B^w(y^s). \tag{5.18}$$

Since $y^w = x^w$, and $y^i \in [x^s, x^w], i \in I$, we conclude that $y^w > y^s$. Then

$$x^s < y^s < y^w = x^w.$$

Because of strict concavity of v^w,

$$v^w(x^w) = v^w(y^s) \geq v^w(x^s). \tag{5.19}$$

Eq. (5.19) contradicts Claim 5.

Step 2. Suppose there exists a babbling equilibrium in Γ. Since announcements are ignored, we will concentrate on the "action" part of the equilibrium strategies. Let $(\hat{x}^s, \hat{x}^w, \hat{x})$ be a actions of type s, type w, and P in this equilibrium. Then

$$\hat{x} = p^s \hat{x}^s + p^w \hat{x}^w,$$

The F.O.C.s for CB are:

$$\begin{aligned} u_1^s(\hat{x}, p^s \hat{x}^s + p^w \hat{x}^w) &= 0 \\ u_1^w(\hat{x}, p^s \hat{x}^s + p^w \hat{x}^w) &= 0. \end{aligned} \tag{5.20}$$

By Lemma 3, $(\hat{x}^s, \hat{x}^w, \hat{x})$ is a Bayesian equilibrium of Γ if and only if $(\hat{x}^s, \hat{x}^w)$ satisfies (5.20). Let us show that system (5.20) has a unique solution. Introduce the functions $f^i : \mathbb{R}^2 \to \mathbb{R}$ by

$$f^i(\zeta^s, \zeta^w) = u_1^i(\zeta^i, p^s \zeta^s + p^w \zeta^w), \quad i \in I.$$

Then (5.20) becomes

$$f(\zeta) = 0,$$

where $\zeta = (\zeta^s, \zeta^w) \in \mathbb{R}^2$, and $f : \mathbb{R}^2 \to \mathbb{R}^2$ is defined by $f = (f^s, f^w)$. It suffices to show that f satisfies the conditions of Lemma 6. To simplify notation let

$$\xi^i = (\zeta^i, p^s\zeta^s + p^w\zeta^w), \quad i \in I.$$

Then

$$\begin{aligned} f_1^s(\zeta) &= u_{11}^s(\xi^s) + p^s u_{12}^s(\xi^s), \\ f_2^s(\zeta) &= p^w u_{12}^s(\xi^s), \\ f_1^w(\zeta) &= p^s u_{12}^w(\xi^w), \\ f_2^w(\zeta) &= u_{11}^w(\xi^w) + p^w u_{12}^w(\xi^w). \end{aligned}$$

Since $u_{11}^s + p^s u_{12}^s$ lies between u_{11}^s and $u_{11}^s + u_{12}^s$, (5.3) and (5.4) imply

$$f_1^s(\zeta) = u_{11}^s(\xi^s) + p^s u_{12}^s(\xi^s) < -\min(c_1, c_2).$$

Thus, condition (i) of Lemma 6 is satisfied. For the same reason,

$$\begin{aligned} &\frac{f_1^s f_2^w - f_2^s f_1^w}{f_1^s} \\ &= \frac{p^s \cdot u_{11}^w(\xi^w)[u_{11}^s(\xi^s) + u_{12}^s(\xi^s)] + p^w \cdot u_{11}^s(\xi^s)[u_{11}^w(\xi^w) + u_{12}^w(\xi^w)]}{u_{11}^s(\xi^s) + p^s u_{12}^s(\xi^s)} \\ &\leq -\min(p^s c_1, p^w c_2). \end{aligned}$$

Thus, condition (ii) of Lemma 6 is also satisfied. ∎

Proof of Proposition 2. We claim that the following strategies σ, π and belief function μ constitute a FRCTE. In the announcement stage, type i sends message i, $i \in I$; after any history $h^t = (s, h_1, \ldots, h_{t-1})$, both P and type s play x_r^s if $h_1 = \ldots = h_{t-1} = x_r^s$; otherwise they play x^w and $B^s(x^w)$ respectively; after any history $h^t = (w, h_1, \ldots, h_{t-1})$, both P and type w play x_r^w if $h_1 = \ldots = h_{t-1} = x_r^w$; otherwise both play x^w. The belief function μ is defined as follows: for any public history h^t and any type $i \in I$,

$$\begin{aligned} \mu(i|h^0) &= p^i, \\ \mu(i|h^t) &= 1 \text{ if } h^t = (i, x_r^i, \ldots, x_r^i), \\ \mu(w|h^t) &= 1 \text{ otherwise.} \end{aligned}$$

Let us prove that this is a perfect Bayesian equilibrium of Γ^∞. We first establish sequential rationality.

Recall our notation:

$$v^i(y) = u^i(B^i(y), y), \qquad i \in I, \ y \in \mathbb{R}.$$

In the announcement stage, if type s follows the prescribed strategy, its discounted expected utility is $u^s(x_r^s, x_r^s)$. If it announces w, the highest discounted expected utility it can receive is the maximum between $(1-\delta)v^s(x_r^w) + \delta v^s(x^w)$ and $u^s(x_r^w, x_r^w)$. By Claim 5, $u^s(x_r^s, x_r^s) \geq v^s(x^s) > v^s(x^w)$. Also, by definition of the Ramsey outcome and Claim 3, $u^s(x_r^s, x_r^s) > u^s(x_r^w, x_r^w)$. Thus, for δ close enough to 1 type s strictly prefers message s.

If type w follows the prescribed strategy, its discounted utility is $u^w(x_r^w, x_r^w)$. If it announces s, the highest possible utility it can obtain is the maximum between $(1-\delta)v^w(x_r^s) + \delta v^w(x^w)$ and $u^w(x_r^s, x_r^s)$. Recall that $v^w(x^w) = u^w(x^w, x^w)$. Since $x_r^w \neq x^w$ by Claim 4, $u^w(x_r^w, x_r^w) > u^w(x^w, x^w)$. And clearly $u^w(x_r^w, x_r^w) \geq u^w(x_r^s, x_r^s)$ with strict inequality when $x_r^w \neq x_r^s$. Therefore, for δ close enough to 1 type w prefers message w. That players action after any history are best responses to the strategies of each other is easily checked. Thus, sequential rationality is established.

The correct initial belief and action-determined belief properties hold by definition of μ. Let us establish the Bayesian updating property. After any history $h^t = (i, h_1, \ldots, h_{t-1})$ there are two possibilities. If $h_1 = \ldots = h_{t-1} = x_r^i$, then $\mu(i|h^t) = 1$, and therefore by definition of μ, $\mu(i|h^t, x, y) = 1$ if $\sigma^i(h^t) = x$. This is consistent with (5.2). If at least one of $h_1, \ldots, h_{t-1}$ is different from x_r^i, then $\mu(w|h^t, x, y) = 1$ for any $x, y \in \mathbb{R}$. This is vacuously consistent with (5.2) since this is a zero-probability event. Thus, the Bayesian updating property also holds.

We have established that $\langle(\sigma, \pi), \mu\rangle$ is a perfect Bayesian equilibrium. It is clear that it is a FRCTE since each type i announces i. Finally, it follows from the construction that this FRCTE supports the Ramsey outcome for each type. ∎

Proof of Proposition 3. In any FRCTE, P knows CB's type after the announcement stage. From that point on the best supportable outcome is the Ramsey outcome for this type. It follows from Proposition 2 that there exists a FRCTE supporting such an outcome. Therefore it is the best FRCTE. The ex-ante utility of this equilibrium is $p^s u^s(x_r^s, x_r^s) + p^w u^w(x_r^w, x_r^w)$.

Recall that we consider only pure strategy equilibria. In any babbling equilibrium after any history only the following two cases are possible:

1. The two types' strategies coincide along the history; in this case P does not know CB's type, and its belief is the initial prior;
2. The two types' strategies differ along the history; in this case P knows CB's type with probability one.

Therefore, in any period in which CB's type is unknown the expected payoff to CB is one of the two forms: $p^s u^s(z, z) + p^w u^w(z, z)$ for some z, or

$p^s u^s(z^s, p^s z^s + p^w z^w) + p^w u^w(z^w, p^s z^s + p^w z^w)$ for some z^s, z^w. This reflects two possibilities: either the actions are the same or they differ in which case P's action is the expected value of the actions. Since the announcement strategies are the same, the first period belief is the initial prior and thus the expected payoff to CB is one of the two forms above. In any period in which the type of CB is known, the expected payoff is no larger than that in the best FRCTE. Therefore, it suffices to show that expected payoffs in any period without knowledge of CB's type are strictly less than a one-period payoff in the best FRCTE. We will show this for both forms of one-period expected payoff.

For the first form, since $x_r^s \neq x_r^w$,

$$p^s u^s(x, x) + p^w u^w(x, x) < p^s u^s(x_r^s, x_r^s) + p^w u^w(x_r^w, x_r^w), \qquad x \in \mathbb{R}.$$

Now consider the second form of one-period payoff. Fix arbitrary $z^s, z^w \in \mathbb{R}$ such that $z^s \neq z^w$. Denote $\bar{z} = p^s z^s + p^w z^w$. We need to show that

$$p^s u^s(z^s, \bar{z}) + p^w u^w(z^w, \bar{z}) < p^s u^s(x_r^s, x_r^s) + p^w u^w(x_r^w, x_r^w). \tag{5.21}$$

Note that

$$p^s u^s(z^s, z^s) + p^w u^w(z^w, z^w) \leq p^s u^s(x_r^s, x_r^s) + p^w u^w(x_r^w, x_r^w).$$

Then it suffices to prove that

$$p^s u^s(z^s, \bar{z}) + p^w u^w(z^w, \bar{z}) < p^s u^s(z^s, z^s) + p^w u^w(z^w, z^w).$$

Let us denote

$$\begin{aligned} \Delta^i &= \bar{z} - z^i, \qquad i \in I, \\ A^s &= [u^s(z^s, \bar{z})] - u^s(z^s, z^s), \\ A^w &= [u^w(z^w, \bar{z})] - u^w(z^w, z^w). \end{aligned}$$

Then we need to prove the following:

$$p^s A^s + p^w A^w < 0. \tag{5.22}$$

Eqs. (5.7) and (5.5) imply that $u_{22}^i < 0$, and thus $u^i(z^i, \cdot)$ is strictly concave. By Assumption 2 and Lemma 2, $u_2^i(z, z)$ is constant: $u_2^i(z, z) = -c_2 < 0$. Then strict concavity of $u^i(z^i, \cdot)$ implies that at least one of the two numbers A^s and A^w is negative. If the other number is nonnegative, then (5.21) holds. Suppose that the other number is positive. Suppose $A^j > 0$ and $A^{j'} < 0$. By concavity of $u^i(x^i, \cdot)$,

$$\begin{aligned} A^j &< c_2 |\Delta^j|, \\ A^{j'} &< -c_2 |\Delta^{j'}|. \end{aligned}$$

Finally, $p^j|\Delta^j| - p^{j'}|\Delta^{j'}| = 0$, and therefore

$$p^s A^s + p^w A^w < p^j|\Delta^j| - p^{j'}|\Delta^{j'}| = 0.$$

This proves (5.22). ∎

Proof of Proposition 5. The proof is similar to that of Propositions 2, 3, and 4. Since the definition of FRCTE does not require a truthful announcement off equilibrium path, for δ close enough to 1, P can support the Ramsey outcome by using the following threat: if after announcing i CB does not play x_r^i, the Ramsey outcome for i, then P will ignore CB's messages forever and play a babbling equilibrium of the one-shot game Γ in every period. In order to prove that this constitute an equilibrium, we need to show that deviations are not profitable. For δ close to one this equivalent to showing that the one-period payoff in a babbling equilibrium is less than the expected one-period payoff when the players play as the equilibrium prescribes:

$$\begin{gathered} p^s u^s(z^s, \bar{z}) + p^w u^w(z^w, \bar{z}) < p^s u^s(x_r^s, x_r^s) + p^w u^w(x_r^w, x_r^w), \\ \bar{z} = p^s z^s + p^w z^w. \end{gathered}$$

But this is equivalent to (5.21) which has already been established in the proof of Proposition 3. This same inequality is used to prove that the best FRCTE is better than the best babbling equilibrium. ∎

Proof of Proposition 4. Since X is compact, u^w and u^s attain their minima on $X \times X$. The proof then follows from the proof of Proposition 1 using continuity arguments. ∎

Appendix C: Proofs in Extensions

Proof of Proposition 6. Recall our notation:

$$\bar{u}^i(x, y) = \bar{u}^i(x + \epsilon, y), \qquad i \in I. \tag{5.23}$$

Let $\bar{u}_b^i$ denote the expected utility of type i in a babbling equilibrium of game G[19] and

$$\bar{u}_b = p^s \bar{u}_b^s + p^w \bar{u}_b^w,$$

i.e., $\bar{u}_b$ is the ex-ante expected utility of CB in a babbling equilibrium of G. Let $W^i(\zeta^s, y^s, \zeta^w, y^w)$ denote the expected discounted utility of type i if in

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[19] As in game Γ, one can show that the equilibrium equilibrium outcome is unique.

normal periods after announcement $i \in I$ P plays y^i, and type j plays $\zeta^j, j \in I$, whereas in reversionary periods CB and P play the babbling equilibrium of G with CB's ex-ante payoff $\bar{u}_b$. For the sake of notational simplicity we will sometime drop the variables. The functions W^i satisfy the following functional equations:

$$\begin{aligned} W^i &= (1-\delta)\bar{u}^i(\zeta^i, y^i) \\ &+ \delta\Big[Prob(\zeta^i + \epsilon \leq \bar{z}^i)[p^s W^s + p^w W^w] + Prob(\zeta^i + \epsilon > \bar{z}^i)\bar{u}_b\Big] \\ &= (1-\delta)\bar{u}^i(\zeta^i, y^i) + \delta\Big[\Phi(\zeta^i + \bar{z}^i)[p^s W^s + p^w W^w] + [1 - \Phi(\zeta^i + \bar{z}^i)]\bar{u}_b\Big] \\ &= (1-\delta)\bar{u}^i + \delta\Big[\Phi^i[p^s W^s + p^w W^w] + [1-\Phi^i]\bar{u}_b\Big], \end{aligned}$$

where $\Phi^i = \Phi(\zeta^i + \bar{z}^i)$. Solving this system yields:

$$\begin{aligned} W^i &= (1-\delta)\bar{u}^i \\ &+ \delta\left(\Phi^i \, \frac{(1-\delta)[p^s\bar{u}^s + p^w\bar{u}^w] + \delta[1 - p^s\Phi^s - p^w\Phi^w]\bar{u}_b}{1 - \delta[p^s\Phi^s + p^w\Phi^w]} + (1-\Phi^i)\bar{u}_b\right). \end{aligned}$$

The equilibrium quantities (y^s, y^w) satisfy the following two first-order conditions:

$$\frac{\partial}{\partial \zeta^i} W^i(y^s, y^s, y^w, y^w) = 0, \qquad i \in I.$$

By appropriate choice of $\bar{z}^i$, we can drive y^i close to the Ramsey outcome $\bar{x}^i_r$. Using the same reasoning as in the proof of Theorem 5, one can show that the incentive constraints hold. When the variance v of Φ is sufficiently small, one can demonstrate that this FRCTE is Pareto superior to babbling equilibrium. ■

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