# The Asymmetric Effects of Monetary Policy Shocks:

# A Nonlinear Structural VAR Approach

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Preliminary version

**Abstract**

The aim of our paper is to investigate the potential asymmetric effects of monetary policy shocks on U.S. economic activity. For that matter, we focus on the business cycle phase in which the shock occurs. We then propose a natural extension of the structural VAR methodology and of its analysis tools to a nonlinear framework. The identification strategy of shocks we adopt is the one proposed by Sims and Zha (1998). At this stage, our findings reveal that a contractionary money supply shock yields asymmetric responses of output, prices and money.

**Keywords:** Asymmetries, Monetary Policy, Nonlinear Structural VAR.

**JEL Classification:** E52, C32, C53.

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# 1 Introduction

There exists a large literature that tries to give an answer to how the economy responds to an exogenous monetary policy shock. Since Sims (1980) seminal work, the Vector Autoregression models (VARs) became a predominant econometric tool in this field of monetary policy research. Later, the will to model as accurately as possible the monetary transmission mechanisms gives place to structural VARs models with some of the more prominent and recent examples being the contributions of Gali (1992), Sims and Zha (1998) and Christiano, Eichenbaum and Evans (2000) *inter alia*. However, this literature fails to take into consideration the existence of eventual asymmetries, either in the underlying theory implicit in the identifying assumptions or in the econometric method used to estimate it. Yet, as far back as the 1930s until our days, there exists a debate among economists on the presence of asymmetries of the monetary policy effects over the business cycle. In this paper, we propose a natural extension of the structural VAR approach to a nonlinear framework. Our aim is to investigate the potential asymmetric effects of monetary policy shocks on the economic activity, depending on the business cycle phase in which the shock occurs.

Our investigation finds its justification in several theoretical representations. Keynes' liquidity trap theory (1936) constitutes an early formalised example of the state dependency of the effectiveness of monetary policy measures[1]. More recently, two main streams of the literature support the hypothesis of this state asymmetry of monetary policy. Theories based on the assumption of a convex short-run aggregate supply curve constitute one of these strands. Convexity implies that the slope of the curve is steeper at higher levels of output and inflation than at lower levels. As a result, a shift in the aggregate demand originated from a shift in monetary policy will have a stronger effect on output and a weaker effect on inflation when the economy is in recession; when in expansion, the inverse is true, that is the effect of the monetary policy shift will have a stronger effect on inflation and a weaker effect on output. An important class of models incorporating a convex short-run aggregate supply curve in their analysis are the models of *S-s* price adjustment[2]. The second strand of the literature is formed by models that suppose asymmetries on financial markets. The presence of these asymmetries on capital markets implies a particular transmission

[1] If nominal interest rates are at such low level that they are not likely to decrease further, expansionary monetary policy through the interest rate channel will be ineffective, like “pushing on a string”.

[2] See Tsiddon (1991) and Caballero and Engel (1992).

mechanism - called *financial factors* by Gertler and Gilchrist (1994) - which may amplify the effects of monetary policy. Two complementary interpretations of the influence of these financial factors may be provided, depending on two classes of models. The first one is an extension of recent financial theories on the real business cycle which emphasises the role played by the financial sheets of borrowers[3]. These models assume that capital market imperfections render the expenses in capital of a certain class of borrowers dependant of their balance sheets through the link existing between the (collateralizable) net worth and the credit contract terms. This hypothesis yields a financial propagation mechanism by which the variations of balance sheets amplify the variations in expenses over the business cycle. The second class of models investigates the bank lending channel in the monetary policy transmission process. These models are based on the ability of monetary policy to regulate the total amount of borrowable funds in the economy through the legal obligations on reserves that apply to the banking system[4]. The bank lending channel is closed in spirit to the financial sheets channel. Both of them suggest that monetary policy should have a disproportional impact on the borrowers who have a limited access to capital markets. In these two classes of models, the financial propagation mechanism should be much powerful in recession periods than in expansion periods since the credit market restrictions should apply to a larger number of firms. The intuition that this financial mechanism is asymmetric over the business cycle is formalised by Bernanke and Gertler (1989).

Empirically, the state asymmetry of monetary policy has been highlighted through nonlinear approaches. Since Garcia and Schaller (1995) original contribution, several researchers have found evidences for this state asymmetry for main OECD economies and with different methodologies[5]. Garcia and Schaller (1995) conduct their study for the United States. Their estimation methodology of state asymmetry has two steps. First, they estimate monetary policy shock series from a vector autoregression model. Second, they introduce this measure of monetary policy shocks as an exogenous variable in a output nonlinear equation modelled as a two-state Markov-switching model. Weise (1999) and Holmes and Wang (2000) look for state asymmetries in the

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[3] See Bernanke and Gertler (1989), Calomiris and Hubbard (1990), Gertler (1992), Greenwald and Stiglitz (1993) and Kiyotaki and Moore (1998).

[4] See Romer and Romer (1990), Bernanke and Blinder (1992) and Kashyap, Stein and Wilcox (1993).

[5] See for instance Kakes (1998), Dolado and Maria-Dolores (1999) and Peersman and Smets (2001) who follow the same methodology to look for state asymmetry of monetary policy in main OCDE economies as well as in the Euro area.

United States and the United Kingdom respectively (in the last three decades), by means of a logarithmic smooth transition vector autoregression (LSTVAR). This autoregressive vector is a reduced form of a simple structural model of aggregate demand - aggregate supply which incorporates asymmetric nominal rigidities. All these studies have found evidences that a monetary policy shock has a stronger effect during recessions than in booms. Yet, while of substantial interest, these contributions display two shortcomings. First, testing the asymmetric effects of monetary policy shocks in a sole nonlinear equation of output fails to consider the monetary transmission mechanisms in a framework as rich as a structural VAR representation. Besides, this isolate testing approach may bias the correct evaluation of this effect. Second, the empirical nonlinear representations used in the studies of Weise (1999) and Holmes and Wang (2000) fail to identify the monetary policy shocks.

This paper proposes to provide an additional support on the state asymmetry of monetary policy overcoming the shortcomings of the existing empirical literature. Our original contribution is to adapt the structural VAR methodology to a nonlinear setup. For that matter, the first step consists in replacing the estimation of the linear propagation system by a non linear one. The second step then consists in generalising the analysis tools of structural VAR, *i.e.* the shocks identification procedure, the impulse-response functions computations and the forecast-error variance decompositions, to this nonlinear setup.

The identification strategy of shocks we adopt is the one proposed by Sims and Zha (1998)[6]. This scheme appears to us as more appealing than the ones encountered in the literature as the non-recursive structure they retain permits to model more convincingly structural contemporaneous restrictions across variables[7]. Indeed, it allows the presence of inertia in the private sector but also some channels of immediate response of the private sector to monetary policy. Such an approach allows to overcome the two “puzzles” generally encountered in closed economies VAR analysis: the “liquidity” puzzle and the “price” puzzle. The “liquidity” puzzle is eluded by distinguishing between money supply and money demand shocks and the “price” puzzle by using price data that capture inflationary expectations.

[6] This scheme has recently been extended by Kim and Roubini (2000) to open economies. See also Peersman and Smets (2001) for an application to the Euro area.

[7] See for instance Bernanke and Blinder (1992), Gordon and Leeper (1994), and Christiano, Eichenbaum and Evans (2000).

Our preliminary results are quite encouraging although much remains to be done. At this stage, our findings reveal that a contractionary money supply shock yields asymmetric responses of output, prices and money. Further, forecast error variance decomposition analysis of real GDP and prices reveals that the monetary policy shock' contribution in explaining this variables' fluctuations, if significant in expansions, almost disappears in recessions.

The structure of this paper is as follows. Section 2 presents the methodology we implement in our analysis. Section 3 discusses the data, the identifying structure as well as estimation results and the IRF analysis. Section 4 offers concluding remarks.

## 2 The methodology

The aim of our approach is to adapt the structural VAR methodology to a nonlinear setup. For that matter, the first step consists in replacing the estimation of the linear propagation system by a non linear one. The second step then consists in generalising the analysis tools of structural VAR, *i.e.* impulse-response functions and the forecast-error variance decompositions, to this nonlinear setup.

### 2.1 Estimation of the model

Two nonlinear approaches prevail in the literature: the Threshold AutoRegressive methodology (Tong (1990), Terasvirta (1995), Weise (1999)) and the Markov Switching methodology (Hamilton (1989)). In both approaches, the estimated parameters are supposed to depend on the state of the system. The difference between these approaches relies on the identification of the state of the system. In the TAR methodology, the identification of the state of the system relies on the selection of a transition function depending on a transition variable through tests. The fact this transition variable is lesser or greater than a threshold $s$ determines if an observation belongs to a regime or the another (in the simplest case of two regimes). The model that is piecewise linear is then estimated for every candidate value of $s$. The retained value for $s$ is the one that provides the highest log-likelihood value. In the Markov-switching methodology, the state of the system is ruled by an unobserved process, supposed to be an one-order Markov chain. The transition probabilities of this process are incorporated in the set of parameters to be estimated. The estimation of the model is then obtained through maximum likelihood thanks to the so-called Hamilton filter, that allows to provide an inference on the parameters and on the state of the system.

In this paper, we retain this latter approach insofar as it seems to present many advantages. First, this approach is quite general and only requires to formulate general assumptions on the process that rules the changes in regimes. Second, it will allow us to dispose of estimated transition probabilities for the exact calculation of forecasting paths that will provide impulse response functions.

The first step of our procedure is to replace the traditional VAR representation by the corresponding MS-VAR model. The estimated model is then:

$$Y_t = \mu_{s_t} + \sum_{i=1}^{p} \Phi_{i,s_t} \cdot Y_{t-i} + u_t \qquad (1)$$

with $Y$ the vector of $k$ variables, $u_t$ is supposed to be gaussian with a variance-covariance matrix $\Omega_{u,s_t}$ and $s_t$ is a discrete process taking its values in [1, $S$]. This is a nonlinear model due to the presence of $s_t$. Every parameter of the model is supposed to be dependant of the state of the system. Indeed, Mac Culloch & Tsay (1994) highlighted that the assumption generally formulated by Hamilton about the equality of autoregressive and variance parameters across regimes is quite too strong, even if particularly interesting for reducing the number of parameters to estimate. This issue is particularly crucial here, insofar as we face the problem of parameters inflation associated to the VAR representation amplified by the nonlinear structure of the model. This remark induces us to retain an estimation approach in two steps. At first, we let the parameters to be different across regimes and estimate the model. Afterwards, we will better the specification by testing equation by equation if some parameters can be equal across regimes or insignificant in order to “clean” the dynamics and to save degrees of freedom.

$s_t$ is the unobservable variable that controls the state of the economy. At every date, it can be equal to 1, 2, … or $S$, with $S$ the number of states supposed. In this preliminary version, we postulate that $S$ is equal to 2, in order to capture the business cycle fluctuations[8]. $s_t$ is a one order Markov chain characterized by transition probabilities $p_{ij}$ between the different states of the system:

$$P(s_t = j \mid s_{t-1} = i, s_{t-2} = k, \ldots) = P(s_t = j \mid s_{t-1} = i) = p_{ij}$$

[8] We will test the model for $S$ equal to 3, as some studies (Sichel [1994], Karamé & Perraudin (1998) or Krolzig & Toro (2000)) point out this possibility.

with $\sum_{j=1}^{S} p_{ij} = 1 \quad \forall i$ .

The set of unknown parameters is $\Theta = \left\{ \mu_{s_t}, \Phi_{1,s_t}, \ldots, \Phi_{p,s_t}, \Omega_{s_t}, P \right\}$, with $P$ the transition probabilities matrix. The model is estimated through maximum likelihood, thanks to the Hamilton (1989) filter (see appendix).

As we postulate that the unobserved process corresponds to the cyclical phases of the economy, an *a posteriori* validation of the estimated model can consist in comparing the inference on the state of the system through filtered and smoothed probabilities to the dates of recessions provided by national agencies.

## 2.2 The identification of shocks

In the traditional SVAR approach, the residual terms are generally correlated (their variance-covariance matrix $\Omega_u$ is not diagonal) and one can then not interpret $u_t$ as primitive innovations. The structural VAR approach aims at introducing some general conditions provided by theoretical representations in order to disentangle the structural economic shocks from estimated residuals. One can then analyse the *a posteriori* coherence between the unconstrained predictions of the structural VAR and the other predictions of theoretical representations.

Whatever the nature of conditions employed to identify the shocks may be (of short-run or long-run), the relation between the residuals from the estimated reduced-form and the economic innovations can be written in a matrix $D$ on the following form:

$$\hat{u}_t = D \cdot \varepsilon_t$$

with $e_t$ the set of structural innovations whose variances-covariances matrix $\Omega_e$ is diagonal. From this relation, one can deduce from the variance of the processes:

$$\Omega_u = D' \cdot \Omega_\varepsilon \cdot D$$

This equality can be considered as a nonlinear system. $D$ contains $k^2$ unknown parameters. $\Omega_e$ is a matrix of $k$ unknown parameters since it is supposed to be diagonal. $\Omega_u$ is an estimated symmetric matrix that provides $k\,(k + 1)/2$ equations. The number of unknown parameters is equal to $k\,(k + 1)$ and the system is then under-identified. One has then to assume at least $k\,(k - 1)/2$ conditions in order to make this system just-identified or over-identified. In this latter case, over-identifying restrictions could be tested.

A drawback of this approach is to neglect the possible presence of nonlinearities in the propagation system. Weise (1999) tries to circumvent this approach by using a VTAR model. However, he does not build a formal link with the theory. Furthermore, our approach differs also from the one of Krolzig & Toro (2000)[9] where the shocks are interpreted as changes in regime in Markov-switching models. In this framework, the number of impulse response functions calculated is then equal to the squared number of the states postulated in the system. It seems to us more natural to identify the shocks and investigate the potential asymmetry of the variables responses with respect to the regime in which the shock occurs.

In our approach, we will suppose the same set of identifying conditions on the innovations across the regimes. However, as $D$ contains either unknown and known parameters, it will also depend on the state of the system. The generalisation of the shocks identification procedure is quite straightforward. We then write:

$$\Omega_{u,s_t} = D_{s_t}' \cdot \Omega_{\varepsilon,s_t} \cdot D_{s_t}$$

The resolution of the $S$ nonlinear systems that provide $D_{s_t}$ and $\Omega_{\varepsilon,s_t}$ is conducted through a numerical method.

## 2.3 Generalised impulse response functions

Once the shocks are identified, one can perform an IRF exercise. For that matter, we use the following rewritten equation:

$$Y_t = \mu_{s_t} + \sum_{i=1}^{p} \Phi_{i,s_t} \cdot Y_{t-i} + D_{s_t} \cdot \varepsilon_t$$

This corresponds to a theoretical forecasting exercise. At date $t = 0$, we suppose a constant value $\overline{Y}$ for the $Y_{t-i}$ . Let's suppose that we know in which regime we are at $t = 1$ (say $s_1 = j$ for instance) and that a shock of one standard-error magnitude occurs at this date. Then, according to the equation, at date $t = 1$, we observe:

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[9] See an application in Artis, Krolzig & Toro (1999).

$$Y_1 = \hat{\mu}_j + \left[\sum_{i=1}^{p} \hat{\Phi}_{i,j}\right] \cdot \overline{Y} + D_j \cdot \varepsilon_1$$

From $t = 2$ to $h$, the horizon of the forecasting exercise, we do not observe the state of the system. We then have to perform an inference on the state of the system thanks to the estimated transition probabilities, knowing the state of the system in $t = 1$. We then compute every possible path of the system (say $2^h$ in this case[10]) and calculate the optimal forecast for the $Y$ process conditionally to each possible path thanks to equation (1). The corresponding path is then the mean of all possible responses weighted by its own probability:

$$\forall t \geq 2, \quad E[Y_t \left| s_1, \varepsilon_1, \hat{\Theta}_T \right.] = \sum_{s_t=1}^{S} \ldots \sum_{s_2=1}^{S} E[Y_t \left| s_{t-1}, \ldots, s_2; s_1, \varepsilon_1, \hat{\Theta}_T \right.] \cdot P(s_{t-1}, \ldots, s_2 \left| s_1, \hat{\Theta}_T \right.)$$

with

$$\forall t \geq 2, \quad E[Y_t \left| s_{t-1}, \ldots, s_2; s_1, \varepsilon_1, \hat{\Theta}_T \right.] = \hat{\mu}_{s_t} + \sum_{i=1}^{p} \hat{\Phi}_{i,s_t} \cdot E[Y_{t-1} \left| s_{t-2}, \ldots, s_2; s_1, \varepsilon_1, \hat{\Theta}_T \right.]$$

and whose associated probability is calculated with:

$$\forall t \geq 2, \quad P(s_t = j, s_{t-1} = i, \ldots, s_2 = k \left| s_1, \hat{\Theta}_T \right.) = \hat{p}_{ij} \cdot P(s_{t-1} = i, \ldots, s_2 = k \left| s_1; \hat{\Theta}_T \right.)$$

As we are in a non linear setup, the same exercise has to be implemented but without the shock occurring at date 1 in order to calculate the baseline ($e_1$ is then set equal to 0). The generalised impulse response function of variables $Y$ to the structural shock $\boldsymbol{e}$ is then obtained from:

$$\forall t \geq 2, \quad E[Y_t \left| s_1, \varepsilon_1, \hat{\Theta}_T \right.] - E[Y_t \left| s_1, 0, \hat{\Theta}_T \right.]$$

It only depends on the state of the system in which the shock occurs. We are then able to present the responses to monetary policy shocks for every state of the system in

[10] Due to the large computational resources needed to calculate these paths, we limit our analysis in this preliminary version to $h = 16$. We also try to develop an approximation based on the collapsing technique (see Kim (1994)) but it presents some systematic bias relatively to the formal calculation.

which it occurs and to discuss the possible asymmetric responses of the studied variables.

## 2.4 The forecast-error variance decomposition

Once the GIRF are calculated, it is easy to compute the forecast-error variance measure $\omega_{j,f,h}\left|s_1\right.$, *i.e.* the contribution of the structural shock *j* to the forecast-error variance of variable *f* at horizon *h*, knowing the state of the system in which the shock occurs:

$$\omega_{j,f,h}\left|s_1\right. = 100 \times \frac{\sum_{i=1}^{h} GIRF_{j,f,i}^{2}}{\sum_{i=1}^{h}\sum_{j=1}^{k} GIRF_{j,f,i}^{2}}$$

# 3 State asymmetries of the effects of monetary policy shocks

## 3.1 Data and estimation results

As the purpose of this study is to extend the analysis of the effects of an unexpected monetary policy shock by investigating potential state asymmetries, we have deliberately chosen a specification - *i.e.* economic time series and identifying restrictions - standard and widely accepted in this kind of VAR-analysis.

Our specification includes two monetary variables represented respectively by the federal funds rate (*FFR*) and the monetary aggregate *M1*, the real GDP (*Y*), the consumer price index (*CPI*) and a commodity price index (*Comm-PI*)[11]. Following Bernanke & Blinder (1992), the federal funds rate is introduced to identify shocks in monetary policy because the deficiencies of money stock growth as a measure of the stance of monetary policy are by now widely accepted in this literature. The price commodity index is included to counteract the “price puzzle” effects, as argued in the introduction of this paper. Finally, the other three variables are well-known variables in monetary business cycle literature.

[11] The final version of this paper will include estimations with other measures of money as M2 and Total Reserves as well as other measures of the policy instrument as the ratio proposed by Strongin (1995) and the Romer’ dates, Romer & Romer (1990).

All variables are seasonally adjusted, taken in logs except for the interest rate series, and in first differences[12]. The sample runs from 1959.Q1 to 2001.Q3[13]. Due to the large system we estimate in two states, we retain one lag in the specification of the model in order to save degrees of freedom. However, in a next version, we will include two lags to check the robustness of our preliminary results.

Unit root tests showed that the variables are I(1). We then specify the MS-VAR model with variables in first differences. This position may be defended by the fact that we will investigate some long-run restrictions on the variables in the identification scheme in a next version of the paper. However, many authors (like Kim & Roubini (2000)) followed Sims, Stock and Watson (1990) who argued that transforming the model to stationary form is unnecessary in many cases since they demonstrated that the statistics of interest have distributions unaffected by nonstationarity. They then perform their analysis by employing variables in level, and with only short-term identifying restrictions. A crucial issue is then to determine whether the treatment of nonstationarity will affect or not our results and conclusions. An analysis has then to be implemented to check the robustness of our results to the specification of the model variables and to the use of short-run or long-run identifying restrictions. Results are in progress.

Table 1 displays the set of estimated parameters corresponding to the MS(2)-VAR(1) presented in equation (1)[14]. Estimated transition probabilities are 0.967 and 0.86 which implies that the estimated regimes are stable and quite persistent. We underline as well the fact that the estimated coefficients are quite different across regimes in most cases, which implies that there exists *de facto* two well differentiated regimes in the dynamics of the data. Before going further in our analysis, we proceed to the interpretation of the regimes estimated by the model. As not imposed, they cannot be interpreted directly as expansion or recession periods. We then compare the estimated regimes from our model with the business cycle phases provided by the NBER. Figure

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[12] Robustness analysis is in progress. It consists in further estimations with the variables taken in levels and / or with two lags as well as with other measures of money and /or monetary instrument as mentioned in footnote (10).

[13] As pointed out by many researchers of American monetary policy, a shift in the conduct of monetary policy occurred in 1979 with the arrival of Chairman P. Volcker at the head of the Federal Reserve. Our sample includes therefore two different regimes of monetary policy. This implies that the Lucas' critique may apply. However, it seems to us to it is the price to pay to implement our approach since it requires a large sample for estimations purposes and to have the possibility to identify different cyclical regimes for the economy.

[14] These estimations were obtained by maximum likelihood through the simplex minimisation method. See appendix for further details.

1 plots the series with the smoothed estimated probabilities of being in state 2. When we compare the NBER recession dates with our smoothed probabilities (figure 2), one may see that they are quite close even if not exactly the same. The main recessions seems then to be captured by the model. This comparison exercise leads us to identify, although cautiously, regime 1 as an expansion regime and regime 2 as a recession regime.

## 3.2 The identifying restrictions

We adopt the identifying restrictions proposed by Sims & Zha (1995)[15]. The identification scheme in each regime is as follows:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ d_{21} & 1 & 0 & d_{24} & 0 \\ d_{31} & 0 & 1 & 0 & d_{35} \\ d_{41} & 0 & 0 & 1 & 0 \\ 0 & d_{52} & d_{53} & d_{54} & 1 \end{pmatrix} \begin{pmatrix} u_t^{comm-pi} \\ u_t^{cpi} \\ u_t^{FFR} \\ u_t^{rgdp} \\ u_t^{m1} \end{pmatrix} = \begin{pmatrix} \varepsilon_t^{comm-pi} \\ \varepsilon_t^{cpi} \\ \varepsilon_t^{FFR} \\ \varepsilon_t^{rgdp} \\ \varepsilon_t^{m1} \end{pmatrix}$$

$u_t^{comm-pi}$, $u_t^{cpi}$, $u_t^{FFR}$, $u_t^{rgdp}$ and $u_t^{m1}$ are the residuals in the reduced form equations. $\varepsilon_t^{comm-pi}$, $\varepsilon_t^{cpi}$, $\varepsilon_t^{FFR}$, $\varepsilon_t^{rgdp}$ and $\varepsilon_t^{m1}$ are the structural disturbances, *i.e.* the commodity price shock, the price shock, the money supply shocks, the real GDP shocks and the money demand shock respectively.

Structural shocks are separated in two blocks. The first two equations define money market equilibrium. The goods market equilibrium is described by the next three equations. The monetary policy feedback rule is defined according to the assumption that the monetary authority sets the interest rate after observing the current money stock and the world commodity price, but not the current values of output and the price level. Sims and Zha's (1995) argument to justify this setting is based on the hypothesis of information delays that impede policymakers to react within the period to price level and economic activity development. This hypothesis regarding prices may be disputed as we use quarterly data[16]. Nevertheless, this flaw is controlled by

[15] Gali' (1992) short-run restrictions are embedded in this identification scheme. More precisely, Gali imposes in the short-run (*i*) no contemporaneous effect of money supply shocks on output, (*ii*) no contemporaneous effect of money supply demand shocks on output, (*iii*) contemporaneous GNP does not enter the money supply rule, and (*iv*) contemporaneous homogeneity in money demand.

[16] GDP data are quarterly and published in the quarter following the relevant one.

including the commodity price index in the monetary policy reaction function which implies the control for current systematic responses of monetary policy to inflationary shocks. The short-run demand equation for real money balances responds as usual to real income and to the opportunity cost of holding money, *i.e.* the nominal interest rate. The next two equations represent the sluggish reaction of the real sector (output and prices) to shocks in the monetary sector (money and interest rates). There is no contemporaneous impact of the monetary policy and the money demand shock on output and prices. This scheme is based on the observation that most types of real economic activity may respond only with a lag to monetary variables because of inherent inertia and planning delays. The commodity price index appears in this block following the assumption that it generates a channel *via* mark-up rules for prices, by which monetary policy exerts a contemporaneous impact in this sluggish block.

This identification scheme is over-identified as we impose twelve restrictions while only ten are needed. In order to check for the robustness of the results, we also use a just-identified scheme. We then relax the restrictions on the monetary policy reaction function and allow for contemporaneous response of the federal funds rate to real GDP and prices. These restrictions appeared to us as the most natural to release since they were the more controversial given the frequency of the data. The identification scheme in each regime then becomes:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ d_{21} & 1 & 0 & d_{24} & 0 \\ d_{31} & d_{32} & 1 & d_{34} & d_{35} \\ d_{41} & 0 & 0 & 1 & 0 \\ 0 & d_{52} & d_{53} & d_{54} & 1 \end{pmatrix} \begin{pmatrix} u_t^{comm-pi} \\ u_t^{cpi} \\ u_t^{FFR} \\ u_t^{rgdp} \\ u_t^{m1} \end{pmatrix} = \begin{pmatrix} \varepsilon_t^{comm-pi} \\ \varepsilon_t^{cpi} \\ \varepsilon_t^{FFR} \\ \varepsilon_t^{rgdp} \\ \varepsilon_t^{m1} \end{pmatrix}$$

Table 2 reports the evaluated contemporaneous coefficients. The estimated values for $d_{31}$ and $d_{35}$ are negative in most cases. That is, the Federal Reserve tightens its policy whenever inflationary pressures appears. As for the equation of money demand, $d_{52}$ and $d_{54}$ are negative and $d_{53}$ is positive. Real balances demand increases with revenue and decreases with positive variations of the opportunity cost of holding money – *i.e.* the federal funds rate. Therefore, the identification scheme, whether over- or just-identified, is able to properly capture the underlying economic behaviour of monetary authorities and of agents' money demand, whatever the state.

## 3.3 Impulse response functions analysis

Figure 4 displays the estimated impulse responses over 16 quarters, to a one-standard deviation positive shock on the federal funds rate, *i.e.* a contractionary monetary shock. Each figure displays the estimated impulse response for a shock occurring in each regime: in state 1, *i.e.* expansions and in state 2, *i.e.* recessions. The scale shows percentage deviations and the GIRF are calculated upon the variables taken in levels. We report in this same figure the GIRF corresponding to the just-identified scheme. Although with slight differences in the response amplitude, the response general pattern is approximately the same as in the over-identified scheme which allows us to accept (for the moment and until further tests) the robustness of the results presented below.

At a very general level, we observe that the estimated responses time pattern is the same for each particular variable whether the system is in state 1 or 2. Nevertheless, the magnitude of these responses, and even the sign in the case of prices and real GDP, are very different from one state to the other one. Thus, the system does not respond in the same way to a monetary contractionary shock occurring in state 1 or in state 2. For this to be a first important result of our paper, we need to analyse more in details the responses of each of the variables in both states.

After a contractionary money supply shock, federal funds rate raises and the monetary aggregate drops, both variations being persistent over the time span considered and true in each regime. This finding is consistent with the presence of a strong liquidity effect. The observed time pattern in the response of these two variables is similar within a state. When the shock occurs in state 2, the interest rate and M1 reach a minimum before the third quarter and remain stable after the sixth quarter. When the shock occurs in state 1, the minimum is reached between the sixth and the eighth quarter to remain at a stable level thereafter. As regards the magnitude of the response (in absolute value), a significant difference appears across the regimes. When the shock occurs in state 1, we observe that the increase in the interest rate is smaller than the fall in money supply. When the shock occurs in state 2, the inverse is true. This has a straightforward explanation. In state 1, we observe a persistent deflation which implies, given the important decrease in the nominal money supply, a small decrease in real balances explaining the small increase in the interest rate. On the contrary, the inflation observed in state 2 coupled with the relatively small drop in the nominal monetary aggregate leads to a strong decrease in real balances and therefore a strong increase in the interest rate.

After a delay of two quarters, there is a timid although sustained decline of real aggregate activity when the shock occurs in state 1, with the maximal decline occurring roughly a year to five quarters after the shock. If we look at the conventional textbook money market representation, this result is conform to the variations observed in real balances and interest rates. Moreover, this small response of output is similar to the findings of Sims and Zha (1998) in the model they estimate with the federal funds rate (and Total Reserves as a measure of money). However, Gali (1992) and Christiano *et al* (2000) found a response much more important in magnitude. With the shock occurring in state 2 (recessions), output shows a quite different response. It first increases very sharply before the second quarter, then falls after the third quarter, and following a new peak around the fifth quarter, gets to a stable positive path. For a shock occurring in state 1, consumer prices and commodity prices immediately fall, reaching the minimum at the tenth quarter and remaining at this level there after. For a shock occurring in state 2, prices increase sharply reaching a peak before the third quarter and fall to attain a stable positive path before the sixth quarter.

In clear, when the shock occurs in expansions periods, the money supply shock effect is coherent with traditional interest rate mechanism. In response to a contractionary policy shock, the federal funds rate rises, monetary aggregates declines, the aggregate price level falls. Aggregate output decreases timidly displaying the conventional hump-shaped form; the money supply shock have little effect on real activity when the economy is in expansions. This is in line with the results of Garcia and Schaller (1995), Kakes (1998), Dolado and Maria-Dolores (1999), Weise (1999) and Peersman and Smets (2001). When the shock occurs in recession periods, the responses of prices and output are non-standard. Following a rise in interest rates, prices increase. Figure 3 displays the inflation rate and the federal funds rate series, together with the NBER' recession periods. One can observe that recessions periods coincide with the sample inflation peaks, and that at these dates, interest rates are at quite high levels. Thus, it seems that the model estimated responses fit into this dynamic when in state 2. Moreover, an important question is to be raised here: Is the interest rate channel still at work when in recessions? In other words, asymmetric effects, if existing, are the expression of the presence of different mechanisms in the transmission of policy. This may be what we find. Nevertheless, we remain very cautious in the interpretation of this result because further robustness analysis has to be done.

### 3.4 The forecast error variance decomposition

Table 3 displays the forecast error variance decomposition of real GDP over the 16 quarters for each state of the model. We see that the federal funds rate shock contribution (as well as the monetary aggregate shock) in real output fluctuations is much lesser when occurring in state 2 than in state 1. That is, the shocks on federal funds rate and on monetary aggregates explain 4.2% of the real GDP fluctuations when they occur in the economy expansions phases. In contrast, when the economy is in recession, shocks on the federal funds rate play almost no role at all. Tables 4 and 5 report the forecast error variance decomposition of the consumer and the commodity prices. As for real activity, shocks from the monetary sector (interest rates and monetary aggregate) are significantly less important in explaining prices fluctuations when occurring in state 2 -recessions- compared to state 1 -expansions.

In summary, there exists a sharp difference in the contribution of a money supply shock on real activity and prices fluctuations whether the economy is in expansion or recession.

## 4 Concluding remarks

This paper tries to find some empirical support for theoretical models forecasting asymmetric effects of monetary policy shocks on economic activity. To do this, we adopt an original strategy which aims at reconciling the structural VAR (SVAR) methodology with the Markov-switching nonlinear approach.

We are heartened by our first results as asymmetries appear in between the two regimes. When the system is in state 2 -state we identified as being a recession after a confrontation to NBER business cycle dating- the standard interest rate channel is at work: money decreases, prices fall and real activity drops. In the state 1 of the system -in expansions- prices and output exhibit a non-standard response. Although we remain very cautious regarding this result, we think that it raises important questions about the transmission mechanism at work during expansions. Forecast error variance decomposition analysis of real GDP and prices again shows asymmetries in between the two states. The federal funds rate shock contribution to these variables fluctuations almost equals zero during recessions while effective in expansions.

Nevertheless, much remains to be done to check the robustness of our findings. We plan to pursue this preliminary study by using different measures of money, different model specifications (*e.g.* changing the number of lags, modifying the form of the data) and investigating alternative identification schemes.

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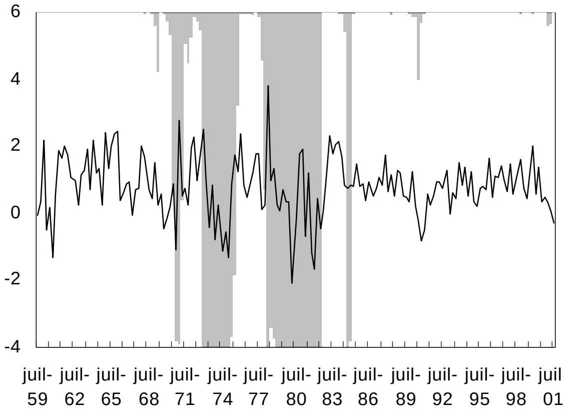
**Table 1:** Estimation results for the MS(2)-VAR(1) model.

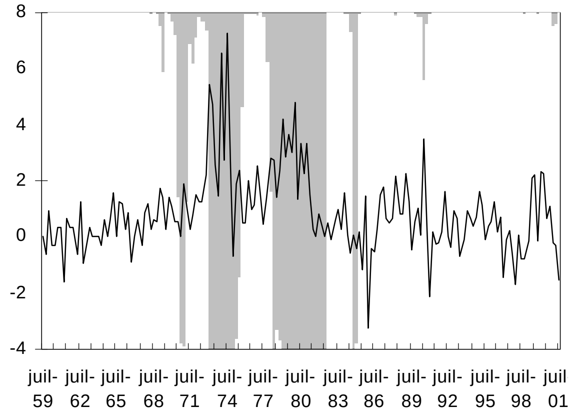
| State of the system | Variables | $\mu_{s_t}$ | $\Phi_{1,s_t}$ | | | | | $\Omega_{u,s_t}$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $s_t = 1$ | *dcomm-pi* | −0.052 | 0.107 | 0.298 | 0.080 | 0.165 | 0.073 | 0.882 | 0.214 | 0.151 | 0.042 | −0.233 |
| | *dcpi* | 0.271 | 0.011 | 0.483 | 0.097 | 0.042 | 0.080 | 0.214 | 0.136 | 0.016 | −0.046 | −0.047 |
| | *dFFR* | −0.280 | 0.086 | −0.052 | 0.192 | 0.262 | 0.034 | 0.151 | 0.016 | 0.243 | 0.087 | −0.051 |
| | *drgdp* | 0.692 | −0.009 | −0.032 | 0.058 | 0.271 | 0.010 | 0.042 | −0.046 | 0.087 | 0.469 | 0.104 |
| | *dm1* | 0.420 | −0.057 | 0.241 | −0.668 | −0.083 | 0.580 | −0.233 | −0.047 | −0.051 | 0.104 | 0.956 |
| $s_t = 2$ | *dcomm-pi* | 1.174 | 0.261 | 0.265 | 0.091 | 0.146 | −0.102 | 2.974 | 0.522 | 1.760 | 0.367 | 0.241 |
| | *dcpi* | 0.726 | 0.110 | 0.391 | 0.130 | −0.090 | 0.225 | 0.522 | 0.312 | 0.586 | 0.011 | 0.035 |
| | *dFFR* | −1.145 | 0.543 | −1.016 | −0.297 | −0.039 | 1.366 | 1.760 | 0.586 | 4.798 | 0.783 | 0.301 |
| | *drgdp* | 1.299 | 0.015 | −0.538 | 0.147 | −0.010 | 0.134 | 0.367 | 0.011 | 0.783 | 1.231 | 0.413 |
| | *dm1* | 2.013 | −0.092 | 0.105 | −0.108 | 0.100 | −0.325 | 0.241 | 0.035 | 0.301 | 0.413 | 0.496 |

$$P = \begin{pmatrix} 0.967 & 0.133 \\ 0.033 & 0.867 \end{pmatrix}, \quad L_T(\hat{\Theta}_T) = -254.19$$

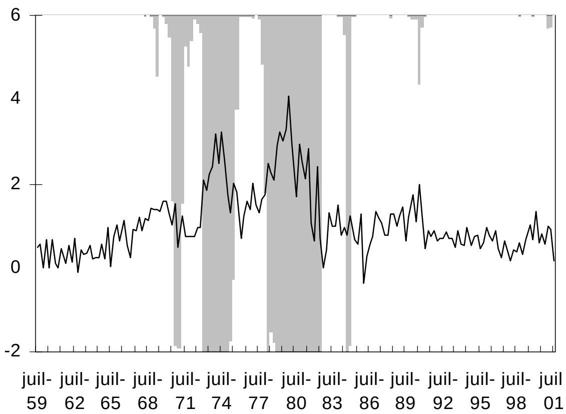
**Figure 1:** Model variables and estimated smoothed probabilities to be in regime 2 (in grey)



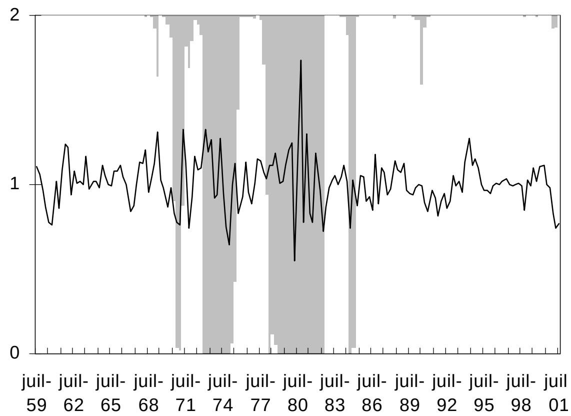

Real GDP (first differences of logarithm)



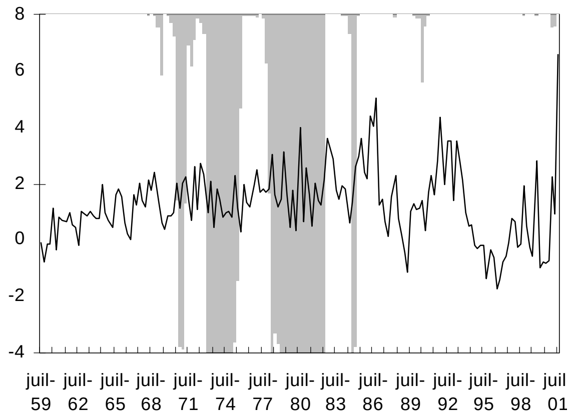

Commodity Price Index (first differences of logarithm)




Consumer Price Index ((first differences of logarithm)




Federal Funds Rate (first differences)




M1 (first differences of logarithm)

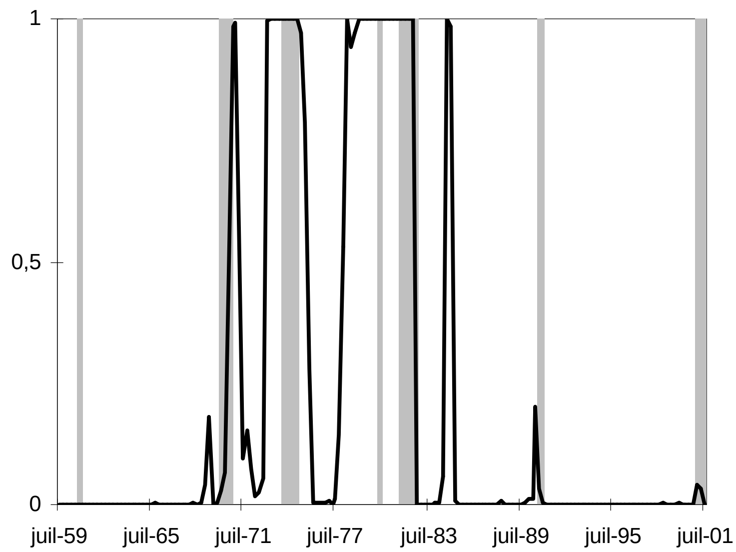



**Figure 2:** Smoothed probabilities (regime 2) and the NBER dates of recessions (in grey)

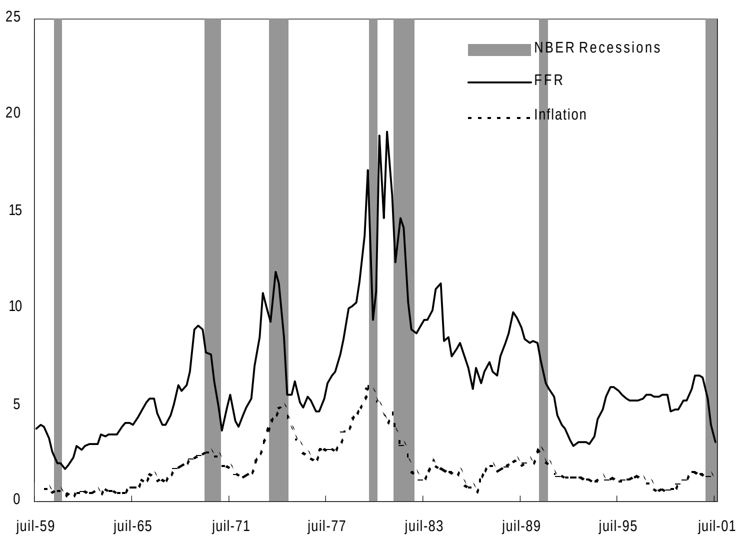
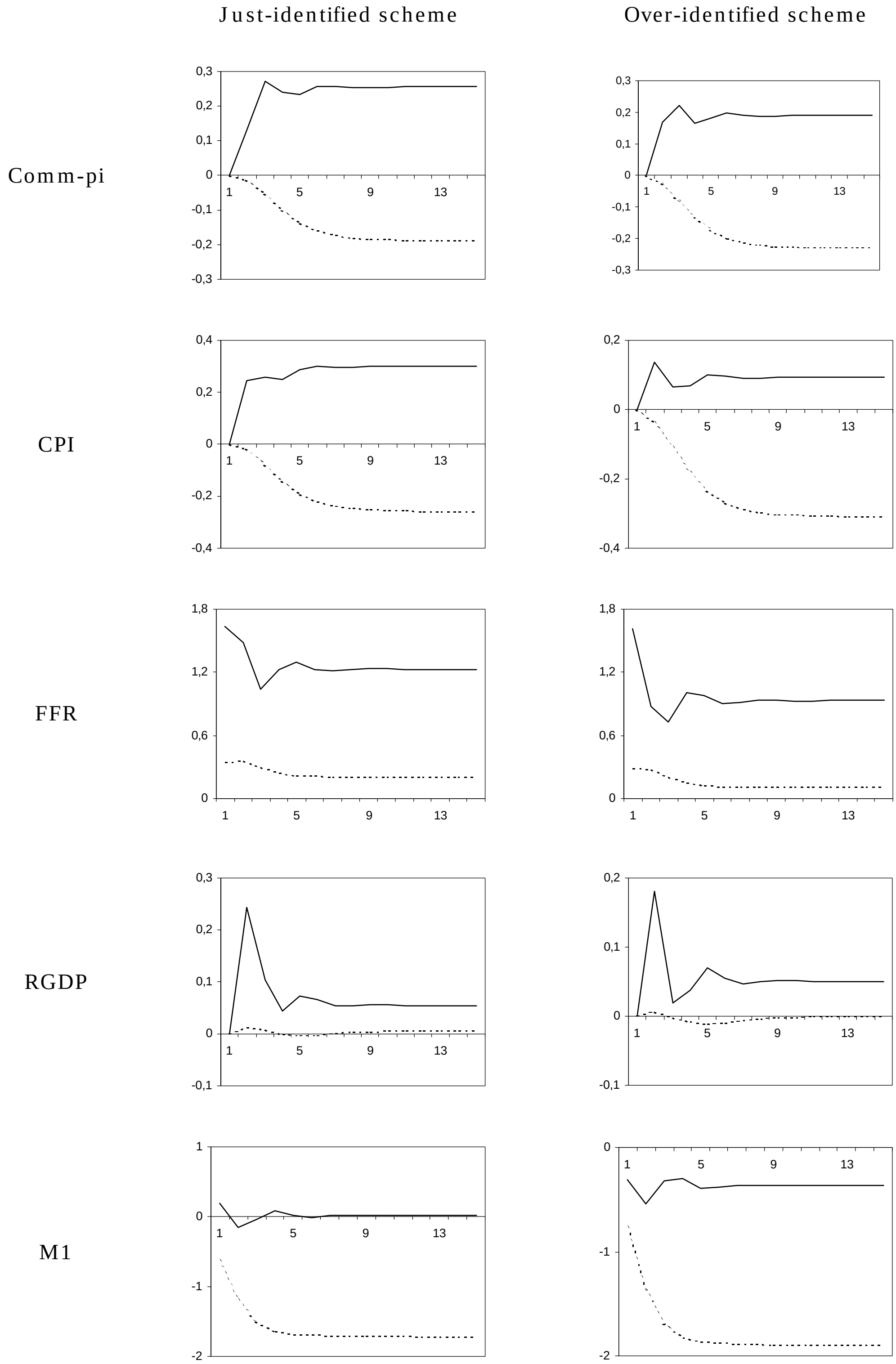



**Figure 3:** Federal funds rates, inflation rate and the NBER dates of recessions (in grey)

**Table 2**: Identification schemes

| State of the system | Variables | over-identified scheme | | | | | just-identified scheme | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $s_t = 1$ | *dcomm-pi* | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| | *dcpi* | –0.244 | 1 | 0 | 0.123 | 0 | –0.249 | 1 | 0 | 0.120 | 0 |
| | *dFFR* | –0.338 | 0 | 1 | 0 | –0.665 | –0.343 | 0.268 | 1 | –0.037 | –0.410 |
| | *drgdp* | –0.050 | 0 | 0 | 1 | 0 | –0.048 | 0 | 0 | 1 | 0 |
| | *dm1* | 0 | –0.562 | 2.658 | –0.764 | 1 | 0 | –0.059 | 1.784 | –0.556 | 1 |
| $s_t = 2$ | *dcomm-pi* | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| | *dcpi* | –0.191 | 1 | 0 | 0.043 | 0 | –0.181 | 1 | 0 | 0.045 | 0 |
| | *dFFR* | –0.464 | 0 | 1 | 0 | –1.727 | –0.318 | –1.436 | 1 | –0.898 | 1.101 |
| | *drgdp* | –0.119 | 0 | 0 | 1 | 0 | –0.123 | 0 | 0 | 1 | 0 |
| | *dm1* | 0 | –0.760 | 0.192 | –0.418 | 1 | 0 | 0.113 | –0.116 | –0.263 | 1 |

**Figure 4:** Responses of variables (in level) to a positive one standard-error shock on the FFR




Dashed line: shock in state 1

Solid line: shock in state 2.

**Table 3:** Forecast error variance decomposition of real GDP (over-identified scheme)

| $h$ | State 1 | | | | | State 2 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $e^{comm\text{-}pi}$ | $e^{cpi}$ | $e^{FFR}$ | $e^{rgdp}$ | $e^{m1}$ | $e^{comm\text{-}pi}$ | $e^{cpi}$ | $e^{FFR}$ | $e^{rgdp}$ | $e^{m1}$ |
| 1 | 3.43 | 0 | 0 | 96.57 | 0 | 3.43 | 0 | 0 | 96.57 | 0 |
| 2 | 3.23 | 1.17 | 2.47 | 91.14 | 1.99 | 3.32 | 0.53 | 1.12 | 94.12 | 0.90 |
| 3 | 4.60 | 2.99 | 4.20 | 86.15 | 2.06 | 2.25 | 2.11 | 0.71 | 94.09 | 0.84 |
| 4 | 6.20 | 3.24 | 4.12 | 83.93 | 2.51 | 1.73 | 3.56 | 0.55 | 93.54 | 0.63 |
| 8 | 6.26 | 3.26 | 4.20 | 83.75 | 2.52 | 1.23 | 6.14 | 0.36 | 91.93 | 0.34 |
| 16 | 6.26 | 3.26 | 4.20 | 83.75 | 2.52 | 1.12 | 7.60 | 0.25 | 90.83 | 0.19 |

**Table 4:** Forecast error variance decomposition of the commodity price index (over-identified scheme)

| $h$ | State 1 | | | | | State 2 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $e^{comm\text{-}pi}$ | $e^{cpi}$ | $e^{FFR}$ | $e^{rgdp}$ | $e^{m1}$ | $e^{comm\text{-}pi}$ | $e^{cpi}$ | $e^{FFR}$ | $e^{rgdp}$ | $e^{m1}$ |
| 1 | 100 | 0 | 0 | 0 | 0 | 100 | 0 | 0 | 0 | 0 |
| 2 | 97.64 | 0.61 | 0.84 | 0.88 | 0.04 | 99.05 | 0.25 | 0.34 | 0.35 | 0.01 |
| 3 | 96.67 | 0.77 | 0.88 | 1.15 | 0.53 | 98.19 | 0.46 | 0.49 | 0.68 | 0.19 |
| 4 | 96.56 | 0.77 | 0.96 | 1.18 | 0.53 | 97.91 | 0.52 | 0.45 | 0.87 | 0.25 |
| 8 | 96.53 | 0.78 | 0.98 | 1.18 | 0.54 | 97.51 | 0.64 | 0.45 | 1.09 | 0.31 |
| 16 | 96.53 | 0.78 | 0.98 | 1.18 | 0.54 | 97.31 | 0.71 | 0.46 | 1.18 | 0.34 |

**Table 5:** Forecast error variance decomposition of the consumer price index (over-identified scheme)

| $h$ | State 1 | | | | | State 2 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $e^{comm\text{-}pi}$ | $e^{cpi}$ | $e^{FFR}$ | $e^{rgdp}$ | $e^{m1}$ | $e^{comm\text{-}pi}$ | $e^{cpi}$ | $e^{FFR}$ | $e^{rgdp}$ | $e^{m1}$ |
| 1 | 32.21 | 67.10 | 0 | 0.69 | 0 | 32.21 | 67.10 | 0 | 0.69 | 0 |
| 2 | 44.73 | 45.96 | 2.93 | 0.53 | 5.85 | 44.30 | 51.77 | 1.26 | 0.15 | 2.52 |
| 3 | 47.32 | 42.65 | 3.35 | 0.69 | 5.99 | 49.69 | 46.16 | 0.71 | 0.09 | 3.34 |
| 4 | 47.42 | 42.60 | 3.33 | 0.68 | 5.97 | 51.7 | 44.38 | 0.53 | 0.08 | 3.31 |
| 8 | 47.51 | 42.36 | 3.44 | 0.68 | 6.01 | 54.35 | 41.61 | 0.44 | 0.07 | 3.52 |
| 16 | 47.51 | 42.36 | 3.44 | 0.68 | 6.02 | 55.43 | 40.48 | 0.40 | 0.08 | 3.61 |

# Appendix

## The estimation procedure

The estimation of parameters is obtained through maximum likelihood:

$$\hat{\Theta}_T = \arg\max_{\Theta} \mathcal{L}_T(\Theta) = \arg\max_{\Theta} \sum_{t=1}^{T} \log f(Y_t \mid \underline{\Psi_{t-1}}; \Theta)$$

with $Y_t$ the set of variables, $\underline{\Psi_t}$ the information set available at date $t$ and $\Theta = \left\{ \mu_{s_t}, \Phi_{1,s_t}, \ldots, \Phi_{p,s_t}, \Omega_{s_t}, P \right\}$ the unknown parameters set. $f(y_t \mid \underline{\Psi_{t-1}}; \Theta)$ is the unconditional distribution of observations. Since they are normally distributed in each regime with parameters depending of the regime, $f(y_t \mid \underline{\Psi_{t-1}}; \Theta)$ is a linear combination of gaussian distributions weighted by the probability to be in the corresponding regime:

$$\begin{aligned} f(y_t \mid \underline{\Psi_{t-1}}; \Theta) &= \sum_{j=1}^{S} P(y_t, s_t = j \mid \underline{\Psi_{t-1}}; \Theta) \\ &= \sum_{j=1}^{S} P(s_t = j \mid \underline{\Psi_{t-1}}; \Theta)\, f(y_t \mid s_t = j, \underline{\Psi_{t-1}}; \Theta) \end{aligned}$$

The log-likelihood function is then computed thanks to an iterative algorithm, the Hamilton filter, for a particular value Θ. The steps of the filter are as follows:

1) At date *t*, the filter is initialised by $P(s_{t-1} = i \mid \underline{\Psi_{t-1}}; \Theta)$.

2) Thanks to the Bayes formula, one can calculate a first inference on the state of the system at date *t* conditionally to the information available at date *t* – 1 and to the transition probabilities:

$$P(s_t = j \mid \underline{\Psi_{t-1}}; \Theta) = \sum_{i=1}^{S} p_{ij}\; P(s_{t-1} = i \mid \underline{\Psi_{t-1}}; \Theta)$$

3) Thanks to the Bayes formula and the conditional distribution of observations, one can compute the following joint distribution:

$$P(y_t, s_t = j \mid \underline{\Psi_{t-1}}; \Theta) = f(y_t \mid s_t = j, \underline{\Psi_{t-1}}; \Theta) \times P(s_t = j \mid \underline{\Psi_{t-1}}; \Theta)$$

4) A second inference on the state of the system a date $t$ conditionally to the information available at date $t$ can then be derived:

$$P(s_t = j \mid \underline{\Psi_t}; \Theta) = \frac{P(y_t, s_t = j \mid \underline{\Psi_{t-1}}; \Theta)}{f(y_t \mid \underline{\Psi_{t-1}}; \Theta)}$$

This probability initialises the filter for the next iteration.

Log-likelihood maximisation is led under the constraints $0 \le p_{ij} \le 1$. The transformation employed to obtain probabilities is $f(x) = \dfrac{x^2}{1 + x^2}$. For $S$ regimes, only $S\ (S - 1)$ transition probabilities are estimated since $\sum_{j=1}^{S} p_{ij} = 1$.

On a practical point of view, the estimation procedure is implemented as follows.

- First, since the estimation procedure heavily relies on numerical methods and due to the large number of parameters to be estimated, a guile is to initialise the procedure with the results obtained from the estimation of the corresponding VTAR model. Since VTAR and MS-VAR models will not provide the same results, there is no need to correctly specify the VTAR model. We are only interested in obtaining a plausible area for initialising the model parameters. The transition variable is then the growth rate of production lagged once, and the transition threshold is arbitrarily set between 0 and 1. In this manner, one can check the robustness to initial conditions of the solution found.

- Second, we use the minimisation simplex method developed by Nelder & Mead (1968)[17]. This method is implemented several times in order to get closer to the solution.

- Third, we use a gradient minimisation method on the last solution in order to nullify all first order conditions on the parameters.

[17] See Press, Flannery, Teukolsky & Vetterling (1992) for more details on this method.

## Inference on the state of the system

Knowing Θ, the next step is to identify the regimes of the economy at each date. Since the process $s_t$ is not directly observable, one examine the probability to be in this regime at date *t*. Two kinds (on the three available from the filter) are generally employed: filtered and smoothed probabilities. The former, noted $P(s_t = j \mid \underline{\Psi_t}; \hat{\Theta}_T)$, are based on all the past information available at date *t* and are calculated by the Hamilton filter a step 4. The latter, noted $P(s_t = j \mid \underline{\Psi_T}; \hat{\Theta}_T)$, are based on the whole sample information. They are calculated in the wake of the estimation procedure in the following way. Knowing $P(s_{t+k}, s_t \mid \underline{\Psi_{t+k}}; \hat{\Theta}_T)$, one iterates on *t* to calculate:

$$P(s_{t+k+1}, s_t \mid \underline{\Psi_{t+k+1}}; \hat{\Theta}_T) = \frac{\sum_{s_{t+k}} P(s_{t+k+1}, s_t \mid \underline{\Psi_{t+k}}; \hat{\Theta}_T) P(s_{t+k+1} \mid s_{t+k}) f(y_{t+k+1} \mid s_{t+k+1}, s_{t+k}, \underline{\Psi_{t+k}}; \hat{\Theta}_T)}{f(y_{t+k+1}, s_t \mid \underline{\Psi_{t+k}}; \hat{\Theta}_T)}$$

When $P(s_T, s_t \mid \underline{\Psi_T}; \hat{\Theta}_T)$ has been calculated, one can then compute the smoothed probabilities from:

$$P(s_t \mid \underline{\Psi_T}; \hat{\Theta}_T) = \sum_{s_T} P(s_T, s_t \mid \underline{\Psi_T}; \hat{\Theta}_T)$$