Price and Quality Regulation with multi-dimensional Heterogeneity

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Abstract

Regulation theory consider commodities with different quality as different goods. However, in many monopolistic industries, services *cannot* be offered at distinct quality level. The standard regulatory toolbox cannot be applied. In this paper, we display the choices of a monopolist facing an heterogeneous population when vertical differentiation is too costly. Optimal tariffs and quality level are computed in the same context. Finally we exhibit a mechanism to implement the later solution. An application is drawn in the transportation sector with a population that is heterogeneous in the valuation of *both* the good and its quality, *i.e.* its value of time.

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1. Introduction

Regulation theory cannot escape the quality issue. While sales give some incentive to the unregulated monopolist to provide quality, there is a question about the existence of similar incentives in a regulated environment. It is indeed feared, and not without any reasons (See e.g. Vickers and Yarrow 1988), that firms might concentrate their efforts on reducing cost at the expense of quality. However, as already shown by Spence (1975), an unregulated monopolist may over- or under-supply quality, depending on the substituability or the complementarity of quantity and quality. Thus, even when quality is not taken into account in the regulatory scheme, the impact of regulation on quality provision is quite unclear.

From an economic point of view, services with different qualities are different commodities. However the regulation over a "bunch of goods" is a very different problem since we consider here that only one quality is provided by the monopolist. The reasons explaining the uniqueness of the quality level are often the same that explains the existence of a monopoly: the costs structure. Like increasing return to scale favour the emergence of monopolist, vertical differentiation may result in decreasing returns, leading to firm to produce only one commodity.
As a result of this uniqueness, multi-product theory of regulation cannot be directly applied. A proper regulation needs to address simultaneously (and explicitly) price and quality issues. This is what we attempt to do in this paper.

This question however was already addressed by many. (For an overview, see Laffont and Tirole 1993). The contribution of this paper lies in the fact that we tackle the issue in a framework with multidimensional heterogeneity. Even if quality is observable by consumers before purchasing,[1] and it is observable and verifiable by the regulator, there is still a question as what is the optimal target when the population (and eventually the quality) displays multi-dimensional heterogeneity while the firm can deliver only one quality level. There is also a problem as whether this multidimensional target can be implemented, and if so, by the means of which instruments. This issue becomes more intricate when the monopolist may adopt

[1] This property give rise to the notion of what is called a *search good* as opposed to the *experienced good*, for which consummers observe quality after purchasing only. In this later case, the provision of quality is linked to reputation effects.

non-linear tariffs. However, we consider it essential in order to address real-world problems that presents indeed such a complexity.

Examples of application are numerous. In particular, almost all network industries can't afford to duplicate the essential facility represented by their network for the only purpose of vertical differentiation. So that differentiation, if any, occur at the margin: In telecommunication, while the operator may offer different quality of services for the connection to the network (the local bundle), it won't offer different networks. Networks are (at least partially) shared across users. Similar phenomena occur for internet, postal services, electricity or gaz transportation etc... So that one can observe that indeed most of the regulated sectors, the main issue consists in the regulation of a monopolist producing one commodity with only one quality level.

When tackling this issue, the problem is often addressed in a one-dimensional framework that all amount to distinguish consumers according to their price-elasticity[2]. We argue that, as soon as quality is under scrutiny, it becomes of essential importance to disentangle the valuation of the good from the valuation of the quality attach to it. Adopting a multi-dimensional approach does not stem here from any fascination of technicalities but from the intrisic nature of the economic phenomena that would be masked otherwise. First, there is a question around the link between the optimal mark-up and the price-elasticity of the consumers. Second, there is a question about how to define the optimal quality provision as a function of the individual valuations. Finally, there is a question about the implementation of the optimal allocation. In particular, if there is no systematic relationship between the valuation of the commodity and the valuation of quality across consumers, it becomes far from obvious that it can be obtained by the standard instruments of modern regulation like price-caps.

In order to illustrate this paper, we have chosen to the transportation sector. Despite the deregulation of the (European) airline industry, competition is still very limited on most of the routes. There is therefore an issue about the regulation of these (natural?) monopolies. In tackling this problem, one cannot reasonably abstract from the quality aspects. A company may offer apparently the same service, say a connection between two cities, with very different "quality" levels, *i.e.* with

[2]There are exceptions like Laffont, Maskin and Rochet, 1987.

different travel time and different frequencies. It is also natural in this context to distinguish the willingness to pay from the value of time, *i.e.* the valuation of the service *per se* from the valuation of quality. A traveller may attach a high value to leave on the first day of vacation and return on the last, while not being ready to pay a high price. Another will see the travel expenses paid by the firm and won't care about timing. Some other display a (more standard) positive correlation between both aspects...

To sum up, we consider a model where the monopolist can choose both the price and the quality of a commodity, given the demand she faces. We first study the unregulated monopolist; then the optimal price(-and quality) structure, with and without out the break-even constraint (Second and First-best). Furthermore we show that these optimal prices can be implemented trough a price-and-frequency cap. After presenting these issues in a simple model with an homogeneous population, all these questions are addressed again when consumers display heterogeneous valuation of the service and an heterogeneous valuation of quality. In particular, we provide a regulation scheme compatible with non-linear pricing like special fares for frequent travellers.

2. The basic model

Consider a situation where there is a unique air transportation company that offers a connection between two cities. Abstracting from all network effect, we assume that the objective of the company is to maximize profits on this connection alone. This may occur because this company does not guaranty any transfer or because simply because the diversity of (individual) transfers makes the overall profits almost independent from the scheduling on the connection considered. Production costs depends on both quantity X and quality $\mathbf{q}$ so that profits are given by:

$$\pi = pX - C\left(X, \mathbf{q}\right)$$

where p denote the unit price of the commodity (the ticket fare). We assume there is no possibility (or no incentive as the result of increasing returns to scale) for the firm to offer the commodity a several quality level.

In order to illustrate the results we might assume that

$$C = f\,\mathcal{C}\,(K)$$

where f is the number of flights (that would play the role of quality) and $K = X/f$ is the capacity of the shuttle. Increasing return to scale would result from an elasticity of $\mathcal{C}\,(K)$ that is lower than one.

Consumers preferences are represented by a quasi-linear utility function $\mathcal{U}_\theta\,(x, \mathbf{q})$ where x denote the individual consumption of the commodity (say the yearly number of flights) and $\mathbf{q}$ the vector of quality parameters (velocity, frequency of connection etc...) that all range, from zero to infinity. The vector $\boldsymbol{\theta}$ is a vector of characteristics. All computations are made with a general utility function $\mathcal{U}_\theta\,(x, \mathbf{q})$ that is increasing in each arguments (concave in x) and monotonic with respect to each component of the characteristic vector $\boldsymbol{\theta}$.
In order to illustrate the various result, we might sometimes use the specific functional form

$$\mathcal{U}_\theta\,(x, q) = \beta U\,(x) - \nu\, x\, g\,(\mathbf{q})$$

that fits pretty well to the transportation sector. The vector of characteristics $\theta = (\beta, v)$ is a pair of positive parameters accounting respectively for the willingness to pay for the commodity and for the valuation for quality. Adopting this functional form of the utility function amount to assume that the marginal benefits of quality are linear in consumption. This assumption that the cross derivative of the utility function $\partial^2 U/\partial q \partial x$ does not depends on x is quite natural in transportation where *e.g.* the benefits of reducing the travel time depends clearly on the number of travels. Remark that, even with this peculiar functional form we neither assume linearity in $\mathbf{q}$ or x.

In this particular case just mentioned, the function $g\,(\mathbf{q})$ can be interpreted as the unitary losses[3] (in monetary terms) incurred by the consumer when the service is delivered at quality $\mathbf{q}$ rather than at the highest quality level. The function $g\,(\mathbf{q})$ is decreasing in (each component of the vector) $\mathbf{q}$ and we will assume without any loss of generality that $\lim_{\mathbf{q}\to\infty} g\,(\mathbf{q}) = 0$ (a normalisation). An example for the function $g\,(\mathbf{q})$ would be $\nu g\,(f) = \nu/\,(2f)$ where ν is the value of time and the vector of quality

[3] According to the assumption made on the utility function, these losses do not depend on x.

$\mathbf{q}$ reduces to the frequency f of connection between two cities (in a model where the desired departure departure time is uniformly distributed across the population).[4] Under this assumption on the utility function $\mathcal{U}$, the variable $\tilde{p}_\nu = p + \nu g(q)$ can be interpreted as the generalised price of the commodity x for a consumer of characteristics (β, ν) whose (inverse) demand is given by:

$$\tilde{p}_\nu = \beta U'(x)$$

As a result, under the assumption made on the utility function, the individual demand function can be written as:

$$x_\theta(p, q) = X\left(\frac{1}{\beta}\left[p + \nu g(q)\right]\right)$$

where X is a standard decreasing demand function.

In this section a very simple model is presented where the population is completely homogeneous, *i.e.* all consumers display the same valuation of the service and of the quality. We will consider that both parameters take the unit value. The use of a representative agent on the consumption side allows to isolate the specific issue of simultaneous price and quality regulation in a basic setting before to address the problem in full generality.

2.1. Unregulated Monopolist

Consider first the price p and the quality $\mathbf{q}$ set by the monopolist if the sector is not regulated. Profits are given by

$$\pi = pX(p, \mathbf{q}) - C(X, \mathbf{q})$$

hence, denoting q_k the various components of the vector $\mathbf{q}$, one gets the following first order conditions:

$$\begin{aligned} X + (p - C_X)\frac{\partial X}{\partial p} &= 0 \\ (p - C_X)\frac{\partial X}{\partial q_k} - C_{q_k} &= 0 \quad \textit{any } k \end{aligned}$$

[4] $1/2f$ being in this case the mean waiting time before to get a connection.

Since, the expressions does not depend on q_k, we will abstract from the multi-dimensionality of $\mathbf{q}$ in what follows and denote q, any of its component. With the standard notations, one can rewrite the F.O.C. as follows:

$$\begin{aligned} \frac{p - C_X}{p} &= \frac{1}{\varepsilon_{X_p}} \\ -q\left(\frac{\partial X}{\partial q}\right) / \left(\frac{\partial X}{\partial p}\right) &= \varepsilon_{C_q} \frac{C}{X} \end{aligned}$$

Thus, the profit of the monopolist are given by:

$$\pi = pX - C(X, q) = \left(\frac{\varepsilon_{C_X}}{\varepsilon_{X_p} - 1} - 1\right) C$$

that gives a range of possible value for the price-elasticity of demand:

$$\varepsilon_{X_p} \in [1, 1 + \varepsilon_{C_X}]$$

In the case where $\mathcal{U}(x, q) = U(x) - xg(q)$ with $g(q) = v/2f$ and $C = f\mathcal{C}(K)$, one would easily establish that $\varepsilon_{C_X} = \varepsilon_{\mathcal{C}_K}$ and $\varepsilon_{C_q} = 1 - \varepsilon_{\mathcal{C}_K}$ hence

$$\begin{aligned} p &= \frac{\varepsilon_{\mathcal{C}_K}}{\varepsilon_{X_p} - 1} \frac{\mathcal{C}(K)}{K} \\ \frac{v}{2f} &= (1 - \varepsilon_{\mathcal{C}_K}) \frac{\mathcal{C}(K)}{K} \end{aligned}$$

2.2. Social Optimum (First Best)

The social welfare is given by the difference between the gross utility and the production costs:

$$W(X, q) = \mathcal{U}(X, q) - C(X, q)$$

2.2.1. Direct method

The maximizing behaviour of the consumer implies that

$$\frac{\partial \mathcal{U}(X, q)}{\partial X} = p$$

hence F.O.C. are:

$$\begin{aligned} p &= \frac{\partial C}{\partial X} \\ \frac{\partial \mathcal{U}(X,q)}{\partial q} &= \frac{\partial C}{\partial q} \end{aligned}$$

or in terms of mean costs:

$$\begin{aligned} p &= \varepsilon_{C_X} \frac{C}{X} \\ -\frac{q}{X} \frac{\partial \mathcal{U}(X,q)}{\partial q} &= \varepsilon_{C_q} \frac{C}{X} \end{aligned}$$

The first equation enhances that, if there are increasing return to scale, profits are negative at the first-best. The second equation determines the optimal quality level. It is of interest to note that, with the specific functional form $\mathcal{U}(X,q) = \beta U(x) - \nu x g(x)$ introduced previously,

$$\frac{\partial \mathcal{U}(X,q)}{\partial q} = -X \left(\frac{\partial X}{\partial q} \right) / \left(\frac{\partial X}{\partial p} \right) = -g'(q) X.$$

As a result the equation that determines quality is identical to the one that we have for the monopolist. However this does not mean that the monopolist provides the optimum level of quality. Indeed the allocation is defined by both equations. One can verify that, as long as the mean costs are lower at the first best and $-qg'(q)$ is decreasing, the quality is higher at first-best than with an unregulated monopolist.

In the case where $g(q) = v/2f$ and $C = f\mathcal{C}(K)$ and $\mu = 0$, one would easily establish that

$$\begin{aligned} p &= \varepsilon_{\mathcal{C}_K} \frac{\mathcal{C}(K)}{K} \\ \frac{v}{2f} &= (1 - \varepsilon_{\mathcal{C}_K}) \frac{\mathcal{C}(K)}{K} \end{aligned}$$

so that, at the optimum, the *generalised price* $\tilde{p} = p + \frac{v}{2f}$ should equate the *mean cost* $\mathcal{C}(K)/K$.

2.2.2. Indirect method

In this section, we compute again the welfare maximizing allocation by using the decision variables p and q. This method is slightly more complex but is the only one that can be extended for the computation of the second best. Now, consider indeed that $X = X(p, q)$:

$$\begin{aligned} \frac{dW}{dp} &= \frac{\partial W}{\partial X}\frac{\partial X}{\partial p} \\ \frac{dW}{dq} &= \frac{\partial W}{\partial X}\frac{\partial X}{\partial q} + \frac{\partial W}{\partial q} \end{aligned}$$

which is equivalent to the previous system:

$$\frac{\partial W}{\partial X} = 0 \qquad and \qquad \frac{\partial W}{\partial q} = 0$$

2.3. Second Best

The first-best allocation is usually considered not to be implementable. We thus attempt to maximize the welfare subject to the constraint that firms profits is positive. The Lagrangien can be written as:

$$L = \mathcal{U}(X, q) - C(X, q) + \lambda [pX - C(X, q)]$$

The first order conditions are now:

$$\begin{aligned} 0 &= \lambda X + (1 + \lambda)(p - C_X)\frac{\partial X}{\partial p} \\ 0 &= \frac{\partial \mathcal{U}}{\partial q} - (1 + \lambda) C_q - \lambda X \left(\frac{\partial X}{\partial q}\right) / \left(\frac{\partial X}{\partial p}\right) \end{aligned}$$

hence, one gets back the standard Ramsey formula:

$$\frac{p - C_X}{p} = \frac{\lambda}{1 + \lambda}\frac{1}{\epsilon_{X_p}}.$$

The quality level is defined by a "mixed" of the previous equation:

$$-\lambda X \left(\frac{\partial X}{\partial q}\right) / \left(\frac{\partial X}{\partial p}\right) + \frac{\partial \mathcal{U}}{\partial q} = (1 + \lambda) C_q$$

So that, for the particular example introduced above, the second equation boils down again to $-g'(q) X = C_q$. As a result, the optimum choice is defined by the system:

$$\begin{aligned} \frac{p - \mathcal{C}_K}{p} &= \frac{\lambda}{1 + \lambda}\frac{1}{\epsilon_{X_p}} \\ \frac{v}{2f} &= (1 - \varepsilon_{\mathcal{C}_K})\frac{\mathcal{C}(K)}{K} \end{aligned}$$

2.4. Price (and quality)-cap

Assume now that the firm maximizes its profits under a generalised cap:

$$\begin{aligned} \pi &= pX(p,q) - C(X,q) \\ s.t & \qquad \alpha p - \gamma q \leq \bar{p} \end{aligned}$$

hence first order conditions are

$$\begin{aligned} X + (p - C_X)\frac{\partial X}{\partial p} + \mu\alpha &= 0 \\ (p - C_X)\frac{\partial X}{\partial q} - C_q - \mu\gamma &= 0 \end{aligned}$$

that is, with the standard notations:

$$\begin{aligned} \frac{p - C_X}{p} &= \left(1 + \mu\frac{\alpha}{X}\right)\frac{1}{\varepsilon_{X_p}} \\ -\mu\gamma - (X + \mu\alpha)\left(\frac{\partial X}{\partial q}\right) / \left(\frac{\partial X}{\partial p}\right) &= C_q \end{aligned}$$

Take $\alpha = X^*$, $\gamma = -\partial U(X^*, q^*)/\partial q$ and $\mu = -1/(1+\lambda)$ where X^* and q^* are the optimal values. The generalised cap imposed to the firm induces a decentralisation of the second-best. Remark that, for this to be the case, the weights α, γ and $\bar{p}$ (which is chosen so that profits are zero so that μ takes the right value) are imposed to the firm. In the example already introduced, this constraint can be written

$$\left(p + \frac{v}{2f}\right)X \leq \bar{p}.$$

3. The general model

We consider now an heterogeneous population with individuals characterised by a (multidimensional) parameter $\boldsymbol{\theta}$. With linear tariffs, the utility function $\mathcal{U}_\theta(x,q)$ give rise to an individual demand $x_\theta(p,q)$ thus an aggregate demand

$$X(p,q) = \int x_\theta(p,q)\, f(\theta)\, d\theta$$

where $f(\boldsymbol{\theta})$ is the density of individuals with characteristic $\boldsymbol{\theta}$. With two-part tariffs, one gets a similar expression. The price p would now be substituted by the marginal tariff b and the participation constraint will require in addition that :

$$\mathcal{U}_\theta(x,q) - (A + bx) \geq 0$$

where A is the access price.

In this section we will first derive again the monopolist choices, the first and second best and a decentralisation mechanism for the later in presence of (multidimensional) heterogeneity. We extend then the results when the firm uses two part tariffs. In order to fits with the transportation sector, we will consider the case where the firm offers a menu of choices to the consumers. A traveller may either pay the price p or access to special discounted tariff b (frequent travellers program) if she pays an access cost A. Thus, in order to set prices and quality the monopolist has to take into account of the incentive constraint. Namely, travellers choose to join the frequent traveller program iff

$$\max_x \{\mathcal{U}_\theta(x,q) - (A + bx)\} \geq \max_x \{\mathcal{U}_\theta(x,q) - px\}$$

Again, all along the paper, the formula are computed for the transportation example introduced at the beginning.

3.1. Unregulated Monopolist

The F.O.C. that follow from the maximisation of profits are unchanged when facing an heterogeneous population:

$$\begin{aligned} \frac{p - C_X}{p} &= \frac{1}{\varepsilon_{X_p}} \\ -q\left(\frac{\partial X}{\partial q}\right) / \left(\frac{\partial X}{\partial p}\right) &= \varepsilon_{C_q}\frac{C}{X} \end{aligned}$$

With the specific functional forms $\mathcal{U}_\theta(x,q) = \beta U(x) - \nu x g(q)$ with $g(q) = 1/2f$ and $C = f\mathcal{C}(K)$ one gets again

$$\begin{aligned} p &= \frac{\varepsilon_{\mathcal{C}_K}}{\varepsilon_{X_p} - 1} \frac{\mathcal{C}(K)}{K} \\ \frac{\hat{v}}{2f} &= \left(1 - \varepsilon_{\mathcal{C}_K}\right) \frac{\mathcal{C}(K)}{K} \end{aligned}$$

where $\hat{v}$ is a weighted sum of the value of time defined by:

$$\hat{v} = \int_{\underline{\nu}}^{\overline{\nu}} \int_{\underline{\beta}}^{\overline{\beta}} \nu \frac{\partial x_{\beta,\nu}/\partial p}{\partial X/\partial p} f(\beta,\nu)\, d\beta d\nu$$

3.2. First best

In the same manner, the maximisation of social welfare

$$W = \int \mathcal{U}_\theta(x,q) f(\theta)\, d\theta - C(X,q)$$

leads to an identical set of conditions:

$$\begin{aligned} p &= \varepsilon_{C_X} \frac{C}{X} \\ -\frac{q}{X} \frac{\partial \mathcal{U}(X,q)}{\partial q} &= \varepsilon_{C_q} \frac{C}{X} \end{aligned}$$

With the specific functional form introduced previously:

$$\begin{aligned} p &= \varepsilon_{\mathcal{C}_K} \frac{\mathcal{C}(K)}{K} \\ \frac{\bar{\nu}}{2f} &= \left(1 - \varepsilon_{\mathcal{C}_K}\right) \frac{\mathcal{C}(K)}{K} \end{aligned}$$

where $\bar{\nu}$ is the mean value of time over the travellers (not the population).

$$\bar{\nu} = \int_{\underline{\nu}}^{\overline{\nu}} \int_{\underline{\beta}}^{\overline{\beta}} \nu \frac{x_{\beta,\nu}}{X} f(\beta,\nu)\, d\beta d\nu$$

3.3. Second Best

The first-best allocation is usually not considered to be implementable since it would imply negative profits for the firm. Like for the homogeneous population, the second-best allocation is defined by:

$$\frac{p - C_X}{p} = \frac{\lambda}{1+\lambda} \frac{1}{\epsilon_{X_p}}$$

$$-\lambda X\left(\frac{\partial X}{\partial q}\right)/\left(\frac{\partial X}{\partial p}\right)+\frac{\partial \mathcal{U}}{\partial q} \quad = \quad (1+\lambda)\,C_q$$

In the particular case considered, this system can be rewritten as:

$$\begin{aligned} \frac{p-\mathcal{C}_K}{p} &= \frac{\lambda}{1+\lambda}\frac{1}{\epsilon_{X_p}} \\ \frac{\tilde{\upsilon}}{2f} &= \left(1-\varepsilon_{\mathcal{C}_K}\right)\frac{\mathcal{C}\,(K)}{K} \end{aligned}$$

where now the value of time to be used is defined as:

$$\tilde{\upsilon}=\left(\frac{\bar{\nu}+\lambda\hat{\upsilon}}{1+\lambda}\right)$$

3.4. Price (and quality)-cap

Given that the formula defining the second-best are identical in the case of an homogeneous population and an heterogeneous one, there is no question as whether it can be decentralised. However, it is far from obvious that the weights that forms the generalised cap present a value that can be computed by the regulator under reasonable assumptions. Suppose that the firm maximizes its profits under the generalised cap proposed here:

$$\begin{aligned} \pi \quad &= \quad pX\,(p,q)-C\,(X,q) \\ s.t \quad . \quad \alpha p-\gamma h\,(q) &\le \quad \bar{p} \end{aligned}$$

where $h\,(q)$ is an increasing function of q. It is easy to establish that the weights $\alpha = X$, $\gamma = -\left(\partial\mathcal{U}/\partial q\right)/h'\,(q)$ and $\mu = -1/\left(1+\lambda\right)$ would induce a decentralisation of the second-best. Of interest is the fact that, in the example introduced above, the constraint can be written:

$$\left(p+\frac{\bar{\nu}}{2f}\right)X \le \bar{p}$$

where $\bar{\nu}$ is the average value of time defined above.

3.5. The two-parts tariffs model

In this section, we assume that consumers may benefit from a discounted price $b < p$ by paying a fee A to access this special tariff. A possibility often encountered to better match the demand of big clients (frequent travellers). This possibility will be used by consumers such that:

$$\max_x \{\mathcal{U}_\theta (x, q) - (A + bx)\} \geq \max_x \{\mathcal{U}_\theta (x, q) - px\}$$

Denote x_θ^+ the consumption of a consumer that choose to access the discounted price and x_θ^- the consumption if she pays standard price p. The hypersurface of critical consumer is defined by

$$H (A, b, p, q, \theta) = \mathcal{U}_\theta \left(x_\theta^+, q\right) - \mathcal{U}_\theta \left(x_\theta^-, q\right) + px_\theta^- - \left(A + bx_\theta^+\right) = 0$$

If $\mathcal{U}_\theta (x, q)$ is strictly monotonic in each component of the vector of characteristics $\boldsymbol{\theta} = (\theta_1, ..., \theta_K)$ this hypersurface ∂S split the set of parameters in two parts corresponding to both categories of consumers. Denote by S^- and S^+ the corresponding sets of parameters to get:

$$\begin{aligned}
X^- (A, b, p, q) &= \int_{S^-} x_\theta^- (p, q) f (\theta) d\theta \\
X^+ (A, b, p, q) &= \int_{S^+} x_\theta^+ (b, q) f (\theta) d\theta \\
N^- (A, b, p, q) &= \int_{S^-} f (\theta) d\theta \\
N^+ (A, b, p, q) &= \int_{S^+} f (\theta) d\theta
\end{aligned}$$

3.5.1. Unregulated Monopolist

Profits are given by

$$\pi = pX^- + AN^+ + bX^+ - C (X, q)$$

hence first order conditions give rise to the system:

$$\begin{aligned}
\frac{\partial \pi}{\partial A} &= N^+ + A\frac{\partial N^+}{\partial A} + (p - C_X) \frac{\partial X^-}{\partial A} + (b - C_X) \frac{\partial X^+}{\partial A} \\
\frac{\partial \pi}{\partial b} &= A\frac{\partial N^+}{\partial b} + (p - C_X) \frac{\partial X^-}{\partial b} + X^+ + (b - C_X) \frac{\partial X^+}{\partial b} \\
\frac{\partial \pi}{\partial p} &= A\frac{\partial N^+}{\partial p} + X^- + (p - C_X) \frac{\partial X^-}{\partial p} + (b - C_X) \frac{\partial X^+}{\partial p} \\
\frac{\partial \pi}{\partial q} &= A\frac{\partial N^+}{\partial q} + (p - C_X) \frac{\partial X^-}{\partial q} + (b - C_X) \frac{\partial X^+}{\partial q} - C_q
\end{aligned}$$

Profit-maximizing access price A **:** The price A is a fixed charge. As a result, it has no impact on marginal consumption. The only effect of a marginal change in A is the change in the set of critical consumers ∂S since some shift from one category to the other. This flux Φ_A^N is given by

$$\Phi_A^N = \int_{\partial S} \left(\frac{\partial \boldsymbol{\theta}}{\partial A} \cdot \mathbf{n} \right) f(\theta)\, ds$$

where $\frac{\partial \boldsymbol{\theta}}{\partial A}$ is the vector of marginal change in the parameter of the critical consumer, $\mathbf{n}$ is the unitarian normal vector of the hyper-surface ∂S:

$$\mathbf{n} = \frac{\frac{\partial H}{\partial \boldsymbol{\theta}}}{\left\| \frac{\partial H}{\partial \boldsymbol{\theta}} \right\|}$$

Remark that ∂S is defined by $H(A, b, p, q, \boldsymbol{\theta}) = 0$ hence

$$\frac{\partial H}{\partial A} + \frac{\partial H}{\partial \boldsymbol{\theta}} \cdot \frac{\partial \boldsymbol{\theta}}{\partial A} = 0$$

As a result:

$$\begin{aligned} \frac{\partial N^+}{\partial A} &= -\int_{\partial S} \left(\frac{\partial \boldsymbol{\theta}}{\partial A} \cdot \mathbf{n} \right) f(\theta)\, ds = \int_{\partial S} \frac{\partial H}{\partial A} \frac{f(\theta)}{\left\| \frac{\partial H}{\partial \boldsymbol{\theta}} \right\|} ds \\ &= -\int_{\partial S} \frac{f(\theta)}{\left\| \frac{\partial H}{\partial \boldsymbol{\theta}} \right\|} ds \end{aligned}$$

so that the density is re-normalised by a coefficient $\left\| \frac{\partial H}{\partial \boldsymbol{\theta}} \right\|$.

Similarly, one can establish that:

$$\begin{aligned} \frac{\partial X^-}{\partial A} &= \int_{\partial S} \left(\frac{\partial \boldsymbol{\theta}}{\partial A} \cdot \mathbf{n} \right) x_\theta^-(p, q)\, f(\theta)\, ds = \int_{\partial S} x_\theta^-(p, q) \frac{f(\theta)}{\left\| \frac{\partial H}{\partial \boldsymbol{\theta}} \right\|} ds \\ \frac{\partial X^+}{\partial A} &= -\int_{\partial S} \left(\frac{\partial \boldsymbol{\theta}}{\partial A} \cdot \mathbf{n} \right) x_\theta^+(b, q)\, f(\theta)\, ds = -\int_{\partial S} x_\theta^+(b, q) \frac{f(\theta)}{\left\| \frac{\partial H}{\partial \boldsymbol{\theta}} \right\|} ds \end{aligned}$$

Define as $\widetilde{x_\theta^+}$ the (generalised) mean consumption over ∂S according to this corrected density:

$$\widetilde{x_\theta^+} = \frac{\int_{\partial S} \frac{1}{\left\| \frac{\partial H}{\partial \theta} \right\|} x_\theta^+(b, q)\, f(\theta)\, ds}{\int_{\partial S} \frac{1}{\left\| \frac{\partial H}{\partial \theta} \right\|} f(\theta)\, ds}$$

and similarly

$$\widetilde{x_\theta^-} = \frac{\int_{\partial S} \frac{1}{\left\|\frac{\partial F}{\partial \theta}\right\|} x_\theta^- (p,q) f(\theta) \, ds}{\int_{\partial S} \frac{1}{\left\|\frac{\partial F}{\partial \theta}\right\|} f(\theta) \, ds}$$

The first order condition can be rewritten as:

$$\begin{aligned} \frac{\partial \pi}{\partial A} &= N^+ + A\frac{\partial N^+}{\partial A} + (p - C_X) \frac{\partial X^-}{\partial A} + (b - C_X) \frac{\partial X^+}{\partial A} \\ &= N^+ + \left[A + (b - C_X) \widetilde{x_\theta^+} - (p - C_X) \widetilde{x_\theta^-} \right] \frac{\partial N^+}{\partial A} \end{aligned}$$

As a result, we find a generalisation of the "standard" mark-up formula:

$$\frac{A + (b - C_X) \widetilde{x_\theta^+} - (p - C_X) \widetilde{x_\theta^-}}{A} = -\frac{N^+}{A\frac{\partial N^+}{\partial A}} = \frac{1}{\varepsilon_{N_A}}$$

The mark-up is equal to the inverse elasticity of demand where the "marginal cost" of production is substituted here by the marginal benefit of one consumer accessing to this tariff.

Profit-maximizing marginal price b **:** A variation in the marginal price b induce a variation in the set of critical consumer ∂S that is *a priori* different from the variation induced by the marginal change of A. The flux of consumers leaving the special program Φ_b^N is given by

$$\Phi_b^N = \int_{\partial S} \left(\frac{\partial \boldsymbol{\theta}}{\partial b} \cdot \mathbf{n} \right) f(\theta) \, ds$$

where $\frac{\partial \boldsymbol{\theta}}{\partial b}$ is the vector of marginal change in the parameter of the critical consumer, $\mathbf{n}$ is the unitarian normal vector of the hyper-surface ∂S. By using

$$\frac{\partial H}{\partial b} + \frac{\partial H}{\partial \boldsymbol{\theta}} \cdot \frac{\partial \boldsymbol{\theta}}{\partial b} = 0$$

one can write

$$\begin{aligned} \frac{\partial N^+}{\partial b} &= -\int_{\partial S} \left(\frac{\partial \boldsymbol{\theta}}{\partial b} \cdot \mathbf{n} \right) f(\theta) \, ds = \int_{\partial S} \frac{\partial H}{\partial b} \frac{f(\theta)}{\left\| \frac{\partial H}{\partial \boldsymbol{\theta}} \right\|} ds \\ &= -\int_{\partial S} \frac{x_\theta^+ (b,q)}{\left\| \frac{\partial H}{\partial \boldsymbol{\theta}} \right\|} f(\theta) \, ds \\ &= \widetilde{x_\theta^+} \frac{\partial N^+}{\partial A} \end{aligned}$$

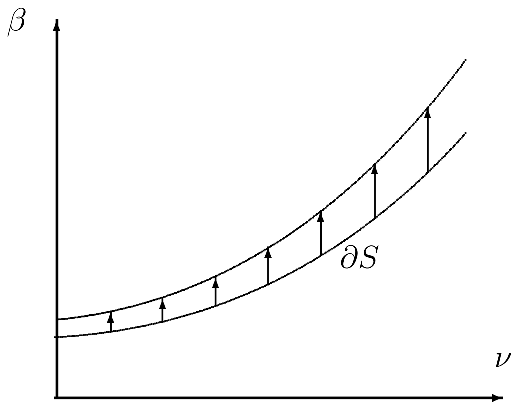
so that the density is re-normalised by a coefficient $x_\theta^+(b,q) / \left\| \frac{\partial H}{\partial \boldsymbol{\theta}} \right\|$.

This leads us to introduce now the mean consumption over ∂S according to this modified density:

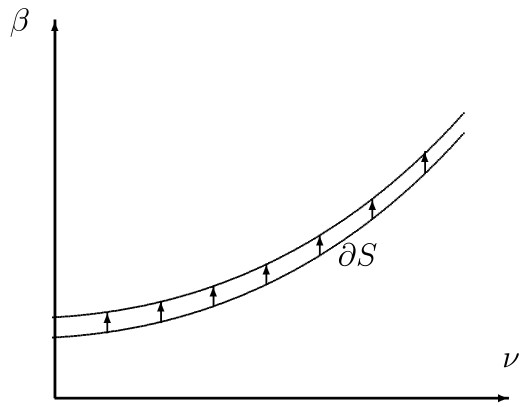
$$\overline{x_\theta^+}^{\wedge} = \frac{\int_{\partial S} x_\theta^+(b,q) \frac{x_\theta^+(b,q)}{\left\| \frac{\partial F}{\partial \theta} \right\|} f(\theta)\, ds}{\int_{\partial S} \frac{x_\theta^+(b,q)}{\left\| \frac{\partial F}{\partial \theta} \right\|} f(\theta)\, ds}$$

and similarly

$$\overline{x_\theta^-}^{\wedge} = \frac{\int_{\partial S} x_\theta^-(p,q) \frac{x_\theta^+(b,q)}{\left\| \frac{\partial F}{\partial \theta} \right\|} f(\theta)\, ds}{\int_{\partial S} \frac{x_\theta^+(b,q)}{\left\| \frac{\partial F}{\partial \theta} \right\|} f(\theta)\, ds}$$



Marginal changes following an increase of A.



Marginal changes following an increase of b.

The marginal changes in the consumption of X can now be rewritten as follows:

$$\begin{aligned}
\frac{\partial X^-}{\partial b} &= \int_{\partial S} \left(\frac{\partial \boldsymbol{\theta}}{\partial b} \cdot \mathbf{n} \right) x_\theta^-(p,q)\, f(\theta)\, ds \\
&= \overline{x_\theta^-}^{\wedge}\, \widetilde{x_\theta^+} \frac{\partial N^+}{\partial A} \\
\frac{\partial X^+}{\partial b} &= \int_{S^+} \left(\frac{\partial x_\theta^+(b,q)}{\partial b} \right) f(\theta)\, d\theta - \int_{\partial S} \left(\frac{\partial \boldsymbol{\theta}}{\partial b} \cdot \mathbf{n} \right) x_\theta^+(b,q)\, f(\theta)\, ds \\
&= \frac{\partial \breve{X}^+}{\partial b} + \overline{x_\theta^-}^{\wedge}\, \widetilde{x_\theta^+} \frac{\partial N^+}{\partial A}
\end{aligned}$$

Denote $\Upsilon^{\wedge}$ the ratio of the average return realised on a marginal consumer leaving the low consumer group to join the high consumer group when b decreases over the same average return, when this shift is obtained by a decrease of A:

$$\Upsilon^{\wedge} = \frac{A + (b - C_X)\,\overline{x_\theta^+}^{\wedge} - (p - C_X)\,\overline{x_\theta^-}^{\wedge}}{A + (b - C_X)\,\widetilde{x_\theta^+} - (p - C_X)\,\widetilde{x_\theta^-}}$$

One can now rewrite the first-order condition in b as follows:

$$\begin{aligned}
\frac{\partial \pi}{\partial b} &= A\frac{\partial N^+}{\partial b} + (p - C_X)\frac{\partial X^-}{\partial b} + X^+ + (b - C_X)\frac{\partial X^+}{\partial b} \\
&= \left[A + (b - C_X)\,\overline{x_\theta^+}^{\wedge} - (p - C_X)\,\overline{x_\theta^-}^{\wedge}\right]\widetilde{x_\theta^+}\frac{\partial N^+}{\partial A} + X^+ + (b - C_X)\frac{\partial \breve{X}^+}{\partial b} \\
&= -\Upsilon^{\wedge}\,\widetilde{x_\theta^+} N^+ + X^+ + (b - C_X)\frac{\partial \breve{X}^+}{\partial b}
\end{aligned}$$

to get a generalised form of the “classical” mark-up formula:

$$\frac{b - C_X}{b} = \left(1 - \Upsilon^{\wedge}\,\frac{\widetilde{x_\theta^+}}{X^+/N^+}\right)\frac{1}{\varepsilon_{X_b^+}}$$

where

$$\varepsilon_{X_b^+} = -\frac{b}{X^+}\frac{\partial \breve{X}^+}{\partial b}$$

Profit-maximizing marginal price p : A variation in the marginal price p leads us again to a specific computation for the flux of consumers joining the special tariffs:

$$\begin{aligned}
\frac{\partial N^+}{\partial p} &= \int_{\partial S}\left(\frac{\partial \boldsymbol{\theta}}{\partial p}\cdot \mathbf{n}\right) f(\theta)\,ds = -\int_{\partial S}\frac{\partial H}{\partial p}\frac{f(\theta)}{\left\|\frac{\partial H}{\partial \boldsymbol{\theta}}\right\|}ds \\
&= -\int_{\partial S}\frac{x_\theta^-(p,q)}{\left\|\frac{\partial H}{\partial \boldsymbol{\theta}}\right\|} f(\theta)\,ds \\
&= \widetilde{x_\theta^-}\frac{\partial N^+}{\partial A}
\end{aligned}$$

so that the density is re-normalised by a coefficient $x_\theta^-(b,q)/\left\|\frac{\partial H}{\partial \boldsymbol{\theta}}\right\|$. We introduce now the mean consumption over ∂S according to this later density:

$$\overline{x_\theta^+}^{\vee} = \frac{\int_{\partial S} x_\theta^+(b,q)\,\frac{x_\theta^-(p,q)}{\left\|\frac{\partial F}{\partial \theta}\right\|} f(\theta)\,ds}{\int_{\partial S}\frac{x_\theta^-(p,q)}{\left\|\frac{\partial F}{\partial \theta}\right\|} f(\theta)\,ds}$$

and similarly

$$\overline{x_\theta^-}^{\vee} = \frac{\int_{\partial S} x_\theta^-(p,q)\,\frac{x_\theta^-(p,q)}{\left\|\frac{\partial F}{\partial \theta}\right\|} f(\theta)\,ds}{\int_{\partial S}\frac{x_\theta^-(p,q)}{\left\|\frac{\partial F}{\partial \theta}\right\|} f(\theta)\,ds}$$

This gives in turn:

$$\begin{aligned}
\frac{\partial X^-}{\partial p} &= \int_{S^-}\left(\frac{\partial x_\theta^-(p,q)}{\partial p}\right) f(\theta)\,d\theta - \int_{\partial S}\left(\frac{\partial \boldsymbol{\theta}}{\partial p}\cdot \mathbf{n}\right) x_\theta^-(p,q)\,f(\theta)\,ds \\
&= \frac{\partial \breve{X}^-}{\partial p} - \overline{x_\theta^-}^{\vee}\,\widetilde{x_\theta^-}\,\frac{\partial N^+}{\partial A} \\
\frac{\partial X^+}{\partial p} &= \int_{\partial S}\left(\frac{\partial \boldsymbol{\theta}}{\partial b}\cdot \mathbf{n}\right) x_\theta^-(b,q)\,f(\theta)\,ds \\
&= \overline{x_\theta^+}^{\vee}\,\widetilde{x_\theta^-}\,\frac{\partial N^+}{\partial A}
\end{aligned}$$

We now introduce $\Upsilon^\vee$ the ratio of the average return realised on a marginal consumer leaving the low consumer group to join the high consumer group when p increases over the same average return, when this shift is obtained by a decrease of A:

$$\Upsilon^\vee = \frac{A + (b - C_X)\,\overline{x_\theta^+}^{\vee} - (p - C_X)\,\overline{x_\theta^-}^{\vee}}{A + (b - C_X)\,\widetilde{x_\theta^+} - (p - C_X)\,\widetilde{x_\theta^-}}$$

to get:

$$\begin{aligned}
\frac{\partial \pi}{\partial p} &= A\frac{\partial N^+}{\partial p} + X^- + (p - C_X)\,\frac{\partial X^-}{\partial p} + (b - C_X)\,\frac{\partial X^+}{\partial p} \\
&= \left[A + (b - C_X)\,\overline{x_\theta^+}^{\vee} - (p - C_X)\,\overline{x_\theta^-}^{\vee}\right]\widetilde{x_\theta^-}\,\frac{\partial N^+}{\partial A} + X^- + (p - C_X)\,\frac{\partial \breve{X}^-}{\partial p} \\
&= -\Upsilon^\vee\,\widetilde{x_\theta^-}\,N^+ + X^- + (p - C_X)\,\frac{\partial \breve{X}^-}{\partial p}
\end{aligned}$$

to get the formula:

$$\frac{p - C_X}{p} = \left(1 - \frac{N^+}{N^-}\Upsilon^\vee\,\frac{\widetilde{x_\theta^-}}{X^-/N^-}\right)\frac{1}{\varepsilon_{X_p^-}}$$

Choice of quality by the profit-maximiser: In order to study the impact of quality change on the set of critical consumers, we use again the fundamental relation:

$$\frac{\partial H}{\partial q} + \frac{\partial H}{\partial \boldsymbol{\theta}}\cdot\frac{\partial \boldsymbol{\theta}}{\partial q} = 0$$

where

$$H(A,b,p,q,\theta) = \mathcal{U}_\theta\left(x_\theta^+, q\right) - \mathcal{U}_\theta\left(x_\theta^-, q\right) + p x_\theta^- - \left(A + b x_\theta^+\right) = 0$$

As a result:

$$\begin{aligned}
\frac{\partial N^{+}}{\partial q} &= -\int_{\partial S}\left(\frac{\partial \boldsymbol{\theta}}{\partial q}\cdot \mathbf{n}\right) f(\theta)\,ds = \int_{\partial S}\frac{\partial H}{\partial q}\frac{f(\theta)}{\left\|\frac{\partial H}{\partial \boldsymbol{\theta}}\right\|}ds \\
&= \int_{\partial S}\left[\frac{\partial \mathcal{U}_{\theta}\left(x_{\theta}^{+},q\right)}{\partial q}-\frac{\partial \mathcal{U}_{\theta}\left(x_{\theta}^{-},q\right)}{\partial q}\right]\frac{f(\theta)}{\left\|\frac{\partial H}{\partial \boldsymbol{\theta}}\right\|}ds \\
&= \int_{\partial S} f^{U}(\theta)\,ds
\end{aligned}$$

where

$$f^{U}(\theta)=\left[\frac{\partial \mathcal{U}_{\theta}\left(x_{\theta}^{+},q\right)}{\partial q}-\frac{\partial \mathcal{U}_{\theta}\left(x_{\theta}^{-},q\right)}{\partial q}\right]\frac{f(\theta)}{\left\|\frac{\partial H}{\partial \boldsymbol{\theta}}\right\|}$$

The impact of quality on demand is given by:

$$\begin{aligned}
\frac{\partial X^{-}}{\partial q} &= \int_{S^{-}}\frac{\partial x_{\theta}^{-}(p,q)}{\partial q}f(\theta)\,d\theta+\int_{\partial S}\left(\frac{\partial \boldsymbol{\theta}}{\partial q}\cdot \mathbf{n}\right)x_{\theta}^{-}(p,q)\,f(\theta)\,ds \\
&= \frac{\partial \breve{X}^{-}}{\partial q}-\int_{\partial S}x_{\theta}^{-}(p,q)\,f^{U}(\theta)\,ds \\
\frac{\partial X^{+}}{\partial q} &= \int_{S^{+}}\frac{\partial x_{\theta}^{+}(p,q)}{\partial q}f(\theta)\,d\theta-\int_{\partial S}\left(\frac{\partial \boldsymbol{\theta}}{\partial q}\cdot \mathbf{n}\right)x_{\theta}^{+}(b,q)\,f(\theta)\,ds \\
&= \frac{\partial \breve{X}^{+}}{\partial q}+\int_{\partial S}x_{\theta}^{+}(b,q)\,f^{U}(\theta)\,ds
\end{aligned}$$

Define the average consumption x_{θ}^{+U} and x_{θ}^{-U}over ∂S and the coefficient Υ^{U} as follows:

$$\begin{aligned}
x_{\theta}^{+U} &= \frac{\int_{\partial S}x_{\theta}^{+}(b,q)\,f^{U}(\theta)\,ds}{\int_{\partial S}f^{U}(\theta)\,ds} \\
x_{\theta}^{-U} &= \frac{\int_{\partial S}x_{\theta}^{-}(p,q)\,f^{U}(\theta)\,ds}{\int_{\partial S}f^{U}(\theta)\,ds} \\
\Upsilon^{U} &= \frac{A+(b-C_{X})\,x_{\theta}^{+U}-(p-C_{X})\,x_{\theta}^{-U}}{A+(b-C_{X})\,\widetilde{x_{\theta}^{+}}-(p-C_{X})\,\widetilde{x_{\theta}^{-}}}
\end{aligned}$$

to get the first order condition:

$$\begin{aligned}
\frac{\partial \pi}{\partial q} &= A\frac{\partial N^{+}}{\partial q}+(p-C_{X})\frac{\partial X^{-}}{\partial q}+(b-C_{X})\frac{\partial X^{+}}{\partial q}-C_{q} \\
&= \left[A+(b-C_{X})\,x_{\theta}^{+U}-(p-C_{X})\,x_{\theta}^{-U}\right]\frac{\partial N^{+}}{\partial q}+(p-C_{X})\frac{\partial \breve{X}^{-}}{\partial q}+(b-C_{X})\frac{\partial \breve{X}^{+}}{\partial q}-C_{q} \\
&= -\varpi\Upsilon^{U}N^{+}+(p-C_{X})\frac{\partial \breve{X}^{-}}{\partial q}+(b-C_{X})\frac{\partial \breve{X}^{+}}{\partial q}-C_{q}
\end{aligned}$$

where the coefficient measure the relative impact of quality versus access price on the partition of consumers:

$$\varpi = \frac{\partial N^{+}}{\partial q} / \frac{\partial N^{+}}{\partial A}$$

Consider now both first order conditions on the marginal prices:

$$\begin{aligned} 0 &= -\Upsilon^{\wedge} \widetilde{x_{\theta}^{+}} N^{+} + X^{+} + (b - C_X) \frac{\partial \check{X}^{+}}{\partial b} \\ 0 &= -\Upsilon^{\vee} \widetilde{x_{\theta}^{-}} N^{+} + X^{-} + (p - C_X) \frac{\partial \check{X}^{-}}{\partial p} \end{aligned}$$

to get:

$$0 = -\varpi \Upsilon^{U} N^{+} - \left[-\Upsilon^{\vee} \widetilde{x_{\theta}^{-}} N^{+} + X^{-} \right] \frac{\partial \check{X}^{-}}{\partial q} / \frac{\partial \check{X}^{-}}{\partial p} - \left[-\Upsilon^{\wedge} \widetilde{x_{\theta}^{+}} N^{+} + X^{+} \right] \frac{\partial \check{X}^{+}}{\partial q} / \frac{\partial \check{X}^{+}}{\partial b} - C_q$$

that can be rearranged by analogy with previous formula as:

$$-\varpi \Upsilon^{U} q \frac{N^{+}}{X} - q \left(\frac{\partial X}{\partial q} / \frac{\partial X}{\partial p} \right)^{agg} = \varepsilon_{C_q} \frac{C}{X}$$

where

$$X \left(\frac{\partial X}{\partial q} / \frac{\partial X}{\partial p} \right)^{agg} = \left[1 - \Upsilon^{\vee} \frac{N^{+}}{N^{-}} \frac{\widetilde{x_{\theta}^{-}}}{X^{-}/N^{-}} \right] X^{-} \frac{\partial \check{X}^{-}}{\partial q} / \frac{\partial \check{X}^{-}}{\partial p} + \left[1 - \Upsilon^{\wedge} \frac{\widetilde{x_{\theta}^{+}}}{X^{+}/N^{+}} \right] X^{+} \frac{\partial \check{X}^{+}}{\partial q} / \frac{\partial \check{X}^{+}}{\partial b}$$

3.5.2. Second best

While the first-best is unchanged when allowing non-linear tariffs, the possibility of two-part tariffs should allow to improve in maximizing welfare subject to the break-even constraint. The Lagrangien can be written as:

$$\begin{aligned} L &= W + \lambda \pi = \int \mathcal{U}_{\theta}(x, q) f(\theta) d\theta - C(X, q) + \lambda \pi \\ &= \int_{S^{-}} \left[\mathcal{U}_{\theta}(x, q) - p x_{\theta}^{-} \right] f(\theta) d\theta + \int_{S^{+}} \left[\mathcal{U}_{\theta}(x, q) - \left(A + b x_{\theta}^{+} \right) \right] f(\theta) d\theta + (1 + \lambda) \pi \end{aligned}$$

where profits are given by

$$\pi = pX^{-} + AN^{+} + bX^{+} - C(X, q)$$

to give, thanks to the condition $H \equiv 0$ along ∂S, the F.O.C.

$$\begin{aligned}
\frac{\partial L}{\partial A} &= -N^{+} + (1+\lambda)\frac{\partial \pi}{\partial A} \\
\frac{\partial L}{\partial b} &= -X^{+} + (1+\lambda)\frac{\partial \pi}{\partial b} \\
\frac{\partial L}{\partial p} &= -X^{-} + (1+\lambda)\frac{\partial \pi}{\partial p} \\
\frac{\partial L}{\partial q} &= -\int \frac{\partial \mathcal{U}_{\theta}(x,q)}{\partial q} f(\theta)\, d\theta + (1+\lambda)\frac{\partial \pi}{\partial q}
\end{aligned}$$

where the first derivative of profits with respect to the decision variables are those computed above:

$$\begin{aligned}
\frac{\partial \pi}{\partial A} &= N^{+} + \left[A + (b - C_X)\widetilde{x_{\theta}^{+}} - (p - C_X)\widetilde{x_{\theta}^{-}}\right]\frac{\partial N^{+}}{\partial A} \\
\frac{\partial \pi}{\partial b} &= \Upsilon^{\wedge}\widetilde{x_{\theta}^{+}}\left[A + (b - C_X)\widetilde{x_{\theta}^{+}} - (p - C_X)\widetilde{x_{\theta}^{-}}\right]\frac{\partial N^{+}}{\partial A} + X^{+} + (b - C_X)\frac{\partial \check{X}^{+}}{\partial b} \\
\frac{\partial \pi}{\partial p} &= \Upsilon^{\vee}\widetilde{x_{\theta}^{-}}\left[A + (b - C_X)\widetilde{x_{\theta}^{+}} - (p - C_X)\widetilde{x_{\theta}^{-}}\right]\frac{\partial N^{+}}{\partial A} + X^{-} + (p - C_X)\frac{\partial \check{X}^{-}}{\partial p} \\
\frac{\partial \pi}{\partial q} &= \Upsilon^{U}\varpi\left[A + (b - C_X)\widetilde{x_{\theta}^{+}} - (p - C_X)\widetilde{x_{\theta}^{-}}\right]\frac{\partial N^{+}}{\partial A} + (p - C_X)\frac{\partial \check{X}^{-}}{\partial q} + (b - C_X)\frac{\partial \check{X}^{+}}{\partial q} - C_q
\end{aligned}$$

Optimal access price A **:** The optimal marginal price is defined by:

$$\frac{\partial L}{\partial A} = \lambda N^{+} + (1+\lambda)\left[A + (b - C_X)\widetilde{x_{\theta}^{+}} - (p - C_X)\widetilde{x_{\theta}^{-}}\right]\frac{\partial N^{+}}{\partial A}$$

with the notations given above. Thus, one gets the following generalisation of the Ramsey formula:

$$\frac{A + (b - C_X)\widetilde{x_{\theta}^{+}} - (p - C_X)\widetilde{x_{\theta}^{-}}}{A} = \frac{\lambda}{1+\lambda}\frac{1}{\varepsilon_{N_A}}$$

Optimal marginal prices: The first-order condition following the maximisation with respect to the marginal prices are:

$$\begin{aligned}
\frac{\partial L}{\partial b} &= -X^{+} + (1+\lambda)\left[\Upsilon^{\wedge}\widetilde{x_{\theta}^{+}}\left[A + (b - C_X)\widetilde{x_{\theta}^{+}} - (p - C_X)\widetilde{x_{\theta}^{-}}\right]\frac{\partial N^{+}}{\partial A} + X^{+} + (b - C_X)\frac{\partial \check{X}^{+}}{\partial b}\right] \\
\frac{\partial L}{\partial p} &= -X^{-} + (1+\lambda)\left[\Upsilon^{\vee}\widetilde{x_{\theta}^{-}}\left[A + (b - C_X)\widetilde{x_{\theta}^{+}} - (p - C_X)\widetilde{x_{\theta}^{-}}\right]\frac{\partial N^{+}}{\partial A} + X^{-} + (p - C_X)\frac{\partial \check{X}^{-}}{\partial b}\right]
\end{aligned}$$

Since at the second-best:

$$\lambda N^{+}+(1+\lambda)\left[A+(b-C_{X})\widetilde{x_{\theta}^{+}}-(p-C_{X})\widetilde{x_{\theta}^{-}}\right]\frac{\partial N^{+}}{\partial A}=0$$

one gets:

$$\begin{aligned}\lambda X^{+}\left(1-\Upsilon^{\wedge}\frac{\widetilde{x_{\theta}^{+}}}{X^{+}/N^{+}}\right)+(1+\lambda)(b-C_{X})\frac{\partial\breve{X}^{+}}{\partial b} &= 0\\ \lambda X^{-}\left(1-\frac{N^{+}}{N^{-}}\Upsilon^{\vee}\frac{\widetilde{x_{\theta}^{-}}}{X^{-}/N^{-}}\right)+(1+\lambda)(p-C_{X})\frac{\partial\breve{X}^{-}}{\partial b} &= 0\end{aligned}$$

to get again generalised forms of the Ramsey formula.

Optimal quality: The optimal quality level when the firm is submit to the break-even constraint is:

$$\begin{aligned}0 &= -\int\frac{\partial\mathcal{U}_{\theta}(x,q)}{\partial q}f(\theta)\,d\theta-(1+\lambda)C_{q}\\ &\quad+(1+\lambda)\Upsilon^{U}\varpi\left[A+(b-C_{X})\widetilde{x_{\theta}^{+}}-(p-C_{X})\widetilde{x_{\theta}^{-}}\right]\frac{\partial N^{+}}{\partial A}\\ &\quad+(1+\lambda)\left[(p-C_{X})\frac{\partial\breve{X}^{-}}{\partial q}+(b-C_{X})\frac{\partial\breve{X}^{+}}{\partial q}\right]\end{aligned}$$

hence, by using the other F.O.C.:

$$\begin{aligned}0 &= -\int\frac{\partial\mathcal{U}_{\theta}(x,q)}{\partial q}f(\theta)\,d\theta-(1+\lambda)C_{q}\\ &\quad-\lambda\Upsilon^{U}\varpi N^{+}-\lambda X^{+}\left(1-\Upsilon^{\wedge}\frac{\widetilde{x_{\theta}^{+}}}{X^{+}/N^{+}}\right)-\lambda X^{-}\left(1-\frac{N^{+}}{N^{-}}\Upsilon^{\vee}\frac{\widetilde{x_{\theta}^{-}}}{X^{-}/N^{-}}\right)\end{aligned}$$

3.5.3. Implementation by price-and-quality cap

Although the computations of two-part tariffs enhances that optimal mark-up are inversely related to the elasticity of demand, the various weights and "correction" over the density of types rise again the question of the possibility to decentralise this optimal allocation. We will assume that now that the firm maximizes its profits under a generalised cap:

$$\alpha_{A}A+\alpha_{b}b+\alpha_{p}p-\gamma q\leq\bar{p}$$

the first order conditions are now:

$$\begin{aligned}
\frac{\partial L}{\partial A} &= N^{+} + \left[A + (b - C_X)\,\widetilde{x_\theta^{+}} - (p - C_X)\,\widetilde{x_\theta^{-}}\right]\frac{\partial N^{+}}{\partial A} + \mu\alpha_A \\
\frac{\partial L}{\partial b} &= \Upsilon^{\wedge}\,\widetilde{x_\theta^{+}}\left[A + (b - C_X)\,\widetilde{x_\theta^{+}} - (p - C_X)\,\widetilde{x_\theta^{-}}\right]\frac{\partial N^{+}}{\partial A} + X^{+} + (b - C_X)\,\frac{\partial \breve{X}^{+}}{\partial b} + \mu\alpha_b \\
\frac{\partial L}{\partial p} &= \Upsilon^{\vee}\,\widetilde{x_\theta^{-}}\left[A + (b - C_X)\,\widetilde{x_\theta^{+}} - (p - C_X)\,\widetilde{x_\theta^{-}}\right]\frac{\partial N^{+}}{\partial A} + X^{-} + (p - C_X)\,\frac{\partial \breve{X}^{-}}{\partial p} + \mu\alpha_p \\
\frac{\partial L}{\partial q} &= \Upsilon^{U}\varpi\left[A + (b - C_X)\,\widetilde{x_\theta^{+}} - (p - C_X)\,\widetilde{x_\theta^{-}}\right]\frac{\partial N^{+}}{\partial A} + (p - C_X)\,\frac{\partial \breve{X}^{-}}{\partial q} + (b - C_X)\,\frac{\partial \breve{X}^{+}}{\partial q} - C_q - \mu
\end{aligned}$$

By taking the exogeneous weights $\alpha_A = N^{+}$, $\alpha_b = X^{+}$ and $\alpha_p = X^{-}$ with a cap $\bar{p}$ such that $\mu = -1/(1+\lambda)$, one finds again that the three F.O.C. deriving from the maximisation in prices are verified. In order to induce the firm to choose the optimal quality level, it is sufficient to impose furthermore

$$\gamma = -\partial\mathcal{U}/\partial q = -\int \frac{\partial \mathcal{U}_\theta(x,q)}{\partial q} f(\theta)\,d\theta$$

Of particular interest is the fact that, despite the complexity of the substitution effects, these weights are relatively simple to compute. In fact, in the particular case where

$$\mathcal{U}_\theta(x,q) = \beta U(x) - \nu x g(q)$$

one gets indeed:

$$\begin{aligned}
\gamma &= -\int \frac{\partial \mathcal{U}_\theta(x,q)}{\partial q} f(\theta)\,d\theta \\
&= g'(q)\left[\int_{S^{+}} \nu\, x_\theta^{+} f(\theta)\,d\theta + \int_{S^{-}} \nu\, x_\theta^{-}\, f(\theta)\,d\theta\right] \\
&= -\bar{\nu}\, g'(q)
\end{aligned}$$

so that the generalised cap is:

$$AN^{+} + bX^{+} + pX^{-} + \frac{\bar{\nu}}{2f} \leq \bar{p}$$

4. References

- Laffont, J.-J., E. Maskin, and J.-C. Rochet, (1987). "Optimal non-linear pricing with two-dimensional characteristics". In *Information, Incentives and Economic Mechanisms: In Honor of L. Hurwicz,* ed. T. Groves, R. Radner, and S. Reiter. Minneapolis: University of Minnesota Press.

- Laffont, J.-J. and J. Tirole, (1993). *A Theory of Incentives in Procurement and Regulation.* Cambridge, MA: MIT Press.

- Spence, A. M., (1975). "Monopoly, quality and regulation". *Bell Journal of Economics* **6**: pp.417-429.

- Vickers, J. and G. Yarrow, (1988). *Privatization: An Economic Analysis.* Cambridge, MA: MIT Press