Strengths of the "Weakest Link"?

Philippe Février*and Laurent Linnemer[†][‡]

January 2002

Abstract

The TV game show the "Weakest Link" involves a kind of paradox. To increase the probability of winning, contestants should eliminate the strongest player. However, if it is anticipated that the best player is to be eliminated, then no one answers and there is nothing to gain. In the game show, players do answer to the questions: does it reveal a lack of rationality? We solve a simple game that illustrates the Weakest Link tradeoffs to show that two kinds of equilibria may coexist: equilibria in which players stay mute and entertaining equilibria in which they give good answers whenever they can. Empirically, we study the first wave of the Weakest Link show broadcasted in France. Contestants vote against the weakest link and answer truthfully to the questions. However, they exhibit myopic behavior as they do not use all the available information. More generally, a coordination problem arises in the voting game. The "mise en scène" of the show prevents the contestants from voting against the strongest and thus the show can go on.

Keywords: Television, game show, vote, field experiment.
***JEL* Classification:** C93, C72, D72, D80.

---

[*] INSEE and CREST-LEI-GRECSTA.

[†] CREST-LEI-GRECSTA and University of Lille 2, linnemer@ensae.fr. Mail address: LEI-ENPC, 28, rue des Saints Pères, 75007 Paris, France.

[‡] We are very grateful to Bernard Caillaud, David Ettinger, Philippe Jéhiel, Anne Perrot and Michael Visser for their useful remarks and especially to Thierry Magnac for meaningful insights.

1 Introduction

The recipe of a good quiz show usually involves one, two or more contestants, questions, a dash of luck and a smack of knowledge. The winner is the one who performed best in answering questions. This general framework embraces games like "Who Wants to Be a Millionaire?", "Jeopardy", "The Price is Right", "The Wheel of Fortune" and many others all around the air. "The Weakest Link " is a new game show from the U.K. which differs in many respects from other shows. First, contestants do not play against one another but behave like a team: any good answer may increase the winning prize. Next, after each round of questions, teammates vote to eject one of their numbers, until the team is reduced to two. Finally, the winner is the one among the remaining two who performs best in a final series of questions. Last but not least, the show is also unusual because of the insistence with which the presenter (a woman most of the time)[1] bosses people around and tries to humiliate the candidates. According to BBC Online[2]: Anne Robinson is "a cross between Cruella de Vil, a dominatrix and a bossy school ma'am..." and that she "has also earned the title of the Rudest Woman on Television."

The voting game inside the Weakest Link creates some kind of a puzzle. Let us try to solve informally the game by backward induction: at the very end the best of the two remaining players wins with a higher probability. To enter this two-player final game the three remaining contestants have to vote one of them out. Who should be singled out? Obviously the strongest![3] Because both weakest players should vote against him/her in order to increase their chance to win the endgame. But how do they know who is the best? As contestants do not know each other before the show, the only way to learn something about the abilities of one player is to look how well he/she answers questions. Therefore if one player is always wrong he/she becomes the best opponent for the final round and nobody would vote him/her out! Contrary to the "You are the Weakest Link, goodbye" which punctuates each elimination, the weakest player should not be eliminated. But if there were an advantage of being thought as the weakest it would imply that in equilibrium

---

[1] Anne Robinson on BBC and NBC, Laurence Boccolini on TF1.

[2] See `http://www.bbc.co.uk/weakestlink/index.shtml`

[3] This is the first reaction that someone has (at least if he/she is an economist!) when watching the show. Moreover it is an equilibrium. Even without describing a game, consider the following strategies: do not answer right to the questions, if someone did, vote against him. If no one answered right, then vote randomly.

no player should answer any question before the endgame!

Of course, such a game would not be very popular and obviously, in real life, people do answer right to some questions. Does it mean that players do not fully understand the strategic implications of the voting game? One could imagine that they have a bounded rationality which does not allow them to link the questions with the vote. In fact, in the show, another component gives them an incentive to answer well when they can. When there is only two players left, a last team game is played by them before the final encounter that decides who takes all the money. Moreover, the gains accumulated during this last team game are multiplied by three, which gives an extra importance to this round. Every player (even if he/she is almost sure to lose in the endgame) should do his/her best as there is no strategic reason to hide his/her capabilities at this stage. Therefore, the higher the abilities of both players are the higher is the gain. So in the three-player voting game there is a tradeoff between voting against the strongest to increase the probability to win in the endgame and voting against the weakest to increase the amount gained in the final team game. However, as shown by the theoretical model, even when this 8-round bonus incentive is not large enough so that people would like to vote against the strongest, a surprising equilibrium exists where the weakest player is eliminated.

Therefore, it is interesting to look in detail what players do in the real show. More generally, it is an opportunity to confront game-theoretical predictions with the behavior of motivated individuals. When economists want to see how well game theory explains people behavior, they have two options: either construct a laboratory experiment or find real-life data. Most of the time, real-life data are not enough detailed to be useful. But when they are, their advantage over laboratory experiments is twofold: first people were not in an had-hoc environment, second monetary incentives can be much larger. For instance, in this paper we compare theoretical predictions with French data on the Weakest Link show. To generate such data in a laboratory would have cost more than 1 million of FF. Many economists have seek such data and game shows[4] have been used in several occasions. In some recent papers, data from game shows are used as a natural experiment to test behavior toward risk (Gertner (1993) and Metrick (1995)) or rationality

---

[4] More generally, the economics of television game shows is not to be neglected. These shows are numerous in all countries and the amount at stake can be huge both in terms of what the participants can win and in terms of advertising revenues for the diffusing channels.

(Berk, Hughson, and Vandezande (1996) and Bennett and Hickman (1993)). In Walker and Wooders (2001) data from professional tennis games are used to study mixed-strategy play and in Chiappori, Levitt, and Groseclose (2002) a similar approach is developed based on penalty kicks in soccer.

Three main questions are at stake about the Weakest Link game show: First, from a theoretical point of view how should contestants play in this game? Next, from an empirical point of view how do contestants play in this game? Finally do contestants play like predicted by theory? Is the behavior of the players in line with a Nash equilibrium or do they obviously play in a manner that is contradictory with rational behavior? In particular do they understand the link between the quiz game and the voting game? To answer these questions we built the simplest game that seemed to get the gist of the Weakest Link tradeoffs. To compare theoretical predictions with facts, we built a database containing an historic of all 36 Weakest Link shows broadcasted in France during the 2001 summer.

Our results are the following: On the theoretical side the game (while simple) exhibits multiple equilibria. In particular one equilibrium in which no one answers right to any question (we call it a "mute equilibrium") and (for many parameter values) one in which people answer right if they can (we call it a "truthful equilibrium") and where the weakest player is eliminated. In consequence, there is a coordination problem on one equilibrium or the other. On the empirical side contestants answer truthfully and the weakest link is eliminated most of the time. Moreover for parameter values close to the real game, both the mute and the truthful equilibrium coexist. *Therefore it seems that the "mise en scène" of the show helps players to coordinate themselves on the most entertaining equilibrium.*[5]

The paper is organized as follows: in section 2 we briefly present how the game developed and what are the rules. Section 3 introduces a simple way to modelize the Weakest Link as a game which equilibria are characterized in section 4. Section 5 describes salient empirical observation that help to understand how people played in the Weakest Link game show in France and confronts the theoretical analysis to the empirical facts. Section 6 concludes.

---

[5] In particular, just before the teammates vote, Robinson shrilly demands: "Who is not up to the standard of the rest of you? Who has lost the plot? Who is least likely to cope with the pressure? Who has stopped you from reaching a decent target? Who is not up to the standard of the team?" Even if it is just "cheap talk" and players can pick who they want, these sentences point in the direction of a focal point in the voting game.

2 History and rules of the Weakest Link

The Weakest Link appeared on BBC Two in the summer of 2000 with 68 daily episodes. Such was its success that BBC One commissioned seven champions' league episodes and one hilarious bad losers' show. These prime time shows hit U.K.'s screens in the autumn. The Weakest Link continued to make a huge impact, and for the second series a staggering 90 episodes were commissioned for daytime and 21 shows for primetime. After its success in the U.K. the game has been exported to other countries. In the U.S. NBC broadcasts the show since April 2001 with Anne Robinson as presenter. The show is also transmitted (with home presenters) in Australia, Belgium, France, Germany, Ireland, Italy, the Netherlands, New-Zealand, Turkey and probably many others.

The structure of the game remains the same across channels but the number of players can vary: for instance 9 in the BBC Two original shows, 7 in subsequent BBC One shows, 8 in NBC shows and 9 in TF1 (France) shows. Contestants stand in a circle around the host. They are strangers to each others but must work as a team to reach the maximum prize money. Each player is asked a general knowledge question, working clockwise around the circle of players. The goal in a round is for the team to answer enough questions correctly to reach the money target within the time limit. Each round ends when players run out of time[6] or reach the money target for that round. Players should take some risks to win more. Imagine that every correct answer is another link on a chain. Each new correct answer increases the value of the chain.[7] If someone get a question wrong he breaks the chain and the team must start building a new chain, starting at zero. But if someone says the word "Bank" before his/her question is asked, the money is safe and a brand new chain starts from zero.

At the end of each round, each teammate votes to eliminate a fellow contestant (officially the one they consider to be the Weakest Link in the chain but they are free to vote against anyone). The player who obtains

---

[6] For the first round 2 minutes and 30 seconds are allocated to the players. Every next round is 10 seconds shorter than the last one. For instance, round 2 lasts 2 minutes and 20 seconds and round 8 lasts 1 minute and 20 seconds.

[7] A one correct answer chain is worth 300 FF, a two correct answer chain 750, and the following are respectively worth 1 500, 3 000, 4 500, 7 500, 9 000, 12 000, and 15 000 FF. For instance, if a player says "bank" after his/her three preceding partners answered right 1 500 FF are won. A round ends if the team earned 15 000 FF.

the more votes is eliminated. In case of a tie, a crucial rule applies which we call the "strongest-link rule". This rule states that the player who is the "strongest-link" chooses among the tied contestants the one he wants out. The "strongest-link" is the player who has the highest rate of good answers.[8]

When only two players remain, they answer a last series of questions (round 8) which allows them to win more money. The amount won in this round is multiplied by three (or only two in some countries) and added to the previous wins. Finally to determine the winner of this jackpot each player answers five questions winning one point for a good answer. The winner is the one who scored best. If after the five questions they have the same score, then they continue answering questions until one answers well while the other is wrong. The winner takes all the jackpot and the losers have nothing.

3 Model

The main strategic effects present inside the Weakest Link can be summarize in the following **four stage three person game**. Each player can be either strong or weak. A strong player knows the answer of a question with probability θ_H, while a weak player knows it with probability θ_L with $0 < \theta_L < \theta_H < 1$. Each player knows his/her own strength but ignores the ability of the others. He/She only knows that a contestant is weak with probability ρ and strong with probability $1 - \rho$. **In the first stage**, one question is asked sequentially to each player. Questions and answers are public. For instance, player 3 (to whom the last question is asked) knows before answering his/her question if player 1 and 2 answered well or not. If a contestant knows the answer he/she decides to give the right answer or to give a wrong answer. Let h denote the history of the responses[9] for instance $h = (1, 0, 0)$ means that the first player answered correctly while the second and third gave a wrong answer. Let $m = f(h)$ denote the accumulated gain for an history h, where f is an increasing function in the number of good answers, with $f((0, 0, 0)) \geq 0$. At the end of stage one, the player who performed best is said to be the "strongest link". In case of equal performances,

---

[8] In case of equality, the "strongest-link" is the one who put more money in the bank. If equality persists the "strongest-link" is randomly selected. In a symmetric way, a "weakest link" is defined but the main difference is that he/she has no prerogative unless being stigmatized by the presenter.

[9] Formally, $h \in \{(0,0,0), (1,0,0), (0,1,0), (0,0,1), (1,1,0), (1,0,1), (0,1,1), (1,1,1)\}$.

one player is randomly chosen among the best ones but (as in the real game) his/her identity is hidden until after the vote. **Stage two** implements a vote. Player vote simultaneously against one of their opponents. The contestant who receives more votes is eliminated. In case every player receives one vote it is to the strongest link to decides who is ejected.[10] **In stage three** the remaining two players try to accumulate the more money they can in the jackpot by answering correctly to questions. At this stage, the expected[11] amount to add in the jackpot is a function $g(\theta_i, \theta_j)$ depending on both abilities. In particular let G_{LL} (resp. G_{LH} and resp. G_{HH}) denote the expected gain when both players are weak (resp. one is weak the other strong and resp. both are strong). It is assumed that (on average) more able players win more, that is, $G_{LL} < G_{LH} < G_{HH}$. **Stage four** determines who wins the jackpot. More precisely, a weak (resp. strong) player wins the jackpot against another weak (resp. strong) player with probability one half, and a weak (resp. strong) player wins against a strong (resp. weak) player with probability K (resp. $1 - K$) with $0 < K < \frac{1}{2}$.

The resolution of the game goes backward. Obviously the best strategy in stages 4 and 3 is to give the right answer every time one knows it. Stage 2 (the voting game) is more tedious. Let x denote the probability of being weak that one player attributes to another one. To understand players' incentives we have to calculate players' expected gains just before the vote. If a player is weak his/her expected gain if he/she reaches the endgame against a player who is weak with probability x is:

$$G_L(x) = \frac{1}{2} x (G_{LL} + m) + K (1 - x) (G_{LH} + m),$$

while if he/she is strong the expected gain is:

$$G_H(x) = (1 - K) x (G_{LH} + m) + \frac{1}{2} (1 - x) (G_{HH} + m).$$

As the above expressions show, we can (without loss of generality) normalize

---

[10] The strongest link can eliminate the player he voted against or any other player.

[11] That is before questions are asked.

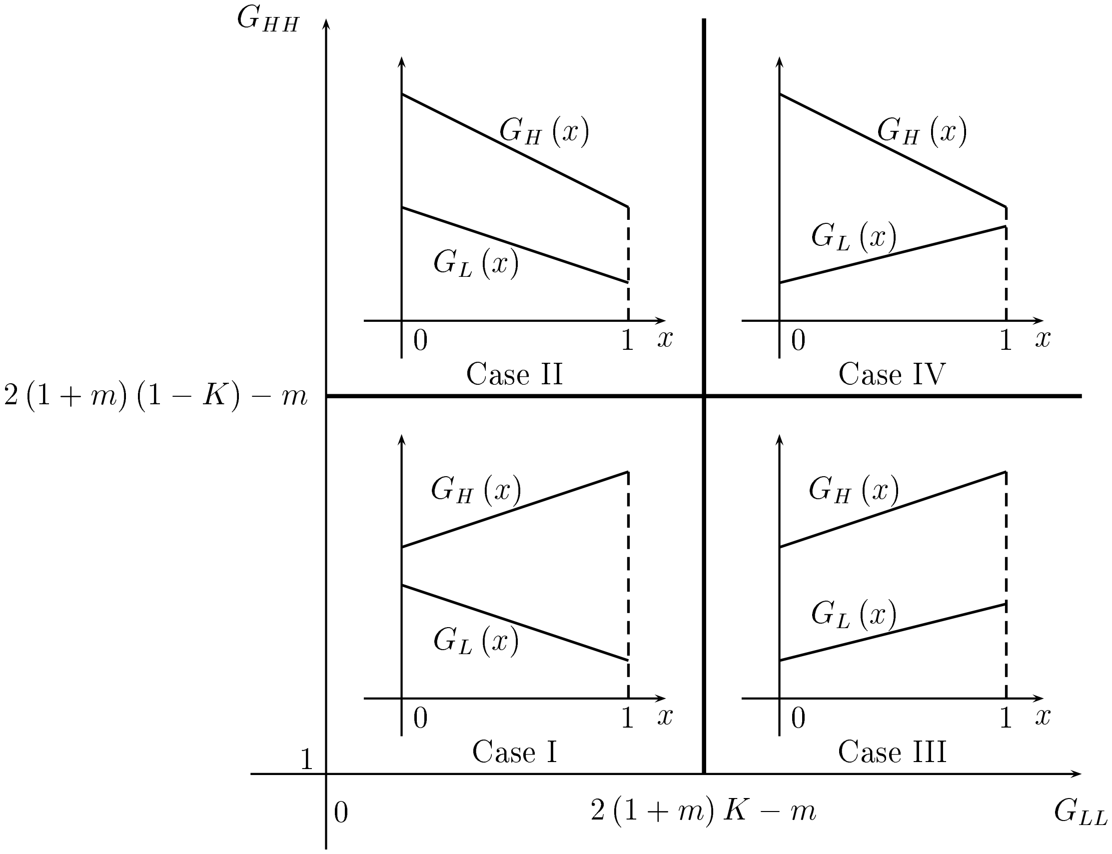

Figure 1: Expected gain for a strong and a weak player

G_{LH} to 1 which leads to the following expected gains[12]:

$$G_L\,(x) = (1+m)\,K + \frac{1}{2}\left[G_{LL} - (2\,(1+m)\,K - m)\right]x,$$
$$G_H\,(x) = \frac{1}{2}\,(G_{HH}+m) + \frac{1}{2}\left[(2\,(1+m)\,(1-K) - m) - G_{HH}\right]x.$$

Both $G_L\,(x)$ and $G_H\,(x)$ are linear functions of x but depending on the parameter values they can be either increasing or decreasing with the probability x that one's opponent in the endgame is weak. However it is always true that $G_H\,(x) > G_L\,(x)$.[13] Figure 1 shows these functions in the (G_{LL}, G_{HH}) space. Depending on the parameter values four cases can occur. When the slope of $G_q\,(x)$ ($q = H$, L) is increasing (resp. decreasing), that means that a player of type q prefers to play the endgame against a type L (resp. H).

First, in the area $0 \leq G_{LL} \leq 2\,(1+m)\,K - m$ and $1 \leq G_{HH} \leq 2\,(1+m)\,(1-K) - m$, which is called case I, players prefer dissimilarity. That is, a strong player

---

[12] To save on notations we do not change the name of the variables after the normalization.

[13] Indeed, it is easy to check that $G_{HH} > 1$ and $K < 1/2$ imply $G_H\,(0) > G_L\,(0)$ and that $G_{LL} < 1$ and $1-K > 1/2$ imply $G_H\,(1) > G_L\,(1)$.

prefers to play the endgame against a weak opponent[14] while a weak player prefers to be matched with a strong player.[15]

Second, in the area $0 \leq G_{LL} \leq 2(1+m)K - m$ and $2(1+m)(1-K) - m \leq G_{HH}$ (denoted case II), players are pro-strong. Indeed, both type of player prefer to play against a strong opponent.

Third, if $2(1+m)K - m \leq G_{LL}$ and $1 \leq G_{HH} \leq 2(1+m)(1-K) - m$ (case III), players are pro-weak. That is, a weak, as well as a strong player, prefers to play against a weak contestant.

Finally, if $2(1+m)K - m \leq G_{LL}$ and $2(1+m)(1-K) - m \leq G_{HH}$ (case IV), both types prefer similarity. Indeed, a weak player prefers to play against a weak opponent, while a strong one prefers to be matched against a strong one.

It is readily confirmed that $K < 1/2$ implies $2(1+m)K - m < 1$ and $2(1+m)(1-K) - m > 1$. That is, case III and IV always exist. However if K is too low ($K < \frac{m}{1+m}\frac{1}{2}$) or equivalently m is too large ($m > \frac{2K}{1-2K}$), then case I and II disappear as $2(1+m)K - m < 0$.

The intuition is that in case II the equilibrium of the vote subgame should be one where the weakest is eliminated (and therefore one where everyone does his/her best to answer right). On the contrary, in case III the strongest is expected to be eliminated in equilibrium (and therefore it is expected that no one answers correctly). However this intuition is only partially true because of the voting game. Indeed, in case III, (surprisingly) equilibria exist where the strongest player is not eliminated. The following complete-information game helps to understand this unexpected result, which relies on the "strongest-link rule". Three players denoted A, B, C, vote in order to eliminate one of them. Player A is a strong player while both B and C are weak. Assume then that A is the strongest-link. As in case three, B and C prefer to play one against the other rather than to play against A. For instance,[16] $U_B(BC) = U_C(BC) = 2$ if A is eliminated. And $U_B(AB) = U_C(AC) = 1$ if A is not eliminated. The utility of an eliminated player is zero. Others payoffs are not useful to show that the following strategies form a Nash equilibrium : B votes against C, C votes against B and A randomly votes against B or C. In case of a tie, A eliminates the player who voted

---

[14] Indeed, G_{HH} is not sufficiently larger than $G_{LH} = 1$ and a type H prefers to maximize the probability of winning rather than the gain in case of a win.

[15] For a type L, G_{LL} is too low compared to $G_{LH} = 1$ and a type L sacrifices the probability of winning in order to win more in case of a win.

[16] Let XY denote the winners, $X, Y \in \{A, B, C\}, X \neq Y$.

against him. If B (resp. C) deviates to vote against A it only creates a tie which leads to his/her elimination. Of course there also exists a more "intuitive" equilibrium where B and C vote against A. The point is that two issues are at stake: on the one hand individual reward matters but on the other hand there is a coordination problem in the voting game.

The above argument extends to our model. In the next section we characterize under which conditions a truthful equilibrium exists (that is an equilibrium where every players answer correctly if he can) and when a mute equilibrium exists (that is an equilibrium where no player answer correctly even when he/she knows the answer).

4 Equilibria

As it is usual in signaling games we should expect to find many equilibria. Moreover our game is a multi-sender multi-receiver game followed by a voting game which should lead to an even greater richness. We restrict our attention to establish when one of the two "extreme" equilibria exist.

Proposition 1. *A mute (perfect bayesian) equilibrium does always exist.*

Proof. The equilibrium strategies are the following: no one answers correctly. If no one gives a correct answer, players believe (using Bayes rule) that each of his/her opponent is weak with probability ρ and vote randomly. If some one deviates and gives a good answer, Bayes rule does not apply and we suppose that the other players do not change their a priori beliefs but vote against him/her. It is readily confirmed that the above strategies form a Nash equilibrium of the game for any value of the parameters. ☐

The existence of a mute equilibrium is both intuitive and striking! It is intuitive and it was our fear when we first show the game. In particular, it seems to be the natural issue of the game when parameters lie in case III (that is when both types are pro-weak). However this equilibrium is striking in case II because for this range of parameter values, both types are pro-strong. In this case, the mute equilibrium depends crucially on the out-of-equilibrium beliefs. Finally, this equilibrium is at odd with observation because players do answer questions!

Next, we characterize when truthful equilibrium exists. Assume that everyone tries to answer correctly. Let λ_0 (resp. λ_1) denote the probability of being weak after a wrong (resp. good) answer.[17] If it is an equilibrium of the voting game to vote against the (apparent) weakest, then it is straightforward that a truthful equilibrium exists. Therefore, the analysis reduces to the analysis of the voting game.

Proposition 2. *A truthful (perfect bayesian) equilibrium does exist if* $G_L(\lambda_1) \geq \frac{3}{4}G_L(\lambda_0)$ *and* $G_H(\lambda_1) \geq \frac{3}{4}G_H(\lambda_0)$*.*

Proof. The equilibrium strategies are the following: everyone answers correctly if he/she knows the answer. In the voting game, four situations can occur. First, everyone answered correctly. Players vote randomly. Second, only one player gave a wrong answer. This player votes randomly with probability $\frac{1}{2}$ against each others, and the others vote against him/her. In case of a tie the strongest-link votes against the other good answerer. Third, two players gave a wrong answer. They vote against each other and the third player votes randomly. In case of a tie, the strongest-link votes against the one that voted against him/her. Fourth, no one answered correctly. Players vote randomly.

It is readily confirmed that the above strategies form a Nash equilibrium of the game for any value of the parameters in cases one, three and four. Case two has to be examined in details.

The player who did not answer has no incentive to deviate. Consider A a good answerer of type T, $T = L, H$. He/she can vote against the wrong answerer (denoted C) which gives him/her an expected gain of:

$$G_T(\lambda_1)$$

Or he/she can vote against the other good answerer (denoted B) which gives him/her an expected gain of:

$$\frac{1}{2}G_T(\lambda_0) + \frac{1}{2}\frac{1}{2}G_T(\lambda_0) = \frac{3}{4}G_T(\lambda_0)$$

Indeed, with probability $\frac{1}{2}$ both A and C vote against B, while with probability $\frac{1}{2}$ a tie occurs as A votes against B, B against C and C against A. In

---

[17] Formally,

$$\lambda_0 = \frac{\rho(1-\theta_L)}{\rho(1-\theta_L) + (1-\rho)(1-\theta_H)} \text{ and } \lambda_1 = \frac{\rho\theta_L}{\rho\theta_L + (1-\rho)\theta_H}$$

that case, A is the strongest link with probability $\frac{1}{2}$ and eliminates B, while B is the strongest link with probability $\frac{1}{2}$ and eliminates A.

Therefore, player A does not deviate if and only if :

$$G_T(\lambda_1) \geq \frac{3}{4} G_T(\lambda_0)$$

Finally, if the above condition is true, players vote against the (apparent) weakest which induces them to answer truthfully. ☐

When is the condition $G_T(\lambda_1) \geq \frac{3}{4} G_T(\lambda_0)$ $(T = L, H)$ satisfied? First, this condition is true when λ_0 is close enough to λ_1 which occurs when the observation of a good or a bad answer is not very informative on players' type. That is either when types are very similar (θ_L close to θ_H) or when the population is "dominated" by one type (ρ close to 0 or ρ close to 1). Next, when λ_0 and λ_1 are not close to each other, this condition (for $T = L$) can still be true if G_{LL} is low enough and (for $T = H$) if G_{HH} is large enough, so both type of players are willing to play against a strong opponent. Corollary 1 makes these existing conditions more precise. Let $\overline{G_{LL}} = 2K\frac{1+3\lambda_0-4\lambda_1}{3\lambda_0-4\lambda_1}$ and $\underline{G_{HH}} = 2(1-K)\frac{3\lambda_0-4\lambda_1}{1+3\lambda_0-4\lambda_1}$.

Corollary 1. *If λ_0 and λ_1 are relatively close, then a truthful equilibrium exists in the extended game for any values of the expected gains G_{HH} and G_{LL}. If not, then a truthful equilibrium exists if G_{HH} is large enough and/or G_{LL} is low enough. More formally:*

1. *If $\lambda_0 \leq \frac{4}{3}\lambda_1 + \frac{2}{3}\frac{K}{1-2K}$, then a truthful (perfect bayesian) equilibrium exists.*

2. *If $\frac{4}{3}\lambda_1 + \frac{2}{3}\frac{K}{1-2K} \leq \lambda_0 \leq \frac{4}{3}\lambda_1 + \frac{1}{3}\frac{1}{2(1-2K)-1}$, then a truthful (perfect bayesian) equilibrium exists for any $G_{HH} \geq 1$ and $G_{LL} \leq (1+m)\overline{G_{LL}} - m$.*

3. *If $\frac{4}{3}\lambda_1 + \frac{1}{3}\frac{1}{2(1-2K)-1} \leq \lambda_0$, then a truthful (perfect bayesian) equilibrium exists for any $G_{HH} \geq (1+m)\underline{G_{HH}} - m$ and $G_{LL} \leq (1+m)\overline{G_{LL}} - m$.*

Proof. See appendix A ☐

In particular a consequence of corollary 1 is that if $K \geq \frac{3}{8}$, then $\frac{2}{3}\frac{K}{1-2K} \geq 1$ and therefore λ_0 is always lower than $\frac{4}{3}\lambda_1 + \frac{2}{3}\frac{K}{1-2K}$ and a truthful equilibrium exists for any values of $G_{HH} \geq 1$ and $G_{LL} \leq 1$.

5 Facts

Is one equilibrium of our stylized model a good approximation of what happens in the show game? First, we exhibited two kinds of equilibria: mute ones in which contestants have an incentive to give a wrong answer and truthful ones in which they give the right answer if they know it. In order to learn how people play this game, we watched carefully all game shows broadcasted in France during 2001 summer.[18]

At the beginning of the show, contestants are invited to briefly present themselves by declining their first name and their age.[19] For each show, we noted all the answers of each player (that is if he answered right, wrong and/or if he/she "banked" as well as the votes and if he/she was the strongest or the weakest link). Table 1 presents summary statistics about the 36 shows.[20][21] The show is equally accessible to women and men, and contestants are distributed on a wide range of age (from 18 to 79 with an average of 40). In all 324 people participated. Over all questions, the average rate of good answers is 0.62 (for a rate of good answers round by round see tables 11 to 18 in appendix D). The average winner received about 30 000 FF (with a minimum gain of 10 000 FF and a maximum of about 54 000 FF while the median gain is 28 000 FF). The total amount distributed for the 36 shows

---

[18] That is, all the shows broadcasted in France in the first wave of the game. In all we watched 36 shows. To watch the shows, we used the facilities provided by the Institut National de l'Audiovisuel (INA) and its department l'Inathèque de France located inside the Bibliothèque nationale de France. This section is responsable for the running of legal deposit of radio and television, defined by law on June 20 1992. It gathers all broadcasts of French radio and television programs. We are very grateful to these institutions for their help. TV programs are recorded on DVD so we have had all the opportunity to comfortably watch them.

[19] They also give a residence area (either a town or a province) and a professional occupation. However we did not record these information as they did not seem relevant for our study.

[20] Sexe=0 for a female and sexe=1 for a male, "Gain" is the amount of money in FF won by the winner, "Rate" is the average rate of good answers for all round.

[21] Tables 11 to 18 in appendix D provide the same information round by round both for the excluded player and for the remaining players.

Table 1: Summary statistics

V	Sexe	Age	Rate	Gain
$\overline{V}$	0.52	39.4	0.62	29 267
σ	0.5	14.5	0.21	12 233
min	0	18	0	10 200
max	1	79	1	54 450
N	324	324	324	36

Gains are in FF, 6.55957FF =1€

comes to more than 1 million (1 053 600) of French Francs. Of course, it would not have been conceivable to generate such data in an experimental context.

As expected, age and gender do not seem to determine the identity of the winner. For example, among 36 shows, 45% ($\sigma = 0.5$) have been won by a woman and 47.5% ($\sigma = 0.5$) of the players are women. In the same spirit, the average player is 39 year old ($\sigma = 14$) when the winner is on average 36 year old ($\sigma = 10$). Tables 11 to 18 in appendix D confirm this round by round and table 7 shows that age and gender do not have a significative influence on the probability of being eliminated after any round.

5.1 Who is culled?

A key moment in the game is the vote. How do players coordinate themselves to vote one of them out? Table 2 presents round by round how many votes the player eliminated in round t received in round t' (with $t' \leq t$).[22] For instance, the line beginning by "Eliminated 2" means that, in average, the player eliminated in round 2 received 0.58 votes in round 1 and 3.97 in round 2. Of course, coordination is not perfect and the eliminated player of one round does not systematically receive all the votes. However, coordination is rather good in the sense that the eliminated player receives significantly more votes than the other players. In particular, only the voted out player receives more than one vote in average. Another way to look at the coordination of the voters is to compute the number of time a tie occurred. Table 3 shows that while most of the time no tie happens, the number of ties is not negligible. In particular, in rounds 5 and 6 where a tie occurred in one third of the shows.

---

[22] It never happened that one player votes against him/herself.

Table 2: Number of votes received in average by the players

Round →	R1	R2	R3	R4	R5	R6	R7
Eliminated 1	5.00	-	-	-	-	-	-
Eliminated 2	0.58	3.97	-	-	-	-	-
Eliminated 3	0.53	0.78	4.10	-	-	-	-
Eliminated 4	0.39	0.61	0.75	3.50	-	-	-
Eliminated 5	0.55	0.53	0.55	0.67	2.86	-	-
Eliminated 6	0.50	0.50	0.52	0.47	0.64	2.34	-
Eliminated 7	0.44	0.69	0.55	0.50	0.52	0.50	1.90
Finalist	0.53	0.50	0.25	0.55	0.58	0.72	0.58
Winner	0.39	0.41	0.25	0.30	0.36	0.44	0.52
Total of the votes	9.00	8.00	7.00	6.00	5.00	4.00	3.00

Table 3: Number of ties for each round

Round →	R1	R2	R3	R4	R5	R6	R7
Number of ties	5	8	6	7	12	11	4
Percentage of ties	13.9	22.2	16.7	19.4	33.3	30.6	11.1

A tie underlines the key role played by the contestant who managed to be the strongest link. The advantage of being the strongest link is stressed by table 4 which shows, for each round, the number of times the strongest link has been voted out. Clearly it does not happen very often. The strongest link position seems slightly less confortable in round 7. However, it should be noted that among the 5 cases where the strongest link has been eliminated in round 7, two times all players answered identically well.

Moreover, table 17 of appendix D as well as figure 2 show that in average the eliminated players in round 7 answered correctly to one question over two, while his/her opponents enjoyed a rate of good answers of 68%.

The position of the weakest player is less enviable as shown in table 5. However he/she is far from being systematically eliminated. By symmetry with the definition of the strongest link one can define a unique weakest player.[23]. However, as the official weakest link as no prerogative, in table 5 a player has been counted as the weakest if he had the weakest rate of right

---

[23] This definition is used by the presenter of the show to stigmatize people. But the identity of the weakest link is not revealed before the votes.

Table 4: Elimination or not of the strongest link

	Eliminated	Not eliminated
round 1	0	36
round 2	0	36
round 3	1	35
round 4	2	34
round 5	3	33
round 6	2	34
round 7	5	31

Table 5: Elimination or not of one weakest link

	Eliminated	Not eliminated
round 1	24	12
round 2	21	15
round 3	21	15
round 4	21	15
round 5	23	13
round 6	24	12
round 7	21	15

answers. However, time is a key element of the game and, for example, a player who answers quickly one time over two can be judged as better by his/her fellow contestants than someone who answers slowly two times over three. Another key element is the "bank": one wrong answer can be very harmful to the team if a large amount of money was at stake, while a wrong answer can more easily go unnoticed if a small amount was at stake. Table 6 presents the average number of votes received in each round by a player weaker (resp. stronger) than the mean contestant.[24] In each round, the players that answered less well than the average received more votes than their fellow contestants who answered better than the average. Moreover, descriptive statistics presented in table 4, 5 and 6 show that a contestant who answers well is less likely to be voted out than a player who answers badly. Good answerers are rewarded but not punished by their fellow contestants.

---

[24] That is, a player is in the weak group if his/her rate of good answers is lower (resp. larger) than (for a given show) the average rate of good answers of the round.

Table 6: Average number of votes received by each type

Round →	R1	R2	R3	R4	R5	R6	R7
Weaker players	7.66	6.61	6.14	4.83	3.64	2.92	1.89
Stronger players	1.33	1.39	0.86	1.17	1.36	1.08	1.11
Sum	9.00	8.00	7.00	6.00	5.00	4.00	3.00

A further insight is given by figure 2 where the (average) rate of good answers is plotted for each round and for each category of eliminated player. That is the "out-2" curve shows the average rate of good answers for the players eliminated after round 2. A striking result emerges: the rate of good answers of a player dramatically decreases the very round in which he/she is eliminated. So clearly, the eliminated player has (in average) the worst performance in his/her fatal round.[25] On contrary, the winner always

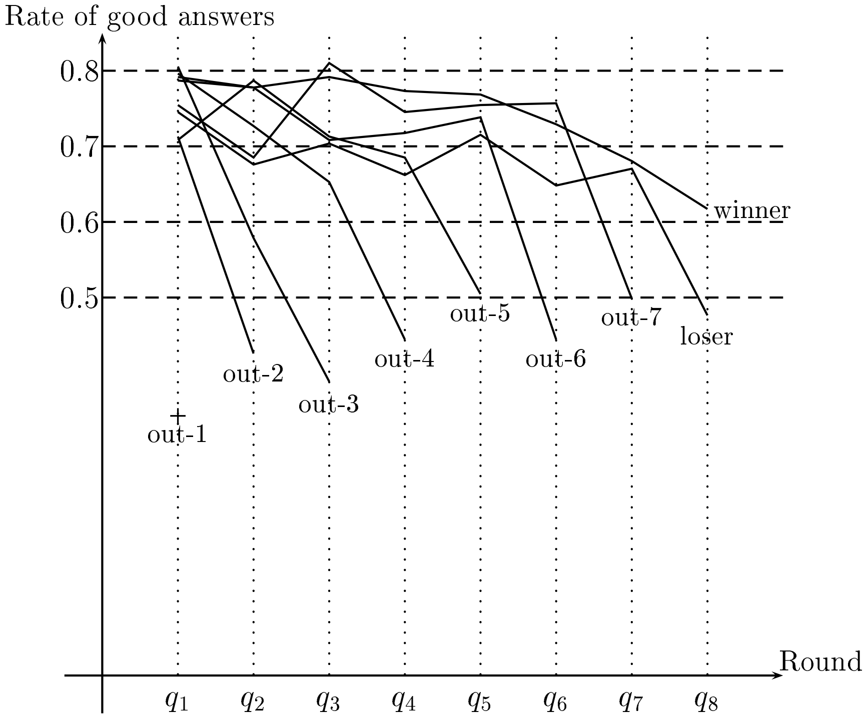

Figure 2: Round by round rate of good answers

---

[25] This fall can be interpreted as a selection bias. If the players vote out the worst answerer, he has to have a significant lower rate of good answers.

performs very well (he/she has, in average, the best rate of good answers in every round). The loser of the final round knows also a dramatic fall in his/her rate of good answers in round 8 from which he/she does not recover in round 9 where he/she loses.[26]

Interestingly, the player eliminated after round 7 seems better than the loser of the final during the first six rounds. This fact may induce people that watch the show to think that the "strongest player" is eliminated before the final round. However, this is not entirely true as the winner of the game is (also) the best one and he/she is not eliminated! And more importantly, the loser of the final performs significantly better than him/her in round 7. To conclude, figure 2 shows that the finalists are the players who did not experience a dramatic fall in their performances.

The previous descriptive analysis is confirmed by the estimation of a multinomial Logit model which shows that **contestants are myopic and that poor answerers are voted out**. In general (following McFadden 1973) when one individual has to chose one alternative among n, the probability that the alternative i is chosen is estimated by a multinomial Logit. In the Weakest Link the team has to chose one player to eliminate among n, which is a very similar situation. The variables used in the modelization are the rates of good answers of each player for each round as well as age and gender. Let n denote the number of players still in the game at the end of round $t = 9 - n + 1$, let r_i denote the history of the rates of good answers for player i, $r_i = (r_{i1}, r_{i2}, \ldots, r_{it})$, let $r = (r_1, \ldots, r_n)$ the history of all players, and finally let a_i denote the age of player i and s_i his/her sex. Let $\beta = (\beta_1, \beta_2, \ldots, \beta_t)$ (resp. α_1, resp. α_2) be the list of t parameters associated to the rates of good answers in rounds 1 to t (resp. age, resp. gender).

The conditional probability p_i of being eliminated, given a certain history r, is:

$$p_i = \frac{e^{\beta.r_i + \alpha_1 a_i + \alpha_2 s_i}}{\sum_{k=1}^{n} e^{\beta.r_k + \alpha_1 a_k + \alpha_2 s_k}}$$

We estimated the value of the parameters β and α by a maximum likelihood method. The results are summarized in table 7 where the significative estimations are typeseted in bold. Clearly age and sex do not influence the

---

[26] The rate of good answers of round 9 are not reported here, as the question are of a different kind in this round.

Table 7: Probability of being eliminated

	round 1	round 2	round 3	round 4	round 5	round 6	round 7
	β (σ)	β (σ)	β (σ)	β (σ)	β (σ)	β (σ)	β (σ)
β_1	−**5.12(1.15)**	−0.63(0.86)	0.73(0.98)	0.84(0.85)	−0.29(0.83)	0.96(1.46)	0.35(1.39)
β_2	-	−**3.8(1.36)**	−**1.99(0.81)**	−0.03(0.87)	0.27(1.07)	0.75(1.24)	−0.10(1.05)
β_3	-	-	−**4.60(1.34)**	−0.97(0.94)	−0.49(0.87)	−0.67(1.01)	1.11(1.02)
β_4	-	-	-	−**3.50(0.93)**	0.13(0.89)	0.24(1.12)	0.71(1.43)
β_5	-	-	-	-	−**4.02(1.11)**	0.99(1.19)	0.26(1.73)
β_6	-	-	-	-	-	−**4.96(1.78)**	1.40(1.14)
β_7	-	-	-	-	-	-	−**3.46(1.45)**
α_1	0.01(0.02)	0.02(0.013)	0.02(0.02)	0.02(0.01)	**0.03(0.01)**	0.03(0.02)	0.00(0.02)
α_2	−0.29(0.44)	0.3(0.48)	−0.15(0.45)	−0.65(0.54)	0.73(0.64)	−0.62(0.59)	−0.22(0.55)

probability (age is only significative in round 5 while sex never is). As the diagonal shows, the probability of being voted out is decreasing with the rate of good answers in the current round. Moreover, (except in round 3) only the rate of good answers in the current round influences the probability which shows that contestants are myopic: In any round a low rate of right answers increases the probability of being eliminated no matter past performances. This is a kind of bounded rationality as the players should use all the available information to evaluate the strength of their opponent.[27] This myopia is maintained by the structure of the game show: in particular all statistics used to stigmatize some players, the definitions of the weakest and strongest links are based on the current round performances.

5.2 Is there a strategic lie?

The structure of the game is such that it could be rewarding not to give a right answer even when one knows it. However our feeling is that it does not happen: **Contestants answer truthfully**. What convinced us of that? *First, casual observation* shows that people are disappointed when they don't give a right answer, that they took time to find an answer,[28] that after their elimination they often say that they couldn't find an easy answer and that they cannot forgive themselves or that they complain another player was clearly weaker and should have been voted out. However, casual observation can be misleading. Our second point is more objective: *The structure of*

---

[27] However, as they are not allowed to take notes, it is only human that they do not remember very well what happened in the previous rounds.

[28] Remember that a fixed number of seconds is given for all the questions of a round, so that if one take more time to answer less time is available for his/her teammates.

the game allows us to perform a statistical test to find if contestants answer truthfully or not. Indeed, during round 8 (when only two players remain) someone who knows the right answer has no incentive to conceal it as there is no further vote. Moreover, gains accumulated during round 8 are multiplied by 3. Therefore we compared the rates of good answers between any two successive rounds for the 2×36 players that reached round 8, for the 36 winners and for the 36 players who lose in round 9.

A statistical test (detailed in appendix B) shows that the rates are not statistically different between round t and $t+1$ for $1 \leq t \leq 6$. Either for the winners, or the losers or the pair. However q_7 and q_8 are statistically different for the population of losers plus winners. But as figure 2 shows this difference is mainly due to the poor performance of the (future) losers. For the winners, q_7 is not statistically different from q_8 but it is for the losers. In any case the fact that q_8 is strictly lower than q_7 is incompatible with the hypothesis that players lie in round 7 and answer truthfully in round 8. The lower rate of good answers in round 8 can be attributable to fatigue. In round 8 players have less time and as they are only two they do have to answer more frequently which means that they do not have time to rest between two questions. Moreover, players who reached round 8 do no longer fear a vote. All these elements allow us to conclude that contestants answer truthfully.

5.3 Test of heterogeneity

The empirical results obtained so far could be consistent with a model where all contestants are homogeneous. That is, a model in which all players have the same ability to know the answer to a question. If all contestants have the same θ, there is no longer any incentives to vote out the best answerer. However, there is still a coordination problem in the voting game. The previous analysis shows that the players coordinate themselves on voting against the weakest answerer.

A key issue is then to determine if the players are heterogeneous or homogeneous in their ability to answer to the questions. After having watch 36 game shows, we are convinced that players are indeed heterogeneous. Some are clearly very weak and others are quite strong. However this kind of casual observation could be misleading. Moreover, if the players are heterogeneous at the beginning of the game, are they still heterogeneous at round 7 after 6 eliminations? One could imagine that after the first 4 or 5 eliminations, the remaining players are rather homogeneous in strength than heterogeneous. In that case the dilemma between voting against the weakest or the strongest would vanish.

One way to prove heterogeneity, is to show that even among the winners there are differences in their abilities. We use a standard panel data approach[29] to show that the hypothesis that all winners have the same θ is rejected by the data. Details can be found in appendix C.

5.4 Parameter values

Data analysis showed that contestants are heterogeneous, answer truthfully and vote out the weakest player. These results are consistent with the truthful equilibrium found in the theoretical analysis of a game with heterogeneity. However, such a truthful equilibrium does not exist for any parameter values. A key issue is then to determine if the parameter values estimated from our data are compatible or not with a truthful equilibrium. Unfortunately, the game played on TV is quite different from ours and the "estimations" given here must be taken as an illustration rather than a statistical test. In real life people ability, θ, to answer right to a random question on "general culture" is distributed between 0 and 1, while in our model we assumed that θ can only take two values θ_L or θ_H. Therefore the main difficulty is to determine two classes of players. In particular, there is no obvious way to do it and we have to make subjective choices. In fact, θ_L and θ_H are not sufficiently different to distinguish them in the data from the first seven rounds. However, in round 8 it is possible to distinguish two groups of players as shown by the histogram of figure 3. We used this information to create a group of (30) weak players who answered less than one question over two and a complementary group of 42 strong contestants. Next, we used data from the first seven rounds to estimate θ_L and θ_H as what matters (in the bayesian revision) is the information revealed by a good answer before round 8. The expected gains G_{HH} and G_{LL} and the probability K are easy to compute.

Therefore, as shown in table 8, $\rho = 30/72 \simeq 0.42$, $\theta_L = 0.65$ and $\theta_H = 0.77$. From which it follows that $\lambda_0 \simeq 0.52$ and $\lambda_1 \simeq 0.38$. Next, we computed the values[30] of m (the accumulated normalized gain from round 1 to 7), G_{LL} and G_{HH} as well as the value of K (the probability with which a L type wins

---

[29] See Hsiao (1986), chapter 2.

[30] These values are normalized by G_{LH}.

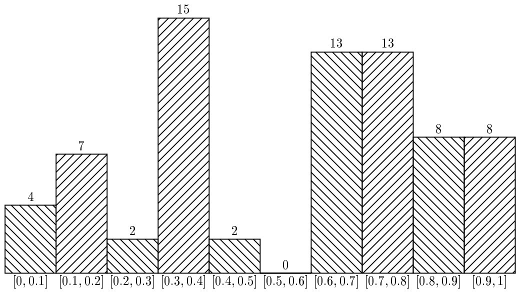

Figure 3: Distribution of θ in round 8

Table 8: Parameter values based on rounds 1 to 7

	ρ	θ_L	θ_H	λ_0	λ_1	K	G_{HH}	G_{LL}	m	Truth
$\widetilde{\theta} = 0.55$	0.42	0.65	0.77	0.52	0.38	0.14	2.38	0.55	8.45	Yes

vs a H type). It appears that with these parameter values, fact K is relatively small while m is rather large which means that any type of player prefers to play the endgame vs a type L (case III of figure 1). However, a truthful equilibrium can still exist because the value of λ_0 and λ_1 are sufficiently close (condition 1 of corollary 1) to one another. That is, the observation of a good answer does not provide enough information about the type of one's opponent.

This section shows that people (despite their myopia) play the entertaining equilibrium which is coherent with theory.

6 Conclusion

The Weakest Link game show is not a trivial quiz show. It raises interesting questions both from a theoretical and an empirical point of view. Most importantly, it allows a confrontation between theoretical prediction and real life behavior. For the parameter values estimated on our data, both a mute and a truthful equilibrium exist. Therefore, players face a coordination problem to select one of these equilibria. Empirically, it appears that they selected the truthful equilibrium. It seems that the environment in which the game is played is indeed favorable to such a coordination: the name of the game, the stigmatization of people who make mistakes, stigmatization of people who vote against a strong opponent. It seems that game theoretical predictions are quite useful to understand how players behave in this game even if it is a complex one.

References

Randall W. Bennett and Kent A. Hickman. Rationality and the 'price is right.'. *Journal of Economic Behavior & Organization*, 21(1):99–105, May 1993.

Jonathan B. Berk, Eric Hughson, and Kirk Vandezande. The price is right, but are the bids? an investigation of rational decision theory. *American Economic Review*, 86(4):954–70, September 1996.

P-A Chiappori, S. Levitt, and T. Groseclose. Testing mixed strategy equilibria when players are heterogeneous: The case of penalty kicks in soccer. *American Economic Review*, Forthcoming 2002.

Robert Gertner. Game shows and economic behavior: Risk-taking on "card sharks.". *Quarterly Journal of Economics*, 108(2):507–21, May 1993.

Cheng Hsiao. *Analysis of Panel Data.* Cambridge University Press, Cambridge, United Kingdom, 1986.

Andrew Metrick. A natural experiment in "jeopardy!". *American Economic Review*, 85(1):240–53, March 1995.

Mark Walker and John Wooders. Minimax play at Wimbledon. *American Economic Review*, 91(5):1521–38, December 2001.

Appendix

A Proof of corollary 1

★ The condition

$$G_L(\lambda_1) \geq \frac{3}{4} G_L(\lambda_0),$$

can be rewritten

$$2K\left(1+m\right)\left(\frac{1}{4}+\frac{3}{4}\lambda_0-\lambda_1\right)\geq\left(\frac{3}{4}\lambda_0-\lambda_1\right)\left(G_{LL}+m\right).$$

First if

$$\frac{3}{4}\lambda_0\leq\lambda_1,\text{ then the condition is always true.}$$

Next, if

$$\frac{3}{4}\lambda_0>\lambda_1,\text{ then the condition can be rewritten as}$$

$$G_{LL}\leq\left(1+m\right)\overline{G_{LL}}-m,$$

where $\overline{G_{LL}}=2K\frac{1+3\lambda_0-4\lambda_1}{3\lambda_0-4\lambda_1}$. Moreover, as G_{LL} is lower than 1 by definition, we must compare $\overline{G_{LL}}$ with 1. Indeed, it comes that

$$\left(1+m\right)\overline{G_{LL}}-m\leq 1\Leftrightarrow\overline{G_{LL}}\leq 1,\text{ and}$$

$$\overline{G_{LL}}\geq 1\text{ if and only if }\lambda_0\leq\frac{4}{3}\lambda_1+\frac{2}{3}\frac{K}{1-2K}.$$

★ The condition

$$G_H\left(\lambda_1\right)\geq\frac{3}{4}G_H\left(\lambda_0\right),$$

can be rewritten

$$\left(\frac{1}{4}+\frac{3}{4}\lambda_0-\lambda_1\right)\left(G_{HH}+m\right)\geq 2\left(1-K\right)\left(1+m\right)\left(\frac{3}{4}\lambda_0-\lambda_1\right)$$

which is obviously true if $\frac{3}{4}\lambda_0\leq\lambda_1$. If $\frac{3}{4}\lambda_0>\lambda_1$, using the fact that $\frac{1}{4}+\frac{3}{4}\lambda_0-\lambda_1>0$, it comes that

$$\text{the condition is true if }G_{HH}\geq\left(1+m\right)\underline{G_{HH}}-m,$$

where $\underline{G_{HH}}=2\left(1-K\right)\frac{3\lambda_0-4\lambda_1}{1+3\lambda_0-4\lambda_1}$. Moreover, as by definition G_{HH} is larger than 1, we must compare $\underline{G_{HH}}$ with 1. Indeed, it comes that

$$\left(1+m\right)\underline{G_{HH}}-m\geq 1\Leftrightarrow\underline{G_{HH}}\geq 1.$$

It follows that

$$\underline{G_{HH}}\leq 1\text{ if and only if }\lambda_0\leq\frac{4}{3}\lambda_1+\frac{1}{3}\frac{1}{2\left(1-K\right)-1}.$$

Finally, it is readily confirmed that

$$\frac{1}{3}\frac{1}{2\left(1-K\right)-1}\geq\frac{2}{3}\frac{K}{1-2K}$$

which ends the proof.

B Statistical test for the strategic lie

Let t denote the number of a given round ($t = 1$ to 7) and i the number of a player ($i = 1$ to N). It is assumed that player i is characterized in round t by a parameter $0 \leq \theta_i^t \leq 1$ which is the probability with which player i knows the answer to any given question. Let $\theta^t = E\left(\theta_i^t\right)$ be the average rate of good answers in round t in the population of contestants.

The test: "Do contestants answer in the same way in rounds t and $t+1$?" can be written as:

$$\mathrm{H}_0 : \quad \theta^t - \theta^{t+1} = 0$$

Let n_i^t denote the number of questions asked in round t to player i and let r_i^t the number of correct answers given in round t by player i. An estimator of $\theta^t - \theta^{t+1}$ is given by

$$\frac{1}{N}\sum_{i=1}^{N}\left(\frac{r_i^t}{n_i^t} - \frac{r_i^{t+1}}{n_i^{t+1}}\right)$$

The variance of this estimator is:

$$\frac{1}{N^2}\sum_{i=1}^{N}\frac{\theta_i^t\left(1-\theta_i^t\right)}{n_i^t} + \frac{1}{N^2}\sum_{i=1}^{N}\frac{\theta_i^{t+1}\left(1-\theta_i^{t+1}\right)}{n_i^{t+1}}$$

which can be rewritten under H_0 as:

$$\frac{1}{N^2}\sum_{i=1}^{N}\left[\theta_i^t\left(1-\theta_i^t\right)\right]\left[\frac{1}{n_i^t} + \frac{1}{n_i^{t+1}}\right]$$

Using

$$\widehat{\theta}_i^t = \frac{r_i^t + r_i^{t+1}}{n_i^t + n_i^{t+1}}$$

as an estimator of θ_i^t, the statistical test is $|T_{t,t+1}| \leq 1.96$ where $T_{t,t+1}$ is given by

$$T_{t,t+1} = \frac{\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\left(\frac{r_i^t}{n_i^t} - \frac{r_i^{t+1}}{n_i^{t+1}}\right)}{\sqrt{\frac{1}{N}\sum_{i=1}^{N}\left[\widehat{\theta}_i\left(1-\widehat{\theta}_i\right)\right]\left[\frac{1}{n_i^t} + \frac{1}{n_i^{t+1}}\right]}}$$

Table 9 shows the different values of the test for $t = 1$ to 7. First for both players who reached the endgame. Next, only for the winner and finally only for the loser.

Table 9: Test

T	$T_{1,2}$	$T_{2,3}$	$T_{3,4}$	$T_{4,5}$	$T_{5,6}$	$T_{6,7}$	$T_{7,8}$
Winner + Loser, $N = 72$	0.96	-0.49	0.72	-0.62	1.38	0.36	3.65
Winner only, $N = 36$	0.17	-0.25	0.34	-0.08	0.75	0.94	1.30
Loser only, $N = 36$	1.21	-0.43	0.64	-0.90	1.19	-0.42	3.80

C Statistical test for the heterogeneity

We restrict ourselves to the winners of the 36 games. The test developed in appendix B allows us to model the rate of good answers in the following way:

$$r_{it} = \theta_i + \varepsilon_{it}$$

where θ_i is the probability with which player i knows the answer, and ε_{it} is the error term with mean zero and variance σ_ε^2. All error terms are independent. To test H_0:

$$\theta_1 = \theta_2 = \ldots = \theta_{36}$$

we follow the methodology presented in the chapter 2 of Cheng Hsiao. The test is equivalent to the ordinary hypothesis test based on sums of squares residuals from linear regression outputs. We note S_1 the unrestricted residual sum of squares in the unrestricted model:

$$S_1 = \sum_{i=1}^{36} \sum_{t=1}^{8} \left(r_{it} - \overline{r_i}\right)^2$$

where $\overline{r_i} = \frac{1}{8} \sum_{t=1}^{8} r_{it}$. Similarly let S_2 denote the restricted residual sum of squares under the homogeneity constrains:

$$S_2 = \sum_{i=1}^{36} \sum_{t=1}^{8} \left(r_{it} - \overline{r}\right)^2$$

where $\overline{r} = \frac{1}{8 \times 36} \sum_{i=1}^{36} \sum_{t=1}^{8} r_{it}$.

The statistic

$$F = \frac{\left(S_2 - S_1\right) / \left(36 - 1\right)}{S_1 / \left(36 \times 8 - 36\right)}$$

is Fisher distributed with 35 and 252 degrees of freedom.

Table 10 shows that the hypothesis of homogeneity is rejected by the data.

Table 10: Test

F	$F_{95\%}\,(35, 252)$	H_0
4.66	1.47	rejected

D Round by round statistics

Table 11: Summary statistics for round 1 which lasts 2 min. 30 sec.

V	Sexe		Age		r_1		t_1	Gain
status	out	in	out	in	out	in		
$\overline{V}$	0.44	0.53	42	39	0.34	0.76	2.25	5 075
σ	0.5	0.5	14	15	0.29	0.3	0.45	3 483
min	0	0	23	18	0	0	1	0
max	1	1	74	79	1	1	3	15 000
N	36	288	36	288	36	288	324	36

Table 12: Summary statistics for round 2 which lasts 2 min. 20 sec.

V	Sexe		Age		r_2		t_2	Gain
status	out	in	out	in	out	in		
$\overline{V}$	0.55	0.53	40	39	0.42	0.71	2.35	4 300
σ	0.5	0.5	16	14	0.29	0.3	0.48	2 890
min	0	0	18	18	0	0	1	300
max	1	1	73	79	1	1	3	12 750
N	36	252	36	252	36	252	288	36

Table 13: Summary statistics for round 3 which lasts 2 min. 10 sec.

V	Sexe		Age		r_3		t_3	Gain
status	out	in	out	in	out	in		
$\overline{V}$	0.55	0.53	40	39	0.39	0.73	2.48	3 800
σ	0.5	0.5	15	14	0.29	0.3	0.51	2 370
min	0	0	18	18	0	0	2	300
max	1	1	72	79	1	1	4	9 000
N	36	216	36	216	36	216	252	36

Table 14: Summary statistics for round 4 which lasts 2 min.

V	Sexe		Age		r_4		t_4	Gain
status	out	in	out	in	out	in		
$\overline{V}$	0.42	0.55	40	38	0.44	0.72	2.65	3 250
σ	0.5	0.5	17	14	0.3	0.27	0.48	1 670
min	0	0	21	18	0	0	2	300
max	1	1	76	79	1	1	3	7 800
N	36	180	36	180	36	180	216	36

Table 15: Summary statistics for round 5 which lasts 1 min. 50 sec.

V	Sexe		Age		r_5		$t-5$	Gain
status	out	in	out	in	out	in		
$\overline{V}$	0.69	0.51	41	38	0.50	0.74	2.90	3 775
σ	0.47	0.5	17	13	0.25	0.28	0.38	2 670
min	0	0	18	18	0	0	2	300
max	1	1	79	76	1	1	4	12 000
N	36	144	36	144	36	144	180	36

Table 16: Summary statistics for round 6 which lasts 1 min. 40 sec.

V	Sexe		Age		r_6		t_6	Gain
status	out	in	out	in	out	in		
$\overline{V}$	0.42	0.55	39	37	0.44	0.71	3.30	2 300
σ	0.5	0.5	14	12	0.25	0.25	0.46	1 450
min	0	0	19	18	0	0	3	0
max	1	1	76	69	1	1	4	7 500
N	36	108	36	108	36	108	144	36

Table 17: Summary statistics for round 7 which lasts 1 min. 30 sec.

V	Sexe		Age		r_7		t_7	Gain
status	out	in	out	in	out	in		
$\overline{V}$	0.52	0.55	38	37	0.50	0.68	3.86	2 400
σ	0.5	0.5	13	12	0.30	0.27	0.48	1 940
min	0	0	19	18	0	0	3	0
max	1	1	68	69	1	1	5	7 500
N	36	72	36	72	36	72	108	36

Table 18: Summary statistics for round 8 which lasts 1 min. 20 sec.

V	Sexe		Age		r_8		t_8	Gain
status	out	in	out	in	out	in		
$\overline{V}$	0.55	0.55	38	36	0.47	0.62	5.06	4 250
σ	0.5	0.5	14	10	0.28	0.23	0.56	3 410
min	0	0	18	21	0	0	3	0
max	1	1	69	58	1	1	6	14 400
N	36	36	36	36	36	36	72	36