Risk Sharing with Formal and Informal Contracts: Theory, Semi-Parametric Identification and Estimation

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(Preliminary and Incomplete Version)

Abstract

We present a theoretical model which shows how households can insure through formal contracts and informal contracts when some verifiable production takes place in an environment of incomplete markets. We construct a theoretical setting which nests the case of complete markets when all risks can be insured by formal contracts (because all states of nature would be verifiable) and the case where only informal agreements are available (agreements specifying informal transfers that needs to be self-enforceable). We derive two equations of interest, an Euler-type equation and an equation of determination of the formal contract. We then study the semi-parametric identification of the model and show how it can be estimated. We estimate both equations using data of village economies in Pakistan. Empirical results are consistent with the model.

JEL Classification: C14, D12, D13, D91, L14, O12, O17, Q12, Q15

Key Words: Risk, Insurance, Contracts, Incomplete Markets, Sharecropping, Informal Transfers, Rural Households, Semi-parametric Identification, Average Derivatives, GMM, Pakistan.

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1 Introduction

Following the seminal paper of Townsend (1994), the empirical testing of whether or not markets are complete in village economies have proved to be a fertile and valuable line of research. It led to a better understanding of market failures and a better targeting of households in the village who are most affected by these failures (Morduch 1995, 1999, Fafchamps, 1997). These results were paralleled by tests of complete markets in developed economies at the aggregate level (see Attanasio and Rios-Rull, 2001). In both literatures, most papers report rejections of the null hypothesis of complete markets and much effort is now put on looking at alternative credible models of partially insured agents. This is where the two literatures, the one in village economies and the other in developed economies, depart (Attanasio and Rios-Rull, 2001, Dubois, 2000). Because village economies seem a priori to be less prone to imperfect information problems, the models that were developed in that literature, put emphasis on contract enforcement. Village economies lack institutions that are able to enforce the whole set of contracts that would permit complete risk sharing. Villagers are bound to enter agreements that are informal. These informal contracts are Pareto-improving because they permit risk sharing but they also need to be self-enforced (Thomas and Worrall, 1988, Coate and Ravallion, 1993). The latter requirement restricts the set of informal agreements which may not be rich enough to lead to complete risk sharing in the village. A few recent papers show the empirical credibility of such alternatives in static or dynamic cases (Ligon, Thomas and Worrall, 2000, 2002).

Although these self-enforcing contracts play their part in sharing risk within extended families or within networks of households formed by kinship, ethnicity and so on, (Grimard, 1999, Fafchamps and Lund, 2000), some contracts may be much easier to enforce. In particular, sharecropping and fixed rent formal contracts are commonly observed in villages of LDCs and their role in allocating risk have been repetitively emphasized. It is why in this paper, we consider the case where both types of contracts co-exist. Risks that households face are of many kinds. Because formal enforceable contracts on production are observed, we take the assumption that formal contracts on production are possible that is we assume that some subset of the set of states of nature are observable and verifiable. Other states of nature however may not be contractible like health-related problems, sickness, or returns to individual activities. Hence, formal contracts are allowed to be contingent on agricultural risk while other risks can only be shared through the use of implicit informal agreements that need to be self-enforced. Of course, informal transfers and contracts can also be contingent on verifiable risks. Informal agreements are decided *given* the formal agreements and would explain why formal contracts are accompanied by informal transfers that can attenuate their effects in bad states of nature (Udry, 1994).

Generally speaking, modeling formal and informal transfers amounts to take seriously the problem of the ex ante diversification of risks that households routinely perform and that, as we will show, makes income endogenous. Random shocks affecting preferences that are observed by the household but unobserved by the econometrician can also determine income. The interpretation of the test of complete markets may therefore be different. It is quite similar to the common case of non separability between leisure and consumption, leisure being determined by random shocks and determining income. It goes however through a different route closer to an insurance mechanism.

We construct a theoretical setting which nests the case of complete markets when all risks can be insured by formal contracts (because all states of nature would be verifiable) and the case where only informal agreements are available. We derive two equations of interest, an Euler-type equation of consumption dynamics and the equation of determination of the formal contract. This theoretical model proves to be quite general and makes a new step in the modelling of incomplete risk sharing with formal (enforceable) and informal (that needs to be self-enforceable) contracts.

Then, we study the semi-parametric identification of the model and implement and show how to implement an empirical estimation. We estimate both equations using data of village economies in Pakistan. This setting yields a richer test of complete markets since we are able to cope with the problem of endogeneity of income using the structural model.

The paper is organized as follows. Section 2 presents the theoretical model and the main propositions obtained. Section 3 studies the theoretical identification and econometric estimation of the model. Section 4 presents the data used and the empirical estimation results. Section 5 concludes and appendices are at the end.

2 Theoretical Model of Risk Sharing with Formal and Informal Contracts

Consider an economy with two agents and states of nature indexed by σ_t for date $t = 1, ..., \infty$. At every date the state of nature σ_t belongs to some finite set Ω, and the distribution of σ_t is i.i.d. We denote by σ a generic element of Ω and by π_σ the probability of state σ. Assume that the income process of agent i is z^i_σ in state σ, the total resources being $z_\sigma = z^1_\sigma + z^2_\sigma$. Agent 2 has a fixed Von-Neuman Morgenstern utility $u_2(.)$. For future estimation purpose we assume that the utility function follows some stochastic process.[1] The utility of agent 1 at date t is equal to $u_t(.) = \eta_t u_1(.)$, where $\eta_1 = 1$, $\eta_t = \widetilde{\varepsilon}_t \eta_{t-1}$, $\widetilde{\varepsilon}_t$ is i.i.d. with mean 1 and positive variance, whose support is an interval. We assume that η_t is observed by the two agents at the very beginning of date t before endowment shocks. The ex-ante utility of agent 1 is then $E\left[\sum_{t=1}^{\infty} \beta^{t-1} \eta_t u_1(c^1_t)\right]$, while it is $E\left[\sum_{t=1}^{\infty} \beta^{t-1} u_2(c^2_t)\right]$ for agent 2.

The ex-ante utility of agent i under autarchy v^a_i is (using $E(\eta_t) = 1$ and $c^i_t = z^i_{\sigma_t}$) :

$$\forall\ i = 1, 2 : v^a_i = \frac{1}{1-\beta} \sum_\sigma \pi_\sigma u_i\left(z^i_\sigma\right)$$

Consider the benchmark case of complete contracts. In this case optimal insurance is achieved. The consumption in date t depends only on the realization of total resources and according to Borch rules, for all states the ratio of marginal utilities is the same. Thus, under a full contracting setting, the stochastic dynamics of consumption is given by

$$\frac{\eta_{t+1}}{\eta_t} \frac{u'_1\left(c^1_{t+1}\right)}{u'_2\left(c^2_{t+1}\right)} = \frac{u'_1\left(c^1_t\right)}{u'_2\left(c^2_t\right)}.$$

[1] As it appears, we will also need it in the theory part to ensure strict concavity of some subprogram.

Now, we introduce limitations on the possibility for agents to sign formal contracts. Incompleteness is modeled here by two restrictions.

- First, contracts are short term, they are signed at the beginning of the period for the on going period. Thus prior to the realization of the period aleas, type two individuals can sign a contract on how resources will be shared. At this stage they are not allowed to contract on the sharing of income for the subsequent periods.

- Second, contracts cannot be contingent to all components of the states of nature but only to some sets of states of nature. There is a set of S events $s \in \{1, ..., S\}$ where $s \subset \Omega$ is interpreted as random shocks affecting the realization of some (say "agricultural") production that is verifiable. We denote π_s the probability of event s: $\pi_s = \sum_{\sigma \in s} \pi_\sigma$. The formal contracts specify a reallocation of resources from agent 2 to agent 1 which can be contingent only on s, and only for the current period. A contract is thus represented by a vector $T = (t_1, ..., t_S)$ of transfers. We assume that T belongs to some set $\mathcal{T} = \times_s [\underline{t}_s, \bar{t}_s]$, where $\underline{t}_s < 0 < \bar{t}_s$.

The model extends Gauthier, Poitevin and Gonzalez (1997) by allowing to account for taste shocks and formal contracting on verifiable production (in Gauthier, Poitevin and Gonzalez (1997), only one ex-ante transfer is allowed such that it corresponds to the case where $S = 1$ and one agent is risk neutral). Because of convexity issues we will allow for some randomization beyond the fundamentals[2]. We assume that at every period, there is a public random variable ε_t that is uniformly distributed and whose realization occurs at the beginning of date t. The precise timing of realization of the various events is the following. The contract for period t is then signed between t and $t + 1/2$:

- Period t

[2] The reason is that the value function may not be concave, see Gauthier, Poitevin and Gonzalez (1997). They assume that the value function is concave because they cannot prove it. However we show that randomization over utilities is enough to obtain concave program even with a non-concave value function.

- t : the taste parameter η_t and ε_t are realized, and the contract $T_t \in \mathcal{T}$ is signed, valid for period t.
- $t+1/2$:the state σ_t is realized and consumptions take place.

With such a timing, current preferences are known when the contract is signed. The contract T_t can thus be contingent on η_t. Moreover the random component ε_t allows a stochastic link between the taste parameter and the contract.
At the date $t+1/2$, the contract T_t can be enforced. However, in some cases, it need not be since the parties are free to complement it by voluntary transfers if they are self-enforcing.

With such a formulation we obtain the standard model of informal risk sharing when no contract is feasible (as a limit) like in Thomas and Worrall (1988), while Gauthier, Poitevin and Gonzalez (1997) corresponds to the case where $S = 1$, and the complete markets hypothesis to $\{1, ..., S\} = \Omega$.

Let $H_t = (\sigma_1, .., \sigma_{t-1}, \eta_1, .., \eta_t, \varepsilon_1, .., \varepsilon_t)$ be the history of the states of nature up to t, and $h_t = (\sigma_1, .., \sigma_t, \eta_1, .., \eta_t, \varepsilon_1, .., \varepsilon_t) = (H_t, \sigma_t)$ the history up to $t+1/2$. An allocation is a random consumption profile c_t^i and contract profile T_t that is measurable with respect to history: $c_t^i = c^i(h_t)$ and $T_t = T(H_t)$. The allocation is feasible if in all states, $c_t^1 + c_t^2 = z_t$ and $T_t \in \mathcal{T}$.
The expected utility of the agents are then

$$
\begin{aligned}
v^1 &= E\left[\sum_{t=1}^{\infty} \beta^{t-1} \eta_t u_1\left(c_t^1\right)\right] \\
v^2 &= E\left[\sum_{t=1}^{\infty} \beta^{t-1} u_2\left(c_t^2\right)\right]
\end{aligned}
$$

The two agents will then coordinate informally on some allocation but they must account for the possibility that at some point one agent may refuse to participate. In the case of bilateral limited commitment, the allocations has to be self sustainable. In order to prevent a party reneging on the agreement, it is optimal to coordinate in such a way that if agent i deviates, the equilibrium that follows is the worst equilibrium for agent i. In other words one should apply an optimal penal code as defined by Abreu (1988) and Abreu, Pearce and Stacchetti (1986, 1990). When no contract is feasible, this means that the agent will receives its autarchic consumption. With short-term contracts however, autarchy may not always be an equilibrium outcome.[3] Fortunately to solve and estimate the model, we don't need to derive the maximal punishment.

Because of the taste parameter our model is not truly a repeated game with hyperbolic discounting. But the same game form is repeated overtime. To see that define the expected utility at the beginning of date t normalized by η_t as v_t^i:

$$\begin{aligned} v_t^1 &= E\left[\sum_{r=1}^{\infty} \beta^{r-1} \frac{\eta_{t+r-1}}{\eta_t} u_1\left(c_{t+r-1}^1\right) \mid H_t\right] \\ v_t^2 &= E\left[\sum_{r=1}^{\infty} \beta^{r-1} u_2\left(c_{t+r-1}^2\right) \mid H_t\right]. \end{aligned}$$

Notice that

$$\begin{aligned} v_t^1 &= E\left[u_1\left(c_t^1\right) + \beta \frac{\eta_{t+1}}{\eta_t} v_{t+1}^1 \mid H_t\right] \\ v_t^2 &= E\left[u_2\left(c_t^2\right) + \beta v_{t+1}^2 \mid H_t\right] \end{aligned}$$

Now consider the subgame starting in date t with η_t known and expected utilities v_t^i. Denote $\hat{\eta}_r^t = \frac{\eta_{t+r}}{\eta_t}$. Given that $\frac{\hat{\eta}_{r+1}^t}{\hat{\eta}_r^t} = \widetilde{\varepsilon}_{t+r+1}$ is i.i.d., the distribution of $\left\{\hat{\eta}_r^t\right\}_{r\geq 1}$ is the same as the distribution of $\left\{\eta_r\right\}_{r\geq 1}$. Thus the subgame starting at date t is identical to the initial game. This means that the sets of equilibria of the two games coincide. In other words, using normalized utilities v_t^i we can solve the game using the same tools as for a repeated game (the difference is that the discount factor is equal to $\widetilde{\delta}_t = \delta^t \eta_t$, where $\frac{\widetilde{\delta}_{t+1}}{\widetilde{\delta}_t}$ is not constant but an i.i.d. random variable).

In particular there are minimal and maximal expected utility levels, denoted $\underline{v}^i$ and $\bar{v}^i$, that can be supported in equilibrium (up to the normalization). For what follows, all that

[3]Suppose for instance that there are two identical agents with two equally probable states of nature with $(z_1^1, z_1^2) = (1, 3)$ and $(z_2^1, z_2^2) = (3, 1)$ while contracts are such that $t_1 = -t_2$. Then a contract $(1, -1)$ yields full insurance. Moreover the two agents must receive the same payoff. Clearly for $\beta < 1$, it is not possible that the autarchy payoff be an equilibrium payoff. **REVOIR**

we need to know is that the minimal utility that a deviant agent can obtain from date t on is $\underline{v}_t^1 = \underline{v}^1$ for agent 1 and $\underline{v}_t^2 = \underline{v}^2$ for agent 2, which may or may not coincide with autarchy levels. We denote also $\bar{v}^i$ the maximal equilibrium payoff. Since the game is one with symmetric information, an allocation can be supported in equilibrium provided that at any point in time both agents prefer to abide to the informal agreement rather than to renege and be punished by receiving his minimal equilibrium utility. Thus at date t, it must be the case that the agent is willing to sign the contract

$$v_t^i \geq \underline{v}_t^i, \tag{1}$$

and at date $t+1/2$, the agent must prefer to make the informal transfer than to enforce the formal contract:

$$u_1(c_t^1) + \beta E\left[\frac{\eta_{t+1}}{\eta_t} v_{t+1}^1 \mid h_t\right] \geq u_1(z_t^1 + t_t) + \beta \underline{v}^1, \tag{2}$$

$$u_2(c_t^2) + \beta E\left[v_{t+1}^2 \mid h_t\right] \geq u_2(z_t^2 - t_t) + \beta \underline{v}^2. \tag{3}$$

Following the standard approach to the problem we derive the set of Pareto optimal equilibria. For a given date 1 expected utility v of agent 2, let $P(v)$ denote the maximal expected utility that the agent 1 can obtain in equilibrium. Then $P(v)$ solves

$$P(v) = \max_{c_t^1, c_t^2, T_t} E\left[\sum_{t=1}^{\infty} \beta^{t-1} \eta_t u_1\left(c_t^1\right)\right] \tag{4}$$

s.t.

$$E\left[\sum_{t=1}^{\infty} \beta^t u_2\left(c_t^2\right)\right] \geq v,\ c_t^1 + c_t^2 = z_t, (1),\ (2),\ (3). \tag{5}$$

A standard argument (see Thomas and Worrall, 1988) shows that the function $P(v)$ is decreasing and continuous. Clearly the optimal contract is such that conditional on H_t, the agent 1 should receive the maximal expected utility given that agent 2 receives at least v_t^2. Notice that under our assumptions on the stochastic processes of σ_t and η_t, the problem of maximizing v_t^1 conditional on H_t and v_t^2 is the same as the problem of maximizing the ex-ante utility of agent 1 subject to giving an ex-ante utility of at least v_t^2 to agent 2. Thus we must have $v_t^1 = P(v_t^2)$.

Then, the standard arguments apply and the allocation of consumption is the solution to the program

$$P(v) = \max_{(c_\sigma^1, c_\sigma^2, t_s, v_{\sigma\eta\varepsilon})} E\left[u_1\left(c_\sigma^1\right) + \beta\eta P\left(v_{\sigma\eta\varepsilon}\right)\right]$$

s.t.

$$\begin{aligned}
u_1(c_\sigma^1) + E\left[\beta P(v_{\sigma\eta\varepsilon}) \mid \sigma\right] &\geq u_1(z_\sigma^1 + t_s) + \beta \underline{v}^1 \;\; \forall\sigma \\
u_2(c_\sigma^2) + E\left[\beta v_{\sigma\eta\varepsilon} \mid \sigma\right] &\geq u_2(z_\sigma^2 - t_s) + \beta \underline{v}^2 \;\; \forall\sigma \\
E\left[u_2\left(c_\sigma^2\right) + \beta v_{\sigma\eta\varepsilon}\right] &\geq v \\
c_\sigma^1 + c_\sigma^2 &= z_\sigma \;\; \forall\sigma \\
v_{\sigma\eta\varepsilon} &\in [\underline{v}^2, \bar{v}^2] \;\; \forall\sigma, \eta, \varepsilon
\end{aligned}$$

In the program, $v_{\sigma\eta\varepsilon}$ is the agent 2 promised utility in date 2, conditional on a realization σ in date $1 + 1/2$, taste parameter η in date 2 and random shock ε in date 2. The expectation operator refers to the joint probability distribution of σ, η and ε. The optimal allocation can thus be described by consumption levels c_σ^i for each agent at date $1 + 1/2$, contract T and continuation expected utility $v_{\sigma n\varepsilon}^i$ at date 2 contingent on the realization of the shocks.

When there is no contract T, it is known (Thomas and Worrall) that the function P is decreasing, concave and differentiable. However, when contracts T are allowed, $P(.)$ need not be concave. Still we show that the problem is concave. Notice that if $P(.)$ is not concave, it is optimal for the agents to randomize between several date 2 utilities. Let us denote $v_{\sigma\eta} = E\left[v_{\sigma\eta\varepsilon} \mid \sigma, \eta\right]$. Given $v_{\sigma\eta}$, choosing $v_{\sigma\eta\varepsilon}$ is equivalent to choosing a distribution of utility on $[\underline{v}^2, \bar{v}^2]$. Denote Δ the set of probability distribution on $[\underline{v}^2,\ \bar{v}^2]$ and F a generic element. Then an optimal allocation is such that conditional on σ and η, the distribution of the future utility solves the program

$$\hat{P}(v_{\sigma\eta}) = \max_{F\in\Delta} E\left[\int P(v^2) dF(v^2)\right] \; s.t. \int v^2 dF(v^2) = v_{\sigma\eta}$$

$\hat{P}(.)$ is a concave decreasing function since the program is linear. $P(.)$ and $\hat{P}(.)$ coincide whenever $P(.)$ is concave.

Consider now the choice of $v_{\sigma\eta}$. Here again, given an expected utility $v_\sigma = E\left[v_{\sigma\eta} \mid \sigma\right]$, it is optimal to choose $v_{\sigma\eta}$ so as to maximize agent 1 utility. Define then

$$Q(v_\sigma) = \max_{v_{\sigma\eta} \in [\underline{v}_2, \bar{v}^2]} E\left[\eta \hat{P}(v_{\sigma\eta}) \mid \sigma\right] \; s.t. \; E\left[v_{\sigma\eta}\right] \geq v_\sigma$$

The function $Q(.)$ is decreasing and concave since $\hat{P}(.)$ is decreasing and concave.

The value function $P(.)$ can then be written as the solution of:

$$P(v) = \max_{(c^1_\sigma, c^2_\sigma, t_s, v_\sigma)} E\left[u_1\left(c^1_\sigma\right) + \beta Q\left(v_\sigma\right)\right] \tag{6}$$

s.t.

$$E\left[u_2\left(c^2_\sigma\right) + \beta v_\sigma\right] \geq v \tag{7}$$

$$u_1\left(c^1_\sigma\right) + \beta Q\left(v_\sigma\right) \geq u_1\left(z^1_\sigma + t_s\right) + \beta \underline{v}^1 \qquad \forall s, \sigma \in s \tag{8}$$

$$u_2\left(c^2_\sigma\right) + \beta v_\sigma \geq u_2\left(z^2_\sigma - t_s\right) + \beta \underline{v}^2 \qquad \forall s, \sigma \in s \tag{9}$$

$$c^1_\sigma + c^2_\sigma = z_\sigma \qquad \forall \sigma \tag{10}$$

$$\underline{v}_2 \leq v_\sigma \leq \bar{v}^2 \qquad \forall \sigma \tag{11}$$

This shows that, for a fixed contract $T = \{t_1, ..., t_S\}$, the program is concave although $P(.)$ may not be concave.

The argument developed in Poitevin et al. (1997) for the case where the contract is a fixed transfer can be used similarly in our case to show that $P(.)$ is continuously differentiable because we proved that $Q(.)$ is concave. This in turn implies that $Q(.)$ and $\hat{P}(.)$ are continuously differentiable. For what follows we need to assume in addition that they are not linear:

Assumption: $\hat{P}'(\underline{v}_2) > \hat{P}'(\bar{v}_2)$.

This ensures that at the solution of $Q(v_\sigma)$, $\underline{v}_2 < v_{\sigma\eta} < \bar{v}^2$ with positive probability. The assumption is thus a non-triviality assumption. It rules out a situation where $P(.)$ is convex everywhere, in which case optimality would require to alternate between corner solutions. This is clearly a degenerate case which is not interesting for estimation purpose nor from a theoretical perspective. We however failed so far to prove that it cannot occur. This is thus a very weak assumption that ensures a rich equilibrium dynamics.

Now, to describe the dynamics of the system, we don't need to describe the whole frontier P but only those points that can occur in equilibrium, in other words the supports of the distribution F that solve $\hat{P}(.)$. It is immediate that (we skip the proof as this is standard): If a point $v^2 \in [\underline{v}_2, \bar{v}^2]$ occurs with positive probability, then $P(v^2) = \hat{P}(v^2)$ and $v^2 = \arg\max_x \left\{ P(x) - \hat{P}'(v^2)x \right\}$.
Let $\mathcal{W}$ be the set of solutions $\Phi(.)$ of the program

$$\Phi(\mu) = \max_{v \in [\underline{v}_2, \bar{v}^2]} P(v) + \mu v$$

when the weight μ varies continuously between $-\hat{P}'(\underline{v}_2)$ and $-\hat{P}'(\bar{v}^2)$. Then the set of utility $v_{\sigma\eta\varepsilon}$ that can obtain with positive probability in equilibrium is included in $\mathcal{W}$. We shall solve this program $\Phi(\mu)$ to derive the equilibrium. This amounts to maximize $E\left[u_1\left(c_\sigma^1\right) + \beta Q\left(v_\sigma\right)\right] + \mu E\left[u_2\left(c_\sigma^2\right) + \beta v_\sigma\right]$ subject to constraints (8) to (11). To solve this program, notice that it is separable between events s. In other words

$$\Phi\left(\mu\right) = \max_{T = \{t_1, .., t_S\}} \sum_s \pi_s \Phi_s\left(\mu, t_s\right)$$

where $\Phi_s(\mu, t_s)$ is the solution for a fixed value of t_s of the maximization of $E\left[u_1\left(c_\sigma^1\right) + \beta Q\left(v_\sigma\right) \mid s\right] + \mu E\left[u_2\left(c_\sigma^2\right) + \beta v_\sigma \mid s\right]$ subject to the constraints (8) to (11) in event s. $\Phi_s\left(\mu, t_s\right)$ is a concave program and we show that due to the preference shock η_t, it is a strictly concave problem with a unique solution. Working with this program we obtain the main result that will be used for estimation:

Proposition 1 *Let's note $r_\sigma = z_\sigma^1 + t_s$ the agent 1 income in state σ. There exists functions $\underline{\mu}\left(z_\sigma, r_\sigma\right) \leq \bar{\mu}\left(z_\sigma, r_\sigma\right)$ with values in $[-\hat{P}'(\underline{v}_2), -\hat{P}'(\bar{v}^2)]$, decreasing in r_σ whenever interior such that:*

When $\bar{\mu}(z_\sigma, r_\sigma) > -\hat{P}'(\underline{v}_2)$ *and* $\underline{\mu}(z_\sigma, r_{\sigma s}) < -\hat{P}'(\bar{v}^2)$*, then* $\bar{\mu}(z_\sigma, r_\sigma) > \underline{\mu}(z_\sigma, r_\sigma)$ *and:*

$$\frac{u_1'(c_\sigma^1)}{u_2'(c_\sigma^2)} = -Q'(v_\sigma) = \bar{\mu}\left(z_\sigma, z_\sigma^1 + t_s\right) \text{ if } \mu \geq \bar{\mu}\left(z_\sigma, z_\sigma^1 + t_s\right) \text{ ((38) binds)} \tag{12}$$

$$\frac{u_1'(c_\sigma^1)}{u_2'(c_\sigma^2)} = -Q'(v_\sigma) = \underline{\mu}\left(z_\sigma, z_\sigma^1 + t_s\right) \text{ if } \mu \leq \underline{\mu}\left(z_\sigma, z_\sigma^1 + t_s\right) \text{ ((39) binds)} \tag{13}$$

$$\frac{u_1'(c_\sigma^1)}{u_2'(c_\sigma^2)} = -Q'(v_\sigma) = \mu \text{ if } \underline{\mu}\left(z_\sigma, z_\sigma^1 + t_s\right) \leq \mu \leq \bar{\mu}\left(z_\sigma, z_\sigma^1 + t_s\right) \tag{14}$$

In addition

$\frac{u_1'(c_\sigma^1)}{u_2'(c_\sigma^2)} \leq -Q'(\underline{v}^2)$ *and* $v_\sigma = \underline{v}^2$ *if* $\bar{\mu}(z_\sigma, r_s) = -\hat{P}'(\underline{v}_2)$

$\frac{u_1'(c_\sigma^1)}{u_2'(c_\sigma^2)} \geq -Q'(\bar{v}^2)$ *and* $v_\sigma = \bar{v}^2$ *if* $\underline{\mu}(z_\sigma, r_\sigma) = -\hat{P}'(\bar{v}^2)$.

Proof. See Appendix A. ■

This result is a generalization of Thomas and Worrall (1988) where one agent is risk neutral or of Gauthier, Poitevin and Gonzalez (1997) where one agent has a constant endowment and T is unidimensional. The proposition thus defines the current and future ratio of marginal utilities as a function of the multiplier μ and ex-post resources (which depend on the contract t_s). Using this we can fully characterize the solution as a function of the contract. The second step is to show that the optimal contract T is monotone in μ. The problem is not concave in T so that there may be multiple solutions for T. Multiple solutions arise when the frontier $P(v)$ is not concave or when no incentive constraint is binding in some event s. However intuition suggests that when μ increases, $T(\mu)$ should decrease as v move along the Pareto frontier toward higher utility for agent 2 (since μ is the slope of the frontier).

Proposition 2 *The mapping* $\bar{T}(\mu) : \mu \to \arg\max_T E\{\Phi_s(\mu, t_s)\}$ *is a monotone decreasing correspondence in* μ *(according to the strong order set).*

Proof. See Appendix B. ■

To summarize, as we move along the frontier $\hat{P}(v)$ toward higher absolute slopes (and thus higher v), the contract becomes uniformly more favorable to the agent 2. Notice that the same holds true for the allocation of consumptions and future utilities (c_σ^2, v_σ).[4]

Let us now turn to the implications of the results for the dynamics of consumption and contracts. For the estimation we assume that corner solution never arise:

Assumption: $prob\{\underline{v}_2 < v_t^2 < \bar{v}^2\} = 1$.

The dynamics can be described by mean of the evolution of the weight $\mu_t = -P'(v_t^2)$ associated with the point in $\mathcal{W}$ chosen after history H_t.

At these stage, agents sign a contract $T_t \in \bar{T}(\mu_t)$. At date $t+1/2$ consumption is given as a function of μ_t, the contract T_t and σ_t by proposition 1. This also defines the slope $Q'(v^2(h_t))$ at this interim stage. Then at date $t+1$, H_{t+1} is realized and thus v_{t+1}^2. This gives the new value of the weight μ_{t+1}. The intertemporal link is provided by the relation $\frac{\eta_{t+1}}{\eta_t}P'(v_{t+1}^2) = \frac{\eta_{t+1}}{\eta_t}\hat{P}'(v_{t+1}^2) = Q'(v^2(h_t))$. The dynamics thus verifies

$$
\begin{aligned}
T_t &= \{t_t(s)\}_s \in \bar{T}(\mu_t) && (15)\\
r_t &= z_t^1 + t_t(s_t) && (16)\\
\frac{u_1'(c_t^1)}{u_2'(c_t^2)} &= \bar{\mu}\left(z_t, r_t\right)\ if\ \mu_t \geq \bar{\mu}\left(z_t, r_t\right) && (17)\\
\frac{u_1'(c_t^1)}{u_2'(c_t^2)} &= \mu_t,\ if\ \underline{\mu}\left(z_t, r_t\right) \leq \mu_t \leq \bar{\mu}\left(z_t, r_t\right) && (18)\\
\frac{u_1'(c_t^1)}{u_2'(c_t^2)} &= \underline{\mu}\left(z_t, r_t\right)\ if\ \mu_t \leq \underline{\mu}\left(z_t, r_t\right) && (19)\\
\mu_{t+1} &= \frac{\eta_t}{\eta_{t+1}}\frac{u_1'(c_t^1)}{u_2'(c_t^2)} && (20)
\end{aligned}
$$

Whenever the Pareto frontier is concave this defines exactly the whole dynamics as $\bar{T}(\mu)$ is singled valued. If $P(.)$ is not concave $\bar{T}(\mu)$ can be multi-valued. Notice that it is single valued for all values μ_t where $\Phi(\mu_t)$ has a unique solution. This corresponds to values where $-\hat{P}'(v_t) = \mu_t$ has a unique solution.[5] But we have shown in the proof of Proposition 1 (in lemma 3) that this occurs with probability 1 due to the effect of the preference shock

[4] This follows from the fact that at the solution of $\Phi_s(\mu, t_s)$, both c_σ^2 and v_σ are non-decreasing with μ and non-increasing with t_s.

[5] $t(s)$ may still be undetermined if the probability that an incentive constraint binds in event s is zero. We rule out such possibility.

η_t. Thus in equilibrium $\bar{T}(\mu_t)$ is single valued with probability 1. We can thus ignore the issue of equilibrium randomization over utilities and contracts for estimation purpose.

Additional noises

In what preceded we assume fixed utilities and a stationary resource process. Suppose that the utility is $u_i(c^i; x_t^i)$ where x_t^i follows a Markov process. Suppose that the resources depend on σ_t and q_t, where q_t follows a Markov process. Suppose also that q_t and x_t^i are learned at the beginning of period t. Let $y_t = (x_t^1, x_t^2, q_t)$ the information at the beginning of period t. Then the value function at date t is a function $P(v, y_t)$. The interim value function is $Q(v; y_t) = \max E\left\{\frac{\eta_{t+1}}{\eta_t}\hat{P}(v(y_{t+1}, \frac{\eta_{t+1}}{\eta_t}); y_{t+1}) \mid y_t\right\}$ subject to $E\left\{v(y_{t+1}, \frac{\eta_{t+1}}{\eta_t}) \mid y_t\right\} \geq v$. The program $P(v; y_t)$ then is the solution of $\max E\left[u_1\left(c^1(y_t, \sigma_t); x_t\right) + \beta Q\left(v(y_t, \sigma_t); y_t\right) \mid y_t\right]$ subject to incentive and participation constraints. In this set-up all the proofs generalize. The functions $\bar{\mu}$ and $\underline{\mu}$ depend only on z_t, r_t and y_t (but not on η_t): $\bar{\mu}(z_t, r_t; y_t)$ and $\underline{\mu}(z_t, r_t; y_t)$. The ratio $\frac{u_1'(c_t^1)}{u_2'(c_t^2)}$ has to be conditioned on x_t^i only: $\frac{u_1'(c_t^1; x_t^1)}{u_2'(c_t^2; x_t^2)}$. The contract depends on μ_t and y_t: $\bar{T}(\mu_t, y_t)$. But the dynamics of the multiplier μ_t is unchanged since $-\frac{\eta_{t+1}}{\eta_t}\hat{P}'(v(y_{t+1}, \frac{\eta_{t+1}}{\eta_t}), y_{t+1}) = -Q'(v(y_t, \sigma_t); y_t) = \frac{u_1'(c_t^1; x_t^1)}{u_2'(c_t^2; x_t^2)}$ with probability 1 and $\mu_{t+1} = \hat{P}'(v(y_{t+1}, \frac{\eta_{t+1}}{\eta_t}), y_{t+1})$.

3 Econometric Specification, Identification and Estimation

We first state the structural form of the econometric model by specifying the two equations of interest: consumption dynamics and the income process. We assume that all functions of interest are linear or log-linear and we investigate identification of the model in the leading case where random shocks are independent of explanatory variables. These restrictions are strong enough to get identification of the main parameters of interest. We estimate the model by GMM using even stronger identifying assumptions that are testable.

3.1 Consumption Dynamics

We start from equations (17, 18, 19, 20) describing the dynamics of the ratios of the marginal utilities of consumption for a pair of households. As we do not observe pairs of households engaged into formal contracts but only individual households, we assume (as is common in this literature, see Ligon, Thomas and Worrall, 2002), that the ratio of marginal utilities between household i and its partner can be written as:

$$\tau(c_{it}, x_{it}) \exp(\delta_{vt}) \tag{21}$$

where c_{it} is household i 's consumption and x_{it} are household demographic variables that affect preferences and where the "partner" household is assumed to be the whole village (or district). Its marginal utility is summarized by a village and period effect δ_{vt}. We also assume that households have constant relative risk aversion, θ:

$$\tau(c_{it}, x_{it}) = \exp(x_{it}\theta\beta).c_{it}^{-\theta}$$

where demographics are permitted to affect the slope of marginal utilities only[6]. The logarithm of marginal utility is therefore assumed to be log-linear. Namely, taking logarithms in equation (21) and including random preference shocks $\frac{\eta_{it-1}}{\eta_{it}}$, we get consumption dynamics in the three regimes, the regimes being defined by whether or not incentive constraints are binding:

$$\ln\tau(c_{it}, x_{it}) + \delta_{vt} = \ln\frac{\eta_{it-1}}{\eta_{it}} + \ln\tau(c_{it-1}, x_{it-1}) + \delta_{vt-1}$$
$$\text{if } \ln\frac{\eta_{it-1}}{\eta_{it}} + \ln\tau(c_{it-1}, x_{it-1}) + \delta_{vt-1} \in [\ln\underline{\mu}(r_{it}, y_{it}), \ln\overline{\mu}(r_{it}, y_{it})] \tag{R1}$$

$$\ln\tau(c_{it}, x_{it}) + \delta_{vt} = \ln\underline{\mu}(r_{it}, y_{it})$$
$$\text{if } \ln\frac{\eta_{it-1}}{\eta_{it}} + \ln\tau(c_{it-1}, x_{it-1}) + \delta_{vt-1} < \ln\underline{\mu}(r_{it}, y_{it}) \tag{R2}$$

[6] The relative risk aversion parameter could also be made a function of observed characteristics as in Dubois (2000). See the empirical section.

$$\ln \tau(c_{it}, x_{it}) + \delta_{vt} = \ln \overline{\mu}(r_{it}, y_{it})$$

$$\text{if } \ln \frac{\eta_{it-1}}{\eta_{it}} + \ln \tau(c_{it-1}, x_{it-1}) + \delta_{vt-1} > \ln \overline{\mu}(r_{it}, y_{it}) \quad \text{(R3)}$$

where r_{it} is agricultural and non agricultural profit net of input costs including labor and where y_{it} is the vector of variables in the information set at the end of period t. This vector comprises any variable that affect current preferences or help to predict future preferences and income processes. In particular, y_{it} includes current taste shifters, x_{it}, asset variables such as owned land and other variables that affect agricultural production and when the contract is signed. All the latter variables are denoted q_{it} so that $y_{it} = (x_{it}, q_{it})$.

Whether incentive constraints are binding or not are not observable events, and the three regimes giving consumption dynamics, are therefore not observable. As a consequence, it is equivalent to write a single equation describing the dynamics of marginal utilities as:

$$\Delta \ln \tau(c_{it}, x_{it}) + \Delta\delta_{vt} + \ln \tilde{\varepsilon}_{it} = \underline{\phi}_{it}.\mathbf{1}\{\underline{\phi}_{it} > 0\} + \overline{\phi}_{it}\mathbf{1}\{\overline{\phi}_{it} < 0\} \quad (22)$$

where $\tilde{\varepsilon}_{it} = \frac{\eta_{it}}{\eta_{it-1}}$ are the random preference shocks and where:

$$\Delta \ln \tau(c_{it}, x_{it}) = \ln \tau(c_{it}, x_{it}) - \ln \tau(c_{it-1}, x_{it-1})$$

$$\underline{\phi}_{it} = \ln \underline{\mu}(r_{it}, y_{it}) + \ln \tilde{\varepsilon}_{it} - \ln \tau(c_{it-1}, x_{it-1}) - \delta_{vt-1}$$

$$\overline{\phi}_{it} = \ln \overline{\mu}(r_{it}, y_{it}) + \ln \tilde{\varepsilon}_{it} - \ln \tau(c_{it-1}, x_{it-1}) - \delta_{vt-1}$$

This is the first equation of the structural model that describes consumption dynamics. The endogenous variables in this equation are the pair of consumption and non labor income (c_{it}, r_{it}) and the explanatory variables are (x_{it-1}, y_{it}). The random shocks unobserved by the econometrician are preference shocks, $\ln \tilde{\varepsilon}_{it}$, revealed at the beginning of the period and income shocks revealed at the end of the period. The specification of the income variable r_{it} is the object of the next subsection. Finally, to conform with our idea of exploring identification under linearity, we assume that the upper and lower constraints are semi-log-linear:

$$\begin{cases} \ln \underline{\mu}(r_{it}, y_{it}) = \underline{\mu}_0 r_{it} + \underline{\mu}_y y_{it} + \underline{\mu}_{vt} \\ \ln \overline{\mu}(r_{it}, y_{it}) = \overline{\mu}_0 r_{it} + \overline{\mu}_y y_{it} + \overline{\mu}_{vt} \end{cases} \quad (23)$$

where $\underline{\mu}_{vt}$ and $\overline{\mu}_{vt}$ are village effects summarizing global resources available at the village level as in the theoretical setting. Parameters $\underline{\mu}_0$ and $\overline{\mu}_0$ are negative as shown in the structural model (consequence of proposition 2). First, note that this specification excludes individual effects that are notoriously difficult to handle in non linear dynamic settings (Magnac and Thesmar, 2001). Second, we do not impose for the moment that for any r_{it}, y_{it}, any village and any period, the constraint $\underline{\mu}(r_{it}, y_{it}) \leq \overline{\mu}(r_{it}, y_{it})$ is verified. We shall return to this point in the section related to identification.

The particular case of complete markets amounts to assume that the incentive constraints never bind in this model, that is $\underline{\phi}_{it} < 0$, $\overline{\phi}_{it} > 0$ so that:

$$\Delta \ln \tau(c_{it}, x_{it}) + \Delta\delta_{vt} + \ln \tilde{\varepsilon}_{it} = 0$$

Two remarks are in order. First, the event $\underline{\phi}_{it} < 0$, $\overline{\phi}_{it} > 0$ can have probability 1 only if all variables appearing in the expressions of $\underline{\phi}_{it}$ and $\overline{\phi}_{it}$ are bounded or if some parameters take infinite values. It does not favor the use of a full parametric test of this assumption but it agrees with the general idea of a model with self-enforcing constraints. In this model, the dynamics is at times consistent with the hypothesis of complete markets and at times not consistent. Secondly, the right hand side of (22) is a function of r_{it} while the left hand side is not. If r_{it} were exogenous, the standard test of a prediction of complete markets consists in looking at the significance of the correlation between the residuals under the null hypothesis and the income process r_{it}. This test is quite robust, since it relies on the assumption that income is excluded from preferences. In the present model however, income r_{it} is endogenous because it depends on formal contracts that depend themselves on random preference shocks. This is why we shall now specify the other equation determining the income process r_{it}. It is clear enough that a test of complete markets can be constructed if there are exogenous variables that affect income and are independent of random preference shocks and therefore of formal contracts[7].

[7] In some papers, the endogeneity issue is considered in a reduced-form setting (Jacoby and Skoufias, 1998, Jalan and Ravallion, 1999, Kochar, 1999).

3.2 Formal Contracts and the Income Process

A formal contract is described by Proposition 2 or equation (22). The vector of formal transfers (*i.e.* for any state s) is a function of the following form:

$$T = T(\ln \tilde{\varepsilon}_{it} - \ln \tau(c_{it-1}, x_{it-1}) - \delta_{vt-1}, y_{it}, y_{it-1})$$

which makes clear that formal contracts are dependent of preference shocks and where y_{it} includes L_{it} the quantity of owned land, for instance. These formal transfers T are supposed to be supported by land-leasing contracts: a sharecropping contract involves M_{it} units of land with an output share α_{it}; a fixed rent-contract concern F_{it} units of land at a fixed price, set at the village level. We freely consider that M_{it} and F_{it} can be positive or negative depending on whether land-leasing is in or out. Moreover, agricultural profits are necessarily a function of these land inputs:

$$\pi_{it} = \pi_{it}(M_{it}, F_{it}, \alpha_{it}, y_{it}, y_{it-1}, z_{it}, \xi_{it})$$

where z_{it}, ξ_{it} are variables or shocks revealed after the signature of the contracts (see below). Depending on available data, we could presumably estimate a production function and input demands including labor in order to derive this profit function. Given the complicated endogenous structure of land exploitation, results will not be robust to specification errors on the production side. This is why we model directly the dependence of profits on the marginal utility of consumption and the information variables, skipping the relationships between the quantities of land under sharecropping and fixed-rent, and the marginal utility[8]. We then write agricultural profits as a linear function:

$$\pi_{it} = \pi_{vt} + \pi_0(\ln \tilde{\varepsilon}_{it} - \ln \tau(c_{it-1}, x_{it-1}) - \delta_{vt-1}) + y_{it}\pi_y + y_{it-1}\hat{\pi}_y + z_{it}\pi_z + \xi_{it}$$

where parameter π_0 is **positive** by Proposition 2. Risks, summarized by state s in the theoretical model, are assumed to be translated by the village intercept π_{vt} and the household

[8] We shall however test in the empirical section that these quantities are related to marginal utilities.

random shock ξ_{it}. Other risks, summarized by state σ in the theoretical model, are described by the random shock ξ_{it} and are also determined by variables z_{it} such as days of sickness and so on. We shall assume that random shocks ξ_{it} are independent across households and that the village effects perfectly take into account any dependence across households.

To close the gap with the income variable that appear in the equation of consumption dynamics, household net income r_{it} is written as the sum of agricultural profits, π_{it}, and non agricultural profits or other exogenous income, π_{it}^e:

$$r_{it} = \pi_{it} + \pi_{it}^e$$

Exogenous income could be other non labor income or exogenous transfers such as exogenous remittances from abroad if they are independent of random preference shocks $\ln \tilde{\varepsilon}_{it}$ and income shocks ξ_{it}. They exclude informal transfers from the extended families studied for instance by Foster and Rosenzweig (2001) because these transfers are linked to the endogenous informal contracts modeled here and obviously dependent on both types of random shocks. This income equation defines the income variable, r_{it}, appearing in (22). We shall below include π_{it}^e into the z_{it} variables above with no loss of generality. We also add some measurement errors, ς_{it}, to agricultural profits to obtain measured profits:

$$\tilde{\pi}_{it} = \pi_{vt} + \pi_0(\ln \tilde{\varepsilon}_{it} - \ln \tau(c_{it-1}, x_{it-1}) - \delta_{vt-1}) + y_{it}\pi_y + y_{it-1}\hat{\pi}_y + z_{it}\pi_z + \xi_{it} + \varsigma_{it} \tag{24}$$

The structural form of the model therefore consists of equations (22) and (24). We now write the reduced form.

3.3 Identifying Restrictions and the Reduced Forms

We state the identifying restrictions and write the reduced form. Recall that all variables x entering preferences are also included in the information set $y = (x, q)$ where q are all other variables known before the signature of the contract (owned land for instance). Recall also that z_{it} denote the variables known after the signature of the contract[9]. We shall

[9] At the identification stage, we can assume that z_{it} and y_{it} are orthogonal. Actually, if they are not orthogonal, we could replace z_{it} by the component of z_{it} independent of y_{it} since we are only interested by identification of π_0 and β. See Kochar (1999).

therefore accordingly denote that coefficients, say, $\pi_y = (\pi_x,\ \pi_q)$ whenever necessary. The two endogenous variables at time t are consumption growth, $\Delta \ln c_{it}$, and agricultural profits, $\tilde{\pi}_{it}$. We also have the following list of (weakly) exogenous variables, $\ln c_{it-1}$, x_{it-1}, x_{it}, q_{it} and z_{it} that appear in both equations.

The identifying restrictions are the following ones. Random shocks are $(\ln \tilde{\varepsilon}_{it}, \xi_{it}, \varsigma_{it})$. We assume first that this triplet of random shocks is independent of $(\ln c_{it-1}, x_{it-1}, x_{it}, q_{it}, z_{it})$. Second, $(\ln \tilde{\varepsilon}_{it}, \xi_{it}, \varsigma_{it})$ is identically distributed and is independent across households and periods.[10]

After using and reshuffling terms in the different equations, the income equation can be written as:

$$\tilde{\pi}_{it} = \tilde{\pi}_{vt} + \pi_0\theta(\ln c_{it-1} - x_{it-1}\beta) + x_{it}\pi_x + q_{it}\pi_q + x_{it-1}\hat{\pi}_x + q_{it-1}\hat{\pi}_q + z_{it}\pi_z + \pi_0 \ln \tilde{\varepsilon}_{it} + \xi_{it} + \varsigma_{it}$$

where the village effect includes preference and income village effects. The reduced form of the income process is therefore:

$$\begin{aligned}\tilde{\pi}_{it} &= \tilde{\pi}_{vt} + \pi_0\theta \ln c_{it-1} + x_{it-1}(\hat{\pi}_x + \pi_x - \pi_0\theta\beta) + \Delta x_{it}\pi_x \\ &\quad + q_{it}\pi_q + q_{it-1}\hat{\pi}_q + z_{it}\pi_z + \pi_0 \ln \tilde{\varepsilon}_{it} + \xi_{it} + \varsigma_{it} \end{aligned} \tag{25}$$

Moreover, the reduced form of the consumption equation (22) can be written as:

$$-\theta(\Delta \ln c_{it} - \Delta x_{it}\beta) + \Delta\delta_{vt} + \ln \tilde{\varepsilon}_{it} = \underline{\phi}_{it}\mathbf{1}\{\underline{\phi}_{it} > 0\} + \overline{\phi}_{it}\mathbf{1}\{\overline{\phi}_{it} < 0\} \tag{26}$$

where:

$$\begin{aligned}\phi_{it} &= \mu_0\left(\pi_{vt} + \pi_0(\ln \tilde{\varepsilon}_{it} - \ln \tau(c_{it-1}, x_{it-1}) - \delta_{vt-1}) + y_{it}\pi_y + y_{it-1}\hat{\pi}_y + z_{it}\pi_z + \xi_{it}\right) \\ &\quad + y_{it}\mu_y + \mu_{vt} + \ln \tilde{\varepsilon}_{it} - \ln \tau(c_{it-1}, x_{it-1}) - \delta_{vt-1}\end{aligned}$$

with implicitly $\phi_{it} \in \{\underline{\phi}_{it}, \overline{\phi}_{it}\}$ and μ_0, μ_y and μ_{vt} are defined accordingly (*i.e.* $\mu_i \in \{\underline{\mu}_i, \overline{\mu}_i\}$ for $i = 0,\ y,\ vt$, see equation (23)). Then, reshuffling:

$$\begin{aligned}\phi_{it} &= \phi_{vt} - (\mu_0\pi_0 + 1)\ln \tau(c_{it-1}, x_{it-1}) + y_{it}\left(\mu_0\pi_y + \mu_y\right) + y_{it-1}\mu_0\hat{\pi}_y + z_{it}\mu_0\pi_z \\ &\quad + (\mu_0\pi_0 + 1)\ln \tilde{\varepsilon}_{it} + \mu_0\xi_{it}\end{aligned}$$

[10] We could relax these assumptions to get identification of some subsets of parameters.

where ϕ_{vt} is the composition of different village effects:

$$\phi_{vt} = \mu_0(\pi_{vt} - \pi_0\delta_{vt-1}) - \delta_{vt-1} + \mu_{vt}.$$

Replacing the marginal utility function and using the different exogenous variables, we get :

$$\begin{aligned} \phi_{it} &= \phi_{vt} + (\mu_0\pi_0 + 1)\theta(\ln c_{it-1} - x_{it-1}\beta) + \\ &\quad + z_{it}\mu_0\pi_z + x_{it}(\mu_0\pi_x + \mu_x) + q_{it}(\mu_0\pi_q + \mu_q) + x_{it-1}\mu_0\hat{\pi}_x + q_{it-1}\mu_0\hat{\pi}_q + u_{it} \end{aligned}$$

where we denote $u_{it} \in \{\underline{u}_{it}, \overline{u}_{it}\}$ the random terms in ϕ_{it} defined by:

$$u_{it} = (\mu_0\pi_0 + 1)\ln\tilde{\varepsilon}_{it} + \mu_0\xi_{it}$$

Linearity implies that unobserved heterogeneity only enters the intercept in indices, ϕ_{it}, which is the central piece of identifying restrictions. A slight generalization of this setting could permit parameters μ_0 and π_0 or other parameters to depend on exogenous variables. Generalizing to functions which slopes depend on some unobserved heterogeneity is a much more difficult task.

If $\underline{\mu}_0 \neq \overline{\mu}_0$, there is a one-to-one mapping between $(\ln\tilde{\varepsilon}_{it},\ \xi_{it})$ and $(\underline{u}_{it},\ \overline{u}_{it})$. It is equivalent to assume that the pair $(\ln\tilde{\varepsilon}_{it},\ \xi_{it})$ is identically and independently distributed conditional on exogenous variables, or that the pair $(\underline{u}_{it},\ \overline{u}_{it})$ is identically and independently distributed conditional on exogenous variables. It is therefore identical at this stage to fix one or the other of these distribution functions.

Indices ϕ_{it} can be written as:

$$\begin{aligned} \phi_{it} &= \phi_{vt} + (\mu_0\pi_0 + 1)\theta\ln c_{it-1} + x_{it-1}(-(\mu_0\pi_0 + 1)\theta\beta + \mu_0\pi_x + \mu_x + \mu_0\hat{\pi}_x) \qquad (27) \\ &\quad + \Delta x_{it}(\mu_0\pi_x + \mu_x) + q_{it}(\mu_0\pi_q + \mu_q) + z_{it}\mu_0\pi_z + q_{it-1}\mu_0\hat{\pi}_q + u_{it} \\ &= \phi^*_{it} + u_{it} \end{aligned}$$

Replacing indices ϕ_{it}, the consumption equation (26) is now given by:

$$\begin{aligned} -\theta(\Delta\ln c_{it} - \Delta x_{it}\beta) + \Delta\delta_{vt} + \ln\tilde{\varepsilon}_{it} &= \qquad (28) \\ (\underline{\phi}^*_{it} + \underline{u}_{it})\mathbf{1}\{\underline{u}_{it} &> -\underline{\phi}^*_{it}\} + (\overline{\phi}^*_{it} + \overline{u}_{it})\mathbf{1}\{\overline{u}_{it} < -\overline{\phi}^*_{it}\} \end{aligned}$$

where the linearity of the intensities of the incentive constraints as a function of heterogeneity has now been made explicit.

The system of equations (25) and (28) defines the endogenous variables as functions of the (weakly) exogenous variables:

$$w_{it} = (\ln c_{it-1}, x_{it-1}, \Delta x_{it}, q_{it-1}, q_{it}, z_{it})$$

3.4 Semi-parametric Identification

The identification analysis proceeds in various steps. First, the parameters of the reduced form of the income equation (25) are trivially identified. Parameters $\tilde{\pi}_{vt}$, $\pi_0\theta$, $\hat{\pi}_x - \beta\pi_0\theta$, π_z, π_x, π_q and $\hat{\pi}_q$ are therefore identified. The conditional independence assumption implies restrictions on the variance for instance and therefore the full independence assumption is testable. We could assume however that the conditional distribution of measurement error ς_{it} depends on covariates in an arbitrary way and in this case, the corresponding identifying restrictions become untestable.

Second, using the consumption equation, it is easy to show that θ is not identified. The transformation from the vector of parameters (θ, π_0, μ_0, μ_x, μ_q, $\Delta\delta_{vt}$, ϕ_{vt}) into (1, $\frac{\pi_0}{\theta}$, $\frac{\mu_0}{\theta}$, $\frac{\mu_x}{\theta}$, $\frac{\mu_q}{\theta}$, $\frac{\Delta\delta_{vt}}{\theta}$, $\frac{\phi_{vt}}{\theta}$) (leaving unchanged the other parameters) and the vector of random shocks ($\ln\tilde{\varepsilon}_{it}$, ξ_{it}, ς_{it}) into ($\frac{\ln\tilde{\varepsilon}_{it}}{\theta}$, ξ_{it}, ς_{it}) is invariant for the two equations of interest. We shall therefore normalize $\theta = 1$ without loss of generality and change the interpretation of other parameters accordingly. It is not a surprise since, in a usual Euler framework, the relative risk aversion or intertemporal substitution parameter is not identified if there is no information on the true interest rate.

The consumption equation becomes:

$$\begin{aligned} \Delta\ln c_{it} &= \beta\Delta x_{it} + \Delta\delta_{vt} + \ln\tilde{\varepsilon}_{it} \\ -(\underline{\phi}^*_{it} + \underline{u}_{it})\mathbf{1}\{\underline{u}_{it} &> -\underline{\phi}^*_{it}\} - (\overline{\phi}^*_{it} + \overline{u}_{it})\mathbf{1}\{\overline{u}_{it} < -\overline{\phi}^*_{it}\} \end{aligned} \tag{29}$$

We can first note that if $\{\underline{\phi}_{it}, \overline{\phi}_{it}\}$ are identified from the consumption equation (as we will show below), then $\underline{\phi}_{vt}$, $\overline{\phi}_{vt}$ are identified. Moreover, as π_{vt} is identified from the income equation and $\Delta\delta_{vt}$ from the consumption equation, there cannot be cross constraints between village effects appearing in the different terms because the structure is sufficiently flexible. The presence of village effects do not bring identification power. We are interested by the identification of the following parameters: π_0, π_x, π_q, π_z, μ_0, μ_x, μ_q, β, $\hat{\pi}_x$, $\hat{\pi}_q$ and the distribution functions of the random shocks. We study the identification of these parameters.

3.4.1 Average Derivatives of the Consumption Equation

Write the conditional expectation of equation (29) conditional on w_{it}:

$$\begin{aligned} E(\Delta \ln c_{it} \quad | \quad w_{it}) = \Delta x_{it}\beta + \Delta\delta_{vt} \\ -E((\underline{\phi}^*_{it} + \underline{u}_{it})\mathbf{1}\{\underline{u}_{it} \quad > \quad -\underline{\phi}^*_{it}\} \mid w_{it}) - E((\overline{\phi}^*_{it} + \overline{u}_{it})\mathbf{1}\{\overline{u}_{it} < -\overline{\phi}^*_{it}\} \mid w_{it}) \end{aligned} \tag{30}$$

The last terms are written as, for instance:

$$E((\underline{\phi}^*_{it} + \underline{u}_{it})\mathbf{1}\{\underline{u}_{it} > -\underline{\phi}^*_{it}\} \mid w_{it}) = \int_{\underline{u}_{it} > -\underline{\phi}^*_{it}(w_{it})} (\underline{\phi}^*_{it}(w_{it}) + \underline{u}_{it}) d\underline{F}(\underline{u}_{it})$$

where $\underline{F}(u_{it})$ is independent of w_{it} under the identifying restrictions. Function $\underline{\phi}^*_{it}(w_{it})$ is the only function that depends on w_{it} in this expression. Therefore, the derivatives of these terms are:

$$\begin{aligned} \frac{\partial}{\partial w_{it}} E((\underline{\phi}^*_{it} + \underline{u}_{it})\mathbf{1}\{\underline{u}_{it} \quad > \quad -\underline{\phi}^*_{it}\} \mid w_{it}) &= \int_{\underline{u}_{it} > -\underline{\phi}^*_{it}(w_{it})} \frac{\partial}{\partial w_{it}} \underline{\phi}^*_{it}(w_{it}) d\underline{F}(\underline{u}_{it}) \\ &= \frac{\partial}{\partial w_{it}} \underline{\phi}^*_{it}(w_{it}). \int_{\underline{u}_{it} > -\underline{\phi}^*_{it}(w_{it})} d\underline{F}(\underline{u}_{it}) = \frac{\partial}{\partial w_{it}} \underline{\phi}^*_{it}(w_{it}).\underline{\Psi}(w_{it}) \end{aligned}$$

and similarly

$$\frac{\partial}{\partial w_{it}} E((\overline{\phi}^*_{it} + \overline{u}_{it})\mathbf{1}\{\overline{u}_{it} < -\overline{\phi}^*_{it}\} \mid w_{it}) = \frac{\partial}{\partial w_{it}} \overline{\phi}^*_{it}(w_{it}).\overline{\Psi}(w_{it})$$

using the smoothness of distribution functions of random terms and where

$$\begin{aligned} \underline{\Psi}(w_{it}) &= \int_{\underline{u}_{it} > -\underline{\phi}^*_{it}(w_{it})} d\underline{F}(\underline{u}_{it}) \\ \overline{\Psi}(w_{it}) &= \int_{\overline{u}_{it} < -\overline{\phi}^*_{it}(w_{it})} d\overline{F}(\overline{u}_{it}) \end{aligned}$$

The derivative of the equation of interest (30) becomes:

$$\frac{\partial}{\partial w_{it}}E(\Delta \ln c_{it} \mid w_{it}) = \frac{\partial \Delta x_{it}\beta}{\partial w_{it}} - \frac{\partial \underline{\phi}^*_{it}}{\partial w_{it}}\underline{\Psi}(w_{it}) - \frac{\partial \overline{\phi}^*_{it}}{\partial w_{it}}\overline{\Psi}(w_{it}) \tag{31}$$

which yields the moment conditions that form the basis of the analysis of identification. First, these derivatives are identified (Pagan and Ullah, 1999). Second, $\underline{\Psi}$ and $\overline{\Psi}$ are functions of linear indices $\phi^*_{it}(w_{it})$ that are left implicit. Note also the implicit dependence on village and period specific indicators.

To proceed, the different relevant derivatives are given in the following table:

w_{it}	$\frac{\partial \Delta x_{it}\beta}{\partial w_{it}}$	$\frac{\partial \phi^*_{it}}{\partial w_{it}}$
$\ln c_{it-1}$	0	$\mu_0\pi_0 + 1$
x_{it-1}	0	$-\beta(\mu_0\pi_0 + 1) + \mu_0\pi_x + \mu_x + \mu_0\hat{\pi}_x$
Δx_{it}	β	$\mu_0\pi_x + \mu_x$
q_{it-1}	0	$\mu_0\hat{\pi}_q$
q_{it}	0	$\mu_0\pi_q + \mu_q$
z_{it}	0	$\mu_0\pi_z$

which shows that the linearity assumptions imply that derivatives are independent of exogenous variables. We can therefore use average derivatives as well. It is well known that the convergence rate of estimates of average derivatives is $\sqrt{n}$ while the convergence of pointwise derivatives (31) is less than $\sqrt{n}$ estimable (Horowitz, 1998, Pagan and Ullah, 1999)[11]. In terms of average derivatives, (31) can be written as:

$$D^w \equiv E(\frac{\partial}{\partial w_{it}}E(\Delta \ln c_{it} \mid w_{it})) = \frac{\partial \Delta x_{it}\beta}{\partial w_{it}} - \frac{\partial \underline{\phi}^*_{it}}{\partial w_{it}}\underline{\Psi} - \frac{\partial \overline{\phi}^*_{it}}{\partial w_{it}}\overline{\Psi} \tag{32}$$

where there is a slight abuse of notations, $\Psi = E(\Psi(w_{it}))$, $\Psi \in \{\underline{\Psi}, \overline{\Psi}\}$. We use this notation in order to emphasize that Ψ could stand for average derivatives or point-wise derivatives as well.

Using this equation only seems to be an interesting way to proceed because the consumption growth equation (29) could include additional random shocks apart from the preference

[11] The average derivatives could be defined holding (v, t) fixed. All the relationships would also be true at this level. Additional tests of the structure could therefore be derived.

shock, $\ln \tilde{\varepsilon}_{it}$, as briefly sketched at the end of the theory section. We can also show that a different specification for the upper and lower constraints (equation 23) lead to the same type of equation (see Appendix C).

3.4.2 From Average Derivatives to Parameters

We know from previous arguments that π_0, π_z, π_x, π_q, $\hat{\pi}_q$ and $\hat{\pi}_x - \pi_0\beta$ are identified from the income equation and we shall treat them as known parameters. As described above, we have five (formal) relationships on average derivatives in order to identify $\underline{\Psi}$, $\overline{\Psi}$, and the seven (formal) parameters of interest, β, $\overline{\mu}_0$, $\underline{\mu}_0$, $\overline{\mu}_x$, $\underline{\mu}_x$, $\overline{\mu}_q$, $\underline{\mu}_q$. The degree of underidentification is at least equal to 4.

Notice that

$$D^x = D^{\Delta x} - \beta(1 + D^c - \frac{\pi_0}{\pi_z}D^z) + \frac{\hat{\pi}_x - \pi_0\beta}{\pi_z}D^z$$

which yields identification of β (and thus $\hat{\pi}_x$):

$$\beta = \frac{D^{\Delta x} - D^x + \frac{\hat{\pi}_x - \pi_0\beta}{\pi_z}D^z}{1 + D^c - \frac{\pi_0}{\pi_z}D^z}.$$

In particular note that under a complete market assumption, $\beta = D^{\Delta x}$.

Moreover

$$D^{q-1} = \frac{\hat{\pi}_q}{\pi_z}D^z$$

which is a structural restriction on average derivatives that does not depend on still unknown parameters.

REMARK: The first may not be robust to a mistake in the choices of the variables put in q and z? Suppose z is observed before the contract (or part is anticipated) but is i.i.d. so that only z_{it} enters in the income equation. Then we should have a coefficient μ_z for z as well, the rest is unchanged apart from the interpretation of π_z. We then get

$$D^{q-1} - \frac{\hat{\pi}_q}{\pi_z}D^z = \underline{\mu}_z\underline{\Psi} + \overline{\mu}_z\overline{\Psi}.$$

The same holds true if z contains some information on future preferences (some type of illness) or future production (land productivity for instance) as it should enter into the functions μ. In this case what is done below remains valid.

The other first order derivatives write as follows:

$$\begin{aligned} -D^c = & \ \pi_0(\underline{\mu}_0\underline{\Psi} + \overline{\mu}_0\overline{\Psi}) + \underline{\Psi} + \overline{\Psi} \\ -D^z = & \ \pi_z(\underline{\mu}_0\underline{\Psi} + \overline{\mu}_0\overline{\Psi}) \\ -D^{\Delta x} + \beta = & \ \pi_x(\underline{\mu}_0\underline{\Psi} + \overline{\mu}_0\overline{\Psi}) + \underline{\mu}_x\underline{\Psi} + \bar{\mu}_x\overline{\Psi} \\ -D^q = & \ \pi_q(\underline{\mu}_0\underline{\Psi} + \overline{\mu}_0\overline{\Psi}) + \underline{\mu}_q\underline{\Psi} + \bar{\mu}_q\overline{\Psi} \end{aligned}$$

The system of average derivatives allows to identify the parameters $\underline{\Psi} + \overline{\Psi}$, $\underline{\mu}_0\underline{\Psi} + \overline{\mu}_0\overline{\Psi}$, $\underline{\mu}_x\underline{\Psi} + \bar{\mu}_x\overline{\Psi}$, $\underline{\mu}_q\underline{\Psi} + \bar{\mu}_q\overline{\Psi}$. As we have four parameters themselves combinations of 8 parameters, the degree of under-identification is equal to four.

To summarize, let $M = (1, \mu_0, \mu_x, \mu_q)$ we have identified the coefficients of the income equation and

$$\underline{M\Psi} + \bar{M}\bar{\Psi}.$$

Testable restrictions on the identified parameters are:

$$\begin{aligned} \pi_0 \ & \geq \ 0 \text{ (monotonicity of the contract)} \\ \underline{\Psi} + \overline{\Psi} \ & \geq \ 0 \text{ (probability that an incentive constraint binds)} \\ \underline{\mu}_0\underline{\Psi} + \overline{\mu}_0\overline{\Psi} \ & \leq \ 0 \text{ (monotonicity of the contract from proposition 2)} \end{aligned}$$

To proceed further, we need to use again the pointwise derivatives defined in (31).

3.4.3 Average Second Derivatives

Differentiate (31) to get:

$$\frac{\partial^2}{\partial w_{it}\partial w_{it}'}E(\Delta \ln c_{it} \mid w_{it}) = -\frac{\partial \underline{\phi}_{it}^*}{\partial w_{it}}\frac{\partial \underline{\phi}_{it}^*}{\partial w_{it}'}\underline{\Psi}'(w_{it}) - \frac{\partial \overline{\phi}_{it}^*}{\partial w_{it}}\frac{\partial \underline{\phi}_{it}^*}{\partial w_{it}'}\overline{\Psi}'(w_{it}) \tag{33}$$

because $\frac{\partial \underline{\phi}_{it}^*}{\partial w_{it}}$ is constant and because $\Psi(w_{it})$ is a function of the linear in w_{it} index ϕ_{it}^*. The average second derivatives are:

$$D^{ww'} \equiv E(\frac{\partial^2}{\partial w_{it}\partial w_{it}'}E(\Delta \ln c_{it} \mid w_{it})) = -\frac{\partial \underline{\phi}_{it}^*}{\partial w_{it}}\frac{\partial \underline{\phi}_{it}^*}{\partial w_{it}'}E\underline{\Psi}'(w_{it}) - \frac{\partial \overline{\phi}_{it}^*}{\partial w_{it}}\frac{\partial \overline{\phi}_{it}^*}{\partial w_{it}'}E\overline{\Psi}'(w_{it}) \tag{34}$$

where $\underline{\Psi}' = E\underline{\Psi}'(w_{it})$ and $\overline{\Psi}' = E\overline{\Psi}'(w_{it})$.

Noting

$$\frac{\partial \phi^*_{it}}{\partial x_{it-1}} = [-\beta(\mu_0\pi_0 + 1) + \mu_0\pi_x + \mu_x + \mu_0\hat{\pi}_x]$$

we have a symmetric matrix of products of derivatives (symmetric terms are not displayed for space reasons):

	c	Δx	q	z	x	q_{-1}
c	$(\mu_0\pi_0+1)^2$	$(\mu_0\pi_0+1)(\mu_0\pi_x+\mu_x)$	$(\mu_0\pi_0+1)\left(\mu_0\pi_q+\mu_q\right)$	$(\mu_0\pi_0+1)\,\mu_0\pi_z$		
Δx		$(\mu_0\pi_x+\mu_x)^2$	$(\mu_0\pi_x+\mu_x)(\mu_0\pi_q+\mu_q)$	$(\mu_0\pi_x+\mu_x)\,\mu_0\pi_z$		
q			$\left(\mu_0\pi_q+\mu_q\right)^2$	$\left(\mu_0\pi_q+\mu_q\right)\mu_0\pi_z$		
z				$(\mu_0\pi_z)^2$		
x	$(\mu_0\pi_0+1)\,\frac{\partial\phi^*_{it}}{\partial x_{it\text{-}1}}$	$(\mu_0\pi_x+\mu_x)\,\frac{\partial\phi^*_{it}}{\partial x_{it\text{-}1}}$	$\left(\mu_0\pi_q+\mu_q\right)\frac{\partial\phi^*_{it}}{\partial x_{it\text{-}1}}$	$\mu_0\pi_z\frac{\partial\phi^*_{it}}{\partial x_{it\text{-}1}}$	$\frac{\partial\phi^*_{it}}{\partial x_{it\text{-}1}}^{2}$	$\frac{\partial\phi^*_{it}}{\partial x_{it\text{-}1}}\mu$
q_{-1}	$(\mu_0\pi_0+1)\,\mu_0\hat{\pi}_q$	$(\mu_0\pi_x+\mu_x)\,\mu_0\hat{\pi}_q$	$\left(\mu_0\pi_q+\mu_q\right)\mu_0\hat{\pi}_q$	$\mu_0^2\pi_z\hat{\pi}_q$		$(\mu_0\hat{\pi}$

Using $D^{zz} = -\pi_z^2(\underline{\mu}_0^2\underline{\Psi}' + \bar{\mu}_0^2\overline{\Psi}')$, we obtain $\underline{\mu}_0^2\underline{\Psi}' + \bar{\mu}_0^2\overline{\Psi}'$. Moreover

$$\begin{cases} \underline{\Psi}' + \overline{\Psi}' = & -\pi_0(\underline{\mu}_0\underline{\Psi}' + \bar{\mu}_0\overline{\Psi}') + D^{cc} - \frac{\pi_0}{\pi_z}D^{cz} \\ \underline{\mu}_0\underline{\Psi}' + \bar{\mu}_0\overline{\Psi}' = & \frac{1}{\pi_z}\left(\frac{\pi_0}{\pi_z}D^{zz} - D^{zc}\right) \end{cases}$$

which gives $\underline{\Psi}' + \overline{\Psi}'$ and $\underline{\mu}_0\underline{\Psi}' + \bar{\mu}_0\overline{\Psi}'$.

Then

$$\begin{cases} \underline{\mu}_x\underline{\mu}_0\underline{\Psi}' + \bar{\mu}_x\bar{\mu}_0\overline{\Psi}' = & -\pi_x(\underline{\mu}_0^2\underline{\Psi}' + \bar{\mu}_0^2\overline{\Psi}') - \frac{D^{z\Delta x}}{\pi_z} \\ \underline{\mu}_q\underline{\mu}_0\underline{\Psi}' + \bar{\mu}_q\bar{\mu}_0\overline{\Psi}' = & -\pi_q(\underline{\mu}_0^2\underline{\Psi}' + \bar{\mu}_0^2\overline{\Psi}') - \frac{D^{zq}}{\pi_z} \end{cases}$$

$$\begin{cases} \underline{\mu}_x^2\underline{\Psi}' + \bar{\mu}_x^2\overline{\Psi}' = & \frac{\pi_x}{\pi_z}D^{z\Delta x} - D^{\Delta x\Delta x} - \pi_x(\underline{\mu}_x\underline{\mu}_0\underline{\Psi}' + \bar{\mu}_x\bar{\mu}_0\overline{\Psi}') \\ \underline{\mu}_q^2\underline{\Psi}' + \bar{\mu}_q^2\overline{\Psi}' = & \frac{\pi_q}{\pi_z}D^{zq} - D^{qq} - \pi_q(\underline{\mu}_q\underline{\mu}_0\underline{\Psi}' + \bar{\mu}_q\bar{\mu}_0\overline{\Psi}') \end{cases}$$

We can thus identify $\underline{\mu}_x\underline{\mu}_0\underline{\Psi}' + \bar{\mu}_x\bar{\mu}_0\overline{\Psi}'$, $\underline{\mu}_q\underline{\mu}_0\underline{\Psi}' + \bar{\mu}_q\bar{\mu}_0\overline{\Psi}'$, $\underline{\mu}_x^2\underline{\Psi}' + \bar{\mu}_x^2\overline{\Psi}'$, $\underline{\mu}_q^2\underline{\Psi}' + \bar{\mu}_q^2\overline{\Psi}'$.

Finally

$$\begin{cases} \underline{\mu}_x\underline{\Psi}' + \bar{\mu}_x\overline{\Psi}' = & -\pi_x\pi_0(\underline{\mu}_0^2\underline{\Psi}' + \bar{\mu}_0^2\overline{\Psi}') - \pi_x(\underline{\mu}_0\underline{\Psi}' + \bar{\mu}_0\overline{\Psi}') - \pi_0(\underline{\mu}_x\underline{\mu}_0\underline{\Psi}' + \bar{\mu}_x\bar{\mu}_0\overline{\Psi}') - D^{c\Delta x} \\ \underline{\mu}_q\underline{\Psi}' + \bar{\mu}_q\overline{\Psi}' = & -\pi_q\pi_0(\underline{\mu}_0^2\underline{\Psi}' + \bar{\mu}_0^2\overline{\Psi}') - \pi_q(\underline{\mu}_0\underline{\Psi}' + \bar{\mu}_0\overline{\Psi}') - \pi_0(\underline{\mu}_q\underline{\mu}_0\underline{\Psi}' + \bar{\mu}_q\bar{\mu}_0\overline{\Psi}') - D^{cq} \end{cases}$$

which identify $\underline{\mu}_x\underline{\Psi}' + \bar{\mu}_x\overline{\Psi}'$ and $\underline{\mu}_q\underline{\Psi}' + \bar{\mu}_q\overline{\Psi}'$.

Note also that

$$\begin{aligned} \underline{\mu}_x\underline{\mu}_q\underline{\Psi}' + \bar{\mu}_x\bar{\mu}_q\overline{\Psi}' &= -\pi_x\pi_q(\underline{\mu}_0^2\underline{\Psi}' + \bar{\mu}_0^2\overline{\Psi}') \\ &\quad -\pi_q(\underline{\mu}_x\underline{\mu}_0\underline{\Psi}' + \bar{\mu}_x\bar{\mu}_0\overline{\Psi}') - \pi_x(\underline{\mu}_q\underline{\mu}_0\underline{\Psi}' + \bar{\mu}_q\bar{\mu}_0\overline{\Psi}') - D^{q\Delta x} \end{aligned}$$

To summarize, we have identified

$$\underline{M}'\underline{M}\Psi' + \bar{M}'\bar{M}\bar{\Psi}' = K'$$

This gives 10 relations (because of the symmetry of $M'M$).

Overall, using first and second average derivatives we have 14 independent relations for 10 parameters. Thus the model is overidentified.

3.5 Estimation using a Generalized Method of Moments

As the previous objects are identified in quite general conditions, we shall further strengthen the identifying assumptions to:

$$\underline{u}_{it} \text{ and } \overline{u}_{it} \text{ are normally distributed conditional to } w_{it}$$

and $(\underline{\sigma}^2, \overline{\sigma}^2)$ are their respective variances. This assumption is testable as shown above. The moment conditions related to the random terms appearing in the consumption equation that we will use are:

$$E(m_{it}' \ln \tilde{\varepsilon}_{it}) = 0$$

if m_{it} are the variables w_{it} and village-period dummies. We will use:

$$E\left[w_{it}'(\underline{\phi}_{it}^* + \underline{u}_{it})\mathbf{1}\{\underline{u}_{it} > -\underline{\phi}_{it}^*\}\right] = E\left[w_{it}'.E\left((\underline{\phi}_{it}^* + \underline{u}_{it})\mathbf{1}\{\underline{u}_{it} > -\underline{\phi}_{it}^*\} \mid w_i\right)\right]$$

where using the normality assumption:

$$E((\overline{\phi}_{it}^* + \overline{u}_{it})\mathbf{1}\{\overline{u}_{it} \quad < \quad -\overline{\phi}_{it}^*\} \mid w_{it}) = \overline{\sigma}[\frac{\overline{\phi}_{it}^*}{\overline{\sigma}}\Phi(\frac{-\overline{\phi}_{it}^*}{\overline{\sigma}}) - \varphi(\frac{\overline{\phi}_{it}^*}{\overline{\sigma}})] = -\overline{\sigma}h(-\frac{\overline{\phi}_{it}^*}{\overline{\sigma}}) \qquad (35)$$

$$E((\underline{\phi}_{it}^* + \underline{u}_{it})\mathbf{1}\{\underline{u}_{it} \quad > \quad -\underline{\phi}_{it}^*\} \mid w_i) = \underline{\phi}_{it}^*.\Phi(\frac{\underline{\phi}_{it}^*}{\underline{\sigma}}) + \underline{\sigma}\varphi(\frac{\underline{\phi}_{it}^*}{\underline{\sigma}}) = \underline{\sigma}h(\frac{\underline{\phi}_{it}^*}{\underline{\sigma}}) \qquad (36)$$

where Φ and φ are the cumulative and density functions of a unit-normal random variable and $h(x) = x.\Phi(x) + \varphi(x)$. Therefore the estimating equation related to consumption dynamics is:

$$E(m_{it}^{'}(\Delta \ln c_{it} - \Delta x_{it}\beta - \Delta\delta_{vt} + \underline{\sigma}h(\frac{\underline{\phi}_{it}^*}{\underline{\sigma}}) - \overline{\sigma}h(-\frac{\overline{\phi}_{it}^*}{\overline{\sigma}}))) = 0 \qquad (37)$$

The other moment conditions related to the profit linear equation is the second estimating equation. The parameters imposing the structural restrictions are estimated in a iterative procedure in two steps. In the first step, we use the weighting matrix corresponding to linear 2SLS. In the second step, we compute an estimator of the weighting matrix using the standard arguments.

4 Empirical Estimation

4.1 Data

The data come from a survey conducted by IFPRI (International Food Policy Research Institute) in Pakistan between 1986 and 1989 (see Alderman and Garcia, 1993). The survey consists of a stratified random sample interviewed 12 times beginning with 927 households from four districts of three regions (Attock and Faisalabad in Punjab, Badin in the Sind, and Dir in the North West Frontier Province, NWFP). For each of the four districts, the villages were chosen randomly from an exhaustive list of villages classified in three sets according to their distances to two markets (*mandis*). In each village, households were randomly drawn from an exhaustive list of village households. The attrition observed in the data (927 households at the beginning and only 887 at the end) seems to come from administrative and political problems rather than from a self selection of households (Alderman and Garcia, 1993). We consider this attrition phenomenon as exogenous. These data are very rich and contain information on household demographic characteristics, on incomes disaggregated in numerous sources, on individual labor supplies, on endowments and owned assets, on agrarian structure, on crops and productions, on land contracts like sharecropping and fixed rent contracts. Income sources are wages, agricultural profits, rents from property rights, pensions, informal transfers (from relatives or others).

To get the variables of interest for this study, we have had to construct some of them from the different available data files. First, the household demographic variables were obtained easily the individual data available. Household food consumption was initially available for each good in quantity and value or quantity with price. Food consumption consist in food expenditures for all members of the household for meals at home including the owned production consumed, the expenditures for meals taken outside but not the value of outside meals due to invitation or rewards in kind because they were not available. Agricultural incomes correspond to cash income from all household agricultural productions, from milk products, from animal poultry and livestock production, net of total agricultural input expenditures including wage costs, feeding costs of productive animals, and all other agricultural inputs like fertilizers and pesticides (and handicraft incomes? check). The wage income correspond to wages received by household members or different agricultural and non agricultural tasks done outside the farm when the households operates one. The rental incomes correspond to property rights rents, fixed pensions regularly received from the government and rentals of different productive assets. Transfers correspond to transfers received from relatives, friends and from solidarity funds of local mosques (*zakat*).

4.2 Empirical Results[12]

First, we present the results obtained using the GMM approach which of course relies on the parametric assumption made. With the normalization of $\theta = 1$, estimating (37) by GMM

$$E(m_{it}^{'}(\Delta \ln c_{it} - \Delta x_{it}\beta - \Delta\delta_{vt} + \underline{\sigma} h(\frac{\underline{\phi}_{it}^{*}}{\underline{\sigma}}) - \overline{\sigma} h(-\frac{\overline{\phi}_{it}^{*}}{\overline{\sigma}}))) = 0$$

where from (27)

$$\begin{aligned} \phi_{it}^{*} &= \phi_{vt} + (\mu_0\pi_0 + 1)\ln c_{it-1} + x_{it-1}(-(\mu_0\pi_0 + 1)\beta + \mu_0\pi_x + \mu_x + \mu_0\hat{\pi}_x) \\ &\quad + \Delta x_{it}(\mu_0\pi_x + \mu_x) + q_{it}(\mu_0\pi_q + \mu_q) + z_{it}\mu_0\pi_z + q_{it-1}\mu_0\hat{\pi}_q \end{aligned}$$

Moreover, from the estimation of the income profit equation (25)

$$\begin{aligned} \tilde{\pi}_{it} &= \tilde{\pi}_{vt} + \pi_0 \ln c_{it-1} + x_{it-1}(\hat{\pi}_x + \pi_x - \pi_0\beta) + \Delta x_{it}\pi_x \\ &\quad + q_{it}\pi_q + q_{it-1}\hat{\pi}_q + z_{it}\pi_z + \pi_0 \ln \tilde{\varepsilon}_{it} + \xi_{it} + \varsigma_{it} \end{aligned}$$

[12] **The empirical results presented here are still preliminary and incomplete.**

The results obtained then allow to test some predictions of the model. According to the theoretical model (proposition 2) and the semi-parametric identification section, a testable restrictions on the identified parameters is that $\pi_0 \geq 0$.

Table 1: Income Profit Equation Results

Parameter		Coeff.	t-stat
π_0		165.1	5.26
π_x			
	household size	1.66	0.06
	number of children	29.26	0.65
$\hat{\pi}_q$			
	land owned in the village	1.84	0.49
	rainfed land owned	5.08	0.61
π_q			
	land owned in the village	23.73	19.91
	rainfed land owned	-29.27	-3.46
π_z			
	household male illness days	56.6	7.32
	household female illness days	-58.4	-4.34
$(\hat{\pi}_x + \pi_x - \pi_0\beta)$			
	household size	10.76	1.71
	number of children	8.24	0.84
$\tilde{\pi}_{vt}$			
All district and time dummies (not shown)			
Observations		8906	

As shown by the estimation results, he first simple prediction that $\pi_0 > 0$ is not rejected on these data. This implies that our theoretical model is actually able to explain empirically the observed pattern of consumption dynamics interaction with the households income processes. Then using the profit equation parameter estimates and the GMM estimates of the consumption dynamics equation, all parameters are identified and Table 2 shows the results.

Table 2: GMM Estimates of the Consumption Dynamics Equation

(to be completed)

Parameter		Coeff.	t-stat
β			
	household size		
	number of children		
$\underline{\mu_0}$			
$\underline{\mu_x}$			
	household size		
	number of children		
$\underline{\mu_q}$			
	land owned in the village		
	rainfed land owned		
$\overline{\mu_0}$			
$\overline{\mu_x}$			
	household size		
	number of children		
$\overline{\mu_q}$			
	land owned in the village		
	rainfed land owned		
$\underline{\sigma}$			
$\underline{\phi_{vt}}$: All district and time dummies (not shown)			
$\overline{\sigma}$			
$\overline{\phi_{vt}}$: All district and time dummies (not shown)			
$\Delta\delta_{vt}$			
All district and time dummies (not shown)			
Observations		8906	

5 Conclusion

In conclusion, we can underline the importance of the structural modelling of alternative assumptions about risk sharing mechanisms. Since, the complete markets hypothesis is generally rejected, the modelling of risk sharing and contracting mechanisms is now necessary to better understand the household behavior in an environment of incomplete markets. Here, we have elaborated a theoretical setting which nests the case of complete markets when all risks can be insured by formal contracts (because all states of nature would be verifiable) and the case where only informal agreements are available. This theoretical model provides two important structural equations of interest, an Euler-type equation of consumption dynamics and the equation of determination of the formal contract. We show that the model is semi-parametrically identified and implemented two estimation methods using either GMM with some parametric assumption or average derivatives estimation allowing to do only semi-parametric assumptions (to be completed). Estimating both equations using data of village economies in Pakistan, we found consistent results with the theoretical model developed.

References

Abreu D. (1988) "On the theory of Infinitely Repeated Games with Discounting", *Econometrica,* 56, 383-396

Abreu D., Pearce D. Stacchetti E. (1986) "Optimal Cartel Equilibria with Imperfect Monitoring", *Journal of Economic Theory,* 39, 251-269

Abreu D., Pearce D. Stacchetti E. (1990) "Toward a Theory of Discounted Repeated Games with Imperfect Monitoring", *Econometrica,* 56, 383-396

Altug, S. and Miller, R. (1990) "Household Choices in Equilibrium", *Econometrica*, 58, 3, 543-570

Arrow K. (1964) "The Role of Securities in the Optimal Allocation of Risk Bearing", *Review of Economic Studies,* 31, 91-96

Atkeson, A. and Lucas R. E. (1992) "On Efficient Distribution with Private Information", *Review of Economic Studies,* 59, 427-453

Attanasio, O. and Davis, S. (1996) "Relative Wage Movements and the Distribution of Consumption", *Journal of Political Economy*, 104, 6, 1227-1262

Attanasio O. and Ríos-Rull J. V. (2001) "Consumption Smoothing and Extended Families.", *Invited Lecture at the World Congress of the Econometric Society, Seattle, 2000.*

Attanasio, O. and Rios-Rull, J. V. (2000) "Consumption Smoothing in Island Economies: Can Public Insurance Reduce Welfare?" *European Economic Review*; 44(7), 1225-58

Attanasio, O. and Weber, G. (1993) "Consumption Growth, the Interest rate and Aggregation", *Review of Economic Studies,* 60, 631-649

Banerjee, A. and Newman A.(1991) "Risk Bearing and the Theory of Income Distribution", *Review of Economic Studies*, 58, 211-235

Bouis, H. and Haddad, L. (1990) *Agricultural Commercialization, Nutrition, and the Rural Poor*, Lynne Rienner Publishers: Washington.

Campbell and Deaton (1989) "Why is Consumption so Smooth?", *Review of Economic Studies*, 56, 357–373

Coate S. and Ravallion M. (1993) "Reciprocity without commitment - Characterization and Performance of Informal Insurance Arrangements", *Journal of Development Economics*, 40, 1-24

Cochrane, J. (1991) "A Simple test of Consumption Insurance", *Journal of Political Economy*, 99, 5, 957-976

Cole H. and Kocherlakota N. (1997) "A Microfoundation for Incomplete Security Markets" WP577 Federal Reserve Bank of Minneapolis

Cole H. and Kocherlakota N. (1997) "Dynamic Games With Hidden Actions and Hidden States" WP583 Federal Reserve Bank of Minneapolis
Cox D. (1987) "Motives for Private Income Transfers", *Journal of Political Economy,* 95(3), 508-46.
Deaton, A. (1990) "On Risk, Insurance and Intra-Village Smoothing", *mimeo*, Woodrow Wilson School, Princeton University
Deaton, A. (1992) *Understanding consumption.* Oxford Clarendon Press
Deaton, A. and Paxson, C. (1994) "Intertemporal choice and inequality", *Journal of Political Economy*, 102, 3, 437-467
Debreu G. (1959) *The Theory of Value* , New York : Wiley
Dercon S; (1998) "Wealth, risk and activity choice: cattle in Western Tanzania", *Journal of Development Economics*, 55, 1-42
Diamond, P. (1967) "The role of stock markets in a general equilibrium model with technological uncertainty", *American Economic Review* 57, 759-776
Dubois P. (2000) "Assurance Parfaite, Hétérogénéité des Préférences et Métayage au Pakistan", *Annales d'Economie et de Statistique, 59, 1-36*
Dubois P. (2002) "Consommation, Partage de Risque et Assurance Informelle: Développements Théoriques et Tests Empiriques Récents", forthcoming *L'Actualité Economique, Revue d'Analyse Economique*
Fafchamps M. (1992) "Solidarity Networks in Pre-Industrial Societies: Rational Peasants with a Moral Economy", *Economic Development and Cultural Change*, 41, 1, 147-174
Fafchamps M. (1997) "Rural Poverty, Risk and Development", *mimeo* Department of Economics, Stanford University
Fafchamps M. and Lund S. (2000) "Risk Sharing Networks in Rural Philippines" *mimeo*
Flavin M. (1993) "The Excess Smoothness of Consumption: Identification and Interpretation", *Review of Economic Studies*, 60, 651–666
Foster A. D. and Rosenzweig M. R. (1996) "Financial Intermediation, Transfers and Commitment: Do Banks Crowd Out Private Insurance Arrangements in Low-Income Rural Areas", *mimeo*, University of Pennsylvania
Foster A., Rosenzweig M. (1997) "Dynamic Savings Decisions in Agricultural Environments with Incomplete Markets" *Journal of Business and Economic Statistics*, 15, 2, 282-292
Foster A. and Rosenzweig M. (2001) "Imperfect Commitment, Altruism and the Family: Evidence from Transfer behavior in Low-Income Rural Areas", *Review of Economics and Statistics*, 83, 3, 389-407
Gauthier C., Poitevin M., Gonzalez P. (1997) "Ex-ante payments in self enforcing risk sharing contracts", *Journal of Economic Theory*, 76, 106-144
Grimard, F. (1997) "Household Consumption Smoothing through Ethnic Ties: Evidence from Côte d'Ivoire", *Journal of Development Economics*, 53:391-422.
Green, E. J. and Oh S. N. (1991) "Contracts, Constraints and Consumption", *Review of Economic Studies*, 58, 883-899

Green, E. J. (1987) "Lending and the Smoothing of Uninsurable Income", in Prescott, E., C. and Wallace, N. (eds.), *Contractual Arrangements for Intertemporal Trade*, Minnesota Studies in Macroeconomics, Vol. 1, University of Minnesota Press, 3-25

Hall, R. E. (1978) "Stochastic Implications of the Life-Cycle Permanent Income Hypothesis: Theory and Evidence" *Journal of Political Economy*, 86, 971-987

Hansen, L. P. (1982) "Large Sample Properties of Generalized Method of Moments Estimators", *Econometrica* 50, 1029-1054

Harris, M. and Townsend, R. (1981) "Resource Allocation Under Asymmetric Information", *Econometrica*, 49, 1, 33-47

Hayashi F., Altonji J. and Kotlikoff L. (1996) "Risk Sharing Between and Within Families", *Econometrica*, 64, 2, 261-294

Horowitz J. and Hardle W. (1996) "Direct Semi-Parametric Estimation of Single-Index Models with Discrete Covariates", *Journal of American Statistical Association*, 91, 436, 1632-1640

Jacoby H. G. and Skoufias E. (1998) "Testing Theories of Consumption Behavior Using Information on Aggregate Shocks: Income Seasonality and Rainfall in Rural India", *American Journal of Agricultural-Economics*, 80, 1, 1-14

Jalan J. and Ravallion M. (1999) "Are the Poor Less Well Insured? Evidence on Vulnerability to Income Risk in Rural China", *Journal of Development Economics,* 58, 1, 61-81

Kimball M. S. (1988) "Farmers' Cooperatives as Behavior Toward Risk", *American Economic Review*, 78, 1, 224-232

Kochar A. (1999) "Smoothing Consumption by Smoothing Income: Hours-of-Work Responses to Idiosyncratic Agricultural Shocks in Rural India", *Review of Economics and Statistics*, 81, 1 50-61

Kocherlakota, N. R. (1996) "Implications of Efficient risk sharing without commitment", *Review of Economic Studies*, 63, 595-609

Kurosaki, T. and Fafchamps, M. (1997) "Insurance Market Efficiency and Crop Choices in Pakistan", *mimeo* Department of Economics, Stanford University

Lambert, S. (1994) "La Migration comme Instrument de Diversification Intrafamiliale des Risques", *Revue d'Economie du Développement*, 2:3-38.

Lehnert, A., Ligon E., Townsend R. (1999) "Liquidity Constraints and Incentive Contracts", *Macroeconomic Dynamics* 3(1):1-47.

Ligon E. (1998) "Risk Sharing and Information in Village Economies", *Review of Economic Studies*, 65, 4, 847-864

Ligon, E., Thomas, J., Worrall, T. (2000) "Mutual Insurance, Individual Savings and Limited Commitment", *Review of Economic Dynamics,* 3, 3, 1-47

Ligon E., Thomas J. and Worrall T. (2002) "Mutual Insurance and Limited Commitment: Theory and Evidence in Village Economies", forthcoming, *Review of Economic Studies*

Lim, Y. and Townsend, R. (1997) "General Equilibrium Models of Financial Systems: Theory and Measurement in Village Economies", CEMFI Working Paper 9716

Lim, Y. and Townsend, R. (1998) "General Equilibrium Models of Financial Systems: The-

ory and Measurement in Village Economies", *Review of Economic Dynamics*, 1, 1, 59-118
Lund, S. and Fafchamps, M. (1997) "Risk Sharing Networks in the Philippines", *mimeo* Department of Economics, Stanford University
Mace, B. (1991) "Full Insurance in the Presence of Aggregate Uncertainty", *Journal of Political Economy*, 99, 5, 928-956
Magnac T. and D. Thesmar (2001) "Identifying Dynamic Discrete Decision Processes", forthcoming *Econometrica*,
Marcet, A. and Marimon, R. (1992) "Communication, Commitment, and Growth", *Journal of Economic Theory*, 58, 219-249
Milgrom P. and Shannon C. (1994) "Monotone Comparative Statics", *Econometrica*, 62, 1, 157-80
Morduch J. (1995) "Income Smoothing and Consumption Smoothing" *Journal of Economic Perspectives*; 9(3), 103-114
Morduch J. (1999) "Between the State and the Market: Can Informal Insurance Patch the Safety Net?" *World Bank Research Observer*; 14(2), 187-207.
Nelson, J. (1994) "On Testing for Full Insurance Using Consumer Expenditure Survey Data", *Journal of Political Economy*, 102, 384-394
Phelan C. (1994) "Incentives and Aggregate Shocks", *Review of Economic Studies*, 61, 681-700
Phelan C. and Townsend R. (1991) "Computing Multi-Period, Information Constrained Optima", *Review of Economic Studies*, 58, 853-881
Prescott E. C. and Townsend R. (1984) "Pareto Optima and Competitive Equilibria with Adverse Selection and Moral Hazard", *Econometrica*, 52,1, 21-45
Prescott E. C. and Townsend R. (1996) "Theory of the Firm: Applied Mechanism Design", Research Department, Federal Reserve Bank of Richmond.
Ravallion, M. and Chaudhuri, S. (1997) "Risk and Insurance in Village India: Comment", *Econometrica*, 65:171-184
Ray, D. (1998) *Development Economics*, Princeton University Press
Rey, P. and Salanié, B. (1990) "Long-Term, Short-Term and Renegotiation: On the Value of Commitment in Contracting", *Econometrica*, 58, 597-619
Rogerson, W. P. (1985-a) "Repeated Moral Hazard", *Econometrica*, 53, 1, 69-76
Rogerson, W. P. (1985-b) "The First-Order Approach to Principal-Agent Problems", *Econometrica*, 53, 6, 1357-1367
Rosenzweig M. (1988a) "Risk, Implicit Contracts and the Family in Rural Areas of Low Income Countries", *Economic Journal*, 98, 1148-1170
Rosenzweig M. (1988b) "Risk, Private Information and the Family", *American Economic Review*, 78, 2, 245-250
Rosenzweig M., Stark O. (1989) "Consumption Smoothing, Migration, and Marriage : Evidence from Rural India", *Journal of Political Economy*, 97, 4,905-926
Runkle, D. E. (1991) "Liquidity constraints and the permanent-income hypothesis", *Journal*

of Monetary Economics, 27, 73-98
Stark, O. (1991) The Migration of Labor, Basil Blackwell:Oxford.
Stiglitz, J. E. (1974) "Incentives and Risk Sharing in Sharecropping", *Review of Economic Studies*, 41, 2, 219-255
Thomas, J., Worrall, T. (1988) : "Self Enforcing Wage Contracts", *Review of Economic Studies*, 55, 541-554
Thomas, J., Worrall, T. (1990) : "Income Fluctuation and Asymmetric Information: An Example of a Repeated Principal-Agent Problem", *Journal of Economic Theory*, 51, 367-390
Townsend, R. (1982) "Optimal Multiperiod Contracts and the Gain from Enduring Relationships under Private Information", *Journal of Political Economy*, 90, 6, 1166-1186
Townsend, R. (1987) "Arrows-Debreu programs as microfundations of macroeconomics", *Advances in Economic Theory*, 5th World Congress, Ch. 11, 379-428
Townsend, R. (1993) "The Medieval Village Economy" Princeton University Press
Townsend, R. (1994) "Risk and insurance in village India", *Econometrica* 62(3), 539-591
Townsend, R. (1995) "Consumption Insurance: An Evaluation of Risk-Bearing Systems in Low-Income Economies" *Journal of Economic Perspectives*, Vol. 9 (3) pp.83-102
Townsend, R. and Mueller, R. (1997) "Mechanism Design and Village Economies: From Credit, to Tenancy, to Cropping Groups", CEMFI Working Paper 9715
Townsend, R. and Mueller, R. (1998) "Mechanism Design and Village Economies: From Credit, to Tenancy, to Cropping Groups", *Review of Economic Dynamics*, 1, 1, 119-172
Udry, C. (1991) "Credits Markets in Northern Nigeria: Credit as Insurance in a Rural Economy", *World Bank Economic Review*, 4:3, 251-271
Udry, C. (1994) "Risk and Insurance in a Rural Credit Market: An Empirical investigation in Northern Nigeria", *Review of Economic Studies*, 61, 3, 495–526
Udry, C. (1995) "Risk and Saving in Northern Nigeria", *American Economic Review*, 85, 5, 1287-1300
Wilson, R. (1968) "The Theory of Syndicates", *Econometrica* 36 (1), 119-132
Zeldes, S. (1989) "Consumption and Liquidity Constraints", *Journal of Political Economy*, 97, 2, 305-346

A Proof of Proposition 1

First we show that the problem is strictly concave because of the preference shock.

Lemma 3 *$Q(.)$ is strictly concave.*

Proof. Consider the set $B \subset \left[\hat{P}'(\bar{v}_2), \hat{P}'(\underline{v}_2)\right]$ of slopes b such that $b = -\hat{P}'(v)$ occurs for more than one value v. The solution of $Q(v)$ is a continuous function v_η of η with

$$\begin{aligned}
\eta \hat{P}'(v_\eta) &= Q'(v) \text{ if } \underline{v}_2 < v_\eta < \bar{v}^2, \\
v_\eta &= \bar{v}^2 \text{ if } \eta \leq \frac{Q'(v)}{\hat{P}'(\bar{v}^2)}, \\
v_\eta &= \underline{v}^2 \text{ if } \eta \geq \frac{Q'(v)}{\hat{P}'(\underline{v}^2)}.
\end{aligned}$$

This defines completely v_η as a function of $Q'(v)$ except at those points where $\frac{Q'(v)}{\eta} \in B$. But $prob\{\frac{Q'(v)}{\eta} \in B\} = 0$ because B is a countable set and η is a continuous random variable. Consider now $v' > v$ with a solution v'_η. It is impossible that $Q'(v') = Q'(v)$ because this would imply $v'_\eta = v_\eta$ with probability one and thus contradicts $E\left\{v'_\eta\right\} = v' > v$. Thus $Q'(.)$ must be decreasing. ∎

The program for given μ and t_s, in the event s, is then:

$$\Phi_s(\mu, t_s) = \max_{(c_\sigma^1, c_\sigma^2, v_\sigma)} E\left[u_1\left(c_\sigma^1\right) + \beta Q\left(v_\sigma\right) \mid s\right] + \mu E\left[u_2\left(c_\sigma^2\right) + \beta v_\sigma \mid s\right]$$

s.t.

$$\left(\frac{\pi_\sigma}{\pi_s}\lambda_\sigma^1\right) \quad u_1\left(c_\sigma^1\right) + \beta Q\left(v_\sigma\right) \geq u_1\left(z_\sigma^1 + t_s\right) + \beta \underline{v}^1 \quad \forall \sigma \in s \tag{38}$$

$$\left(\frac{\pi_\sigma}{\pi_s}\lambda_\sigma^2\right) \quad u_2\left(c_\sigma^2\right) + \beta v_\sigma \geq u_2\left(z_\sigma^2 - t_s\right) + \beta \underline{v}^2 \quad \forall \sigma \in s \tag{39}$$

$$\left(\frac{\pi_\sigma}{\pi_s}\psi_\sigma\right) \quad c_\sigma^1 + c_\sigma^2 \leq z_\sigma \quad \forall \sigma \in s \tag{40}$$

$$\left(\frac{\pi_\sigma}{\pi_s}\beta\bar{\gamma}_\sigma\right) \quad v_\sigma \leq \bar{v}^2 \tag{41}$$

$$\left(\frac{\pi_\sigma}{\pi_s}\beta\underline{\gamma}_\sigma\right) \quad \underline{v}^2 \leq v_\sigma \tag{42}$$

The terms in brackets are Lagrange multipliers. The Lagrangian of the program is:

$$\sum_{\sigma \in s} \frac{\pi_\sigma}{\pi_s} \left\{ \begin{array}{c} u_1\left(c_\sigma^1\right) + \beta Q\left(v_\sigma\right) + \mu\left[u_2\left(c_\sigma^2\right) + \beta v_\sigma\right] + \lambda_\sigma^1\left[u_1\left(c_\sigma^1\right) + \beta Q\left(v_\sigma\right)\right] \\ + \lambda_\sigma^2\left[u_2\left(c_\sigma^2\right) + \beta v_\sigma\right] - \psi_\sigma\left[c_\sigma^1 + c_\sigma^2\right] + (\underline{\gamma}_\sigma - \bar{\gamma}_\sigma)\beta v_\sigma \end{array} \right\}$$

As the program is strictly concave, the first order conditions of this program as necessary and sufficient for optimality. After elimination of ψ_σ, $\underline{\gamma}_\sigma$, $\bar{\gamma}_\sigma$, they reduce to:

$$\begin{aligned}
\frac{u_1'\left(c_\sigma^1\right)}{u_2'\left(c_\sigma^2\right)} &= \frac{\mu+\lambda_\sigma^2}{1+\lambda_\sigma^1} \\
-Q'\left(v_\sigma\right) &= \frac{\mu+\lambda_\sigma^2}{1+\lambda_\sigma^1} \text{ if } \underline{v}^2 < v_\sigma < \bar{v}^2 \\
v_\sigma &= \bar{v}^2 \text{ if } -Q'\left(\bar{v}^2\right) \leq \frac{\mu+\lambda_\sigma^2}{1+\lambda_\sigma^1} \\
v_\sigma &= \underline{v}^2 \text{ if } -Q'\left(\underline{v}^2\right) \geq \frac{\mu+\lambda_\sigma^2}{1+\lambda_\sigma^1}
\end{aligned}$$

along with complementary slackness conditions.

Let $\phi(.)$ be the inverse function of $-Q'(.)$ (which is increasing). Notice that $\underline{v}_2 = \phi(\underline{v}_2)$, $\bar{v}^2 = \phi(\bar{v}^2)$, and $\underline{v}_2 < \phi(\mu) < \bar{v}^2$ if $-\hat{P}'(\underline{v}_2) < \mu < -\hat{P}'(\bar{v}^2)$. Define $\psi^i(z,\mu)$ as the solution of

$$\begin{aligned}
\frac{u_1'\left(\psi^1\right)}{u_2'\left(\psi^2\right)} &= \mu, \\
\psi^1+\psi^2 &= z.
\end{aligned}$$

The solution coincide with $c_\sigma^i = \psi^i(z_\sigma,\mu)$ and $v_\sigma = \phi(\mu)$ in all states where

$$u_1\left(\psi^1\left(z_\sigma,\mu\right)\right)+\beta Q(\phi(\mu)) \geq u_1\left(z_\sigma^1+t_s\right)+\beta\underline{v}_1, \tag{43}$$
$$u_2\left(\psi^2\left(z_\sigma,\mu\right)\right)+\beta\phi(\mu) \geq u_2\left(z_\sigma-z_\sigma^1-t_s\right)+\beta\underline{v}_2. \tag{44}$$

The LHS of the first condition decreases with μ while the LHS of the second condition increases with μ. Thus there exists $\bar{\mu}\left(z_\sigma, z_\sigma^1+t_s\right)$ and $\underline{\mu}\left(z_\sigma, z_\sigma^1+t_s\right)$ such that the two conditions are verified if

$$\underline{\mu}\left(z_\sigma, z_\sigma^1+t_s\right) \leq \mu \leq \bar{\mu}\left(z_\sigma, z_\sigma^1+t_s\right).$$

Lemma 4 *$\underline{\mu}\left(z_\sigma, z_\sigma^1+t_s\right) < \bar{\mu}\left(z_\sigma, z_\sigma^1+t_s\right)$ or $\underline{\mu}\left(z_\sigma, z_\sigma^1+t_s\right) = \bar{\mu}\left(z_\sigma, z_\sigma^1+t_s\right) \in \{-\hat{P}'(\underline{v}_2), -\hat{P}'(\bar{v}^2)\}$.*

Proof. It suffice to notice that it is not possible that

$$\begin{aligned}
u_1\left(\psi^1\left(z_\sigma,\mu\right)\right)+\beta Q(\phi(\mu)) \leq u_1\left(z_\sigma^1+t_s\right)+\beta\underline{v}_1, \\
u_2\left(\psi^2\left(z_\sigma,\mu\right)\right)+\beta\phi(\mu) \leq u_2\left(z_\sigma-z_\sigma^1-t_s\right)+\beta\underline{v}_2,
\end{aligned}$$

given that $\psi^1+\psi^2 = z_\sigma$, $\phi(\mu)+Q(\phi(\mu)) > \underline{v}_1+\underline{v}_2$, $\phi(\mu) \geq \underline{v}_2$, $Q(\phi(\mu)) \geq \underline{v}_1$. ■

Moreover $\underline{\mu}\left(z_\sigma, z_\sigma^1+t_s\right)$ and $\bar{\mu}\left(z_\sigma, z_\sigma^1+t_s\right)$ decreases in their second argument as the RHS of (43) increases and the RHS of (44) decreases with $z_\sigma^1+t_s$.

Now suppose that $\mu \geq \bar{\mu}\left(z_\sigma, z_\sigma^1+t_s\right) > -\hat{P}'(\underline{v}_2)$. Then $\frac{u_1'\left(c_\sigma^1\right)}{u_2'\left(c_\sigma^2\right)} = -Q'(v_\sigma) = \bar{\mu}\left(z_\sigma, z_\sigma^1+t_s\right)$ verifies the first order conditions and thus is the solution.

Suppose that $\bar{\mu}\left(z_\sigma, z_\sigma^1+t_s\right)=-\hat{P}'(\underline{v}_2)$. In this case $u_1\left(\psi^1\left(z_\sigma,\mu\right)\right)+\beta Q(\phi(\mu))<u_1\left(z_\sigma^1+t_s\right)+\beta\underline{v}_1$ for all μ. which implies that $c_\sigma^1>\psi^1\left(z_\sigma,\mu\right)$. The solution verifies

$$
\begin{aligned}
u_1\left(c_\sigma^1\right)+\beta Q\left(v_\sigma\right) &= u_1\left(z_\sigma^1+t_s\right)+\beta\underline{v}^1 \\
\frac{u_1'\left(c_\sigma^1\right)}{u_2'\left(c_\sigma^2\right)} &< \mu \\
\frac{u_1'\left(c_\sigma^1\right)}{u_2'\left(c_\sigma^2\right)} &= -Q'(v_\sigma) \text{ if } v_\sigma>\underline{v}_2 \\
\frac{u_1'\left(c_\sigma^1\right)}{u_2'\left(c_\sigma^2\right)} &\leq -Q'\left(\underline{v}^2\right) \text{ if } v_\sigma=\underline{v}^2
\end{aligned}
$$

But $\frac{u_1'\left(c_\sigma^1\right)}{u_2'\left(c_\sigma^2\right)}=-Q'(v_\sigma)$ is not possible as this would imply $u_1\left(\psi^1\left(z_\sigma,\hat{\mu}\right)\right)+\beta Q(\phi(\hat{\mu}))=u_1\left(z_\sigma^1+t_s\right)+\beta\underline{v}_1$ for $\hat{\mu}=-Q'(v_\sigma)$. Thus we have

$$
\begin{aligned}
v_\sigma &= \underline{v}^2 \\
\frac{u_1'\left(c_\sigma^1\right)}{u_2'\left(c_\sigma^2\right)} &\leq -Q'\left(\underline{v}^2\right)
\end{aligned}
$$

The reverse holds for the threshold $\underline{\mu}\left(z_\sigma, z_\sigma^1+t_s\right)$.

B Proof of Proposition 2

The result follows from Milgrom and Shannon (1994), Theorem 4. Given the separability in t_s, it is also quasi-supermodular in T. the following lemma shows that it also verifies the single crossing condition in $(T;\mu)$.

Lemma 5 $\frac{\partial\Phi_s(\mu,t_s)}{\partial t_s}$ *is non-increasing with* μ*, decreasing if at least one incentive constraint binds.*

Proof. From the envelop theorem, $\frac{\partial\Phi_s}{\partial t_s}$ is equal to :

$$E\left[\lambda_\sigma^2 u_2'\left(z_\sigma-z_\sigma^1-t_s\right)-\lambda_\sigma^1 u_1'\left(z_\sigma^1+t_s\right)\mid s\right]$$

Now

$$-\lambda_\sigma^1=\inf\{1-\frac{\mu}{\bar{\mu}\left(z_\sigma, z_\sigma^1+t_s\right)},0\} \text{ if } \bar{\mu}\left(z_\sigma, z_\sigma^1+t_s\right)>-\hat{P}'(\underline{v}_2),$$

while

$$-\lambda_\sigma^1=1-\mu\frac{u_2'(c_\sigma^2)}{u_1'(c_\sigma^1)} \text{ if } \bar{\mu}\left(z_\sigma, z_\sigma^1+t_s\right)=-\hat{P}'(\underline{v}_2)$$

where $\frac{u_2'(c_\sigma^2)}{u_1'(c_\sigma^1)}$ is independent of μ (given by (38) and $v_\sigma = \underline{v}_2$).
Similarly

$$\lambda_\sigma^2 = \max\{\underline{\mu}\left(z_\sigma, z_\sigma^1 + t_s\right) - \mu, 0\} \text{ if } \underline{\mu}\left(z_\sigma, z_\sigma^1 + t_s\right) < -\hat{P}'(\bar{v}_2),$$

while

$$\lambda_2^1 = \frac{u_1'(c_\sigma^1)}{u_2'(c_\sigma^2)} - \mu \text{ if } \underline{\mu}\left(z_\sigma, z_\sigma^1 + t_s\right) = -\hat{P}'(\bar{v}_2)$$

where $\frac{u_1'(c_\sigma^1)}{u_2'(c_\sigma^2)}$ is independent of μ (given by (39) and $v_\sigma = \bar{v}_2$).
Both are non-increasing in μ, decreasing if the constraint is binding. ■

C Extension to the Constrained Case

The extension parallels the development of section 3 and we highlight differences only. Instead of equations (23) we assume that:

$$\begin{cases} \ln \underline{\mu}(r_{it}, y_{it}) = \underline{\mu}_0 r_{it} + y_{it}\underline{\mu}_y + \underline{\mu}_{vt} \\ \ln \overline{\mu}(r_{it}, y_{it}) = \ln \underline{\mu}(r_{it}, y_{it}) + \exp(\overline{\mu}_0 r_{it} + y_{it}\overline{\mu}_y + \overline{\mu}_{vt}) \end{cases} \tag{45}$$

and the constraint $\underline{\mu}(r_{it}, y_{it}) \leq \overline{\mu}(r_{it}, y_{it})$ is naturally satisfied.
Notice that we still have $\underline{\mu}_0 < 0$ but we loose the condition $\bar{\mu}_0 < 0$.
The arguments leading to equations (27) carry over. The lower bound is written as:

$$\underline{\phi}_{it} = \underline{\phi}_{it}^* + \underline{u}_{it}$$

while the upper bound is slightly modified and is:

$$\underline{\phi}_{it}^* + \underline{u}_{it} + \exp(\tilde{\phi}_{it} + \tilde{u}_{it})$$

where:

$$\tilde{\phi}_{it} = \overline{\mu}_0((\pi_{vt} - \pi_0\delta_{vt-1}) + \pi_0(\ln c_{it-1} - x_{it-1}\beta) + y_{it}\pi_y + z_{it}\pi_z) + y_{it}\overline{\mu}_y + \overline{\mu}_{vt}$$

$$\tilde{u}_{it} = \overline{\mu}_0\pi_0 \ln \tilde{\varepsilon}_{it} + \overline{\mu}_0\xi_{it}$$

and the consumption equation is modified accordingly.

The arguments leading to the consumption equation (30) carry over and we get:

$$\begin{aligned} E(\Delta \ln c_{it} \quad &| \quad w_{it}) = \Delta x_{it}\beta + \Delta\delta_{vt} \\ -E((\underline{\phi}_{it}^* + \underline{u}_{it})\mathbf{1}\{\underline{u}_{it} \quad &> \quad -\underline{\phi}_{it}^*\} \mid w_{it}) \\ -E((\underline{\phi}_{it}^* + \underline{u}_{it} + \exp(\tilde{\phi}_{it} + \tilde{u}_{it}))\mathbf{1}\{\underline{u}_{it} \quad &< \quad -(\underline{\phi}_{it}^* + \exp(\tilde{\phi}_{it} + \tilde{u}_{it}))\} \mid w_{it}) \end{aligned} \tag{46}$$

The derivatives of the last term are modified into:

$$\begin{aligned}
&\int_{\underline{u}_{it}<-(\underline{\phi}^*_{it}+\exp(\tilde{\phi}_{it}+\tilde{u}_{it}))} (\frac{\partial}{\partial w_{it}}\underline{\phi}^*_{it}(w_{it}) + (\frac{\partial}{\partial w_{it}}\tilde{\phi}^*_{it}(w_{it}))\exp(\tilde{\phi}_{it}+\tilde{u}_{it}))dF(\underline{u}_{it},\tilde{u}_{it}) \\
=\;& \frac{\partial}{\partial w_{it}}\underline{\phi}^*_{it}(w_{it}). \int_{\underline{u}_{it}<-(\underline{\phi}^*_{it}+\exp(\tilde{\phi}_{it}+\tilde{u}_{it}))} dF(\underline{u}_{it},\tilde{u}_{it}) \\
&+\frac{\partial}{\partial w_{it}}\tilde{\phi}^*_{it}(w_{it}). \int_{\underline{u}_{it}<-(\underline{\phi}^*_{it}+\exp(\tilde{\phi}_{it}+\tilde{u}_{it}))} \exp(\tilde{\phi}_{it}+\tilde{u}_{it})dF(\underline{u}_{it},\tilde{u}_{it})
\end{aligned}$$

As in the text, consider the derivative of (46):

$$\frac{\partial}{\partial w_{it}}E(\Delta \ln c_{it} \mid w_{it}) = \frac{\partial \Delta x_{it}\beta}{\partial w_{it}} - \frac{\partial \underline{\phi}^*_{it}}{\partial w_{it}}\underline{\Lambda} - \frac{\partial \tilde{\phi}^*_{it}}{\partial w_{it}}\tilde{\Lambda} \tag{47}$$

where:

$$\underline{\Lambda} = \int_{\underline{u}_{it}>-\underline{\phi}^*_{it}(w_{it})} dF(\underline{u}_{it}) + \int_{\underline{u}_{it}<-(\underline{\phi}^*_{it}+\exp(\tilde{\phi}_{it}+\tilde{u}_{it}))} dF(\underline{u}_{it},\tilde{u}_{it})$$

$$\tilde{\Lambda} = \int_{\underline{u}_{it}<-(\underline{\phi}^*_{it}+\exp(\tilde{\phi}_{it}+\tilde{u}_{it}))} \exp(\tilde{\phi}_{it}+\tilde{u}_{it})dF(\underline{u}_{it},\tilde{u}_{it})$$

The average derivative equation then becomes:

$$D^w \equiv E(\frac{\partial}{\partial w_{it}}E(\Delta \ln c_{it} \mid w_{it})) = \frac{\partial \Delta x_{it}\beta}{\partial w_{it}} - \frac{\partial \underline{\phi}^*_{it}}{\partial w_{it}}\underline{\Lambda} - \frac{\partial \tilde{\phi}_{it}}{\partial w_{it}}\tilde{\Lambda} \tag{48}$$

and the derivatives are given in the following table ($\frac{\partial \underline{\phi}^*_{it}}{\partial w_{it}}$ remains the same):

w_{it}	$\frac{\partial \tilde{\phi}_{it}}{\partial w_{it}}$
$\ln c_{it-1}$	$\overline{\mu}_0 \pi_0$
x_{it-1}	$-\beta\overline{\mu}_0\pi_0 + \pi_x\overline{\mu}_0 + \overline{\mu}_x$
Δx_{it}	$\pi_x\overline{\mu}_0 + \overline{\mu}_x$
q_{it}	$\pi_y\overline{\mu}_0 + \overline{\mu}_q$
z_{it}	$\pi_z\overline{\mu}_0$

Only the form of the first two average derivatives are affected by imposing constraints. The restriction:

$$D^x = -\beta D^c + D^{\Delta x} + \beta$$

remains unaffected.