The Non Transferable Utility Bargaining Model with Two Privately Informed and Patient Players

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Abstract

For a NTU-bargaining game with two privately informed and patient players and an exogenously fixed disagreement outcome, consider any ex-ante efficient social choice function which is truthfully implementable in Bayesian Nash equilibrium with a fixed number of messages. Conditions are derived under which this social choice function can be implemented in Perfect Bayesian equilibrium of an escalation game. In this game, the players choose simultaneously their demands and then decide in turn whether to accept the opponent's proposal or to escalate until a deadline is reached. Escalation stops the game with an appropriate exogenously imposed probability of the disagreement outcome.

Keywords: Mechanism design, Bargaining models, Risk limits.

1. Introduction

In the literature on sequential bargaining models with asymmetric information and with transferable utility, impatience is the driving force in resolving conflicts. If the players have the same discount factor, players expecting larger gains lose more by delay and are therefore willing to pay more in a compromise which ends the conflict at an earlier stage. This contrasts with the early literature on bargaining in which risk limits determine whether a player will submit to the opponent's demand. Risk limits were introduced by Harsanyi (1956) in his interpretation of the bargaining model of Zeuthen (1930) for which delay of an agreement is not penalized. In the choice between submitting to the opponent's demand and a lottery involving the own demand and the disagreement outcome, the risk limit represents the highest probability of disagreement that a player would be willing to face and still insist on obtaining his own terms, rather than accept her opponent's terms. The risk limit, therefore, divides the loss of making a concession by the loss from ending in disagreement and is independent of linear transformations of the utility function. The Nash bargaining solution selects the outcome from the bargaining set which, once proposed by either of two players and given their risk limits at that point, excludes further concessions. The Kalai-Smorodinsky solution is the efficient outcome which equalizes the risk limits when uncompromising players opt for their dictatorial outcomes.

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Risk limits have fallen in oblivion since Rubinstein's (1982) discovery that impatience or the potential loss of delay induces players to accept a compromise immediately in models with complete information. Subsequently, Pareto inefficiencies, which are essential to induce players to reveal their private information, and appear under the form of missing some valuable trades in static bargaining models (Meyerson and Sattertwaithe(1983)), reappear as the actual cost of delay borne by impatient players in sequential bargaining models (Aussubel and Deneckere (1989)). The predictions are Coasian dynamics and cream skimming with less demanding players as more information becomes available as time proceeds.

This paper returns to the original argument stating that risk limits determine the nature of a compromise in bargaining. If different preferences result in different risk limits, then the tenacity of a player to her demand is evidence for her resolve. In a sequential bargaining model, players' types with higher risk limit will submit at a later stage because they are willing to run a higher risk on a low payoff. This revelation mechanism also works when players do not discount the future or when they have to reach an agreement before a commonly known deadline. The predictions are stubbornness of players and unexploited gains of cooperation. The frequently observed stubbornness of players sticking to their initial demands was explained by Schelling (1980), Crawford (1982) and in a series of subsequent papers as a strategic commitment to increase bargaining power. Schelling argues that if a bargainer is placed in the position which is impossible for her to retreat, this will persuade the opponent to concede. Knowing this, bargainers purposely place themselves in the position which is impossible for them to withdraw, known as the tactic of "Burning one's bridge". Crawford presents a demand game with two-stages. In case the demands made in the first stage are incompatible, players can choose in the second stage either to commit or to concede depending on whether an exogenously given and privately known cost of conceding is sufficiently high not to retreat from the incompatible demands. A player's demand will be compatible if she believes that the opponent's revoking cost is significantly high. The only influence a player can exert on her costs is by exercising her option not to attempt commitment by making compatible demand. But, in a two-stage model, there is no feedback in which the exposure to risk is used to acquire information on the preferences of the players needed to solve a bargaining problem.

This paper focuses on the transmission of private information in the several stages of an escalation game after the players have determined their demands. As in Crawford's analysis, the choice of the initial demands will depend on the final outcome of the subsequent bargaining game and therefore on the extensive form of the bargaining game. By ruling out monetary transfers which induce the opponent to submit to the own demand, we are able to describe the effects of private information in the classical bargaining problem with nontransferable utility. In contrast the existing literature on sequential bargaining restricted the analysis of bargaining with private information to the case of transferable utility.

We first study ex-ante efficient social choice functions which are truthfully implementable in Bayesian Nash equilibrium with a fixed number of messages. For any simultaneous report of the risk limits, a direct mechanism without monetary transfers determines the probabilities of three outcomes: each of the two outcomes proposed by the players and the disagreement outcome. We compare these outcomes with the outcomes of an escalation game. At each stage before reaching the deadline of an escalation game, one of the players has to decide whether she accepts her opponent's project or escalates, provided that neither of the players submitted before. As a justification for accepting disagreement by rational players in a dynamic game, Schelling (1980) argues that negotiators, as a bargaining tactic, purposely create uncertainty by reducing the scope of their own authority. It implies that an escalating player runs the risk of an undesired outcome, such as a strike or a war. As a result, player types with low risk limits will capitulate early to avoid disaster. The escalating player reveals indirectly by her resolve that her risk limit exceeds the next value of an increasing sequence.

The revelation principle states that the outcome of any Perfect Bayesian equilibrium of an escalation game can be implemented in a direct mechanism. We show that, under some mild restrictions, Pareto undominated direct truthtelling mechanisms can be implemented in Bayesian Perfect equilibrium of an escalation game. This result shows that in a variety of situations players, bargaining games with private information can be resolved without having to call on a third party which enforces a particular allocation. It suffices to select a profile for the probabilities of the disagreement outcome at each stage of the escalation game. Moreover, the strategies of the players in an escalation game are a sequence of 0-1 decisions which in practice are more easily understood than the report of the own type.

The previous analysis justifies an extensive form of bargaining games without discounting and constant disagreement outcome which is implemented in two parts. In the initial stage, the players propose their demands chosen from a feasible set of alternatives. In subsequent stages, patient players stick to their initial demands until they believe from their experience that their opponent is too strong. In that case, they prefer to submit as the probability of disagreement occuring before the opponent is expected to submit is deemed too large.

The paper is organized as follows. In the next section we analyze truthtelling direct mechanisms determining the collective choice in the case of three alternatives. By giving positive probability to the Pareto dominated disagreement outcome, each of the two players reveal their risk limit. In sections 3 and 4, we derive properties of Pareto undominated mechanisms with a finite number of messages. We show that the matrix representatation of a monotone mechanism, in which players choose rows or columns, has the dichotomous alternating herringbone pattern: it is possible to delete leading columns and rows with identical allocations in an alternating way until the matrix is reduced to one element. In each allocation, one player wins with probability zero.

The proof is in two steps. In a generalization of Harsanyi's argument, a player reveals her risk limit in her choice between submitting to the demand of the opponent and a lottery with three outcomes: winning, submitting at a later stage and disagreeing. Disagreement distorts the allocation. In screening models extracting information of one player along one dimension, there is no distortion at the top. It is shown that this result also holds in a screening model with two players. The disagreement outcome, therefore, does not occur in the first round of the screening process. Monotonicity of an allocation requires that the first round results in a unique allocation which calls one of the players as the winner when her opponent reports a low risk limit, independently of the report of the winner. This is the first step taken in section 3. A low risk limit will be reported only if, in the continuation game, the payoff of the lowest risk limit of the player who did not win in the first round does not exceed some bound. In section 4, it is shown in a second step that a relaxation of this bound increases the probabilities of winning and submitting at all rounds in the continuation game by the same proportion. The outcome of the continuation game is therefore obtained by dividing the probabilities of winning and submitting at all rounds in the screening model from which the types submitting in the first round have been removed. If this game were played, there would be no distortion at the top and one of the players would have won with probability one in the first roun, after her opponent reported a low risk limit. It follows that, in the second round of the original screening model, the player who did not submit in the first round wins independently of her report if a low risk limit was reported by her opponent and if the disagreement outcome did not come up. The screening process continues until the deadline is reached. We also derive a condition under which the monotonicity restriction is not binding. The link between direct mechanisms with alternating herringbone pattern and escalation games is made in section 5 and the nature of the Perfect Bayesian equilibrium (PBE) is characterized in section 6. In section 7, the escalation game is augmented with a demand game and shown to be a solution of an NTU-bargaining game in which patient players have private information. We compare this game with the problems of dividing a Euro, the double auction (Chatterjee and Samuelson (1983)) and the expected externality mechanism (d'Aspremont and Gérard-Varet (1979)). We conclude with some of the policy implications of the model.

2. Direct mechanisms

We start by analyzing when two players must make a collective choice from the set $\Omega \cup \{\delta\}, \Omega = \{\omega_1, \omega_2\}$, where ω_1 is the project or the outcome preferred by player 1, ω_2 is the project or the outcome preferred by player 2, and δ is the disagreement outcome. Later on we extend the model when Ω is a set with a finite number of Pareto efficient social outcomes, as in a bargaining problem. Prior to this choice, type θ_i of player i observes her Bernoulli utility function $u_i(x, \theta_i), x \in X, \theta_i \in \Theta_i$. Each player is assumed to be an expected utility maximizer. For type θ_i of player $i = 1, 2$, $w_i = u_i(\omega_i, \theta_i)$ is the payoff when player i wins, $s_i = u_i(\omega_{-i}, \theta_i)$ is the payoff when player i submits to the demand of the other player $-i$, and $d_i = u_i(\delta, \theta_i)$ is the payoff when no agreement can be reached. We assume that

$$w_i > s_i \geq d_i \text{ for almost all } \theta_i \in \Theta_i$$

Since $w_i > d_i$ almost always, define the risk limit for type θ_i as

$$k(\theta_i) = \frac{w_i - s_i}{w_i - d_i}$$

which is independent of linear transformations of the utility functions. Let Σ_3 be the three-dimensional simplex. In the allocation $\pi = (\pi_1, \pi_2, \pi_3) \in \Sigma_3$, ω_1 is selected with probability π_1, ω_2 is selected with probability π_2, and δ is selected with probability π_3. For this allocation, the expected utility of type θ_1 player 1 is

$$\begin{aligned}\widetilde{W}_1(\theta_1;\pi) &= \pi_1 w_1 + \pi_2 s_1 + \pi_3 d_1 \\ &= d_1 + (w_1 - d_1)\widetilde{U}_1(k(\theta_1);\pi) \\ \widetilde{U}_1(k(\theta_1);\pi) &= \pi_1 + \pi_2(1 - k(\theta_1))\end{aligned}$$

$\widetilde{W}_2(\theta_2;\pi)$ is derived in a similar way. In a direct revelation mechanism, the allocation depends on the report of the two types $\theta = (\theta_1, \theta_2) \in \Theta_1 \times \Theta_2$. Since the expected utility of some type of any player is a positive linear transformation of the expected utility of another type of the same player with the same risk limit, two types of the same player with the same risk limit will send the same report. We assume that the risk limits of the two players are independently distributed and that k_i is distributed according to the absolutely continuous distribution function $F_i(k_i)$ with support $\left[\underline{K}_i, \overline{K}_i\right] \subset [0,1]$, where $f_i(k_i)$ represents the associated density.

A direct mechanism is a mapping π from $\left[\underline{K}_1, \overline{K}_1\right] \times \left[\underline{K}_2, \overline{K}_2\right]$ into Σ_3: for the report $k \in \left[\underline{K}_1, \overline{K}_1\right] \times \left[\underline{K}_2, \overline{K}_2\right]$ the expected utilities for type $\overline{k}_i$ of player $i = 1,2$ are respectively

$$\begin{aligned}U_1(\overline{k}_1;k) &= \pi_1(k) + \pi_2(k)(1 - \overline{k}_1) \\ U_2(\overline{k}_2;k) &= \pi_2(k) + \pi_1(k)(1 - \overline{k}_2))\end{aligned}$$

For the mechanism π to be truthfully implementable in Bayesian Nash equilibrium it is required that, for almost all $k \in \left[\underline{K}_1, \overline{K}_1\right] \times \left[\underline{K}_2, \overline{K}_2\right]$,

$$\begin{aligned}E_{k_2}\left[U_1(k_1;k_1,k_2\right] - E_{k_2}\left[U_1(k_1;k_1',k_2\right] \geq 0 \text{ for all } k_1' \in \left[\underline{K}_1, \overline{K}_1\right] \\ E_{k_1}\left[U_2(k_2;k_1,k_2\right] - E_{k_1}\left[U_2(k_2;k_1,k_2'\right] \geq 0 \text{ for all } k_2' \in \left[\underline{K}_2, \overline{K}_2\right]\end{aligned}$$

If the mechanism π satisfies the truthtelling constraints, then the mechanism is said to be Bayesian incentive compatible. For a Bayesian incentive compatible mechanism

$$\overline{U}_1(k_1) = E_{k_2}\left[U_1(k_1;k_1,k_2)\right] \qquad \overline{U}_2(k_2) = E_{k_1}\left[U_2(k_2;k_1,k_2)\right]$$

Proposition 1 *If π is truthfully implemented in Bayesian equilibrium, then for each player $i = 1,2$, $\overline{U}_i(.)$ is convex on $\left[\underline{K}_i, \overline{K}_i\right]$.*

Proof. Truthtelling in Bayesian Nash equilibrium implies that

$$\overline{U}_1(k_1) = \sup_{k_1' \in \left[\underline{K}_1, \overline{K}_1\right]} E_{k_2}\left[U_1(k_1;k_1',k_2)\right] = E_{k_2}\left[U_1(k_1;k_1,k_2)\right]$$

The pointwise supremum of a family of linear functions is convex. ■

It follows from the envelope theorem that, in a Bayesian incentive compatible mechanism π, -$E_{k_{-i}}\pi_{-i}(.,k_{-i})$ belongs to the subdifferential of the expected payoff function $\overline{U}_i(.)$. The Proposition establishes that $E_{k_{-i}}\pi_{-i}(.,k_{-i})$, the expected probability of submitting of each player i, is non-increasing in her type.

The outcome $(\overline{U}_1'(.), \overline{U}_2'(.))$ Pareto dominates the outcome $(\overline{U}_1(.), \overline{U}_2(.))$ if for $i = 1,2$ and for all $k_i \in \left[\underline{K}_i, \overline{K}_i\right]$, $\overline{U}_i'(k_i) \geq \overline{U}_i(k_i)$ and, for some i and for all k_i in some open interval in the support of F_i, $\overline{U}_i'(k_i) > \overline{U}_i(k_i)$. A mechanism π yielding $(\overline{U}_1(.), \overline{U}_2(.))$ is Pareto undominated if it is a Bayesian incentive compatible mechanism and if there does not exist another Bayesian incentive compatible mechanism π' yielding $(\overline{U}_1'(.), \overline{U}_2'(.))$ which

Pareto dominates $(\overline{U}_1(.), \overline{U}_2(.))$. For $i = 1, 2$, let

$$W_i = E_k \overline{U}_i(k_i) \widetilde{f}_i(k) dk$$

where $\widetilde{f}_i(k)$ is the density function of an absolutely continuous distribution function $\widetilde{F}_i(k_i)$ with support $\left[\underline{K}_i, \overline{K}_i\right] \subset [0, 1]$. A mechanism π is Pareto undominated if there exists $\widetilde{F} = (\widetilde{F}_1, \widetilde{F}_2)$ and $\xi = (\xi_1, \xi_2) \in \Sigma_2$ such that $\xi_1 W_1 + \xi_2 W_2$ is maximized over the set of Bayesian incentive compatible mechanisms. The incentive constraints depend on the the type distributions $F = (F_1, F_2)$. There are two reasons why one might distinguish the weight distributions $\widetilde{F}$ from the type distributions F. Firstly, the expected utility of some type of player i is given by $\widetilde{W}_i = d_i + (w_i - d_i)\left(\pi_i + \pi_{-i}(1 - k_i)\right)$, so that it may desirable in the evaluation of a mechanism to correct the distributions of the risk limits when $E(\,w_i - d_i|\,k_i)$ depends on k_i. Secondly, one may want to discriminate between types of the same player.

3. Coarse Direct Mechanisms

In order to link direct mechanisms to escalation games defined below, we restrict the number n_i of messages sent by each player. Of course, the outcome selected in a direct mechanism with a restricted number of possible outcomes approaches the outcome of a completely revealing direct mechanism when n_1 and n_2 increase without bound. Rank the groups according to the expected probability of submitting in the mechanism π. By Proposition 1, all the members of a group with higher rank have lower risk limits than the members of all groups with lower rank. Consider therefore the two increasing sequences

$$\mathcal{K}_i = \{\underline{K}_i, K_i(1), ..., K_i(n_i - 1), \overline{K}_i\}$$

partitioning $\left[\underline{K}_i, \overline{K}_i\right]$ into n_i intervals $I_i(1) = [\underline{K}_i, K_i(1)], ..., I_i(t) = (K_i(t-1), K_i(t)], ..., I_i(n_i) = (K_i(n_i - 1), \overline{K}_i]$. In the direct revelation mechanism, all types belonging to an interval $I_i(t)$ send the same message $s_i(t)$, so that the direct mechanism can be represented by the matrix

			$player$	2	
		$\mathbf{s}_2(\mathbf{1})$	$\mathbf{s}_2(\mathbf{2})$	...	$\mathbf{s}_2(\mathbf{n}_2)$
	$\mathbf{s}_1(\mathbf{1})$	$a(1,1)$	$a(1,2)$	...	$a(1,n_2)$
$player$	$\mathbf{s}_1(\mathbf{2})$	$a(2,1)$	$a(2,2)$	...	$a(2,n_2)$
1	...	...	...	...	...
	$\mathbf{s}_1(\mathbf{n}_1)$	$a(n_1,1)$	$a(n_1,2)$	...	$a(n_1,n_2)$

An entry $a(r, m)$ of the matrix A is a pair $(a_1(r, m), a_2(r, m))$ defining the allocation $(a_1(r, m), a_2(r, m), 1 - a_1(r, m) - a_2(r, m)) = \pi(k_1, k_2), k_1 \in I_1(r)$ and $k_2 \in I_2(m)$. If two neighbouring columns or rows are identical, they are merged. The probability of $I_i(t)$ is $\mu_i(t) = F_i(K_i(t)) - F_i(K_i(t - 1))$.

Proposition 2 *The coarse direct mechanism π is Bayesian incentive compatible and partitions the types in $n_1 \times n_2$ classes if and only if, for $r = 1, ..., n_1 - 1$ and for $m =$*

$1,...,n_2-1,$

$$E_m\left[\left(a_2(r,m)-a_2(r+1,m)\right)\left(1-K_1(r)\right)+\left(a_1(r,m)-a_1(r+1,m)\right)\right]=0$$
$$E_r\left[\left(a_1(r,m)-a_1(r,m+1)\right)\left(1-K_2(m)\right)+\left(a_2(r,m)-a_2(r,m+1)\right)\right]=0$$

where $a(r,m)$ is an entry of A, $K_1(r)$ belongs to an increasing sequence $\mathcal{K}_1$ and $K_2(m)$ belongs to the increasing sequence $\mathcal{K}_2$.

Proof. The result follows from taking the pointwise supremum of a finite set of linear functions, generating the functions $\overline{U}_1(k_1)$ and $\overline{U}_2(k_2)$, which are piecewice linear, convex and decreasing. Assume that the mechanism partitions the types in $n_1 \times n_2$ classes. The kinks of $\overline{U}_1(.)$ occur at $K_1(r)$, $r=1,...,n_1-1$, when $E_{k_2}U_1(K_1(r);K_1(r),k_2)=E_{k_2}U_1(K_1(r);K_1(r+1),k_2)$. Since $\overline{U}_1(.)$ is decreasing, $K_1(r)>K_1(r+1)$. The same holds for player 2. ■

From Proposition 1, $E_m\left[\left(a_2(r,m)-a_2(r+1,m)\right)\right]>0$ and $E_r\left[\left(a_1(r,m)-a_1(r,m+1)\right)\right]>0$. It follows from Proposition 2 that types with higher risk limits, who are less likely to submit, are more likely to win in a contest against the same opponent with unknown risk limit. It also follows that

$$\begin{aligned}E_m\left[a_3(r+1,m)-a_3(r,m)\right]&=E_m\left[a_2(r,m)+a_1(r,m)-\left(a_2(r+1,m)+a_1(r+1,m)\right)\right]\\&=E_r\left[a_2(r,m)-a_2(r+1,m)\right]K_1(r)>0\end{aligned}$$

Additional information about one player's type is obtained only by increasing the expected probability of disagreement.

We next impose monotonicity ex-post which is stronger than the monotonicity ex-ante which is required to hold in any truthtelling mechanism.

Assumption A1

$\pi_3(.,.),\pi_1(.,k_2)$ and $\pi_2(k_1,.)$ are non-decreasing and $\pi_1(k_1,.)$ and $\pi_2(.,k_2)$ are non-increasing.

A mechanism π satisfying A1 is said to be monotone. The subsequent analysis is in two steps. We first analyze monotone mechanisms which do not allow for a Pareto improvement within the class of monotone mechanisms. Mechanisms which are constrained to be monotone are not necessarily undominated. We then proceed by deriving an assumption under which Pareto undominated mechanisms which are monotone ex-ante are also monotone ex-post. The next Lemma shows that in an incentive compatible monotone mechanism, none of the types of some player ever gains if her opponent becomes stronger ex-post.

Lemma 3 *If a coarse mechanism does not allow for a Pareto improvement within the class of monotone mechanisms, then $U_1(k_1;k_1,.)$ and $U_2(k_2;.,k_2)$ are non-decreasing.*

Proof. Assume that $k_1\in I_1(r)$ sends the report r, that $k_2\in I_2(m)$ sends the report m and $k_2'\in I_2(m+1)$ sends the report $m+1$. By Proposition 2, $k_2'>k_2$. By A1,

$$\begin{aligned}U_1(k_1;k_1,k_2)&=a_1(r,m)+a_2(r,m)(1-k_1)\\&=1-a_3(r,m)-a_2(r,m)k_1\\&\geq 1-a_3(r,m+1)-a_2(r,m+1)k_1=U_1(k_1;k_1,k_2')\end{aligned}$$

Similarly for player 2. ■

The probability of the disagreement outcome reveals the risk limit in a mechanism.

This distorts the efficiency of an allocation. It is well known that in screening models, there is no distortion at the top. The probability of disagreement is therefore equal to zero if $k \in I_1(1) \times I_2(1)$. The next Proposition extends this result by showing that in the two dimensional model, which screens the risk limit of two players, for reports in at least one of the intervals $I_1(1)$ or $I_2(1)$, there will be no distortion. In that case, monotonicity requires that the allocations in that interval, say $I_1(1)$, are identical. But then, a mechanism is a lottery in which player 1 always wins in one case and in which player 2 wins if player 1 reports in $I_1(1)$ and the screening process continues in the other case. ppears only on the lefthandside of the incentive compatibility constraints. We therefore expect that $a_1(1,1) + a_2(1,1) = 1$. But, if the mechanism dominates dictatorship of the first player, the probability that the first player wins must be zero. It follows that there is no distortion if one of the two players reports in the interval with the lowest risk limits, in which case her opponent always wins.

Proposition 4 *If a coarse mechanism Pareto dominates dictatorship and does not allow for a Pareto improvement within the class of monotone mechanisms, then* $a_1(1,m) = 1$ *for* $m = 1, ..., n_2$ *or* $a_2(r,1) = 1$ *for* $r = 1, ..., n_1$ *in the matrix representation* A*.*

Proof. By A1, $\nu = a_1(1,1) + a_2(1,1) \geq a_1(r,m) + a_2(r,m)$. If $\nu < 1$, consider the mechanism $A' = A/\nu$. This monotone mechanism satisfies incentive compatibility and Pareto dominates π. Therefore $\nu = 1$ and $a_1(1,1) + a_2(1,1) = 1$. Assume that both n_1 and n_2 exceed 1 and that in the partitioned matrix

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

$a_1(r,m) + a_2(r,m) < 1$ for all entries of A_{12}, A_{21} and A_{22}, so that it is feasible to increase $a_1(r,m)$ or $a_2(r,m)$ to obtain, for sufficiently small changes, a monotone mechanism π'. By A1, such a partitioning of A exists if and only if both the first column and the first row of A have an entry for which $\pi_3 > 0$. If $F_1(K_1(r)) \leq F_2(K_2(m))$, increase $a_1(r,m)$ in all entries of A_{12} by $d\alpha$ and increase $a_1(r,m)$ in all entries of A_{21} by $d\beta$ such that $F_1(K_1(r))d\alpha = (1 - F_1(K_1(r)))d\beta$. Incentive compatibility is restored in π' after adjusting $K_2(m)$ by

$$dK_2(m) = -\frac{(1 - F_2(K_2(m)))d\alpha - F_2(K_2(m))d\beta}{E_r\left[\pi_1(r,m) - \pi_1(r,m+1)\right]} > 0$$

The inequality follows since, by Proposition 2, $E_r\left[\pi_1(r,m) - \pi_1(r,m+1)\right] > 0$ and, by assumption,

$$\begin{aligned} &(1 - F_2(K_2(m)))d\alpha - F_2(K_2(m))d\beta \\ = &\left(1 - F_2(K_2(m)) - \tfrac{1 - F_1(K_1(r))}{F_1(K_1(r))} F_2(K_2(m))\right) d\beta \leq 0 \end{aligned}$$

By A1 and by Lemma 3, all types of player 1 gain by a switch of some types of player 2 from the report $m+1$ to the report m. If $F_1(K_1(r)) > F_2(K_2(m))$, increase $a_2(r,m)$ in all entries of A_{21} by $d\alpha$ and increase $a_2(r,m)$ in all entries of A_{12} by $d\beta$ such that $F_2(K_2(m))d\alpha = (1 - F_2(K_2(m)))d\beta$ resulting in a change of reports for the types $[K_1(r), K_1(r) + dK_1]$. Obtain in both cases a Pareto improvement. If Pareto improvements in the class of monotone mechanisms are excluded, one leading row or column has allocations for which all entries have $\pi_3 = 0$. By A1, these entries must be identical.

Without loss of generality, consider the mechanism π which Pareto dominates dictatorship and for which the entries of the first column of A are $(\alpha_1, 1 - \alpha_1)$. By A1, $a_2(r,m) \geq \alpha_2 = 1 - \alpha_1$. Note that the mechanism π' with $a_2'(r,m) = a_2(r,m) - \alpha_2$ and $a_1'(r,m) = a_1(r,m)/\alpha_1$ is monotone and Bayesian incentive compatible. Moreover, the lottery which implements the preferred outcome of player 2 with probability α_2 and the mechanism π' with probability α_1 is payoff equivalent to the mechanism π. Since, by assumption, π Pareto dominates dictatorship of player 2, the mechanism π is Pareto undominated only if the lottery is degenerate or if $\alpha_2 = 1 - \alpha_1 = 0$. ■

4. Mechanisms with herringbone pattern

The reduced matrix, obtained after deleting the column or row with identical elements in the matrix representation of an undominated monotone mechanism, satisfies Bayesian incentive compatibility. We next show that a leading column or row of the reduced matrix must also have identical elements.

Definition

A direct coarse mechanism π has the herringbone pattern if by iterated elimination of leading columns or rows with identical elements, the associated $n_1 \times n_2$ matrix A can be reduced to a column or a row. The herringbone pattern is alternating if columns are deleted after rows and rows after columns. The herringbone pattern is dichotomous if for each entry $a_1(r,m)a_2(r,m) = 0$.

It follows that for a direct Bayesian incentive compatible mechanism which has the herringbone pattern, the reduced matrix, obtained after deleting the first row or the first column, has also the herringbone pattern. In a mechanism with alternating herringbone pattern in which player 2's types with low risk limit submit, player 1 cannot gain by overreporting when player 2 sends her first message and player 2's report is decisive: her report determines whether or not the allocation of the first column is implemented. If player 2 therefore does not send the first message, then player 2 cannot gain by overreporting when player 1 sends her first message and player 1's report is decisive in the selection of the first row of the reduced matrix. This feature is shared by the second-price sealed-bid auction, in which the winner cannot gain and the allocation does not change by overreporting. It follows that if the player sending the message $s_i(t)$ would know that the type of her opponent belongs to the interval $I_{-i}(t'), t' < t$, she would be unable to increase her payoff by having such information. Therefore, as in the English auction with ascending bids, each one of the players may be asked in turn whether her risk limit exceeds the next treshold in one of the two increasing sequences of $\mathcal{K}_i$, starting with $K_i(1)$.

From Proposition 4, the types of some player i belonging to $I_i(1)$ submit in a monotone, Pareto undominated, mechanism π. After these types have been removed, the monotone, Pareto undominated, mechanism π' for the surviving types requires that the interval of types with risk limit below some critical level also submit. However, the outcome in the continuation game determines the initial willingness to submit. Committing to an inefficient mechanism π' for the screening of the survivors, which does not have the herringbone pattern, could therefore facilitate submission at an earlier stage. The next Proposition shows that, for monotone mechanisms, the desirable way to induce earlier submissions is to reduce proportionally the survivors' probability of winning. The Pareto undominated monotone direct mechanisms have the herringbone pattern and can be derived as the outcome of a recursive maximization problem. At each stage, after the disagreement outcome has been imposed with an appropriately selected probability, the designer chooses the Pareto undominated monotone mechanism for the surviving types. A plan which is optimal for a set of types remains optimal if this set is augmented with new types which have lower risk limits and submit at an earlier stage, provided that the probabilities of winning of the former are proportionally reduced in order to induce submission of the latter. Dynamic inconsistency can be ruled out in the present model with asymmetric information[2]

If the lowest risk limits of player 2 never win in the first round, it is required that the critical type $K_2(1)$ is indifferent between truthful reporting, when she will receive $(1-K_2(1))$ as a normalized payoff and overreporting, when she will receive $\overline{U}_2(K_2(1))$ as a normalized payoff. Note that $\overline{U}_2(K_2(1))$ is homogeneous of degree 1 in the allocation. If the payoff of a truthful report of the critical type $K_2(1)$ is increased to C, dividing the probabilities of winning of the surviving types after $K_2(1)$'s truthful report by $\varphi = (1 - K_2(1))\,/C$, yields a mechanism which still satisfies the incentive compatibility and monotonicity constraints. If, initially, the social welfare of the survivors was $\widehat{W}$, then after the transformation it is $\widehat{W}/\varphi$. By Proposition 4, one of the players should wins with probality 1 in the mechanism from which the types belonging to $I_2(1)$ have dropped out. If this is player 2, then type $K_2(1)$ gets $C = 1$ as a normalized payoff in the mechanism maximizing the social welfare of the survivors. It follows that the optimal mechanism for the survivors is optimal for the the whole population if the probability of winning for the survivors in the former is multiplied by $\varphi = (1 - K_2(1))$ in the latter.

Proposition 5 *A coarse mechanism which does not allow for a Pareto improvement within the class of monotone mechanisms can be implemented as a mechanism having the dichotomous alternating herringbone pattern*

Proof. For an undominated mechanism π, there exists a pair $\left(\xi, \widetilde{F}\right)$ such that, given the maximum number $n_1 \times n_2$ of possible outcomes and the type distributions F, the associated welfare function $W(.)$ is maximized over the set of monotone incentive compatible mechanisms. In the set of maximizers, we retain only those mechanisms having the dichotomous alternating herringbone pattern and show that this is a non-empty subset. Let $W_i^d = W(\pi^i)$ in the mechanism π^i in which player i always wins. If $\max_\pi W = \max\left[W_1^d, W_2^d\right]$, the Proposition is satisfied in a trivial way.

We consider therefore the case when $\max_\pi W > \max\left[W_1^d, W_2^d\right]$. For suitable labeling of the players, player 1 always wins for $k_2 \in I_2(1)$ in the monotone undominated

[2] A similar problem was addressed in Malcomson and Spinnewyn and in Chiappori, Rey, Salanié and Sanchez for the principal-agent model with hidden action. It was shown in these papers that commitment of the principal in the multiperiod model cannot result in a better outcome.

mechanism π and $\overline{U}_2(K_2(1)) = 1 - K_2(1)$. Let A' be the matrix representation of the monotone and Bayesian incentive compatible mechanism π', in which $\underline{K}_1$ and $K_2(1)$ have the lowest risk limit within their class and which is obtained after deleting the first column in the matrix representation A of π and dividing all entries of the reduced matrix by $\nu = a_1(1,2)+a_2(1,2)$. Since $a_1(r,m+1)+a_2(r,m+1) \leq a_1(1,2)+a_2(1,2)$, it follows that $a_1'(r,m) + a_2'(r,m) \leq a_1'(1,1) + a_2'(1,1) = 1$. That A' inherits monotonicity and incentive compatibility from A is established as follows. Firstly, $a(r,1) = (1,0)$ implies that all $a(r,1)$ drop from the incentive compatibility constraints in the mechanism for the survivors. Secondly, the monotonicity and incentive compatibility remain satisfied after dividing $a(r,m)$ by the same scalar.

We show that the social welfare function $\widehat{W}(.)$ for the survivors associated with $\left(\xi, (\widetilde{F}_1, \widetilde{F}_2/(1-\widetilde{F}_2(K_2(1)))\right)$ with $(F_1, F_2/(1-F_2(K_2(1))))$ and with $n_1 \times (n_2 - 1)$ as the maximal number of possible outcomes, is maximized over the set of monotone incentive compatible mechanisms for π'. Firstly, note that the original mechanism π is Pareto undominated if and only if it maximizes $\widehat{W}(.)$ for the survivors under the same incentive and monotonicity constraints and under the additional constraint

$$\overline{U}(K_2(1);\pi) = 1 - K_2(1)$$

Consider next a relaxation of the additional constraint by increasing $1 - K_2(1)$ to C and let $\varphi = (1 - K_2(1))/C$. Try the mechanism π^0 for which $a^0 = a/\varphi$ when $a_1^0(r,m) + a_2^0(r,m) < 1$. Since $\overline{U}(K_2(1);\pi)$ is homogeneous in a, simplify the system of binding constraints and recover the original system of constraints. The objective becomes $\widehat{W}(\pi)/\varphi$. Since the solution of a maximization problem is invariant to linear transformations of the objective function, it follows that π^0 is the maximizer of the relaxed problem. Furthermore, $\overline{U}(K_2(1);\pi^0) < 1$ when $a_1^0(r,m) + a_2^0(r,m) < 1$. For some $C \leq 1$, the additional constraint becomes non-binding, and $a^0 = a' = a/\nu$.

If $\max_\pi \widehat{W} = \widehat{W}_1^d$, then $\max_\pi W = W_1^d$, a case which has already been covered. If $\max_\pi \widehat{W} = \widehat{W}_2^d$, then $a(1,2) = (0, 1 - K_2(1)$ and π is a mechanism with two outcomes having the alternating herringbone pattern. If, therefore, $\max_{\widehat{\pi}} \widehat{W} > \max\left[\widehat{W}_1^d, \widehat{W}_2^d\right]$, then by Proposition 4, the maximum of $\widehat{W}$ is reached in a mechanism π' for which the entries of the first column are equal to $(1,0)$ or the entries of the first row are $(0,1)$. In the former, the first and the second column of A can be merged and the simplified matrix has $n_1 \times (n_2 - 1)$ outcomes. In the latter case, delete the first row. In both cases, repeat the same argument as before, until a matrix is obtained which is a column or a row. This column or row is irreducible if it consists of at most two entries. A Pareto undominated, monotone and incentive compatible mechanism has therefore the alternating dichotomous herringbone pattern. ■

We conclude the analysis of undominated mechanisms by deriving a condition under which the monotonicity constraints of a monotone mechanism are not binding.

Proposition 6 *Assume that $a(r,m)$ or $a(m,r)$ is the first entry of a leading column or row of a reduced matrix obtained after deleting the maximal number of identical rows or columns without deleting that element. Assume that player i has a positive probability of* *winning. If in all such cases, for* $K_i(r) \leq K_i(r')$,$K_i(r)$ *and* $K_i(r')$ *belonging to* $\mathcal{K}_i$,

$$J_i(K_i(r), K_i(r')) = \int_{K_i(r)}^{\overline{K}_i} k_i d\widetilde{F}_i(k_i) - \int_{K_i(r)}^{K_i(r')} k_i \frac{d\widetilde{F}_i(k_i)}{F_i(K_i(r))} \geq 0 \qquad \text{(A2)}$$

then a Pareto undominated, incentive compatible coarse mechanism can be implemented as a mechanism with alternating herringbone pattern.

Proof. It suffices to consider the case when $n_i > 1, i = 1, 2$. Assume that the mechanism π maximizes W for $(\xi, \widetilde{F}, F)$ on the set of monotone mechanisms. By Proposition 5, it has the alternating dichotomous herringbone pattern. If $J_i(K_i^*) \geq 0,$ we show that the monotonicity constraints do not bind. Assume that the entries of the first column of the matrix representation A are equal to $(1, 0)$. Replace the entries by the allocation $(1 - d\alpha, d\alpha)$. In evaluating dW, we can neglect the induced changes in $\mathcal{K}_i$ for restoring incentive compatibility by the envelope theorem. Since by this change, monotonicity remains satisfied,

$$\Delta(1) = \frac{dW}{d\alpha} = -\xi_1 \int_{\underline{K}_1}^{\overline{K}_1} k_1 d\widetilde{F}_1(k_1) F_2(K_2(1)) + \xi_2 \int_{\underline{K}_2}^{K_2(1)} k_2 d\widetilde{F}_2(k_2) \leq 0$$

Restrict the replacement to the entries $a(r', 1), r' \geq r$. In that case

$$\Delta(r) = \frac{dW}{d\alpha} = -\xi_1 \int_{K_1(r)}^{\overline{K}_1} k_1 d\widetilde{F}_1(k_1) F_2(K_2(1)) + \xi_2 \int_{\underline{K}_2}^{K_2(1)} k_2 d\widetilde{F}_2(k_2)(1 - F_1(K_1(r)))$$

Since $\Delta(1) \leq 0$,

$$\xi_2 \frac{\int_{\underline{K}_2}^{K_2(1)} k_2 d\widetilde{F}_2(k_2)}{F_2(K_2(1))} \leq \xi_1 \int_{\underline{K}_1}^{\overline{K}_1} k_1 d\widetilde{F}_1(k_1)$$

so that

$$\Delta(r) \leq \xi_1 F_2(K_2(1)) \left(\int_{\underline{K}_1}^{K_1(r)} k_1 d\widetilde{F}_1(k_1) - \int_{\underline{K}_1}^{\overline{K}_1} k_1 d\widetilde{F}_1(k_1) F_1(K_1(r)) \right)$$

If, therefore, $J_1(\underline{K}_1, K_1(r)) \geq 0$, it follows that $\Delta(r) \leq 0$ and that the monotonicity constraint is not binding for $a(1, r)$. Since by Proposition 5, the mechanism π is a maximizer of the welfare function for the survivors after deleting identical columns and rows, the same argument can be repeated. Finally, for mechanisms having the herringbone pattern, monotonicity between the different entries of columns and rows and of $a_1(.,.) + a_2(.,.)$ is always satisfied. ■

5. The escalation game

Let t be the round at which a column or row is deleted in an undominated incentive compatible mechanism having a dichotomous alternating herringbone pattern. $b = 1$ if a row is first deleted and $b = 2$ if a column is first deleted in the matrix representing this mechanism and

$$m(t) = \begin{cases} 1 \text{ if } t - b \in 2IN \\ 2 \text{ if } t - b \in (2IN + 1) \end{cases}$$

Merge the two sequences $\mathcal{K}_1$ and $\mathcal{K}_2$ of critical players in the sequence $\mathcal{K}$, such that $K(t) \in \mathcal{K}$ is the critical type of player $m(t)$ who is indifferent between reporting in the column or row which is deleted at round t and at round $t+2$. By Proposition 5, the entries of the leading column or row of the reduced matrix are of the form $\varphi(t)(1,0)$ for $m(t) = 2$ and of the form $\varphi(t)(0,1)$ for $m(t) = 1$. Note that $\varphi(t)$ is the probability that a type wins when a column or row is deleted at round t and that $\varphi(.)$ is decreasing. The relevation mechanism can therefore be casted in a dynamic form. If $\varphi(t+1)/\varphi(t)$ is the conditional probability that the game did not stop in disagreement before round t, then $r(t) = 1 - \varphi(t+1)/\varphi(t)$ is the conditional probability of disagreement in round t. Assume that round t is reached. If type $K(t)$ of player i submits at t, she receives the normalized payoff $1 - K(t)$ and if she escalates, she receives the normalized payoff

$$\frac{\overline{U}_{m(t)}(K(t))}{\varphi(t)} = \left[\begin{array}{c} \frac{F_{m(t+1)}(K(t+1))}{1-F_{m(t+1)}(K(t-1))} \frac{\varphi(t+1)}{\varphi(t)} + \\ \frac{1-F_{m(t+1)}(K(t+1))}{1-F_{m(t+1)}(K(t-1))} \frac{\varphi(t+2)}{\varphi(t+1)} \frac{\varphi(t+1)}{\varphi(t)} (1 - K(t)) \end{array} \right]$$

Consider the profile $\mathcal{R}$ =$\{r(1), ..., r(n)\}$ of conditional probabilities of stopping the revelation game in disagreement, with $r(n) = 1$ in the last round n. For this profile, consider the sequence $\mathcal{K}(n)$ solving, for 1,..., $n-1$, the second order difference equation

$$\begin{array}{c} K(t) = \frac{r(t)+(1-r(t))r(t+1)e(t+1)}{1-(1-r(t))(1-r(t+1))e(t+1)} \\ e(t+1) = \frac{1-F_{m(t+1)}(K(t+1))}{1-F_{m(t+1)}(K(t-1))} \end{array} \tag{K}$$

In this solution, $K(t)$ is indifferent between a truthful report and overreporting, because $1 - K(t)$ is equal to

$$\overline{U}_{m(t)}(K(t))/\varphi(t) = (1-e(t+1))(1-r(t)) + e(t+1)(1-r(t+1))(1-r(t))(1-K(t))$$

In this expression, $1 - e(t+1)(1 - r(t))$ is the conditional probability that the opponent submits at round $t+1$, given that $K(t)$ escalates and the revelation game did not stop in disagreement at round t and $e(t+1)(1-r(t+1))(1-r(t))$ is the conditional probability that type $K(t)$ would submit at round $t+2$, given that the opponent escalated at round t and that the game the revelation game did not stop in disagreement at round $t+1$ and round $t+2$.

Note that $K(n) = \overline{K}_{m(n)}$, so that $e(n) = 0$ when the deadline is reached. Also, $K(t) = \overline{K}_{m(t)}$ and $e(t) = 0$ whenever $e(t+1) = 1$. In the period preceding t^0, in both cases, $K(t^0) = r(t^0)$. If, at stage t, the types of player $m(t)$ with risk limit exceeding $K(t-2)$ did not submit before t and the types with risk limit not exceeding $K(t)$ submit at t, then for the uninformed opponent, $e(t) = (1 - F_{m(t)}(K(t)))/(1 - F_{m(t)}(K(t-2)))$ is the probability of escalation at stage t.

In a direct mechanism, one derives $\mathcal{K}$ and $\mathcal{R}$ maximizing some social welfare function W. In fact, an undominated mechanism is a pair ($\mathcal{K}$,$\mathcal{R}$). The sequence $\mathcal{K}$ implemented in a direct mechanism can also be obtained in the escalation game $\Gamma \equiv \{n, b, \mathcal{R}, F\}$, where at each of at most n stages, the players decide in turn whether they submit or escalate, starting with player b, $\mathcal{R}$ is the profile of the exogenously imposed probabilities of the disagreement outcome and F are the distributions of the risk limits. Since strategies need only to be defined if the game did not stop before a stage of the game has been reached, the history of the game is completely defined by the stage of game t. The strategy of the type k player moving at stage t is given by

$$\sigma_{m(t)}(k,t) = \begin{cases} 1 \text{ if type } k \text{ of player } m(t) \text{ chooses } E \\ 0 \text{ if type } k \text{ of player } m(t) \text{ chooses } S \end{cases}$$

We assume that a player submits if she can not gain by escalating. Since F defines continuous probability distributions, ties almost never occur. If $\rho_{m(t-1)}\left(k_{m(t)}, t-1\right)$ is the measure of the player $m(t-1)$'s belief on her opponent's risk limit at stage $t-1$, then

$$e(t) = \int_0^1 \sigma_{m(t)}(k,t)\,\rho_{m(t-1)}\left(k_{m(t)}, t-1\right) dk$$

is this player's forecast that player $m(t)$ will escalate at stage t. Since, player $m(t+1)$ does not move at stage t, it follows that $\rho_{m(t+1)}\left(k_{m(t)}, t\right) = \rho_{m(t+1)}\left(k_{m(t)}, t+1\right)$ and, by Bayes rule, that

$$\rho_{m(t+1)}\left(k_{m(t)}, t\right) = \frac{\sigma_{m(t)}\left(k_{m(t)}, t\right)\rho_{m(t+1)}\left(k_{m(t)}, t-1\right)}{\int_0^1 \rho_{m(t+1)}\left(k_{m(t)}, t-1\right) dk} \tag{BU}$$

A PBE (Perfect Bayesian Equilibrium) of an escalation game $\Gamma \equiv \{n, 1, \mathcal{R}, F\}$ reaching stage $n-1$ with positive probality[3] is a pair (σ, ρ), where

$$\begin{aligned} \sigma &= \left(\sigma_1(.,1), ..., \sigma_{m(t)}(.,t), ..., \sigma_{m(n)}(.,n)\right) \\ \rho &= \left(\rho_1(.,1), ..., \rho_{m(t)}(.,t), ..., \rho_{m(n)}(.,n)\right) \end{aligned}$$

such that, σ is sequentially optimal for ρ and ρ satisfies Bayesian updating for σ. Let $\widehat{v}_{m(t)}(k,t)$ be the continuation payoff to type k of player $m(t)$ when she survives until stage t, then in a PBE for an escalation game

$$v_{m(t)}(k,t) = \frac{\widehat{v}_{m(t)}(\theta,t) - s_{m(t)}}{\left(w_{m(t)} - d_{m(t)}\right)} \geq 0$$

$$v_{m(t)}(k,t)\left(1 - \sigma_{m(t)}(k,t)\right) = 0$$

6. The PBE of an Escalation Game

Lemma's 7-9 establish properties which are needed in the characterization of a PBE of an escalation game.

Lemma 7 *A two stage escalation game $\Gamma = \{2, 1, \{r(1), 1\}, F\}$ has a unique PBE, in which*

$$v_1(k_1) = \max\left[k_1 - r(1), 0\right]$$

[3] In that case, out of equilibrium beliefs need not be defined.

$$v_2(k_2) = -r(1)(1-k_2)(1-F_1(r(1))) + k_2 F_1(r(1)))$$

are the normalized continuation payoffs.

Proof. Since $r(2) = 1$ and $s_2 \geq d_2$, player 2 submits in the second stage. Therefore, player 1 receives the payoff of $r(1) d_1 + (1 - r(1)) w_1$ if she escalates and she receives s_1 if she submits. Player 1 escalates if $r(1) d_1 + (1 - r(1)) w_1 > s_1$ or $\frac{w_1 - s_1}{w_1 - d_1} > r(1)$. Therefore she always escalates if $k_1 - r_1 > 0$, yielding the normalized payoffs $(v_1, v_2) = (k_1 - r_1, -r_1(1-k_2))$. If $k_1 - r_1 < 0$, she always submits, yielding the normalized payoffs $(v_1, v_2) = (0, k_2)$. If $k_1 - r_1 = 0$, by assumption, player 1 submits. Take expectations to obtain the expected normalized payoff of player 2. ■

Lemma 8 *For any PBE of the escalation game $\Gamma = \{n, b, \mathcal{R}, F\}$, $n > 1$, the normalized continuation payoff of the escalating player satisfies*

$$\begin{aligned}
0 \quad < \quad & v_{m(t)}(k,t) = \\
& r(t)(k-1) + \\
& (1 - r(t)) \begin{pmatrix} (1 - e(t+1)) k + e(t+1) \\ \left(r(t+1)(k-1) + \left(1 - r(t+1) v_{m(t)}(k, t+2)\right)\right) \end{pmatrix} \\
\leq \quad & k - r(t)
\end{aligned} \tag{1}$$

and $v_{m(t)}(., t)$ *is non decreasing.*

Proof. The proof is done by induction. For $n = 2$, the Lemma follows from Lemma 7. Assume that the Lemma holds for any escalation game whose deadline does not exceed $n - 1$. Consider $\Gamma = \{n, 1, \mathcal{R}, F\}$. If player 1 escalates, her continuation payoff is

$$\widehat{v}_1(\theta, 1) = r(1) d_1 + (1 - r(1)) \begin{pmatrix} (1 - e(2)) w_1 + e(2) \\ (r(2) d_1 + (1 - r(2)) \widehat{v}_1(\theta, 3)) \end{pmatrix}$$

Without discounting, the coefficients of the payoffs on the right-hand side add to 1. Subtract s_1 from both sides and divide by $w_1 - d_1 > 0$. The normalised continuation payoffs are related by

$$v_1(k, 1) = \frac{\widehat{v}_1(\theta, 1) - s_1}{w_1 - d_1}$$

$$= r(1)(k-1) + (1 - r(1)) \begin{pmatrix} (1 - e(2)) k + \\ e(2)(r(2)(k-1) + (1 - r(2)) v_1(k, 3)) \end{pmatrix}$$

Since, by assumption, the normalised continuation payoff in the $(n-2)$-escalation game $v_1(., 3)$ is non-decreasing, $v_1(., 1)$ is increasing. If player 1 submits, her normalized payoff is 0 and independent of her type. It follows that $v_1(., 1)$ is non-decreasing. At the best, the escalating player wins in stage 2. Therefore, $\widehat{v}_1(\theta, 1) \leq r(1) d_1 + (1 - r(1)) w_1$ implying that $v_1(k, 1) \leq k_1 - r(1)$. Since the submitting player obtains $s_1$as a payoff, for the escalating player, $\widehat{v}_1(\theta, 1) > s_1$ or $v_1(k, 1) > 0$. It now follows that the relation 1 holds for player $m(t)$ who escalates in stage t.■

Note that the condition 1 relies on the assumption of no discounting. It is therefore implicitly assumed that the delay in reaching an agreement is very short.

Definition 1 *The escalation game* $\Gamma = \{n, b, r, F\}$ *is essential if* $n = 1$ *or if for* $n > 1, r(t) < \overline{K}_{m(t)}$ *and* $(1 - r(t-1))(1 - r(t)) < 1$ *for* $t = 1, ..., n-1$.

The first condition ensures that the essential escalation game does not necessarily stop before the deadline. Otherwise, we can advance the deadline such that the condition is satisfied without changing the PBE. The second part excludes uninformative delay.

Lemma 9 *Consider an essential escalation game* $\Gamma = \{n, b, \mathcal{R}, F\}$ *in which players submit in a tie. There exists a PBE of* Γ *reaching* n *with positive probability, such that the types* $k \in (K(t-2), K(t)]$ *of player* $m(t)$ *submit and the types* $k \in \left(K(t), \overline{K}_{m(t)}\right]$ *escalate in stage* $t < n$. $K(t)$ *is an elements of the sequence* $\mathcal{K}$ *satisfying*

$$K(t) = \arg\max_{k} \left[v_{m(t)}(k, t) \leq 0\right]$$

and the subsequences $\mathcal{K}_1$ *and* $\mathcal{K}_2$ *are strictly increasing.*

Proof. In a PBE, all types k of player $m(t)$ with $v_{m(t)}(k, t) > 0$ escalate and, by assumption, all other types submit. Suppose $K(t) = \arg\max_k \left[v_{m(t)}(k, t) \leq 0\right]$. By Lemma 8, $v_{m(t)}(., t)$ is non-decreasing. Therefore, for some type k of player $m(t)$ reaching stage t, all types $k < K(t)$ of player $m(t)$ submit in stage t and the players $k > K(t)$, if any, escalate in a PBE. If $t < n$, by assumption, $K(t) < \overline{K}_{m(t)}$. At each stage before n, in an essential escalation game some types leave. Consider type $K(t)$ who is indifferent between submitting and escalating at stage t. If she would have escalated at t, then, by Lemma 3, she would strictly prefer to submit at $t+2$. But then, in equation 1, $v_{m(t)}(K(t), t) = 0$ determines type $K(t)$ so that $K(t)$ is an element of $\mathcal{K}$. Since, at each stage some types leave, $\mathcal{K}_1$ and $\mathcal{K}_2$ are strictly increasing. ■

Proposition 10 *For the essential escalation game* $\Gamma = \{n, b, \mathcal{R}, F\}$,

$$\sigma_{m(t)}(k, t) = \begin{cases} 1 \text{ if } k_{m(t)} \in (K(t-2), K(t)\,] \\ 0 \text{ if } k_{m(t)} \in (K(t), \overline{K}_{m(t)}\,] \end{cases}$$

and

$$\rho_{m(t)}(k, t) = \begin{cases} \frac{dF_{m(t-1)}(k)}{1 - F_{m(t-1)}(K(t-1))} & \text{for } k \geq K(t-1) \\ 0 & \text{for } k \leq K(t-1) \end{cases}$$

where $K(t)$ *are the elements of* $\mathcal{K}$ *satisfying (K), is a PBE which reaches the deadline with positive probability*

Proof. An immediate consequence of the previous Lemma's. ■

We note that for game $\Gamma = \{n, b, \mathcal{R}, F\}$ there may exist perfect equilibria stopping before the deadline. Of course, the sequence $\mathcal{K}(t)$ satisfying (K) must define a PBE for the game $\Gamma = \{t, b, \mathcal{R}, F\}$. The next Proposition derives the condition for which this is the case.

Proposition 11 *For the essential game* $\Gamma = \{n, b, \mathcal{R}, F\}$ *there exists a PBE stopping with probability 1 at* $t < n - 1$ *if and only if*

$$\sum_{\tau=t}^{n} D(\tau) \begin{pmatrix} (1 - r(\tau))(1 - e^0(\tau + 1)) - \\ (1 - (1 - r(\tau))(1 - r(\tau + 1)))(1 - \overline{K}_{m(t)}) \end{pmatrix} \leq 0 \qquad \text{(A3)}$$

where

$$D(\tau) = \begin{cases} 1 \text{ if } \tau = t \\ (1 - r(\tau))(1 - r(\tau + 1))e(\tau + 1) \text{ if } \tau > t \end{cases}$$

and

$$e(\tau) = \frac{1 - \max[F_{m(t-1)}(r(\tau)), F_{m(t-1)}(r(\tau - 2))]}{1 - F_{m(t-1)}(r(\tau - 2))}$$

Proof. Let $i = m(t)$. Since $v_i(., t)$ is decreasing, an essential game stops with probability 1 at t in a PBE if and only if $v_i(\overline{K}_i, t) = 0$. In that case, from (K), $K(t - 1) = r(t - 1) \leq \overline{K}_{m(t-1)}$. If $\overline{K}_i$ submits at t on the equilibrium path, Bayes updating rule cannot be used if $\overline{K}_i$ would escalate off the equilibrium path. The out of equilibrium beliefs can be arbitrarily chosen in the continuation game Γ^0 with $n - t$ stages in which player $m^0(1) = i$ makes the first move and the disagreement probabilities are those of the remaining stages in the original game. Since in the PBE of this continuation game $v_i^0(\overline{K}_i, 1) = 0$, $v_i^0(\overline{K}_i, 3) = 0$. It follows from (K) that $K^0_{m^0(2)} = r(t+1)$, defining the escalation probability $e^0(t+1)$ of player $m^0(2)$. The sum of the expected payoffs in the first two periods in the continuation game are

$$r(t)d_i + (1 - r(t)) \left((1 - e^0(t + 1))w_i + e^0(t + 1) \left(r(t + 1)d_i + (1 - r(t + 1))s_i\right)\right)$$

or after normalization

$$1 - e^0(t + 1) - (1 - (1 - r(t))(1 - r(t + 1)))(1 - \overline{K}_i)$$

Unexpected escalation starts a new escalation game in the same way as before with similar payoffs in which $v^0_{m(t)}\left(\overline{K}_{m(t)}, 2\right) = 0$ in the continuation game. Unexpected escalation takes place if and only if the disagreement outcome did not come up and the opponent escalates. The probability of this event is $D(t)$. Add these payoffs and conclude that stopping with probability 1 can occur at t if and only if (A3) is satisfied. ■

Note that condition (A3) is always satisfied if the disagreement probabilities are non-increasing. In that case, one can extend the strategy space by allowing that the players propose a compromise. In that case, there exists a PBE in which none of the players ever compromise, expecting that her opponent will always escalate thereafter. On the other hand, if the profile of the disagreement probabilities is sufficiently steep, then the PBE stopping with positive probability at $t > n - 1$ is unique.

7. The Demand Game with Private Information

It has been shown in the previous sections that an outcome of a Pareto undominated incentive compatible direct mechanism for a binary social choice problem satisfying (A2) is a pair $(\mathcal{K}, \mathcal{R})$ which can be implemented in a perfect Bayesian equilibrium of some escalation game in which $\mathcal{R}$ is exogenously imposed. In that case, for an appropriate choice of $\mathcal{R}$, players with private information do not need an intermediary to resolve their conflict. Many conflicts of interest fit in this framework. For a successful merger of two firms, one firm may have to take up the standards or culture of the other firm. Either choice has advantages and disadvantages, but only one of the two alternatives can be adopted so that one firm has to submit. The negotiators postponing an agreement run the risk of failure initiated by a third party with opposed interests, such as possibly one of the shareholders calling off the operations. Other examples are leadership contests between two players which may end a lifelong friendship, matrimonial conflicts on educational choices for the childeren with the risk of separation or child custody after separation with the risk of legal battles.

In many cases, more alternatives are available other than the two alternatives for which the players fight. One frequently observes that players do not want to change their initial demand and wait until the other party submits or submit themselves when they recognize eventually that their opponent is too strong and the expected loss of escalating is too high. In order to avoid signalling problems, we assume that players decide on on a menu of demands before knowing their types. The incentive for the players to compromise or for choosing outside the menu when they know their type disappears if the former is interpreted as a sign of weakness or the latter raises suspicion. In the latter case, play the escalation game in which the unorthodox player moves first and the probability of disagreement is 1 in the first round. In the former case, as explained above under condition (A3), there exists a PBE in which the compromising player has no other choice than to submit immediately thereafter. If immediate submission is the best response off the equilibrium path, then a player knowing her type will choose the best demand on the menu chosen before she knew her type and stick to it for the remainder of the game. It follows that the outcome of a Pareto undominated incentive compatible direct mechanism with support $\{\omega_1, \omega_2, \delta\}$ satisfying (A2) and (A3) is a pair $(\mathcal{K}, \mathcal{R})$ which can be implemented in perfect Bayesian equilibrium of a demand game augmented with an escalation game in which $\mathcal{R}$ is exogenously imposed.

In a two-step game in which the players play the escalation game after choosing their demand ω_i from a menu Ω_i, the distribution of the risk limits will depend on the selected demand pair $\omega = (\omega_1, \omega_2) \in \Omega_1 \times \Omega_2$. This puts the demand game and the importance of risk limits as opposed to impatience in the resolution of conflicts into new perspective.

7.1 Pooling Equilibria of the Demand Game

We first clarify the effect of the choices in the demand game on the outcome of the escalation game, when in the equilibrium all types of the same player make the same demand. In the direct mechanism, the designer of the mechanism is constrained to choose one project for each of the players and, depending on the reports, assign probabilities on each project. For the demand pair $\omega = (\omega_1, \omega_2) \in \Omega_1 \times \Omega_2$ choosen in the pooling equilibrium, the strategies of players with the same risk limit coincide in the escalation game. F_i depends on the pair ω which has been selected. From Proposition 1, it follows that $\overline{U}_i(.; \omega)$ is a decreasing convex function of k_i. As a result, the expected payoff in the demand game of type θ_1 is given by

$$\widetilde{W}_1(\theta_1, \omega) = u_1(\delta, \theta_1) + (u_1(\omega, \theta_1) - u_1(\delta, \theta_1))\, \overline{U}_i(k(\theta_1); \omega)$$

Assume that

$$\omega \in \arg\max_{\omega'} E_\theta \left[\widetilde{W}_1(\theta_1, \omega) + \widetilde{W}_2(\theta_2, \omega) \right]$$

the pair $(\mathcal{K}, \mathcal{R})$ satisfying (A2) and (A3) defines a Pareto undominated incentive compatible direct mechanism for ω, then the demand game augmented by an escalation game implements the direct mechanism for $\mathcal{R}$

Consider the following simple example:

$$u_1(\omega, \theta_1) - d_1 = \sigma_1 - \omega\theta_1 \qquad u_2(\omega, \theta_2) - d_2 = \sigma_2 + \omega\theta_2$$

and θ_i is uniformly distributed with support $[0, 1]$. If θ is public information, this specification defines a bargaining problem (S, d) for which

$$u_2 = \sigma_2 + d_2 + \frac{\theta_2}{\theta_1} (\sigma_1 + d_1) - \frac{\theta_2}{\theta_1} u_1$$

is the linear Pareto frontier of S. For $d = (0, 0)$, $\sigma = (1/2, 1/2)$ and $\theta_1 = \theta_2$, we obtain a symmetric bargaining game. If θ is private information and ω is a money transfer for player 1 to player 2, then θ_i measures the (locally constant) marginal utility of income or, alternatively, in a quasi-linear environment, θ measures altruism, increasing in θ_1 for player 1 and decreasing in θ_2 for player 2. The choice of ω has been addressed in bargaining models studying the division of an Euro (Rubinstein) and in the double auction model (Chatterjee and Samuelson). Take $d = (0, 0)$ and $\sigma = (1/2, 1/2)$ in the former case and take $d = (0, -\varepsilon)$ and $\sigma = (1, \varepsilon)$ in the latter case[4].

The distributions for the types and their weights are

$$F_i(k_i; \omega_1, \omega_2) = \frac{\omega_2 k_i - \omega_1 \sigma_i}{\omega_2 - \omega_1 - (-1)^i \omega_i k_i}$$

and

$$\widetilde{F}_i(k_i; \omega_1, \omega_2) = \frac{k_i \sigma_i^2 \left(2 (\omega_2 - \omega_1) - (-1)^i \omega_i k_i\right)}{(2\sigma_i + (-1)^i \omega_i) (\omega_2 - \omega_1 - (-1)^i \omega_i k_i)^2}$$

defined on the supports

$$\left[0, \frac{\omega_2 - \omega_1}{\sigma_i + (-1)^i \omega_i}\right]$$

In this specification, there is no discrimination between types and the utilitarian welfare function takes $\xi_i = 1 + (-1)^i \omega_i / 2$ as weights for the two players. These distributions satisfy (A2) guaranteeing that Pareto-undominated mechanisms can be implemented as an escalation game. It has furthermore been checked that, for some demands, for $\left(\xi, \widetilde{F}\right)$ the outcome of an escalation game is preferred to the dictatorial outcome.

By increasing her demand, a player increases her own payoff. But, since distributions of the risk limits with high demands first order stochastically dominate distributions with low demands, this player faces less types of her opponent with low risk limits. As a result, the probability of winning is decreased.

[4] A good which has net value $-\varepsilon$ (after deducting shipping cost) to player 2, who is moving and value 1 to player 2.

7.2 Separating Equilibria for the Demand Game

In the separating equilibria, players with types $\theta_i \in \Theta_{i,\omega_i} \subset \Theta_i$ select $\omega_i \in \Omega_i$ from the menu. The demand ω_i in the demand game reveals Θ_{i,ω_i} to the opponent, who knows her type in the escalation game. The distribution of types in the escalation game depend on the demand pair $\omega \in \Omega_1 \times \Omega_2$ which has been selected. The strategies of all types in Θ_{i,ω_i} with the same risk limit coincide. From Proposition 1, it follows that $\overline{U}_i(.;\omega)$ is a decreasing convex function of k_i. As a result, the expected payoff in the demand game of type θ_1 choosing ω_1 is given by

$$\widetilde{W}_1(\theta_1, \omega_1) = E_{\omega_2}\left[\begin{array}{c} u_1(\delta, \theta_1) + \\ (u_1(\omega_1, \theta_1) - u_1(\delta, \theta_1))\,\overline{U}_1(k(\theta_1); \omega_1, \omega_2) \end{array}\right]$$

Incentive compatibility requires that

$$\begin{array}{c} \widetilde{W}_1(\theta_1, \omega_1) \geq \widetilde{W}_1(\theta_1, \omega_1') \\ \forall \theta_1 \in \Theta_{1,\omega_1}, \forall \omega_1' \in \Omega_1, \forall \omega_1 \in \Omega_1 \end{array}$$

Similarly for player 2. It follows that, in an equilibrium,

$$\Theta_{i,\omega_i} = \{\, \theta_i \in \Theta_i |\, \widetilde{W}_i(\theta_i, \omega_i) \geq \widetilde{W}_i(\theta_i, \omega_i'), \forall \omega_i' \in \Omega_i\}$$

Extending the social choice problem selecting one project out of a pair of efficient projects to a problem with a a fixed number of efficient projects can therefore be viewed as a two step procedure. In the formulation of her demands in the first step, each player reveals the element Θ_{i,ω_i} of a partition of the types Θ_i which contains her type. In the second step, the players reveal their risk limit. If $\Theta_{1,\omega_1} \times \Theta_{2,\omega_2}$ is a singleton, no further screening is needed in the second step. If $u_1(\omega_1(\theta_1), \theta_1) + u_2(\omega_1(\theta_1), \theta_2) > u_1(\omega_2(\theta_2), \theta_1) + u_2(\omega_2(\theta_2), \theta_2)$, then player 1 wins with probability 1 in the first and unique round of the screening process of the risk limits. Otherwise, player 2 wins with probability 1. Since, in that case,

$$\overline{U}_i(k(\theta_i); \omega_1, \omega_2) = 1 - \pi_{-i} k(\theta_i) = 1 - \pi_{-i} \frac{u_i(\omega_i, \theta_i) - u_i(\omega_{-i}, \theta_i)}{u_i(\omega_i, \theta_i) - u_i(\delta, \theta_i)}$$

it follows that

$$\widetilde{W}_i(\theta_i, \omega_i) = \begin{cases} u_i(\omega_i, \theta_i) \text{ if } \pi_{-i} = 0 \\ u_i(\omega_{-i}, \theta_i) \text{ if } \pi_{-i} = 1 \end{cases}$$

If all the outcomes of the demand game are singletons, the incentive constraints of the demand game determine the allocations, as in the familiar implementation in Bayesian Nash equilibrium. In the other case, the escalation game allows for further screening. Fine screening is possible only if the probability of the disagreement outcome is increased. Coarse screening, on the other hand, implements the project of one player in some cases for which the project of the other player is desirable from the social point of view.

Assume that, for a fixed number of choices,

$$\{\omega_1^1, ..., \omega_m^1, \omega_1^2, ..., \omega_m^2\} \in \underset{\overline{\omega}_1^1, ..., \overline{\omega}_m^1, \overline{\omega}_1^2, ..., \overline{\omega}_m^2}{\arg\max} \sum_{k=1,m; k'=1,m} E_{\theta \in \Theta_{1,\overline{\omega}_k^1} \times \Theta_{2,\overline{\omega}_{k'}^2}} \left[\widetilde{W}_1(\theta_1, \overline{\omega}_k^1) + \widetilde{W}_2(\theta_2, \overline{\omega}_{k'}^2)\right]$$

Assume that the pairs $(\mathcal{K}(\omega_k^1, \omega_{k'}^1), \mathcal{R}(\omega_k^1, \omega_{k'}^1))$ define a Pareto undominated incentive compatible coarse direct mechanism for each pair $(\omega_k^1, \omega_{k'}^1)$ satisfying (A2) and (A3). Then the family of demand games augmented by an escalation game with $\mathcal{R}(\omega_k^1, \omega_{k'}^1)$ is an NTU-bargaining game implementing the direct mechanism.

NTU-bargaining games with asymmetric information and without money transfers have not been considered in the literature, which focuses on mechanisms with quasi-linear utility functions. We briefly discuss the effect of introducing money transfers in bargaining games in order to compare our result with the expected externality mechanism of d'Aspremont and Gérard-Varet. Let $\chi = 1$ if the winner pays a transfer $m_i(\omega_i)$ to her opponent and let $\chi = 0$ if the winner pays this transfer to a third party. If utility is additive in the project and income, the payoffs ex post are

$$u_i^f(\omega_i, m_i(\omega_i), \theta_i) = u_i(\omega_i, \theta_i) - u_i^m(m_i(\omega_i), \theta_i)$$

for the winner and

$$u_{-i}^f(\omega_i, m_i(\omega_i), \theta_{-i}) = u_{-i}(\omega_i, \theta_{-i}) + \chi u_i^m(m_i(\omega_i), \theta_{-ii})$$

Moreover,

$$k_i^f(\theta) = k_i(\theta) - \frac{\chi u_i^m(m_{-i}, \theta_i)\left(u_i(\omega_i, m_i, \theta_i) - u_i(\delta, \theta_i)\right) + u_i^m(m_i, \theta_i)(u_i(\omega_{-i}, \theta_i) - u_i(\delta, \theta_i))}{\left(u_i^f(\omega_i, m_i, \theta_i)) - u_i(\delta, \theta_i)\right)\left(u_i(\omega_i, m_i, \theta_i)) - u_i(\delta, \theta_i)\right)}$$

so that increasing the transfer of the winner reduces her risk limit if $u_i(\omega_{-i}, \theta_i) > u_i(\delta, \theta_i)$ or $\chi = 1$. The expected externality mechanism of d'Aspremont and Gérard-Varet (1979), restricted to two players, takes $\chi = 1$ and risk neutral players. In that case, there exist money transfers which implements an ex-post efficient outcome in Bayesian Nash equilibrium. The escalation games following the demand game yield dictatorial outcomes and do not have any positive contribution. However, if the players have different degrees of risk aversion, then the demand game is unable to partition the player types into singletons. The escalation games following the demand game may improve on the allocation. In auctions with private values, $u_i(\omega_{-i}, \theta_i) = u_i(\delta, \theta_i)$ or $\chi = 0$ and $k_i^f(\theta) = k_i(\theta) = 1$ and the escalation games do not have any positive contributions. Nevertheless, escalation games following undecisive bidding games may be welfare improving under less restrictive assumptions.

8. Concluding Remarks

Demand games followed by an escalation game were proposed as a solution to NTU- bargaining games with asymmetric information, whether or not monetary transfers are available. The choice of the menu from which the players can choose in the demand game and the choice of the profile of $\mathcal{R}$ determining the outcome for the escalation game can also be viewed as the outcome of a preliminary cooperative bargaining game, before the types are revealed which, ex-ante, cannot be Pareto improved in the subsequent escalation game played after the types have been revealed. This is the task of those preparing the demands of the officials at the negotiation table of a conference with a strict time table. Due to unpredictable developments, the preferences of the official negotiators cannot be determined exactly at the start of a conference. Schelling (1980) argues that negotiators, as a bargaining tactic, purposely create uncertainty by reducing the scope of their own authority: union officials stir up excitement and determination on the part of the membership, government decisions are not entirely predictable due to internal disputes. It implies that an escalating player runs the risk of an undesired outcome, such as a strike or a war.

But in an effort to contain the conflict, at least in the long run, one may try to regulate the profile of $\mathcal{R}$ so as to obtain the sequence $\mathcal{K}$ which maximizes the social welfare function. $\mathcal{R}$ can be considered as a measure of the impact of third parties who are able, with some probability, to impose the disagreement outcome which is disliked by both players. In another interpretation it takes into account the possibility that the preferences of the players change (negotiators may be replaced by another team favouring another outcome, one of the players gets another opportunity which she prefers to the continuation of the current relationship, players get excited). If, as an example, some of the outside parties are too powerful and are able to impose early submission on a player whose project is desirable from a social point of view by threatening with a deadlock after her move, they may be eventually restrained by new legislation or rules and conversely. Similarly, governments may try to influence the outcome of wage negotiations by taxing the winner or superpowers may try to speed up peace settlements by subsidizing the party who is willing to submit.

The paper shows that when the optimal direct mechanism satisfies conditions (A2) and (A3), then the social choice problem with two players can be decentralized and the constrained optimal social choice can be attained without an impartial intermediator. If this is the case, then it does not matter whether the players have information which is not provided to the intermediator, provided that the players have to agree if one of the two Pareto undominated solutions is implemented. The present model is therefore helpful in understanding the wisdom of Solomon in his judgement.

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