Optimal Risk Taking and Information Policy to Avoid Currency and Liquidity Crises

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Abstract

This paper reconsiders the principal's problem of determining the optimal combination of risk taking and information dissemination, when threatened with a coordinated speculative attack on the fixed exchange rate by traders, respectively a coordinated withdrawal of credits by a group of lenders. In a global game approach, we find that optimal risk and economic transparency are contingent on the prior expected mean of fundamentals. Whenever the prior mean of economic performance is below a certain threshold, the central bank respectively the firm should commit to maximal risk and to disclosing private information of maximal precision. For good prior expectations, in contrast, optimal policy requires the principal to avoid any risks and to disseminate private information of lowest possible precision.

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1 Introduction

Recently, the global game approach, developed by Carlsson and van Damme (1993) and Morris and Shin (1998, 2000), has gained a lot of attention, because it allows unique predictions of the failure point for coordination games such as currency and liquidity crises, whereas traditional approaches could only identify regions of fundamental data with multiple equilibria.

Morris and Shin (1999) and Hellwig (2000) have shown that uniqueness requires private information to be sufficiently precise in comparison with public information about fundamentals. Besides their influence on uniqueness versus multiplicity, little is known about the effects of private and public information in global games. For the typical examples of currency or liquidity crises, however, one would like to identify conditions, for which increases in the precision of these two kinds of information increase or reduce the probability of a speculative attack or of an inefficient liquidation. Morris and Shin (1999) argued that the effects of the precision of private and public information are ambiguous at best. Heinemann and Illing (2002) have shown that increasing the precision of private information reduces the probability of a speculative attack in the absence of public information. Metz (2001) has demonstrated that public and private information may have opposite effects on the prior probability of a crisis, contingent on the prior mean of fundamentals.

In this paper we reconsider the information problem. On basis of the currency crisis model by Morris and Shin (1998, 1999), we analyze optimal risk taking and information policy for a central bank or government that intends to minimize the prior probability of a currency crisis. We do also apply our results to a firm with multiple lenders that tries to minimize the probability of early liquidation. We use a three stage game, in which first, the principal (central bank or firm) decides on parameters concerning the distribution of economic fundamentals and on the precision of posterior private information on fundamentals. In a second stage, traders/lenders get private information on the realized state and decide whether to speculate on devaluation or roll over credits, respectively. In a third stage, the principal decides on whether to devaluate the currency or continue the project. This decision is based on knowledge of the fundamental state and on the proportion of traders who attack the currency or withdraw credits, respectively. The fundamental state, in this sense, does not need to be the true state of the economy, but rather the information of the principal on which she bases her decisions.

Risk taking behavior of the authorities refers to the accepted variability of the fundamental state around an exogenously given mean. The chosen distribution of parameters is assumed to be common knowledge among market participants. According to former work on this subject by Morris and Shin (2000), the prior mean can also interpreted as 'public information'. Public information thus comprises the common belief concerning the principal's reaction to market positions. Observable policy conducted in order to increase or decrease fundamental risk, therefore changes the precision of public information. The lower the risk chosen by the authorities, the closer is the realized fundamental state to its commonly expected value and, as such, the more precise public information will be.

Information policy is conducted by disseminating individual signals on the realized fundamental state to the agents. Note, however, that information is 'private', in the sense that individual signals might and will almost certainly differ from each other. Precision of private information is measured by the conditional variance of signals for given fundamentals. The principal's choice is described as a commitment to provide markets with private signals of a determined precision about her own information on fundamentals. The variance of private signals indicates how precisely the principal disseminates her own information to agents. Thus, high precision of private information may be interpreted as transparency chosen by the principal.

Our analysis shows that optimal risk taking behavior and information policy depend on the prior expected mean of fundamentals. There is a unique threshold θ_0^*, such that it is optimal to choose maximal risk and maximal precision of private information if the prior expected mean of economic performance is below this threshold, while for higher means optimal policy is just reversed, avoiding any economic risks and limiting precision of private information to the minimum level that is needed to guarantee a unique equilibrium.

In case of good prior expectations, avoiding risks locks in the good state and thus reduces the probability of bad outcomes. Low transparency prevents markets to become aware of bad outcomes, if they happen to occur, though. Together, the choice of low risk and low transparency reduces the probability of an attack or early liquidation to levels close to zero.

In case of bad prior expectations, by taking high risks in the economic performance, the principal gambles for resurrection and increases the probability of good outcomes. A commitment to provide markets with most precise private information on the outcome, whatever it may be, ensures that private agents become aware of good states and abstain from attacking or withdrawing credits. If the outcome turns out to be bad, private agents would attack anyway and precise private information does not facilitate unwanted market positions. Together, high risk and transparency reduce the probability of an attack or early liquidation close to 1/2, which is the best to achieve in case of bad prior expectations.

The remainder of the paper is structured as follows. Chapter 2 is concerned with currency crises only. The model outlined in section 2.1 is taken from Morris and Shin (1998a, 1998b, 1999). Section 2.2 derives the optimal policy with a subsequent interpretation of results. Chapter 3 shows that similar results concerning optimal risk exposure and information dissemination also arise in models with multiple source lending. Section 3.1 briefly delineates the model, which is based upon Morris and Shin (1999) and Hubert and Schäfer (2001). The results for the firm's optimal policy are investigated in section 3.2. Chapter 4 concludes. Proofs are given in the Appendix.

2 Currency Crises

2.1 The Model

In this chapter we refer to a reduced form currency crisis model developed by Morris and Shin (1999a)[1]. The central bank has pegged the exchange rate to a fixed parity. There is a continuum of risk neutral speculators, indexed by $[0, 1]$. Each speculator disposes of one unit of domestic currency that he may hold or sell. A speculator short-selling the currency faces transaction costs as well as costs stemming from the interest rate differential between domestic and foreign currency. Both are comprised in parameter t. If the attack is successful, i.e. the fixed parity is abandoned and the domestic currency is devalued, selling yields a fixed revenue $D > t$. In situations of currency crises, transaction costs are rather low compared with potential gains from a devaluation. In this paper, we assume that $t < D/2$, which seems reasonable for currency crises.

The fundamental state of the economy is given as an index denoted by θ. A high (low) value of θ represents a good (bad) fundamental state. In accordance with usual second-generation currency crisis models we assume that in order to force the central bank to abandon the peg, the proportion of attacking speculators, denoted by l, must be large if θ is high, whereas for bad fundamentals a small proportion of market participants is sufficient to enforce a devaluation. Normalizing θ, we assume that the central bank abandons the peg whenever $l \geq \theta$.

Whenever a speculator knows that $\theta < 0$, it is a dominant strategy to attack, because the peg will be abandoned anyway. If an agent knows that $\theta > 1$, the dominant strategy is not to attack, because an attack cannot be successful. With common knowledge of the fundamental state, there are two symmetric equilibria in pure strategies for all $\theta \in [0, 1]$. Either all agents attack and get a reward of $D - t > 0$, or no agent attacks, since any single agent who would attack the parity receives a negative payoff of $-t$.

[1] For correction of a faulty expression in Morris and Shin (1998) see Heinemann (2000).

Morris and Shin (1998a,b, 1999) have shown that there is a unique equilibrium, if θ is not common knowledge, but agents get sufficiently precise private information on θ. In their model, the fundamental state is normally distributed around a mean of y with variance $\frac{1}{\alpha}$. The central bank is supposed to exclusively know the exact value of θ. Thus, θ may also be interpreted as information of the central bank when it decides on the exchange rate.

Speculators cannot observe the true fundamental value, though. Instead, they individually receive private signals $x_i = \theta + \varepsilon_i$. Noise terms ε_i are random and subject to independent normal distributions with $\mathrm{E}(\varepsilon_i) = 0$, $\mathrm{Var}(\varepsilon_i) = \frac{1}{\beta}$ and $\mathrm{E}(\varepsilon_i \, \theta) = 0$, so that $x_i|\theta \sim N(\theta, \frac{1}{\beta})$. While signals are private information, the common distribution of state and signals is common knowledge, including the prior mean y and precisions α and β. Based on their noisy information on θ, speculators simultaneously decide whether or not to attack the fixed parity. The central bank then observes the proportion l of attacking speculators and devalues the peg, if l is higher than or equal to θ.

Since traders are uncertain about the information of other agents even in equilibrium, they hold probabilistic beliefs on the proportion of attacking agents and on the ultimate success of an attack. In order to find the optimal action in this game, each speculator compares the payoff from an attack weighted with the probability of the attack being successful with the cost of such an action. This brings an additional equilibrium condition into the game and identifies the equilibria.

An equilibrium is described by two thresholds x^* and θ^*, such that

1. each agent attacks if and only if his signal is below x^*,
2. at state θ^* the proportion of attacking agents is l,
3. the currency is devalued if and only if the fundamental state is below θ^*,
4. an agent with signal x^* is indifferent to attack.

As derived by Morris and Shin (1999), the equilibrium condition for the critical fundamental state θ^* is given by

$$\theta^* = \Phi\left(\frac{\alpha}{\sqrt{\beta}}[\theta^* - y] - \sqrt{1+\frac{\alpha}{\beta}}\,\Phi^{-1}\left(\frac{t}{D}\right)\right), \tag{1}$$

where Φ is the cumulative density function of the standard normal distribution. There is a unique solution to this equation for all y, if and only if

$$\beta \geq \frac{\alpha^2}{2\pi}. \tag{2}$$

Thus, uniqueness requires the conditional variance of private signals $1/\beta$ to be sufficiently small relative to the variance of the fundamental state $1/\alpha$.

For $\beta/\alpha \to \infty$, the equilibrium threshold θ^* converges to

$$\theta_0^* = 1 - t/D. \tag{3}$$

This limit point has the property that it is a best response to each agent believing that the proportion of attacking agents has a uniform distribution in $[0, 1]$ (Morris and Shin, 2000).

Metz (2001) has shown that θ^* rises with increasing precision of private information β, if and only if

$$\theta^*(\alpha, \beta, y) < y + \frac{1}{\sqrt{\alpha+\beta}}\Phi^{-1}(\frac{t}{D}). \tag{4}$$

Moreover, θ^* rises with increasing α, if and only if

$$\theta^*(\alpha, \beta, y) > y + \frac{1}{2\sqrt{\alpha+\beta}}\Phi^{-1}(\frac{t}{D}). \tag{5}$$

Solving for y, Metz demonstrated that there are three regions of prior fundamentals: Define y_β as the solution to (4) with equality and y_α as solution to (5) with equality. For $y < \min\{y_\alpha, y_\beta\}$, increasing the precision of private signals, β, and decreasing α reduces the probability of an attack. For $y > \max\{y_\alpha, y_\beta\}$, these effects are reverse. For y between y_α and y_β, she found that the precision of both types of information have the same influence on the probability of a speculative attack. However, she recommends to stop short at the condition for uniqueness, since otherwise multiple equilibria might destabilize the economy.

Yet, since θ^* depends itself on α and β, it cannot be concluded from these results that a central bank should commit to accurate private and no public information under bad prior fundamentals and the reverse if prior fundamentals are good. In order to derive the optimal policy which minimizes the probability of a currency crisis, exogenous conditions have to be found for the influence of α and β on θ^*, whereas the above findings by Metz rely on endogenous conditions only.

Even more problematic is an interpretation that the central bank should choose the precision of its released information posterior to the realization of state θ. The equilibrium, as derived, relies on constant precisions for all posterior realizations. If these would change across states, conditional probabilities would have to take account of that, which changes the whole equilibrium and basically makes it algebraically untractable.[2] In the following, we therefore analyze the optimal policy for government or central bank, based on the common prior y.

2.2 Optimal Risk Taking and Information Policy

We extend the model of Morris and Shin (1999a) by assuming that central bank and government can influence the distribution of fundamentals and the precision of private information. In particular, we analyze a game with the following structure:

At stage 1, the government can decide on parameters α and β. Its goal is to minimize the probability of a successful speculative attack. While the prior mean of fundamentals is assumed to be given exogenously, government policy can in- or decrease real uncertainty by choosing more or less risky strategies in developing the economy (i.e. low or high α). The chosen precision β of private information binds the central bank to provide certain kinds of

[2] In this respect, see also Sbracia and Zaghini (2001). They point out that changes in information precision have to be mean-preserving in order to induce a unique equilibrium.

information to private agents at stage 2. While selecting optimal values of α and β, the government in any case wants to avoid instabilities arising from multiple equilibria. This restricts its choice to parameters for which uniqueness condition (2) holds.

At stage 2 of the game, speculators get private information and decide on either attacking or not. At stage 3, the central bank keeps the peg or devalues, depending on the proportion of attacking speculators. Stages 2 and 3 are the Morris–Shin game as described above. Our addendum is the endogenous choice of the fundamental's variance, $\frac{1}{\alpha}$, and of the precision of private information, β.

Since the prior probability of a currency crisis is given by

$$\text{Prob}(\theta < \theta^*) = \Phi\left(\sqrt{\alpha}\,(\theta^* - y)\right), \tag{6}$$

the government's optimization problem at stage 1 is

$$\min_{\alpha,\beta} \Phi\left(\sqrt{\alpha}\ [\theta^*(\alpha,\beta,y) - y]\right) \qquad \text{s.t. (1) and (2) hold.} \tag{7}$$

The government's choice on β is a commitment to supply private agents with well specified kinds of information at stage 2. A large β may be viewed as economic transparency. The higher β, the more reliable are private signals and the better can private agents infer the information held by the central bank. Government's choice on α may be interpreted as fiscal and monetary policy, influencing the real economy, and leading to more or less risk for the economy.

The government's selected values of α and β are common knowledge. This is a natural assumption for β, since information policy must be determined and information must be reliable to allow Bayesian updating. Common knowledge of α can be justified by rational expectations: Even if agents were not informed on the riskiness of economic policy, they would be able to deduce the government's choice of α from solving for the optimal strategy of the government, as the rules of the game are common knowledge. This is even more plausible, since we will find a unique solution for the government's optimization problem for every combination of exogenous parameters. Hence, we assume complete transparency on the model and on the government's objective to minimize the probability of a crisis.

The prior mean of the fundamental state, y, is treated as an exogenous variable, because we are only interested in optimal risk taking behavior and optimal information policy. In any case, as long as we assume expectations to be rational, a policy that intends to change the mean of the fundamental's distribution would be foreseeable and thus lead to the true mean becoming common knowledge. The prior mean in our model should therefore be interpreted as the solution of such a political process with rational expectations. For a similar reason, we exclude cheating on the information policy by the central bank. Any possible way and incentive to provide one–sided information within the bands that commitments allow, is anticipated by private agents, who correct their posterior beliefs for any such information bias.[3]

In the solution to (7), risk taking behavior and the commitment to provide information interact in different ways for different cases of the remaining exogenous parameters t, D and y. Keep in mind that the limit point of the equilibrium threshold for $\beta/\alpha \to \infty$ is given by $\theta_0^* = 1 - t/D$. The government can always approach this default point by choosing a sufficiently large precision of private information β, and hence $\theta^* \leq \theta_0^*$.

By analyzing optimal risk taking and information policy we proceed in two steps. First, we solve for the optimal precision of private information β for any given α. Afterwards, we search for the optimal risk parameter, given that information policy is chosen optimally.[4]

The first step thus requires to find solutions to

$$\min_{\beta} \Phi\left(\sqrt{\alpha}\,[\theta^*(\alpha, \beta, y) - y]\right) \qquad \text{s.t. (1) and (2) hold.} \tag{8}$$

The results are summed up in the following proposition:

[3] Cheli and Della Posta (2001) analyze the impact of biased information if the public cannot detect the bias.

[4] The sequence of solution steps is chosen due to simplicity reasons. The results do not change by first solving for the optimal α and then searching the optimal β given the already optimal risk.

Proposition 1 *Assume $t < D/2$. The precision of information that minimizes the probability of a speculative attack for given variance of fundamentals $1/\alpha$ is*

$$\beta^*(\alpha) \begin{cases} = \frac{\alpha^2}{2\pi} & \text{if} \quad y > 1 - t/D \text{ and } \alpha \geq \tilde{\alpha} \\ = \tilde{\beta}(\alpha) & \text{if} \quad y > 1 - t/D \text{ and } \alpha < \tilde{\alpha} \\ \to \infty & \text{if} \quad y < 1 - t/D \end{cases}$$

where $\tilde{\beta}(\alpha)$ is defined by

$$\theta^*(\alpha, \tilde{\beta}, y) = y + \frac{1}{\sqrt{\alpha + \tilde{\beta}}} \Phi^{-1}(t/D).$$

and $\tilde{\alpha}$ is defined by

$$\theta^*(\tilde{\alpha}, \tilde{\beta}(\tilde{\alpha}), y) = \theta^*(\tilde{\alpha}, \beta^{min}(\tilde{\alpha}), y).$$

The proof to proposition 1 can be found in the appendix. In the following we will rather give an informal reasoning for the solution to the optimal information precision. Here, we distinguish between prior fundamentals being good, i.e. $y > \theta_0^* = 1 - t/D$, or bad, i.e. $y < \theta_0^* = 1 - t/D$. We assume that $D > 2t$, so that speculators would want to attack, if posterior probability of an attack being successful is $1/2$.

The derivative of the prior probability of a crisis with respect to precision β is given by

$$\frac{d\Phi(\sqrt{\alpha}[\theta^*(\alpha, \beta, y) - y])}{d\beta} = \phi(\sqrt{\alpha}[\theta^*(\alpha, \beta, y) - y]) \cdot \sqrt{\alpha}\frac{\partial\theta^*(\alpha, \beta, y)}{\partial\beta}. \quad (9)$$

From (9) we can see that the optimal choice of β only depends on the partial derivative of θ^* with respect to β, because the normal density $\phi(\cdot)$ always takes on positive values. Note that the government can always achieve θ_0^* by choosing an almost infinite precision of private information.

If the prior mean of fundamentals is better than this state, $y > \theta_0^* = 1 - \frac{t}{D}$, the prior incentive to attack is rather low. However, given that the state has a normal distribution, there is always some probability for the state turning out to be far worse than the prior mean. In those cases, high precision of private information allows traders to realize that the state is bad and lead them to attack. A lower precision of private information increases uncertainty on the part of the traders and reduces the incentive to attack even at those states that are worse than the good prior mean. Thus, it may be advantageous to withhold information in order to avoid attacks in case of bad outcomes. Consequently, the optimal precision of private information depends on the variance of fundamentals. If this variance is small ($\alpha \geq \tilde{\alpha}$), the prior probability of bad states is small and the best information policy is to provide private information with the minimal precision needed for a unique equilibrium (to avoid self–fulfilling attacks). With low β, private signals are very much dispersed and some agents may get very bad signals and attack. If the variance of fundamentals is larger ($\alpha < \tilde{\alpha}$), there is a rather large probability that state θ turns out to be bad and a small proportion of attacking agents is sufficient for devaluation. In this case, too little private information induces a high risk of an attack triggered by those agents who happen to get bad signals. Hence, for small α there is an interior optimum for the precision of private information.

If the prior mean of fundamentals is bad already, $y < \theta_0^* = 1 - \frac{t}{D}$, the prior incentive to attack is rather high and agents attack the currency if they do not get any posterior information that leads them to reconsider this action. The more precise private information is, the larger is the set of states, at which agents realize that an attack is not promising. The bad prior mean can be outweighed by good private information if private signals are reliable. Therefore, with a bad prior mean, private information should be as precise as possible, $\beta \to \infty$. This leads agents to abstain from an attack for all states above θ_0^*, while for lower states, an attack is inevitable no matter how precise private information is.

In order to fully solve optimization problem (7), we next look for the precision of the fundamental's distribution, α^*, that minimizes the probability of a speculative attack, given that information policy will be adjusted to

$\beta^*(\alpha)$. Hence, we minimize over α:

$$\text{prob}(\theta < \theta^*(\alpha, \beta^*(\alpha), y)) = \Phi\left(\sqrt{\alpha}\,[\theta^*(\cdot) - y]\right), \tag{10}$$

subject to conditions (1) and (2). The solution to optimization problem (7) is given by the following proposition.

Proposition 2 *Optimal risk taking behavior and associated optimal precision of private information that minimize the probability of a crisis are*

$$\alpha^* \begin{cases} \to \infty & \text{if} \quad y > 1 - t/D \\ \to 0 & \text{if} \quad y < 1 - t/D \end{cases}$$

and

$$\beta^*(\alpha^*) \begin{cases} = \frac{\alpha^{*2}}{2\,\pi} & \text{if} \quad y > 1 - t/D \\ \to \infty & \text{if} \quad y < 1 - t/D \end{cases}$$

The complete proof can be found in the appendix. At this point, again, let us rather give an informal interpretation of the solution. Note that

$$\frac{\mathrm{d}\,\text{prob}(\theta < \theta^*(\cdot))}{\mathrm{d}\,\alpha} = \phi\left(\sqrt{\alpha}\,[\theta^* - y]\right)\left(\frac{1}{2\sqrt{\alpha}}\,[\theta^* - y] + \sqrt{\alpha}\,\frac{\mathrm{d}\,\theta^*}{\mathrm{d}\,\alpha}\right). \tag{11}$$

Thus, in contrast to β, risk parameter α does not only influence the probability of a currency crisis by affecting θ^*, which corresponds to the last term in parenthesis, but also by directly changing the variance of the distribution of fundamentals. If the prior mean y is below [above] θ^*, increasing variance of fundamentals decreases [increases] the probability of the state being below θ^*, where a speculative attack occurs.

For good priors, $y > \theta_0^* = 1 - t/D$, it is optimal to minimize risk ($\alpha \to \infty$) and to disseminate as imprecise private information as possible ($\beta = \beta^{\min} = \frac{\alpha^2}{2\pi}$). By choosing a low risk the government tries to lock in the good state. Now, the distribution of fundamentals is very dense around the good expected fundamental state. The probability of bad states is low and agents have a low prior incentive to attack except for those cases, where they become aware of an extraordinary bad outcome by private information.

With low precision of private information, traders attach a large weight to the good prior mean and abstain from an attack even in those cases where a bad state happens to materialize, because they do not become aware of this event.

In the case of bad priors, $y < \theta_0^* = 1 - t/D$, speculators have the highest prior incentive to attack, so that the fixed parity is very vulnerable. By choosing a risky policy, $\alpha \to 0$, the government gambles for resurrection. If the variance of fundamentals is almost infinite, the probability of the fundamental state being better than θ_0^* is almost $1/2$. High precision of private information ensures that traders become aware of these good outcomes and abstain from an attack, while they cannot be kept from attacking in case of bad states, no matter how precise private information is. Thus, maximal risk taking and a commitment to provide private information as precise as possible reduce the probability of devaluation to $1/2$, which is the best one can achieve in this case.

3 Liquidity Crises

Questions concerning the optimal combination of risk and information dissemination also arise in the context of multiple source lending. Inefficient coordination of lenders may force a firm into bankruptcy that can be avoided if all lenders decide to roll over their loans. The structure of this coordination game is very similar to the speculative attacks model described above. At some intermediate stage of a project, lenders may roll over their debt contracts or call in collateral at some loss. To the firm, refinancing foreclosed credits is more costly than repayment of extended credits. Thus, the profitability of a project may be sufficient to repay credits but too low to bear the costs of switching lenders. If expected returns are sufficiently large, the project will be successful and rolling over debt is a dominant strategy. If returns are low, the project fails and loans cannot be repayed. At intermediate states there are two equilibria: Either all lenders roll over loans and the project succeeds, or lenders foreclose and refinancing costs drive the firm into bankruptcy.

Morris and Shin (1999b) and Hubert and Schäfer (2001) analyzed this coordination problem using the global game approach.[5] They showed that with private information there is a unique equilibrium at which the project is terminated, whenever the expected return is below some threshold that depends on the distribution of returns and private signals. We extend this theory by analyzing which risk exposure and information policy minimize the probability of a project being terminated.

3.1 The Model

Let us consider the model of multiple source lending, as introduced by Hubert and Schäfer (2001). A group of creditors, indexed by $i \in [0,1]$, is financing a project. At stage 1 of the game, each creditor may decide to foreclose on his loan, i.e. seize in the collateral and sell it, so that he receives the liquidation value of collateral, $K > 0$. Otherwise he rolls over the loan, promising him a face value of repayment $L > K$ at maturity. We assume that the present value of collateral is at least half the value of repayment at maturity, i.e. $K > L/2$.

Let T be the fraction of loans called in. At stage 2, the firm decides whether to put in effort V (observable but not verifiable by lenders) and continue the project, or to terminate the project. If the firm decides to continue the project, she needs to refinance the fraction T of terminated loans from new lenders, who require a promised return of W at period 1. It is assumed that refinancing is costly, $W > L$, so that switching lenders is disadvantageous to the firm. If continued, the project yields a return of R. Otherwise, if the project is terminated, the remaining creditors and the manager are left with a payoff normalized to 0.

The firm's payoff from continuation depends on the fraction of capital that

[5] A similar issue has been analyzed by Chui, Gai and Haldane (2002) as well for the case of a sovereign liquidity crisis. However, they mainly concentrate on the welfare effects of different policies by the principal.

must be refinanced, T, and on the return R. It is given by

$$\pi(T,R) = R - V - L - T(W - L). \tag{12}$$

The manager therefore decides to continue, if and only if

$$\pi(T,R) \geq 0 \;\Leftrightarrow\; T \leq \frac{R - V - L}{W - L} \tag{13}$$

Hence, the project fails if the proportion of lenders seizing in their loans is larger than $\frac{R-V-L}{W-L}$. In order to facilitate on notation, define $\theta = R - V - L$, $z = W - L$ and $\lambda = K/L$, which is the notation chosen by Morris and Shin (1999). State variable θ then gives the net value of the project at maturity. Variable z measures the severity of disruption caused by early liquidation, i.e. refinancing costs. The project fails, if $T > \theta/z$. λ is the ratio of the value of collateral to repayment at maturity. As appropriate for lending to firms, we assume that $\lambda > 1/2$.

At stage 1 creditors have to decide simultaneously whether or not to roll over the loan. If they have perfect information on returns, they roll over loans if and only if they expect the firm to succeed. For $R < V + L \;\Leftrightarrow\; \theta < 0$, the firm fails even if all credits are extended. Here, it is a dominant strategy to foreclose the loans. In contrast, if $R > V + W \;\Leftrightarrow\; \theta > z$ the project is so profitable, that it succeeds even if all creditors withdraw their loans. Here, it is a dominant strategy to roll over. However, for $R \in [V + L, V + W] \;\Leftrightarrow\; \theta \in [0, z]$ the model renders multiple equilibria with the outcome depending on creditors' beliefs.

If lenders cannot perfectly observe returns but have private information on R or θ, there may be a unique equilibrium, provided that the precision of private information is sufficiently large. The argument follows the same logic as in the speculative attacks model.

Let us assume that θ follows a normal distribution with mean r and variance $1/a$. Each creditor gets a private signal $x_i = \theta + \epsilon_i$, with ϵ_i being independently normal distributed with mean zero and variance $1/b$. An equilibrium is a pair of thresholds x^* and θ^*, where a creditor forecloses on her loans if and only if she gets a signal below x^* and the project is terminated if and only if $\theta < \theta^*$. At critical state θ^* the fraction of foreclosed debt contracts is $T = \theta^*/z$. At the critical signal x^* creditors are indifferent.

Morris and Shin (1999) have shown that the equilibrium condition for the critical state θ^* is given by

$$\theta^* = z\,\Phi\left(\frac{a}{\sqrt{b}}\,[\theta^* - r] + \sqrt{1 + \frac{a}{b}}\,\Phi^{-1}(\lambda)\right). \tag{14}$$

θ^* is the critical state up to which the firm fails. For states $\theta < \theta^*$ the proportion of creditors who get bad signals and foreclose on their loans is so large that the firm cannot cover refinancing costs and fails. For states $\theta > \theta^*$ the proportion of creditors who withdraw credit is small enough for the firm to cover refinancing costs and proceed the project. There is a unique equilibrium for all r if and only if

$$b \geq \frac{(za)^2}{2\pi}. \tag{15}$$

For $b/a \to \infty$ the equilibrium threshold converges to

$$\theta_0^* = z\,\lambda. \tag{16}$$

The structure of this game is very similar to the speculative attacks model and we proceed by asking the same question as before: What risk exposure and information policy minimize the probability of liquidation?

3.2 Optimal Risk Exposure and Information Policy

We extend the game by introducing a preceding stage at which the firm chooses riskiness of the project and commits to provide information of a certain precision to lenders. The complete structure is as follows:

1. Nature selects a prior mean of states r that is common knowledge among firm and creditors. Interpret r as prior information on the ability of the firm's manager to earn profits.

2. The firm engages in a risky project. The manager chooses a risk that is associated with a variance of resulting profits of $1/a$. Furthermore, the firm commits to a precision of private information b. It is restricted by the condition for uniqueness (15).

3. Nature selects state θ, which is the profitability of the project, and private signals x^i for all creditors according to normal distributions.

4. Creditors decide whether to roll over or withdraw loans on basis of public information r and private information x_i. The project is continued if and only if $\theta \geq \theta^*(a, b, r)$ as defined by (14).

The firm's optimization problem is

$$\min_{a,b} \text{prob}(\theta < \theta^*(a, b, r)) \quad \text{s.t. (14) and (15),} \tag{17}$$

which, due to the assumed distributional restrictions, can be represented as

$$\min_{a,b} \; \Phi(\sqrt{a}(\theta^*(a, b, r) - r)). \tag{18}$$

Following the same reasoning as in the case of currency crises, we find again that the optimal combination of risk-taking and information dissemination depends on the commonly expected value r of the project. Whenever the prior expected value of the project is sufficiently high, i.e. $r > z\,\lambda$, the probability of premature liquidation can be minimized by avoiding risks and disseminating posterior information about the project with the lowest precision needed to avoid multiplicity. With minimal risk exposure the firm is trying to close in expected success. If profitability turns out to be low anyway, poor private information prevents creditors from becoming aware of the bad outcome. In contrast, if the commonly expected value of the project is low, i.e. $r < z\,\lambda$, the firm can minimize the probability of liquidation by taking large risks (gambling for resurrection) and committing to provide precise private information on the outcome to creditors. This maximizes the probability of good outcomes and ensures that lenders become aware of good results, if they occur. If results turn out to be bad, liquidation is inevitable and not facilitated by precise private information.

The results can be summarized as follows:

Proposition 3 *Assume $\lambda > 1/2$. The optimal combination of risk taking and information policy which minimizes the probability of premature liquidation of the project is given as*

$$a^* \begin{cases} \to 0 & \text{if} \quad r < z\,\lambda \\ \to \infty & \text{if} \quad r > z\,\lambda \end{cases}$$

and

$$b^*(a^*) \begin{cases} \to \infty & \text{if} \quad r < z\,\lambda \\ = \frac{(za)^2}{2\pi} & \text{if} \quad r > z\,\lambda \end{cases}$$

The formal proof follows the same logic as for Propositions 1 and 2 and is given in the appendix.

A firm trying to finance a project through multiple source lending must be very careful in selecting the optimal strategy. If the project is threatened by early liquidation through lenders, it is of utmost importance for the management to bear in mind the common belief of the market about this project. In order to illustrate the impact of a correct perception of the market sentiment, consider the following example: assume that the management mistakes the market sentiment to be in favor of the project's development, $r > z\,\lambda$, and decides to minimize risk and disseminate only few and imprecise information about the project to creditors, while actually the market is pessimistic, i.e. $r < z\,\lambda$. In such a case, the chosen combination of risk and information policy may increase the probability of a premature liquidation up to 100 per cent, so that the firm will go bankrupt with almost certainty. In contrast, if the firm had chosen the 'correct' policy combination as appropriate to the case of a pessimistic common belief towards the project, the management would have decided on a more risky strategy and would have endowed creditors with very precise information. Optimal risk and information policy reduce the probability of early liquidation to a level close to $\frac{1}{2}$. Figure 1 shows the probabilities of early liquidation for different prior expected values of a project under the two regimes of risk and information policy.

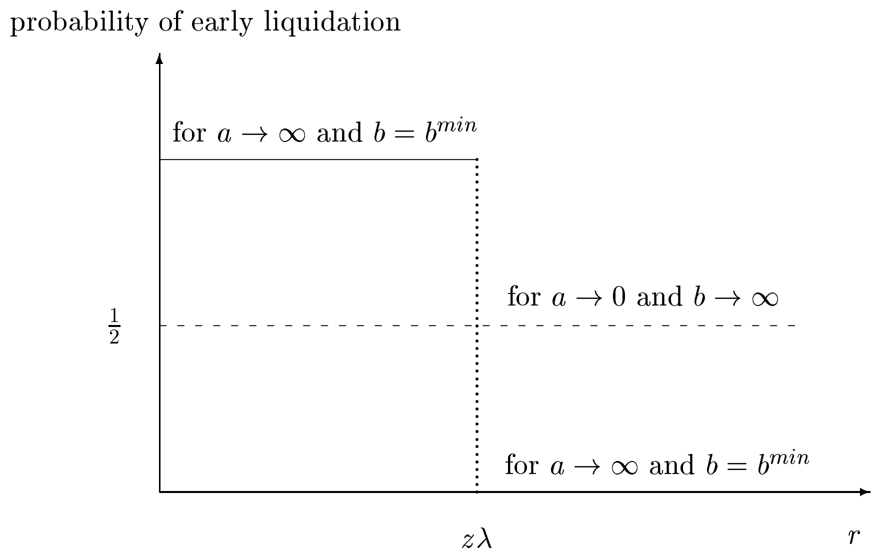

Figure 1: Probability of early liquidation for two policy regimes

4 Conclusion

Analysing the optimal policy for a principal threatened by inefficient coordination failure on the part of a continuum of agents, delivered the following results. First, we find that the optimal combination of public- and private-information policy strongly depends on the common belief of the market. In this respect, we were able to show that whenever the market sentiment concerning the payoff-relevant variable, i.e. the common prior, is sufficiently strong, the principal can minimize the probability of a crisis by deciding on the lowest possible fundamental risk while at the same time disseminating the least precise private information about the fundamental state. Due to this strategy, agents are forced to neglect their private information and believe in the market sentiment, i.e. rely solely on public information. If fundamental risk is kept low, an additional argument for this behaviour is that public information in this case is known to be very reliable. Hence, the incidence of an inefficient crisis can be reduced to the lowest possible value. Note that in this case with a strong market sentiment, the danger of a crisis is diminished on the one hand by reducing fundamental risk, on the other hand by shifting the range of coordination to the 'efficient' action (i.e. 'not-attack' respectively 'roll-over'). Thus, it is made easier for agents to coordinate on the 'good' equilibrium action, so that the risk of a coordination failure is reduced as well.

However, for a weak market sentiment concerning the underlying fundamental state we find that the opposite combination of policy is optimal. In this case, the principal will want to decide on the highest possible risk concerning payoffs, but send very precise private signals to the agents. By doing so, agents are forced to neglect the common information and rely heavily on their own private information. Consequently, this policy is characterized by a large fundamental risk for creditors, while the risk of premature liquidation out of lacking coordination of lenders is rather low. The case of a weak market sentiment therefore displays a strong risk-shifting from the principal towards private agents.

Appendix

Proof of Proposition 1

Starting from the partial derivative of θ^* with respect to β we know that the threshold value θ^* increases in precision β whenever $\theta^*(\alpha, \beta, y) < y + \frac{1}{\sqrt{\alpha+\beta}}\Phi^{-1}(\frac{t}{D})$, as given in (4). Moreover, we know that for infinitely precise private signals the equilibrium value θ^* converges to the constant $\theta^*_0 = 1 - \frac{t}{D}$. The right-hand-side of inequality (4), however, converges to y for infinitely high β. Convergence is from below, since $D > 2t \Leftrightarrow \Phi^{-1}(t/D) < 0$. We therefore need to distinguish the two cases of either $y > \theta_0^* = 1 - \frac{t}{D}$ or $y < \theta_0^* = 1 - \frac{t}{D}$, in order to find the value of β which minimizes θ^* for given α.

Case 1: $y > \theta_0^*$

As θ^* approaches θ_0^* for $\beta \to \infty$, inequality (4) holds for large values of β. Reducing β from high levels lowers threshold θ^* but also reduces the r.h.s. of (4). If there is some $\tilde{\beta} > \beta^{\min} = \frac{\alpha^2}{2\pi}$, at which (4) holds with equality, a further reduction in β increases threshold θ^* while the r.h.s. continues to fall. Hence, $\tilde{\beta}$ is uniquely determined by (4) as equality. As indicated by Figure 2, the optimal precision of private information in this case is given by

$$\beta^*(\alpha) = \max\left\{\tilde{\beta},\, \beta^{\min}\right\}.$$

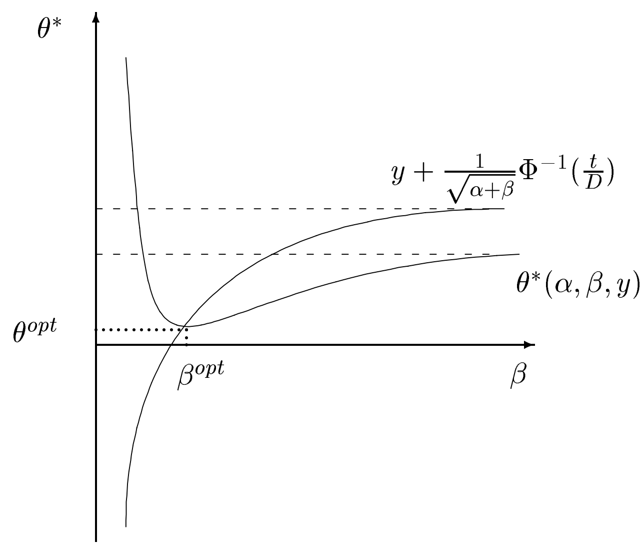

Figure 2: The critical state as a function of precision of private information in case of good prior expectations, $y > \theta_0^*$

Inserting (4) as equality into (1), we get

$$y + \frac{\Phi^{-1}(t/D)}{\sqrt{\alpha + \tilde{\beta}}} = \Phi\left(-\sqrt{\frac{\tilde{\beta}}{\alpha + \tilde{\beta}}}\,\Phi^{-1}(t/D)\right). \tag{19}$$

Total differentiation of (19) delivers

$$\frac{\mathrm{d}\,\tilde{\beta}}{\mathrm{d}\,\alpha} = \frac{\sqrt{\tilde{\beta}} + \tilde{\beta}\,\phi(\cdot)}{\alpha\,\phi(\cdot) - \sqrt{\tilde{\beta}}}. \tag{20}$$

Since the denominator of equation (20) is negative for $\tilde{\beta} > \beta^{\min}$, the interior solution $\tilde{\beta}$ decreases in α. As $\beta^{\min}$ is increasing in α, there must be a unique $\tilde{\alpha}$, such that $\tilde{\beta}(\tilde{\alpha}) = \beta^{\min}(\tilde{\alpha}) = \frac{\tilde{\alpha}^2}{2\pi}$. For $\alpha \geq \tilde{\alpha}$, the optimal precision of private information is $\beta^{\min}$, because here the interior solution $\tilde{\beta}$ is not feasible. For $\alpha < \tilde{\alpha}$, the optimum is at $\tilde{\beta}(\alpha)$.

Case 2: $y < \theta_0^*$

For $\beta \to \infty$, inequality (4) is violated in this case. Reducing β raises threshold θ^* and lowers the r.h.s. of (4). Therefore, threshold θ^* unambiguously rises with decreasing precision of private information. In this case, the best information policy is to choose the highest possible β, i.e. $\beta^*(\alpha) \to \infty$. QED

This can also be seen from Figure 3.

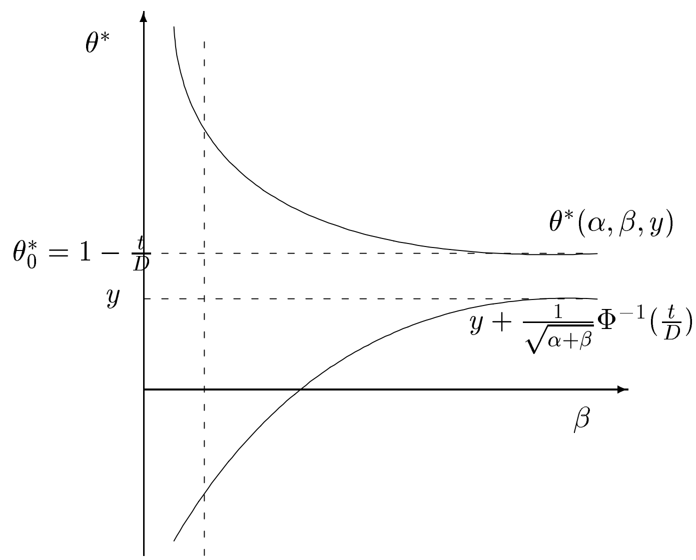

Figure 3: The critical state as a function of precision of private information in case of bad prior expectations, $y < \theta_0^*$

Proof of Proposition 2

To solve (7), we look for the risk parameter α^* that minimizes the probability of a speculative attack, given that information policy is given by $\beta^*(\alpha)$. Thus we minimize over α:

$$\text{prob}(\theta < \theta^*(\alpha, \beta^*(\alpha), y)) = \Phi\left(\sqrt{\alpha}\,[\theta^*(\cdot) - y]\right), \tag{21}$$

subject to (1) and (2).

The derivative of (21) with respect to α is given by

$$\frac{\mathrm{d}\,\text{prob}(\theta < \theta^*(\cdot))}{\mathrm{d}\,\alpha} = \phi\left(\sqrt{\alpha}\,[\theta^* - y]\right)\left(\frac{1}{2\sqrt{\alpha}}\,[\theta^* - y] + \sqrt{\alpha}\,\frac{\mathrm{d}\,\theta^*}{\mathrm{d}\,\alpha}\right). \tag{22}$$

The optimal value of α does not only depend on the influence of α on the threshold θ^*, but also on whether θ^* is higher or lower than y.

In case 1 with $y > \theta_0^* = 1 - \frac{t}{D}$, the optimal precision of private information is either $\beta^{\min}(\alpha) = \frac{\alpha^2}{2\,\pi}$ for $\alpha < \tilde{\alpha}$ or $\tilde{\beta}(\alpha)$ otherwise. Inserting $\beta^{\min}(\alpha)$ in equation (1), delivers:

$$\theta^*(\alpha, \beta^{\min}(\alpha), y) = \Phi\left(\sqrt{2\,\pi}\,(\theta^* - y) - \sqrt{1 + 2\,\pi/\alpha}\;\Phi^{-1}(t/D)\right). \qquad (23)$$

For $\alpha \to 0$, this threshold converges to $\Phi(-\infty) = 0$. Total differentiation of (23) yields

$$\frac{\mathrm{d}\,\theta^*(\alpha, \beta^{\min}(\alpha), y)}{\mathrm{d}\,\alpha} = \frac{\phi(\cdot)\,\pi\,\sqrt{\alpha}}{\alpha^2\,\sqrt{\alpha + 2\,\pi}\,(1 - \sqrt{2\,\pi}\,\phi(\cdot))}\;\Phi^{-1}\,(t/D)\,. \qquad (24)$$

Since $\Phi^{-1}(\frac{t}{D}) < 0$, due to the assumption of low transaction costs, we find that $\theta^*(\alpha, \beta^{\min}(\alpha), y)$ is decreasing in α.

Plugging in $\tilde{\beta}$, instead, and using the implicit function theorem, we find that

$$\frac{\mathrm{d}\,\theta^*(\alpha, \tilde{\beta}(\alpha), y)}{\mathrm{d}\,\alpha} = \frac{\partial\theta^*(\alpha, \beta, y)}{\partial\alpha}\bigg|_{\beta=\tilde{\beta}(\alpha)} = \frac{\phi(\cdot)}{\sqrt{\tilde{\beta}} - \alpha\,\phi(\cdot)} \cdot \frac{\Phi^{-1}(t/D)}{2\,\sqrt{\alpha + \tilde{\beta}}}, \qquad (25)$$

which is negative for $\tilde{\beta} > \beta^{\min}$, i.e. for $\tilde{\beta}$ being feasible. Thus, for both solutions increasing α reduces θ^*. Since the government can always approach θ_0^* by high precision of private information, $\theta^*(\alpha, \beta^*(\alpha), y)$ can never exceed θ_0^* and therefore, $\theta^* \leq \theta_0^* < y$. Hence, (22) implies that the probability of a speculative attack is decreasing with rising α, and optimal government policy is given by minimizing economic risks and choosing $\alpha \to \infty$.

In case 2 with $y < \theta_0^* = 1 - \frac{t}{D}$, proposition 1 tells that it is optimal to choose the highest precision of information, $\beta^*(\alpha) \to \infty$. This leads to the critical threshold $\theta_0^* = 1 - \frac{t}{D}$, which is independent of α. Hence, $\frac{d\,\theta^*}{d\,\alpha} = 0$. Since $y < \theta^* = \theta_0^*$ in this case, (22) is negative, so that $\frac{d\,\mathrm{prob}(\theta < \theta^*(\cdot))}{d\,\alpha} > 0$. The optimal risk-policy for the central bank therefore prescribes to choose the lowest possible value of α, i.e. $\alpha \to 0$, so that the fundamental variance is maximized. QED

Proof of Proposition 3

This proof follows the previous two proofs. Changes in the probability of early liquidation that are induced by the precision of private information are given by

$$\frac{\mathrm{d}\,\mathrm{prob}(\theta < \theta^*(a,b,r))}{\mathrm{d}\,b} = \phi(\sqrt{a}[\theta^*(\cdot) - r]) \cdot \sqrt{\alpha}\,\frac{\partial\theta^*(a,b,r)}{\partial b}, \qquad (26)$$

where

$$\frac{\partial\theta^*}{\partial b} = \frac{-a}{2\,b}\,\frac{z\,\phi(\cdot)}{\sqrt{b} - a\,z\,\phi(\cdot)}\left[\theta^* - r + \frac{\Phi^{-1}(\lambda)}{\sqrt{a+b}}\right]. \qquad (27)$$

Changes in the precision of the prior distribution are given by

$$\frac{\mathrm{d}\,\mathrm{prob}(\theta < \theta^*(a,b,r))}{\mathrm{d}\,a} = \phi(\sqrt{a}[\theta^*(\cdot) - r]) \cdot \left[\frac{1}{2\sqrt{a}}\,(\theta^* - r] + \sqrt{a}\,\frac{\mathrm{d}\theta^*(\cdot)}{\mathrm{d}a}\right] \qquad (28)$$

If $r < \theta_0^* = z\,\lambda$, threshold θ^* falls with increasing b and the optimal precision of private information is $b^* \to \infty$. The associated equilibrium threshold is $\theta_0^* > r$ and the probability for states above θ_0^* increases in the variance of fundamentals. Therefore, the optimal precision of the prior distribution is $\alpha^* \to 0$.

If $r > z\,\lambda$, θ^* is a U-shaped function of b for any given a. Setting (27) equal to zero gives the optimal precision $\tilde{b}$, provided that it is greater than the minimal precision needed for uniqueness. $\tilde{b}$ is the unique solution to

$$\theta^*(a, \tilde{b}, y) = r - \frac{\Phi^{-1}(\lambda)}{\sqrt{a + \tilde{b}}}. \qquad (29)$$

Using (14), this is equivalent to

$$r - \frac{\Phi^{-1}(\lambda)}{\sqrt{a + \tilde{b}}} = z\,\Phi\left(\sqrt{\frac{\tilde{b}}{a+\tilde{b}}}\;\Phi^{-1}(\lambda)\right). \qquad (30)$$

Total differentiation of (30) shows that $\tilde{b}$ decreases for increasing α as long as it is larger than $b^{\min} = (z\,a)^2/(2\,\pi)$. There is a unique $\tilde{a}$ at which $\tilde{b}(\tilde{a}) = (z\,\tilde{a})^2/(2\,\pi)$. For higher precisions of the fundamental's distribution, the optimal information policy is $b^{\min} = (z\,a)^2/(2\,\pi)$.

By the implicit function theorem,

$$\frac{\mathrm{d}\,\theta^*(a, \tilde{b}(a), r)}{\mathrm{d}\,a} = \frac{\phi(\cdot)}{\sqrt{\tilde{b}} - a\,z\,\phi(\cdot)}\,\frac{-\,\Phi^{-1}(\lambda)}{2\sqrt{a + \tilde{b}}}, \qquad (31)$$

which is negative for $\tilde{b} > \beta^{\min}$ and $\lambda > 1/2$. Furthermore,

$$\frac{\mathrm{d}\,\theta^*(a, b^{\min}(a), r)}{\mathrm{d}\,a} = \frac{z\,\phi(\cdot)}{1 - \sqrt{2\,\pi}\,\phi(\cdot)} \cdot \frac{-\,\pi\,\Phi^{-1}(\lambda)}{z\,a^2\,\sqrt{z^2 + 2\,\pi/a}}, \tag{32}$$

which is also negative for $\lambda > 1/2$. Thus, θ^* is decreasing in a for both information policies. Furthermore, $\theta^* \leq \theta_0^* < r$ and hence, (28) shows that the probability of premature liquidation is decreasing with rising a. Hence, the optimal precision of the prior distribution is $a^* \to \infty$ and $b^* = b^{\min}$. QED

References

[1] Carlsson, Hans and Erik van Damme (1993). Global Games and Equilibrium Selection. *Econometrica*. 61, 989-1018.

[2] Cheli, Bruno and Pompeo Della Posta (2001). Self-fulfilling Currency Attacks with Biased Signals: Morris and Shin's Model Reconsidered. Mimeo.

[3] Chui, Michael, Prasanna Gai and Andrew G. Haldane (2002). Sovereign liquidity crises: Analytics and implications for public policy. *Journal of Banking and Finance* 26, 519-546.

[4] Heinemann, Frank (2000). Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks: Comment. *American Economic Review*. 90(1), 316-318.

[5] Heinemann, Frank and Gerhard Illing (1999). Speculative Attacks: Unique Sunspot Equilibrium and Transparency. Forthcoming in *Journal of International Economics*

[6] Hellwig, Christian (2000). Public Information, Private Information and the Multiplicity of Equilibria in Coordination Games. Forthcoming in *JET*.

[7] Hubert, Franz und Dorothea Schaefer (2001). Coordination Failure with Multiple-Source Lending, the Cost of Protection Against a Powerful Lender. Forthcoming in *JITE*.

[8] Metz, Christina E. (2001). Private and Public Information in Self-Fulfilling Currency Crises. Forthcoming in *Journal of Economics*.

[9] Morris, Stephen and Hyun Song Shin (1998a). Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks. *American Economic Review*. 88, 587-597.

[10] Morris, Stephen and Hyun Song Shin (1999a). A Theory of the Onset of Currency Attacks. Agenor, P.-R., Miller, M., Vines, D., We-

ber, A. (Eds.) *The Asian Financial Crisis: Causes, Contagion and Consequences*, 230-255, Cambridge: Cambridge University Press,

[11] Morris, Stephen and Hyun Song Shin (1999b). Coordination Risk and the Price of Debt. Mimeo.

[12] Morris, Stephen and Hyun Song Shin (2000). Global Games: Theory and Applications. Forthcoming in the *Volume of the Eighth World Congress of the Econometric Society*.

[13] Sbracia, Massimo and Andrea Zaghini (2001). Expectation and Information in Second Generation Currency Crises Models. *Economic Modelling* 18, 203-222.