Abstract

"Strategic R&D investment, competition toughness and growth."

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We study the relationship between toughness of competition on the product market, strategic R&D investment and growth. We develop an overlapping generations model, where firms as well as consumers have a two-period life. Firms invest in R&D during the first period and compete on the product market in the second period. The number of firms is endogenously determined through capital market clearing, not through a zero profit condition. We exhibit the non-monotonicity of the relation between the toughness of the competition regime and the incentives to innovate.

Strategic R&D investment, competition toughness and growth

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February 12, 2002

1 Introduction

There is apparently a paradox in trying to assess, both empirically and theoretically, the impact of competitive pressure on innovation and growth. On one hand, in the Schumpeterian tradition, the reward provided by monopoly rent is required to stimulate sufficient R&D investment and technological progress, and the means and profitability of innovation are jeopardized by intense compettiton. On the other hand, the incentives to innovate are weaker when there is a monopoly than in a less concentrated industry (Arrow 1962, Dasgupta and Stiglitz, 1980). When competition is intense on the product market, innovation may be seen as the only way for a firm to survive.

In the neo-Schumpeterian models of endogenous growth (Segerstrom et al., 1990, Aghion and Howitt, 1992, and Grossman and Helpman, 1991) innovation allows a firm in an industry to take the lead and gain profit. But the monopoly rent enjoyed by the winner is only temporary, and a new innovator, capitalizing on accumulated knowledge, is always able to "leapfrog" the leader. In such a patent-race model (Reinganum, 1985), although the identity of the monopolist in each industry keeps varying over time, the market structure remains the same. This is a model of growth through creative destruction, stressing competition at the research level, and not at the production level. More recently Aghion, Harris and Vickers (1997), supposing a duopoly in each sector, both at the research and the production levels, have introduced what they call "step-by-step innovation" according to which technological progress allows a firm to

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take the lead, but with the lagging firm remaining active and eventually capable of catching up. This model has been extended by Encaoua and Ulph (2000), allowing for the possibility that the lagging firm leapfrogs the leader, without driving it out of the market. As suggested by Thompson and Waldo (1994), these tournament models represent a way to combine the two kinds of innovative capitalisms described by Schumpeter (1928), "competitive capitalism" and its creative-destructive process, and "trustified capitalism", for which research competition takes place among active firms in the same industry. The advantage of these new tournament models is to permit an analysis of the effects of varying the product competition regime (implying different intensity of rivalry between firms) on innovation and growth. But in these analyses, the number of firms, even though different from monopoly, remains fixed.

Our purpose is to study the relationship between toughness of competition on the product market, strategic R&D investment and growth, in a model where the number of firms is endogenously determined. As usually understood, "tougher competition" means lower prices and markups for any given number of firms. We develop an overlapping generations model, where firms as well as consumers have a two-period life (in contrast with the previous tournaments models where firms are infinitely-lived). Firms invest in R&D during the first period and compete on the product market in the second period. Competition at the research level is still modelled as a tournament under uncertainty: it is a patent race but including the possibility of several simultaneous winners and imperfect patent protection (there are spillover effects). For a firm, the strategically chosen probability of innovating, and thus of increasing technological know-how, is produced by simple labor, as supplied by young consumers, expecting some share of profits when old. The number of firms is endogenously determined through capital market clearing, not through a zero profit condition. Our main conclusion will be the non-monotonicity of the relation between the toughness of the competition regime and the incentives to innovate. The equilibrium level of R&D effort might be decreasing or hump-shaped with increasing competition toughness.

This conclusion reverses the results reached by the tournament models above, at least in comparable situations (*i.e.*, when all firms in an industry have the same knowledge level *ex ante*). There, more intense product market competition - such as under Bertrand competition with respect to Cournot competition - enhances innovation and growth. But it also contradicts the results obtained in non-tournament models dealing with the same issue. For instance, van de Klundert and Smulders (1997) also show that Bertrand competition implies a higher rate of innovation than Cournot competition. The key to their result is that tougher competition implies more concentration, *i.e.* firm destruction through product market competition, not through research competition. But in their model, technological progress is non-stochastically and competitively determined by the R&D investments of each firm, taking as given the capital market rate of interest, and the number of firms is determined by the zero-profit condition. In a similar model, studying the interdependence of market structure and growth, the same positive association between competition toughness and growth is obtained by Peretto (1999), also through industry concentration. Our own conclusion is more akin to recent results in industrial organisation (Boone, 2001) showing, in standard partial equilibrium settings, but with an external R&D sector, the non-monotone relation between intensity of competition and the incentives to innovate. However, there, the non-monotonicity is due to firm asymmetries: the identity of the firm willing to bid the highest price for the externally produced innovation may change. In our model, firms are all identical initially and compete for innovation individually.

In the next section, we introduce our model with a continuum of oligopolistic industries and characterize consumption and production. Then, in section 3, we specify different competition regimes with varying degree of toughness and analyze strategic investment. We derive the relations between a winner's expected gains under various regimes. In section 4, we define general equilibrium and construct our example to establish the non-monotonicity of the incentives to innovate as a function of competition toughness.

2 Producers and consumers

We consider an overlapping generations model with two types of agents: firms and consumers. Both types live for two periods.

On the producers side, there is a continuum of uniformly distributed oligopolistic industries of mass 1. In each industry i, there is, in each period t, a small (and varying) number N_{it-1} ($N_{it-1} \geq 2$) of firms, created in the preceding period, producing good i for immediate consumption, with labour supplied by young consumers. Capital, to be seen as a form of knowledge, results from the stock of knowledge K_{it-1} accumulated in the industry, adjusted for current period innovation. Innovation is a random variable depending upon the levels of R&D investment, decided by each firm in period $t-1$. Investment is in labour provided by young consumers.

On the consumers side, there is a continuum of identical consumers of constant unit mass, for each one of the two coexisting generations. We assume that labour is inelastically supplied by young consumers either to old producing firms or to new investing ones, at the same wage (normalized to 1). In period t, each young household can allocate its wage income (equal to 1, by so normalizing the labour endowment) either to current consumption x_t, at prices p_t, or to saving[1] $1 - \langle p_t, x_t \rangle$. Saving is supposed to be invested in funds that allow to cancel out idiosyncratic risks, but not aggregate risk, so that the young consumer anticipates a return factor on capital $\widetilde{r}_{t+1}$, which is a random variable depending upon the success of the innovative efforts by all investing firms.

Young consumers have to choose present consumption $x_t \in \mathbb{R}_+^{[0,1]}$ and saving $z_t \in \mathbb{R}_+$, under the budget constraint $\langle p_t, x_t \rangle + z_t = 1$, and also anticipated future consumption $\widetilde{x}_{t+1}$, which is a random variable induced by $\widetilde{r}_{t+1}$ and $\widetilde{p}_{t+1}$ according to the budget constraint $\langle \widetilde{p}_{t+1}, \widetilde{x}_{t+1} \rangle = \widetilde{r}_{t+1} z_t$. For simplicity we

[1]Given y and z belonging to $\mathbb{R}^{[0,1]}$, we let $\langle y, z \rangle$ denote the inner product $\int_0^1 y_i z_i di$.

assume symmetric log-linear sub-utility functions, so that we obtain the optimal present and future consumptions of any good i: $x_{it} = (1 - z_t) / p_{it}$ and $\widetilde{x}_{it+1} = \widetilde{r}_{t+1} z_t / \widetilde{p}_{it+1}$.

We further assume intertemporal preferences of young consumers represented by the utility function:

$$U(x, \widetilde{x}) = \alpha \int_0^1 \ln x_i di + (1 - \alpha) \int_0^1 \ln \widetilde{x}_i di, \text{ with } \alpha \in]0, 1[.$$

Hence, taking x_t and $\widetilde{x}_{t+1}$ as functions of z_t, the problem of the young consumer is equivalent to maximizing $\alpha \ln (1 - z_t) + (1 - \alpha) \ln z_t$, leading to the solution $z_t = 1 - \alpha$.

Clearly, the old consumers at period t optimally choose, given r_t (the actual return factor) and p_t, consumptions $x'_{it} = (1 - \alpha) r_t / p_{it}$. Thus, adding consumptions by youngs and olds, we obtain the aggregate demand for good i:

$$d(p_{it}, A_t) = \frac{A_t}{p_{it}},$$

with aggregate expenditure

$$A_t = \alpha + (1 - \alpha) r_t. \tag{1}$$

Coming back to the producers side, we may now consider for each industry i the set of N_{it} firms which will be potentially active in period $t + 1$, according to the intertemporal equilibrium of the economy (to be defined later). For these firms, the situation appears as a two-stage non-cooperative game $\Gamma(N_{it}, \widetilde{A}_{t+1})$, parameterized in N_{it} and the random variable $\widetilde{A}_{t+1}$. Observe that, as we have assumed a continuum of industries, this random variable is unaffected by sectoral idiosyncratic variations. In the first stage of the game, corresponding to the investment period t, each firm j, producing good i, chooses strategically a committing level of R&D investment leading to a probability θ_{ijt} of success. This level of R&D investment corresponds to the cost $C(\theta_{ijt})$, in labour, of ensuring that probability of success. We assume a quadratic cost function:

$$C(\theta) = \phi + (c/2) \theta^2,$$

with a fixed cost $\phi > 0$. In the second stage, uncertainty on the aggregate expenditure is supposed to be resolved, and each firm j accordingly chooses a quantity y_{ijt+1}. We will consider subgame perfect equilibria of each game $\Gamma(N_{it}, \widetilde{A}_{t+1})$. Later, in section 4, using the solutions to the family of two-stage games $\left\{\Gamma(N_{it}, \widetilde{A}_{t+1})\right\}$ we will define equilibrium for the whole economy, and thus fix parametrically the number of firms in each industry at each period.

We assume a Cobb-Douglas production function F, linear in both labour l_{ijt+1} and the accumulated stock of knowledge in the industry K_{it}. In addition, production is multiplicatively affected by the *innovation step* $k > 1$ if the firm is successful. Otherwise, it is subject to spillover effects coming from innovation by any other successful firm. To be explicit, we suppose:

$$F\left(\delta_{ijt+1}, l_{ijt+1}, K_{it}\right) = K_{it} k^{\delta_{ijt+1}} l_{ijt+1},$$

with $\delta_{ijt+1} = 1$, if j is successful (with probabilty θ_{ijt}), $\delta_{ijt+1} = \delta \in [0,1[$ (the spillover coefficient, supposing one successful firm at least), and $\delta_{ijt+1} = 0$ if no firm succeeds.

We also have to introduce some characterization of the knowledge accumulation process. For that purpose, we shall assume that the growth rate of the stock of knowledge in industry i is proportional to the percentage of successful firms, the proportion factor being the innovation increment $(k-1)$:

$$\frac{K_{it+1}}{K_{it}} - 1 = (k-1)\,\frac{n_{it}}{N_{it}}. \tag{2}$$

In the next section, we analyze successively (and backwards) the two stages of the game $\Gamma(N_{it}, \widetilde{A}_{t+1})$ under different competition regimes.

3 Toughness of competition regimes

At the second stage, competition among firms may be more or less tough, according to the type of coordination that they succeed in enforcing. A standard distinction is between Cournot and Bertrand competition. In the present context of homogeneous products in each sector and R&D tournament with possibly multiple winners, the Bertrand solution appears as rather extreme. Indeed, except in the case of a single winner, any firm, whether successful or not, will earn zero profit. A less extreme solution, which we call the *limit-pricing equilibrium*, consists in assuming that successful firms coordinate on the highest price allowing elimination of the unsuccessful, if any, and that they play Bertrand if they are either all successful or all unsuccessful[2].

These three equilibrium concepts can be viewed as three instances of a relatively large and flexible concept of oligopolistic equilibrium, the "min-pricing equilibrium", allowing for multiple solutions that can be ranked in increasing competition toughness, according to a decreasing market price level from Cournot to Bertrand equilibrium price (see d'Aspremont *et al.,* 1991). The equilibrium market price is the minimum of the prices announced by the producers, each facing a residual demand. This is a way to formalise price formation in various oligopolistic contexts and it includes the use of different kinds of "facilitating practices", namely advance notice and matching policies together with parallel rationing.

[2]Bertrand competition is considered in van de Klundert and Smulders (1997) and Peretto (1999), but industry products are assumed to be differentiated goods. Bertrand competition without product differentiation is introduced by Encaoua and Ulph (2000), but in duopoly, where it coincides with limit-pricing, as defined here.

Formally, a *min-pricing equilibrium* is a simultaneous solution $(p_{ijt+1}, y_{ijt+1})_j$ to the following family of individual programmes (with $j = 1, ..., N_{it}$):

$$\max_{(p_{ijt+1}, y_{ijt+1})} \left\{ \left(\min_{j'} \{p_{ij't+1}\} - \frac{1}{K_{it} k^{\delta_{ijt+1}}} \right) y_{ijt+1} \right\},$$

$$\text{s.t. } y_{ijt+1} \leq \max \left\{ d(\min_{j'} \{p_{ij't+1}\}, A_{t+1}) - \sum_{j' \neq j} y_{ij't+1}, 0 \right\}.$$

There is a large indeterminacy in solving these programmes, both in prices between the Cournot and the Bertrand equilibrium levels, and in quantities (since there is arbitrariness in the way the market is shared). But we will focus on the three mentioned solutions, with equal treatment among equal cost firms.

3.1 Cournot equilibria

At the Cournot equilibrium, each producer chooses the same optimal price against a residual demand, without being blocked by the min-scheme[3]. This gives the well-known solution (with $n_{it} \geq 1$ successful firms and $(N_{it} - n_{it})$ unsuccessful firms):

$$y^C_{ijt+1} = \left[1 - \frac{(N_{it} - 1) k^{1-\delta_{ijt+1}}}{n_{it} + (N_{it} - n_{it}) k^{1-\delta}} \right] Y^C_{it+1}, \text{ with} \tag{3}$$

$$Y^C_{it+1} = \sum_{j'} y^C_{ij't+1} = A_{t+1} K_{it} \frac{(N_{it} - 1) k}{n_{it} + (N_{it} - n_{it}) k^{1-\delta}}, \tag{4}$$

provided $n_{it} < k^{1-\delta} / \left(k^{1-\delta} - 1\right)$, ensuring that all firms are active[4]. The corresponding price is:

$$p^C_{it+1} = \frac{n_{it} + (N_{it} - n_{it}) k^{1-\delta}}{(N_{it} - 1) k} \frac{1}{K_{it}}.$$

If no firm is successful, we get for every j

$$y^C_{ijt+1} = A_{t+1} K_{it} \frac{1 - 1/N_{it}}{N_{it}}, \; p^C_{it+1} = \frac{N_{it}}{(N_{it} - 1) K_{it}}.$$

The second stage equilibrium profit *per unit of sectoral expenditure* A_{t+1}:

$$\Pi^C (\delta_{ijt+1}, n_{it}, N_{it}) = \left[1 - \frac{(N_{it} - 1) k^{1-\delta_{ijt+1}}}{n_{it} + (N_{it} - n_{it}) k^{1-\delta}} \right]^2, \tag{5}$$

$$\text{if } 1 \leq n_{it} \leq k^{1-\delta} / \left(k^{1-\delta} - 1\right),$$

$$\Pi^C (0, 0, N_{it}) = \frac{1}{(N_{it})^2}, \text{ if } n_{it} = 0, \tag{6}$$

[3] For the complete argument, see d'Aspremont, Dos Santos Ferreira and Gérard-Varet (1991).

[4] For brevity, we shall ignore the case $n_{it} \geq k^{1-\delta} / \left(k^{1-\delta} - 1\right)$.

where δ_{ijt+1} takes the values 1 or δ (resp. 0), according to firm j being successful or unsuccessful with $n_{it} \geq 1$ (resp. $n_{it} = 0$).

3.2 Bertrand and limit-pricing equilibria

At the Bertrand equilibrium, unsuccessful firms remain inactive, and we assume that successful firms equally share sectoral demand at a price $p_{it+1}^B = 1/K_{it}k$ equal to their production cost (if $1 < n_{it} \leq N_{it}$):

$$y_{ijt+1}^B = \frac{A_{t+1}K_{it}k}{n_{it}} \text{ if successful, } y_{ijt+1}^B = 0, \text{ otherwise.} \tag{7}$$

At the limit-pricing equilibrium, successful firms coordinate in choosing a price $p_{it+1}^L = 1/K_{it}k^\delta$ equal to the production cost of the unsuccessful firms (if $1 \leq n_{it} < \min\left\{N_{it}, k^{1-\delta}/\left(k^{1-\delta}-1\right)\right\}$)[5]. We suppose that the latter are accordingly eliminated and that the former share the total quantity equally:

$$y_{ijt+1}^L = \frac{A_{t+1}K_{it}k^\delta}{n_{it}} \text{ if successful, } y_{ijt+1}^L = 0, \text{ otherwise.} \tag{8}$$

If $n_{it} = 1$, the Bertrand equilibrium gives the limit-pricing outcome. And, if $n_{it} = 0$ or $n_{it} = N_{it}$, we assume that the Bertrand solution applies in the limit-pricing regime, with $p_{it+1}^L = 1/K_{it}$ or $p_{it+1}^L = 1/K_{it}k$, respectively.

The second stage limit-pricing equilibrium profit *per unit of sectoral expenditure* A_{t+1} is, for $1 \leq n_{it} < \min\left\{N_{it}, k^{1-\delta}/\left(k^{1-\delta}-1\right)\right\}$:

$$\Pi^L(1, n_{it}, N_{it}) = \left(\frac{1}{K_{it}k^\delta} - \frac{1}{K_{it}k}\right) K_{it}k^\delta \frac{1}{n_{it}} = \frac{1}{n_{it}}\left(1 - k^{\delta-1}\right), \tag{9}$$

$$\Pi^L(\delta, n_{it}, N_{it}) = 0. \tag{10}$$

If $n_{it} = 0$ or $n_{it} = N_{it}$,

$$\Pi^L(0, 0, N_{it}) = \Pi^B(0, 0, N_{it}) = \Pi^L(1, N_{it}, N_{it}) = \Pi^B(1, N_{it}, N_{it}) = 0, \tag{11}$$

where Π^B denotes the Bertrand equilibrium profit function. Notice also that $\Pi^B(1, 1, N_{it}) = 1 - k^{\delta-1}$.

Whatever the type of subgame considered at the second stage, Cournot, Bertrand or limit-pricing, we shall denote $p_{it+1}^* = p^*(n_{it}, N_{it})/K_{it}$ the equilibrium min-price and $y_{ijt+1}^* = y^*(\delta_{ijt+1}, n_{it}, N_{it})\, A_{t+1}K_{it}$ the associated output. Also, we define the corresponding labour demand from producing firms:

$$\begin{aligned} L^*(n_{it}, N_{it})\, N_{it}A_{t+1} &= \left(\frac{n_{it}}{N_{it}}\frac{y^*(1, n_{it}, N_{it})}{k}\right) N_{it}A_{t+1} \\ &\quad + \left(1 - \frac{n_{it}}{N_{it}}\right)\frac{y^*(\delta, n_{it}, N_{it})}{k^\delta} N_{it}A_{t+1}, \\ \text{if } n_{it} &\geq 1, \text{ otherwise} \\ L^*(0, N_{it})\, N_{it}A_{t+1} &= y^*(0, 0, N_{it})\, N_{it}A_{t+1}. \end{aligned}$$

[5] We ignore the case $n_{it} > k^{1-\delta}/\left(k^{1-\delta}-1\right)$, in which the limit price becomes unsustainable as a min-pricing equilibrium, successful firms preferring to set a lower price.

If we want to specify the type of equilibrium the star will be replaced by C, B or L, accordingly. The same convention will be applied to the profit function Π^*.

It is now easy to verify:

Lemma 1 *The three solutions can be ranked according to "competition toughness", i.e.*

$$\begin{aligned} p^B(n_{it}, N_{it}) &< p^L(n_{it}, N_{it}) < p^C(n_{it}, N_{it}), \ \textit{if } 1 < n_{it} < N_{it}, \\ p^B(n_{it}, N_{it}) &= p^L(n_{it}, N_{it}) < p^C(n_{it}, N_{it}), \ \textit{if } n_{it} = 0, 1, N_{it}. \end{aligned}$$

Not surprisingly, we see that competition under Cournot is less intense than under limit-pricing, which is never more intense than under Bertrand[6].

3.3 Strategic investment under different regimes

Investment in R&D is determined by the choice by firm j of the (independent) probability of success θ_{ijt}. This is decided in the first stage, by maximizing with respect to this variable the expectation of future profit $E_t\tilde{\Pi}\left(\theta_{ijt}, \theta_{i(-j)t}\right)$ (with E_t denoting the conditional expectation operator). Denoting by Π^* the second stage profit per unit of expenditure at the selected min-pricing equilibrium, and by $\overline{A}_{t+1} = E_t\widetilde{A}_{t+1}$ the expected value of the random variable $\widetilde{A}_{t+1}$ conditional on information available in period t, we may write:

$$\begin{aligned} & E_t\tilde{\Pi}\left(\theta_{ijt}, \theta_{i(-j)t}\right) = \\ & \theta_{ijt} \sum_{\nu=0}^{N_{it}-1} \Pr\left\{\nu \mid \theta_{i(-j)t}\right\} \Pi^*(1, \nu+1, N_{it}) \overline{A}_{t+1} + (1-\theta_{ijt}) \times \\ & \left[\Pr\left\{0 \mid \theta_{i(-j)t}\right\} \Pi^*(0,0,N_{it}) + \sum_{\nu=1}^{N_{it}-1} \Pr\left\{\nu \mid \theta_{i(-j)t}\right\} \Pi^*(\delta, \nu, N_{it})\right] \overline{A}_{t+1} \\ & -C(\theta_{ijt}). \end{aligned}$$

where, assuming $\theta_{i(-j)t} = (\theta_{it}, ..., \theta_{it})$,

$$\Pr\left\{\nu \mid \theta_{i(-j)t}\right\} = \frac{(N_{it}-1)!}{(N_{it}-1-\nu)!\nu!} \theta_{it}^{\nu} (1-\theta_{it})^{N_{it}-1-\nu}. \tag{12}$$

The expectation of future profit is clearly strictly concave in the strategy variable θ_{ijt}. Taking the (necessary and sufficient) first order condition for an interior maximum:

$$\begin{aligned} & C'(\theta_{ijt}) = \Pr\left\{0 \mid \theta_{i(-j)t}\right\} \left[\Pi^*(1,1,N_{it}) - \Pi^*(0,0,N_{it})\right] \overline{A}_{t+1} + \\ & \sum_{\nu=1}^{N_{it}-1} \Pr\left\{\nu \mid \theta_{i(-j)t}\right\} \left[\Pi^*(1, \nu+1, N_{it}) - \Pi^*(\delta, \nu, N_{it})\right] \overline{A}_{t+1} \end{aligned}$$

[6] Boone (2001) defines a notion of "intensity of competition" by some axioms on profit functions. The three axioms that are relevant in this model are compatible with the profit functions Π^* and the ranking we just verified.

for each j, we can derive the symmetric equilibrium R&D investment (with $\theta_{ijt} = \theta_{it}$, for any j). For simplicity of notation, we denote

$$G^*(\nu) = \Pi^*(1, \nu+1, N_{it}) - \Pi^*(\delta, \nu, N_{it}) \text{ for } \nu \geq 1 \quad (13)$$

$$G^*(0) = \Pi^*(1, 1, N_{it}) - \Pi^*(0, 0, N_{it}) \text{ for } \nu = 0 \quad (14)$$

the gain in profit when switching from a state of failure to a state of success. If we take into account that this gain depends upon the number ν of successful competitors, which is a random variable $\tilde{\nu}$ following a binomial law with parameters $N_{it} - 1$ and θ_{it}, we can write the first order condition in the symmetric case as

$$C'(\theta_{it}) = E[G^*(\tilde{\nu}) \mid N_{it} - 1, \theta_{it}] \overline{A}_{t+1} \equiv \overline{G}^*(N_{it}, \theta_{it}) \overline{A}_{t+1}, \quad (15)$$

that is, the equality of the marginal cost of R&D and of the expected value of the gain in profit if the R&D effort succeeds. Of course, if $C'(1) = c < \overline{G}^*(N_{it}, 1) \overline{A}_{t+1}$, the corner solution $\theta_{it} = 1$ applies. As C' is increasing, with $C'(0) = 0$, there is always a value of θ_{ijt} satisfying equation (15).

In the Cournot case, we may compute from (13), (14) and (15), using (4), (5), (6) and (12), that the expected value of the gain, per unit of expenditure, of successful R&D is:

$$\overline{G}^C(N, \theta) = \sum_{\nu=0}^{N-1} \frac{(N-1)!}{(N-1-\nu)!\nu!} \theta^\nu (1-\theta)^{N-1-\nu} \times$$
$$\left(\left(\frac{1 + (N-1-\nu)\left(k^{1-\delta} - 1\right)}{1 + \nu + (N-1-\nu) k^{1-\delta}} \right)^2 - \left(\frac{\nu - (\nu-1)k^{1-\delta}}{\nu + (N-\nu) k^{1-\delta}} \right)^2 \right).$$

In the limit pricing case (by referring to equations (8) and (9)-(11)), we obtain:

$$\overline{G}^L(N, \theta) = \sum_{\nu=0}^{N-2} \frac{(N-1)!}{(N-1-\nu)!\nu!} \theta^\nu (1-\theta)^{N-1-\nu} \frac{1 - k^{\delta-1}}{1+\nu}.$$

Finally, in the Bertrand case, we simply get:

$$\overline{G}^B(N, \theta) = (1-\theta)^{N-1} \left(1 - k^{\delta-1}\right).$$

From these expressions, we may immediately derive the following properties:

Lemma 2 *(i)* $0 < \overline{G}^C(N, 0) < \overline{G}^B(N, 0) = \overline{G}^L(N, 0)$ *and* $0 = \overline{G}^B(N, 1) = \overline{G}^L(N, 1) < \overline{G}^C(N, 1)$*;*

(ii) $\overline{G}^B(N, \theta) \leq \overline{G}^L(N, \theta)$*, for any* $\theta \in [0, 1]$*, with strict inequality for* $\theta \in]0, 1[$ *and* $N > 2$*.*

Proof. See Appendix. ■

These relations between the winner's expected gains under various regimes are the key to compare incentives to innovate as they vary when competition toughness increases. From this lemma we see that, by continuity, the relation between competition toughness and the winner's expected gain is non-monotone for low values of the probability of success of his competitors[7]. For high values, the relation is monotone but R&D incentives decrease with competition toughness. The same is true when comparing the limit-pricing and the Bertrand regimes for all values of that probability. We shall now examine the consequences of these observations in a general equilibrium intertemporal model.

4 General Equilibrium

The construction of a general equilibrium concept will be based on the preceding analysis, distinguishing the three cases corresponding to the three degrees of competition toughness. We will proceed in two steps, first defining temporary and intertemporal equilibria, and then focusing on quasi-stationary symmetric ones.

4.1 Temporary and intertemporal equilibria

Using the solutions to the family of two-stage games $\left\{\Gamma(N_{it}, \widetilde{A}_{t+1})\right\}$ associated with any particular type of sectoral min-pricing equilibria at the second stage, we can now define our concept of equilibrium for the whole economy. In particular, this will fix parametrically the number of firms in each industry at each period. Because of the symmetry of the model, we shall only define equilibria treating symmetrically each category of firms (successful or unsuccessful).

An *intertemporal stochastic equilibrium* is a sequence $(\tilde{A}_t, \tilde{r}_t, \left(\tilde{p}_{it}, \tilde{\theta}_{it}, \tilde{N}_{it}, \tilde{K}_{it}\right)_i)$ of functions of the random variables $(\tilde{n}_{it-1})_i$ following a binomial law of parameters θ_{it-1} and N_{it-1} such that, for every t and for every realisation of these random variables,

(1) $\left(\tilde{A}_t, \tilde{r}_t, \left(\tilde{p}_{it}, \tilde{\theta}_{it}, \tilde{N}_{it}\right)_i\right)$ is a *temporary equilibrium* given the past realisations $(K_{it-1}, N_{it-1}, n_{it-1})_i$, and the expected value $\overline{A}_{t+1} = E_t\widetilde{A}_{t+1}$, as defined by:

(i) min-pricing equilibrium in each industry i

$$p_{it} = p^*\left(n_{it-1}, N_{it-1}\right)/K_{it-1},\ A_t = \alpha + (1-\alpha)\, r_t \tag{16}$$

(ii) labour market clearing

$$\int_0^1 \left[N_{it-1}L^*\left(n_{it-1}, N_{it-1}\right)A_t + N_{it}C\left(\theta_{it}\right)\right]di = 1, \text{ for } n_{it-1} \geq 1, \tag{17}$$

$$\int_0^1 \left[N_{it-1}L^*\left(0, N_{it-1}\right)A_t + N_{it}C\left(\theta_{it}\right)\right]di = 1, \text{ otherwise.} \tag{18}$$

[7]A similar non-monotonicity is exhibited by Boone (2001) but for other reasons, in a deterministic context, with ex ante cost asymmetries and research done by some independent laboratory.

(*iii*) capital market clearing

$$\int_0^1 N_{it} C(\theta_{it})\, di = 1-\alpha, \text{ with } C'(\theta_{it}) = \overline{G}^*(N_{it}, \theta_{it})\, \overline{A}_{t+1}. \tag{19}$$

(2) $g_{it-1} \equiv K_{it}/K_{it-1} - 1 = (n_{it-1}/N_{it-1})(k-1)$, for all i (the knowledge accumulation rule is verified).

4.2 Quasi-stationary symmetric equilibria

Quasi-stationary symmetric equilibria are intertemporal stochastic equilibria with random variables $(\tilde{n}_{it})_i$ following the same[8] binomial law of constant parameters (N, θ), thus inducing constant expected values for:

$$C'(\theta) = \overline{G}^*(N,\theta)\, A \tag{20}$$
$$r = (A-\alpha)/(1-\alpha) \tag{21}$$
$$n = N\theta \tag{22}$$
$$g = \theta(k-1). \tag{23}$$

From the investment optimality condition, together with the factor market clearing conditions, we obtain an equation in N and θ stressing investors' behaviour, and exhibiting accordingly a different specification for each one of the three types of min-pricing equilibria we have to consider (Cournot, Bertrand and limit pricing):

$$C'(\theta) = \overline{G}^*(N,\theta)\, A^*(N,\theta) = \overline{G}^*(N,\theta)\, \frac{\alpha}{NL^*(\theta N, N)}, \tag{24}$$

The quasi-stationary symmetric equilibrium of the economy is determined by this equation, together with a second relation between N and θ, the capital market clearing condition, which stresses savers' behaviour:

$$NC(\theta) = 1-\alpha. \tag{25}$$

From this equation, we see that some restrictions have to be imposed on the parameters of the cost function (ϕ and c) and on the propensity to save $(1-\alpha)$, ensuring that capital demand and supply can be equalized for $N > 2$ (but not too large, in order to remain in our simple case: $N < k^{1-\delta}/\left(k^{1-\delta}-1\right)$). To illustrate, the following proposition shows that such restrictions are sufficient for a quasi-stationary symmetric equilibrium to exist in the three competitive regimes.

Proposition 3 *Assume:*

$$\phi \leq \min\left\{(1-\alpha)/3, (1-\alpha-c)/2\right\}. \tag{26}$$

[8] Strictly speaking, this is incorrect, since we will in the following treat N as a continuous variable, for simplicity. More precisely, we use linear interpolations of the functions $\overline{G}^*(\cdot,\theta)$, which are defined on $\mathbb{N}$ only, and abusively treat N as a real number otherwise.

Then there exists a quasi-stationary symmetric equilibrium, with $N \in \left]2, (1-\alpha)/\phi\right[$ *in the limit-pricing regime, and in the Cournot regime if* $k^{1-\delta}$ *is larger but close enough to* 1*, with all firms active. In the Bertrand regime, a quasi-stationary equilibrium exists for* c *positive but close enough to* 0.

Proof. Consider the value of θ as determined by the capital market clearing condition:

$$\theta = \Theta(N; c, \phi) \equiv \sqrt{\frac{2}{c}\left(\frac{1-\alpha}{N} - \phi\right)},$$

a decreasing function of N, namely $\Theta(\cdot; c, \phi) : \left[2, \overline{N}\right] \to [0, 1]$, where $\overline{N} = (1-\alpha)/\phi$. We now show that, for $\underline{N} = (1-\alpha)/(\phi + c/2) \in \left[2, \overline{N}\right[$, such that $\Theta(\underline{N}; c, \phi) = 1$,

$$\begin{aligned} \frac{\overline{G}^*\left(\underline{N}, \Theta(\underline{N}; c, \phi)\right) A^*\left(\underline{N}, \Theta(\underline{N}; c, \phi)\right)}{\Theta(\underline{N}; c, \phi)} &< c \\ \text{and } \frac{\overline{G}^*\left(\overline{N}, \Theta\left(\overline{N}; c, \phi\right)\right) A^*\left(\overline{N}, \Theta\left(\overline{N}; c, \phi\right)\right)}{\Theta\left(\overline{N}; c, \phi\right)} &> c. \end{aligned}$$

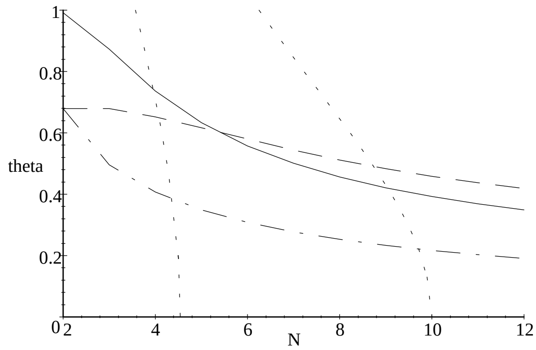
The second inequality is satisfied since $\Theta\left(\overline{N}; c, \phi\right) = 0$, and $\overline{G}^*\left(\overline{N}, 0\right) A^*\left(\overline{N}, 0\right) > 0$ (by Lemma 2), entailing infinity of its right hand side. As to the first inequality, it is verified if its left hand side is equal or close to zero. By Lemma 2, $\overline{G}^L(\underline{N}, 1) = \overline{G}^B(\underline{N}, 1) = 0$ and, by (**??**),

$$\overline{G}^C(\underline{N}, 1) = \left(\frac{1}{\underline{N}}\right)^2 - \left(\frac{1 - (\underline{N} - 2)\left(k^{1-\delta} - 1\right)}{\underline{N} + k^{1-\delta} - 1}\right)^2,$$

which tends to 0 as $k^{1-\delta}$ tends to 1. Then, by continuity, equation (24) must be satisfied for some N^* in the interval $\left]\underline{N}, \overline{N}\right[$ and for $\theta^* = \Theta(N^*; c, \phi)$. Also, for $k^{1-\delta}$ close enough to 1, $\overline{N} < k^{1-\delta}/\left(k^{1-\delta} - 1\right)$, so that all firms will be active at Cournot equilibrium. In the Bertrand regime, since $\overline{G}^B(N, \theta) A^B(N, \theta) > 0$ for $\theta \in [0, 1[$, the equation $C'(\theta) = \overline{G}^B(N, \theta) A^B(N, \theta)$ always has a solution for c low enough. ■

5 An example

We shall now come back to our main purpose, namely to further explore the relationship between competition toughness and R&D investment. In the following example (see Figure 1), we compare the three regimes of competition, represented by the three curves determined by the investment optimality condition (24), namely Cournot (solid curve), limit-pricing (dashed curve) and Bertrand (dotted-dashed curve). The corresponding equilibria result from the intersection of each one of these curves with one of two steep dotted curves (under two values of the fixed cost), computed from the capital market clearing condition (25).



$\alpha = .75,\ k = 1.1,\ \delta = .15,\ c = .03,\ \phi \in \{.025, .055\}$

Figure 1

The interesting feature of this example is the non-monotonicity of the relationship between the intensity of competition and the R&D effort (and the resulting growth rate). We observe that, for the higher fixed cost (corresponding to the left steep dotted curve), the equilibrium level of R&D effort decreases with competition toughness, from Cournot ($\theta \simeq .74$) to Bertrand ($\theta \simeq .38$) through limit-pricing ($\theta \simeq .66$), with an increasing number N of investing firms close to 4. However, for the lower fixed cost (corresponding to the right steep dotted curve), this ordering is upset, the highest equilibrium level of R&D effort being associated to limit-pricing ($\theta \simeq .49$), with Cournot in an intermediate position ($\theta \simeq .42$) and Bertrand in the lowest one ($\theta \simeq .22$), the number of investing firms fluctuating between 8 and 10.

This contrasts dramatically with the result obtained by van de Klundert and Smulders (1997) in a non-tournament model, showing that the (differentiated) Bertrand equilibrium always implies a higher rate of innovation (and more concentration) than the Cournot equilibrium.[9] In their model, tougher competition, meaning lower markups and prices, enlarges the market and weakens the relative weight of R&D costs, increasing the attractiveness of R&D investment, but reduces the number of firm, implying larger firm size and more means devoted to R&D activity.

Our result also contrasts with Encaoua and Ulph (2001). Of course in our model firms live only for two periods and are all identical *ex ante*, so that the different leader-follower situations that they analyse at the investment stage do not occur here. However, if we consider in their model the situations where all firms are at the same level of knowledge at the investment stage, Bertrand

[9]Notice that their ZP (zero profit) curve corresponds in fact to our capital market clearing curve, whereas their CME (capital market equilibrium) curve - resulting in particular from (non-strategic) investment optimality conditions - plays in their model a role equivalent to our investment optimality curve.

competition generates higher innovation and higher growth than Cournot competition. In our model, tougher (but not too tough) competition strengthens the incentive to innovate only when successful innovations are expected for just a small proportion of the investing firms, that is, when N is large and θ low (corresponding in our example to the high fixed cost case). Otherwise, with a large prospective proportion of winners, when N is small and θ high (corresponding to the low fixed cost case), the winner's gain is larger when competition is softer, as in the Cournot regime, and the R&D investment becomes more attractive then.

A Proof of Lemma 2

Statement (ii) is a trivial consequence of (**??**) and (**??**), and so is the equality $\overline{G}^L(N,\theta) = \overline{G}^B(N,\theta)$ for $\theta \in \{0,1\}$. As $\overline{G}^B(N,1) = 0$, the last part of statement (i) immediately results, by (**??**), from

$$\frac{1+(N-1-\nu)\left(k^{1-\delta}-1\right)}{1+\nu+(N-1-\nu)\,k^{1-\delta}} > \frac{\nu-(\nu-1)k^{1-\delta}}{\nu+(N-\nu)\,k^{1-\delta}},$$

when $\nu = N-1$, that is,

$$\frac{1}{N} > \frac{1-(N-2)\left(k^{1-\delta}-1\right)}{N+\left(k^{1-\delta}-1\right)}.$$

Thus, the proof is complete once we show that $0 < \overline{G}^C(N,0) < \overline{G}^B(N,0)$, that is,

$$0 < \left(\frac{1+(N-2)\left(1-k^{\delta-1}\right)}{N-\left(1-k^{\delta-1}\right)}\right)^2 - \left(\frac{1}{N}\right)^2 < 1-k^{\delta-1}.$$

The first inequality is clearly true. Using the notation $x \equiv 1-k^{\delta-1} \in \,]0,1[$, the last inequality can be written:

$$N^2\left(1+(N-2)\,x\right)^2 - (N-x)^2 < N^2\,(N-x)^2\,x,$$

or

$$f(x;N) \equiv -N^2x^2 + \left(N^2(N-2)^2 + 2N^3 - 1\right)x - N\left(N^3 - 2(N-1)^2\right) < 0,$$

which is always satisfied for $x \in \,]0,1[$, since $f(1;N) = -(N-1)^2 < 0$ and $f'(1;N) = N^2(N-2)^2 + 2N^2(N-1) - 1 > 0$. ■

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