Mechanism Design of Interactive Quiz Shows: Are Simple Questions Optimal?

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Abstract

In german TV, there recently have appeared a number of interactive quiz shows. A - usually very simple - question is asked and spectators may make a costly phone call. One of the callers is selected randomly and, if he knows the answer, is allocated a prize. The TV channel's revenue from these calls is the expected number of callers times the price of the phone call. Simple questions increase the number of phone calls since more spectators might know the answer. On the other hand, the likelihood of winning for any caller decreases with each additional caller. I analyze this trade-off in a Bayesian setting and find that the most simple questions are optimal indeed.

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1 Introduction

In german television, and probably elsewhere, there recently have appeared a number of interactive quiz shows: *Call TV* on channel RTL2, *People, Time out* or *Paradies* on NeunLive (the former channel TM3). [1] These quiz shows all work in roughly the same way.[2] The presenter asks a question. Then anyone from the audience at home may phone. One of the callers is chosen randomly and, if he knows the answer, is allocated a prize. Each caller pays a price per call which is way above the rate for a regular phone call. The price for the phone call is paid in any case - no matter if a caller gets connected or not. Some share of the money made with phone calls goes to a phone company, but the rest is the basis for the channel's revenue from the show.

The puzzling fact of these shows - besides the high phone rates - is that questions are very simple as the following examples suggest.

1. Which secret agent has the number 007?
2. Which character appearing in fairy tales rides on a broom?
3. In which city is the Arc de Triomphe?

This paper analyzes if such simple questions are optimal for the TV channel. The channel's expected revenue is the expected number of callers times the price for a phone call (from which some share goes to a phone company). The TV channel faces a trade-off: on the one hand, simple questions increase the number of people knowing the answer to a question. Hence simple questions should increase the expected number of callers. On the other hand, spectators' expected benefits decrease in the number of other spectators calling since the presence of each additional caller decreases the likelihood of winning for any other caller. Simple questions might hence decrease the expected number of callers. I study this trade-off applying the tools of game theory

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[1] see http://www.NeunLive.de.

[2] I describe the shows more detailed in section 2.

and mechanism design and find that it is optimal to ask the most simplest questions indeed.

The model displays the quiz situation as a Bayesian game among the spectators of a quiz show. Each spectator i is risk neutral and is privately informed about his ability θ_i to answer a given question. He is able to answer a question if and only if $\theta_i \geq \widetilde{\theta}$, where $\widetilde{\theta}$ indicates the difficulty of the question. The distribution function of abilities is common knowledge and abilities are drawn independently from each other. The TV channel's revenue is given by the expected number of caller times the price for a phone call. The TV channel awards a prize to the spectator who correctly answers the question. If there is more than one correct answer, a winner is chosen randomly among all callers with equal probability. The TV channel sets the degree of a question's difficulty, $\widetilde{\theta}$, and the price per call c. Each spectator decides whether to make a phone call or. He bases his decision on his valuation of the prize, the price per call, his ability to answer a question of difficulty $\widetilde{\theta}$ and his expectation about the other spectators' calling decisions. First, the presenter of the quiz show announces $\widetilde{\theta}$ and the price for the phone call. Given $(\widetilde{\theta}, c)$ and his private information θ_i, each caller decides whether to call or not. The second stage yields a Bayes Nash equilibrium in participation decisions among the spectators. In this equilibrium spectators call if and only if they are able to answer the question. Given this equilibrium I analyze the TV channel's optimal choice of c and $\widetilde{\theta}$. It turns out that it is optimal to induce calls from all spectators by asking the most simple questions! I then analyze a different class of equilibria in which not all spectators who know the answer call. Given *these* equilibria in the spectators' subgame, it is still optimal to ask simple questions. The result remains also true if the TV channel cannot influence the cost for the telephone call.

The paper is organized as follows. In the following section I describe the quiz shows in more detail. I introduce the model in section 3 and also provide a short motivation from a theoretical perspective (section 3.1.). I derive the equilibrium and study optimal quiz design in section 4. In section 5 I analyze a different class of equilibria in the calling game, equilibria in which not all spectators knowing the answer call. I study the case of a fixed cost for the phone call in section 6. In section 7 I conclude with a simple extension of the basic model and show that most simple questions are not optimal anymore if the number of spectators is endogenous.

2 Interactive Quiz Shows in German Television

There are several interactive quiz shows, mainly on channel NeunLive, for example *Greif an!, Paradise* or *People.* The name of the show stands for the field from which the questions are taken. In *People* questions focus on the world of the famous and the rich, in *Paradise*, it's all about holidays and foreign countries. In *Greif an!*, the interactive quiz is part of a larger quiz show which is more close to the famous *Who wants to be a Millionaire.* Actually, most NeunLive shows have this interactive calling feature.

In each of the shows, the presenter asks a question which is also visible on the TV screen. Also visible is the price for the phone call, which is about 0.50 € for the above mentioned shows. This price is the same for all shows. Then spectators may call. Usually he presenter asks a question over and over again. Moreover, he makes more or less helpful and more or less entertainining comments on the question. Presumably, there is a certain number of callers per question which is required to break even. Sometimes, the presenter announces the time remaining for a phone call for a particular question. On average, each question is asked for about about 7 to 10 minutes on the table. Then, one lucky spectator gets connected and answers the question. The prize for the winner varies between 50 and 150 € which the winner obtains for sure. Moreover, the winner sometimes participates in a lotterie: out of three letters, T, O and P, a combination is selected randomly. If TOP turns up the caller wins another 5.000 €. Hence the total expected gain for a 50 € question is, for example, up to 50 €+ 5.000€/6 $\approx$ 950 €.

The shows make their revenue from the phone calls (50 %) and advertisements (50 %).[3] Hence, phone calls are a pretty important resource of revenue for these shows. Shows like *People* or *Paradies* have regularly less than 10.000 spectators and a market share of about 0,2 to 0,2 %.[4] Although the market share of these shows is not huge, it seems that interactive calling shows could become more prominent in the future: in the US, there are several similar formats where individuals may participate through the internet.

The quiz format described appears also elsewhere. The same type of quiz can be found on various videotext pages of NeunLive.[5] or on the channel's homepage, http://www.neunlive.de. The story is the same: a pretty simple question is asked, people may make a pretty costly phone call and a winner is chosen randomly. The format appears also in other shows which focus on other contents but organize such interactive quizzes as some additional feature of the show. In the (at least in Germany...) well known comedy show *TV Total* on channel Pro7, for example, the showmaster Stefan Raab conducts one interactive quiz per show. The same possibility to participate for spectators at home is offered by showmaster Günter Jauch in the 10 Millionen Mark Show. Moreover, the famous RTL show *Who Wants to be a Millionaire?* - makes most of its revenues from entry phone calls! To make it into the main show, an applicant has to phone the TV channel and is quizzed. This situation is again very similar to the one described above. The prize in that case is the participation in the quiz show plus the expected benefit from winning some amount of money there.

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[3] Private communication to NeunLive.

[4] Sueddeutche Zeitung from June 2001.

[5] See NeunLive videotext page 399, for example.

3 The Model

The players are one presenter of the quiz show and $N \in \mathbb{N} \backslash \{0\}$ spectators of the quiz show. Each spectator is characterized by a type θ_i. Types are independently distributed. Spectator i's type is private information and takes values in $[\underline{\theta}, \overline{\theta}] \subset \mathbb{R}_+$ and has cumulative distribution F and density f. Type θ_i stands for the degree of difficulty spectator i is able to manage. If a question is of difficulty $\widetilde{\theta}$, spectator i knows the answer to the question if and only if $\theta_i \geq \widetilde{\theta}$. Each spectator has valuation $v \in \mathbb{R}_+$ for the prize. The TV channel is represented by a the presenter who has valuation $v_{TV} \leq v$ for the prize. Only one question is asked.

The timing of the game is as follows.

1. Spectators' types are drawn.
2. The presenter announces the degree of difficulty $\widetilde{\theta}$ and the price c for the phone call.
3. Spectators decide whether to phone or not.
4. Among all those who call, one spectator i is selected randomly (with equal probability) and θ_i gets known to the presenter.
5. The presenter allocates the prize to spectator i if and only if $\theta_i \geq \widetilde{\theta}$.

The presenter's strategy is to choose $\widetilde{\theta} \in \mathbb{R}_+$ and $c \in \mathbb{R}$. A spectator's (pure) strategy is a function $\sigma_i : [\underline{\theta}, \overline{\theta}] \times \mathbb{R}_+ \times \mathbb{R} \to \{0, 1\}$, where $\sigma_i(\theta_i, \widetilde{\theta}, c) = 1$ means that spectator i makes the phone call, while $\sigma_i(\theta_i, \widetilde{\theta}, c) = 0$ means that spectator i does not make the phone call. I many cases, I shall omit c from the argument in the strategy function of a spectator.

It is clear, that calling is a dominated strategy if $\theta_i < \widetilde{\theta}$. Either i gets chosen, in that case, i does not get the prize and has to pay c. Or he does not get chosen in the first place and in that case he also has to pay c. I shall therefore omit these cases when describing spectators' utilities and the profits for the TV channel.

To get some insight into spectators' utilities, suppose that $N = 2$.[6] Then, spectator i's expected utility U_i is

$$
\begin{aligned}
&U_i(\sigma_i(\theta_i), \sigma_j(\theta_j), \theta_i, \widetilde{\theta}) = \\
&\sigma_i(\theta_i, \widetilde{\theta} | \theta_i \geq \widetilde{\theta}) \cdot \left[-c + \int_{\widetilde{\theta}}^{\overline{\theta}} (v(1 - \sigma_j(\theta_j, \widetilde{\theta})) + \frac{v}{2} \sigma_j(\theta_j, \widetilde{\theta})) f(\theta_j) d\theta_j \right]
\end{aligned}
\tag{1}
$$

If i calls and if $\theta_i \geq \widetilde{\theta}$, his expected utility is either v (if j does not call) or $v/2$ (if j does call). Spectator j, on the other hand, calls only if $\theta_j \geq \widetilde{\theta}$.

The TV channels expected revenue π is, for $N = 2$,

$$
\begin{aligned}
&\pi(\sigma_1(\theta_1), \sigma_2(\theta_2), \widetilde{\theta}, c) = \\
&\int_{\widetilde{\theta}}^{\overline{\theta}} \sigma_1(\theta_1, \widetilde{\theta}) c f(\theta_1) d\theta_1 + \int_{\widetilde{\theta}}^{\overline{\theta}} \sigma_2(\theta_2, \widetilde{\theta}) c f(\theta_2) d\theta_2 \qquad (2) \\
&- \int_{\widetilde{\theta}}^{\overline{\theta}} \int_{\widetilde{\theta}}^{\overline{\theta}} \left[1 - (1 - \sigma_1(\theta_1, \widetilde{\theta}))(1 - \sigma_2(\theta_2, \widetilde{\theta})) \right] v_{TV} f(\theta_1) f(\theta_2) d\theta_1 d\theta_2 \\
&- \int_{\widetilde{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\widetilde{\theta}} \sigma_1(\theta_1, \widetilde{\theta})) v_{TV} f(\theta_1) f(\theta_2) d\theta_1 d\theta_2 - \int_{\widetilde{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\widetilde{\theta}} \sigma_2(\theta_2, \widetilde{\theta})) v_{TV} f(\theta_1) f(\theta_2) d\theta_1 d\theta_2
\end{aligned}
$$

The TV channel makes c from every spectator who calls. Moreover, the TV channel has to pay v_{TV} in all cases in which at least one spectator calls and knows the answer.

I shall look for a symmetric Bayes Nash equilibrium in the N-player game among the spectators. The equilibrium is characterized by a threshold value $\widetilde{\theta}$ such that everybody calls phone if his type is above $\widetilde{\theta}$ while nobody calls if his type is below $\widetilde{\theta}$. Then, I compute the values for $\widetilde{\theta}$ and c which maximize the TV channel's revenue.

3.1 Theoretical Motivation

Why did I choose a relatively complicated model with private information to answer the - seemingly simple - question of how much participation is optimal? Suppose that

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[6] $N = 2$ is chosen here for the sake of exposition. I shall perform the main analysis for any N.

we have a much simpler model: there are n spectators and no private information whatsoever. Which number $n^* \leq n$ of callers is optimal for the TV channel? An answer to that question would also indicate if simple questions are optimal. If $n^* = n$, then the most simple questions were optimal. If n^* people call, one would have to set the phone cost c equal to v/n^* since each of the n^* callers wins with equal probability. Then, it is a Nash equilibrium for n^* callers to call. The TV channel's revenue is $\pi = n^* c$ (assume $v_{TV} = 0$). But then we have $\pi = v$, which does not depend on n^*! Hence, from a theorist's perspective, such a simple model does *not* allow to analyze the question of optimal participation. As we shall see, this changes in my model with private information in a world where the TV channel has an additional instrument, the degree of difficiculty, to decide how much participation is optimal.

4 Equilibrium and Optimal Quiz Design

Let σ_i^* denote the equilibrium strategy for spectator i.

Proposition 1. *Suppose $v \geq c \geq v/N$. Let, for all c,*

$$\widetilde{\theta} = \left\{ \theta \in [\underline{\theta}, \overline{\theta}] : \frac{v}{N} \frac{1 - F(\theta)^N}{1 - F(\theta)} = c. \right\} \tag{3}$$

Then, for all $i = 1, ..., N$

$$\sigma_i^*(\theta_i, \widetilde{\theta}, c) = \begin{cases} 1 & \text{if } \theta_i \geq \widetilde{\theta}, \\ 0 & \text{if } \theta_i < \widetilde{\theta}. \end{cases}$$

The value $\widetilde{\theta}$ determined by (3) is unique.

Proof. Given the proposed equilibrium strategies, spectator i makes a phone call if and only if

$$U_i(\sigma_i^*(\theta_i), \sigma_{-i}^*(\theta_{-i}), \widetilde{\theta}, c, \theta_i | \theta_i \geq \widetilde{\theta}) =$$
$$\sum_{j=0}^{N-1} \binom{N-1}{j} (1 - F(\widetilde{\theta}))^j F(\widetilde{\theta})^{N-j-1} \frac{v}{j+1} - c \geq 0. \tag{4}$$

Let $q = F(\widetilde{\theta})$ and $x = (1-q)/q$. Then, if $\sigma_i^*(\theta_i|\theta_i \geq \widetilde{\theta}) = 1$,

$$U_i = \sum_{j=0}^{N-1} \binom{N-1}{j} q^{N-1} x^j \frac{v}{j+1} - c.$$

Noting that[7]

$$\sum_{j=0}^{N-1} \binom{N-1}{j} q^{N-1} x^j \frac{v}{j+1} = \frac{\left(1+\frac{1-q}{q}\right)^N - 1}{N\left(\frac{1-q}{q}\right)} q^{N-1} = \frac{1-q^N}{1-q},$$

we obtain

$$U_i = \frac{v}{N}\frac{1-F(\widetilde{\theta})^N}{1-F(\widetilde{\theta})} - c.$$

Since $U_i = v/N - c$ for $\widetilde{\theta} = \underline{\theta}$ and $U_i = v - c$ for $\widetilde{\theta} = \overline{\theta}$ and since U_i is continuous in $\widetilde{\theta}$ there exists at least one $\widetilde{\theta}$ such that phoning is a best respone.

To show uniqueness, we need the expression

$$\frac{1-q^N}{1-q}$$

strictly increasing in $\widetilde{\theta}$. The derivative of this with respect to $\widetilde{\theta}$ is strictly positive if

$$\frac{-Nq^{N-1}(1-q)f(\widetilde{\theta}) + (1-q)^N f(\widetilde{\theta})}{(1-q)^2} > 0$$

or

$$q^N + Nq^{N-1}(1-q) < 1. \tag{5}$$

The proof is now by induction on N. For $N = 2$, (5) holds. Then,

$$q^{N+1} + (N+1)q^N(1-q) = q(q^N + (N+1)q^{N-1}(1-q).$$

Using the induction hypothesis,

$$q(q^N + Nq^{N-1}(1-q) + q^{N-1}(1-q)) < q(1 + q^{N-1}(1-q)) < 1.$$

If $\theta_i < \widetilde{\theta}$ it is always a best response not to call, $\sigma_i^*(\theta_i|\theta_i < \widetilde{\theta}) = 0$, since the expected benefit is zero: even if i gets chosen to answer, he is not allocated the prize. Since any caller has to pay c in any case, calling is not worthwile if $\theta_i < \widetilde{\theta}$.

$\square$

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[7] Follows from integrating the identity $\sum_{j=0}^{N-1}\binom{N-1}{j}x^j = (1+x)^{N-1}$ with respect to x.

Corollary 1. *Moreover, $\widetilde{\theta}$ is*

- *increasing in c*
- *decreasing in v*
- *increasing in N.*

Corollary 2. *If negative externalites from full particiption are small ($Nc < v$) then any $\widetilde{\theta}$ induces calls from all spectators with $\theta_i \geq \widetilde{\theta}$.*

Proposition 1 states that only a question which is not as simple as possible ($\widetilde{\theta} > \underline{\theta}$) induces phone calls from all spectators with $\theta_i \geq \widetilde{\theta}$ when negative externalities from full participation are large ($Nc \geq v$). The interpretation of the comparative statics on $\widetilde{\theta}$ is straightforward. Corollary 2 is also easy to show and to interpret. In particular, even a most difficult question (high $\widetilde{\theta}$), induces calls from all spectators.

Next I derive the combination of $(\widetilde{\theta}, c)$ which maximizes the TV channel's revenue for the interesting case $Nc \geq v$. It is convenient to express a spectator's equilibrium condition as a condition on c: if

$$c \leq c(\widetilde{\theta}) := \frac{v}{N} \frac{1 - F(\widetilde{\theta})^N}{1 - F(\widetilde{\theta})}, \tag{6}$$

then all spectators call.

Let θ^* and c^* denote the TV channel's revenue maximizing choices of a question's difficulty and of the price per phone call. I shall assume $v_{TV} = 0$, the following Proposition holds for all $v_{TV} < v$.

Proposition 2. *Suppose $Nc \geq v$ and $v_{TV} = 0$. The TV channel maximizes its revenue at $\theta^* = \underline{\theta}$ and $c^* = v/N$.*

Proof. Given the equilibrium in the game among spectators, the TV channel's maximizes his expected revenue

$$\pi(\widetilde{\theta}, c) \quad = \quad \sum_{i=0}^{N} \binom{N}{i} (1 - F(\widetilde{\theta}))^i F(\widetilde{\theta})^{N-i}\, i \cdot c \tag{7}$$

subject to

$$c \leq c(\widetilde{\theta}) := \frac{v}{N} \frac{1 - F(\widetilde{\theta})^N}{1 - F(\widetilde{\theta})}. \tag{8}$$

Note that the TV channel's revenue is strictly increasing in c. Hence, a spectator's participation constraint (8) binds. Letting $q = F(\widetilde{\theta})$ and $x = (1-q)/q$, rewrite π as

$$\pi = c(\widetilde{\theta}) q^N \sum_{i=0}^{N} \binom{N}{i} x^i \cdot i. \tag{9}$$

Note that $\sum_{i=0}^{N} \binom{N}{i} x^i = (1+x)^N$. Differentiating this identity with respect to x yields

$$\sum_{i=0}^{N} \binom{N}{i} i \cdot x^i = N(1+x)^{N-1} x. \tag{10}$$

Hence the TV channel's expected revenue is

$$\pi = c(\widetilde{\theta}) q^N N \left(1 + \frac{1-q}{q}\right)^{N-1} \frac{1-q}{q}. \tag{11}$$

Plugging in $c(\widetilde{\theta})$ from (8), we obtain

$$\pi = \frac{v}{N} \frac{1-q^N}{1-q} q^N N \left(1 + \frac{1-q}{q}\right)^{N-1} \frac{1-q}{q} - v = v(1-q^N), \tag{12}$$

which is strictly decreasing in $\widetilde{\theta}$. So it is optimal to set $\widetilde{\theta} = \underline{\theta} := \theta^*$. Using this in (8) yields $c^* = v/N$. The TV channel's revenue is then $\pi = 0$ for all N. □

The TV channel faces the following trade off. A low $\widetilde{\theta}$ induces phone calls from all spectators. So the TV channel's expected revenue increases if $\widetilde{\theta}$ decreases. However, since extractable spectators expected utility from calling decreases if all spectators call, the cost for the phone call c decreases if $\widetilde{\theta}$ decreases and has to be reduced appropriately. As it turns out, the first effect dominates the second effect: it is optimal to induce spectator's full participation! The intuition for the result is easily seen from the case with only one spectator: he is willing to spend all his valuation, v to participate. But he participates only if his type θ_i is above $\widetilde{\theta}$. To maximize this probability, it is optimal to set $\widetilde{\theta}$ equal to $\underline{\theta}$. This observation carries over to the case of N spectators. The result matches the above mentioned observations. In TV shows like *Paradies* or *people!* in NeunLive, questions *are* extremely simple. By choosing $\theta^* = \underline{\theta}$, the TV channel basically induces a simple lottery among all spectators where each specatator makes a costly phone call and a winner is chosen randomly among all callers.

As N tends to infinity the probability that no spectator has a type above $\widetilde{\theta}$ gets small and the degree of the question's difficulty plays no role. If $v_{TV} = v$, the degree of difficulty $\widetilde{\theta}$ plays no role.

5 Revenue-equivalent Equilibria

In the last section, I focused on equilibria in the spectator subgame in which all spectators called if they know the answer to a question, that is, if $\theta_i \geq \widetilde{\theta}$. However, one presumably observes many spectators who watch these shows, know the answer to a question and still don't call. I shall now present other equilibria in which some spectators do not call even if they know the answer. In all these equilibria, the TV channel makes the same revenue as in the previous section. I argue also that it is optimal again to ask simple questions, that is, to set $\widetilde{\theta} = \underline{\theta}$.

The argument is simple. Fix a number $M \leq N$ and set $c^* = v/M$ and $\widetilde{\theta} = \underline{\theta}$. Then, all N spectators know the answer. The utility of any of the $N - M$ non-callers is 0. If anyone of these non-callers calls, his utility is $v/(M + 1)$ while the cost is v/M. Hence, he has no incentive do deviate. The TV channels revenue is zero, as in the previous section. Is it worthwile to set $\widetilde{\theta} > \underline{\theta}$? No. In that cases, with probability $(F(\widetilde{\theta}))^N$ nobody knows the answer to the question and hence nobody calls and the channel would make positive revenue from phone calls only with probability $1 - (F(\widetilde{\theta}))^N$. Moreover, consider all these cases where somebody knows the answer to the question. Then the revenue must be less than the revenue in all equivalent cases with the spectators' equilibrium derived in section 4 since not everybody calls even if he knows the answer. But the revenue in section 4 in turn is equal to the revenue for the TV channel given an equilibrium in the game among spectators described in this section, given that $\widetilde{\theta} = \underline{\theta}$.

Clearly, there is a multiplicity of such equilibria which all yield the same payoff from the TV channel. This consideration can be seen as an argument for the equilibrium in the game among spectators in the previous section: this equilibrium was attractve since it clearly divided the set of spectators in a group of people who call (all those who know the answer) and in a group of people who don't call (all those who do not know the answer).

6 Fixed Price for Phone Calls

The equilibria discussed in section 5 are interesting if we assume that c is fixed and cannot be chosen by the TV channel. This could be the case, for example, if the phone company who is also involved in the production of the show, has all the bargaining power in a game which divides the revenues from the calling game between the TV channel and the phone company. Then, for any $c \equiv \overline{c}$, there is an equilibrium in the spectators' subgame, where $M = v/\overline{c}$ spectators call and $N - v/\overline{c}$ spectators do not call. Again, it is optimal for the TV channel to set $\overline{\theta} = \underline{\theta}$, to avoid the case that nobody knows the answer to the question. Hence, the result that simple questions are optimal does not depend on the assumption that the TV channel as the mechanism designer has two instruments at hand.

7 Conclusion and Preference for Difficulty: An Example

I presented a model of an interactive quiz show and used the tools of mechanism design to show that it is optimal to ask very simple questions which we also observe in reality. It is in the TV channel's interest to induce basically a simple lottery among all spectators: by asking the most simple questions, the TV channel basically induces a game of chance among all spectators. Since everybody knows the answer to a question, everybody may phone and it is a pure matter of chance and not of knowledge who wins. Officially, games of chance are forbidden in TV shows. Hence the TV channel sort of overcomes this legal restrictions optimally by asking the most simple question.

As a next step one might ask how robust this result is if the number of spectators is endogenous. After all, the number of spectators also influences the winning probability for a caller. Moreover, revenues from advertisement are influenced by the number of spectators a given show has.

Suppose that there are $N = 2$ individuals, where N is now the *potential* number of spectators.[8] Each individual has opportunity cost k_i for watching the show. The cost k_i is private information and drawn from the uniform distribution on $[0, 1]$. Moreover, θ_i is uniformly distributed on $[0, 1]$ for each spectator i.

Let V_i denote the expected utility an individual has from watching the show. I assume

$$V_i = \widetilde{\theta} + U_i,$$

where U_i is the expected benefit from making the phone call and $\widetilde{\theta}$ is the degree of difficulty of the question. Hence each individual prefers to watch a show with difficult questions. This could be since shows with difficult questions simply are more entertaining, since people have the feeling of learning something, since spectators like solving difficult problems or since it is more entertaining to observe how others deal with difficult questions.

An individual watches the show only if

$$V_i \geq k_i.$$

Given that an individual has become a spectator and watches the show, a spectator calls only if

$$U_i \geq 0.$$

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[8] $N = 2$ is chosen for the sake of exposition.

The timing is as follows.

1. For each i, k_i and θ_i are drawn (independent draws, independent distribution etc.).
2. TV channel announces the degree of difficulty of a question, $\widetilde{\theta}$, and c.
3. Individuals switch on the telly and decide if to watch the show or not.
4. Individuals who watch the show decide if to make the phone call or not. Spectator i does not know if spectator j watches as well.
5. Among all those who call, one spectator i is selected randomly (with equal probability) and θ_i gets known to the presenter.
6. The presenter allocates the prize to spectator i if and only if $\theta_i \geq \widetilde{\theta}$.

I look for an equilibrium in which, given $\widetilde{\theta}$ and c, an individual watches the show if and only if $V_i \geq k_i$ and makes a phone call if and only if $\theta_i \geq \widetilde{\theta}$ and $U_i \geq 0$.

I start with the last stage of the game: let μ_i denote the belief of by i that individual $j \neq i$ watches the show. This belief has to be $\mu_i = \Pr(k_j \leq V_j)$, given the proposed equilibrium. Then, given that $\theta_i \geq \widetilde{\theta}$ and given the proposed equilibrium strategies

$$U_i = \mu_i \left(\frac{v}{2}(1 - \widetilde{\theta}) + v\widetilde{\theta} \right) + (1 - \mu_i)v - c. \tag{13}$$

Given that an individual watches the show already, given the proposed equilibrium behavior of j and given that $\theta_i \geq \widetilde{\theta}$, she calls if and only if

$$\mu_i \left(\frac{v}{2}(1 - \widetilde{\theta}) + v\widetilde{\theta} \right) + (1 - \mu_i)v \geq c.$$

In any equilibrium, suppose that the TV channel sets c such that this inequality holds with equality, hence we have

$$\mu_i \left(\frac{v}{2}(1 - \widetilde{\theta}) + v\widetilde{\theta} \right) + (1 - \mu_i)v = c.$$

Let β denote the equilibrium probability that an individual watches the show. The objective for the TV channel is then

$$\begin{aligned}\pi &= \beta^2(2(1-\widetilde{\theta})\widetilde{\theta}c + (1-\widetilde{\theta})^2 2c) + 2\beta(1-\beta)((1-\widetilde{\theta})c) - v \\ &= 2c\beta(1-\widetilde{\theta}) - v. \qquad (14)\end{aligned}$$

The channel's problem is to maximize this expression with respect to $\widetilde{\theta}$ and c subject to

$$\mu_i\left(\frac{v}{2}(1-\widetilde{\theta}) + v\widetilde{\theta}\right) + (1-\mu_i)v = c, \qquad i = 1, 2.$$

Since the constraint binds, each individual watches if and only if $V_i = \widetilde{\theta} \geq k_i$ or if $k_i \leq \widetilde{\theta}$ which happens, in case of the uniform distribution of k_i, with probability $\widetilde{\theta}$. Hence, $\beta = \widetilde{\theta}$ and $\mu_i = \Pr(k_i \leq \widetilde{\theta}) = \widetilde{\theta}$. Using this in the constraint and in the TV channel's objective (14), one obtains $\theta^* = 1/2$ as the unique real solution[9] to the foc's and $c^* = 7/8v$.

Hence, if the number of spectators is endogenous and if spectators prefer difficult questions (this makes watching such shows more interesting) the most simple questions are not optimal anymore. Suppose that in addition to revenues from phone calls the TV channel also makes revenues from advertisement and that these revenues depend on the number of spectators. Then, given the result of this section, it might be optimal to choose θ^* even higher.

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[9] There are also two complex solutions.