No Logo? Global and Local Players, and the Contest for Managerial Skills*

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Abstract

This paper develops a simple general equilibrium, multi-market model in which firms set up head departments by hiring managers from a heterogeneous distribution of skills in order to create and position a trademark. Despite the ex ante symmetry of potential entrants into the economy, the endogenous sorting process of managers into head departments typically generates asymmetric free-entry equilibria with a non-degenerate distribution of profits. Compared to their lower-profit rivals, high-profit firms are shown to have typically (i) a higher valuation of its trademark by consumers, (ii) a wider range of markets in which they operate, and (iii) a wider range of product varieties.

Key words: Asymmetric equilibrium; Endogenous sunk costs; Managerial skills; Market power; Multi-product firms; Trademark.

JEL classification: F23; L10; L13.

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1 Introduction

Firms operating at more than one market location (e.g. multinationals) are networks which are often characterized by a "logo", "label", "brand" or "trademark", respectively.[1] A trademark serves for consumers as signal for the quality of a firm's product(s) wherever its products are available. From the point of view of a firm, it thus is a *common asset*. Prominent examples include firms like McDonalds, Nike, Microsoft, Douglas, H&M, BMW, Nestlé, etc. It is important to note, however, that in contrast to these examples many brands are available only locally, say, within a single country or a region. In fact, to understand the coexistence of "global players" and only locally operating firms as equilibrium phenomena under free entry into the economy is one goal of this paper.

The following questions are examined. First, what determines the total number of trademarks in the economy and the distribution of profits among firms? Second, why do we observe networks operating "globally" (i.e. in many markets) and others operating only "locally" (i.e. in a few markets)? Third, is there a relationship between the number of varieties supplied by a single firm and the valuation of its trademark by consumers? So far, surprisingly little has been said on these issues from a general equilibrium point of view.[2]

To answer these questions, this paper develops a model in which a network is characterized by three endogenous dimensions: the valuation of its trademark by consumers, the number (or range) of markets in which it is active (e.g. has branches or sells products, respectively), and its respective the number of varieties

[1]There is a huge literature on multinational enterprises, focusing on a wide range of aspects which are different to the present paper. For instance, these include foreign direct investment (e.g. Schnitzer, 1999; Konrad and Lommerud, 2001), transfer-pricing (e.g. Schjelderup and Weichenrieder, 1999; Holmstrom and Tirole, 1991), technology choice (e.g. Norback, 2001), ownership issues (e.g. Katrak, 1983), tax competition (Kehoe, 1989; Huizinga and Nielsen, 1997), and general equilibrium trade effects (e.g. Helpman, 1985; Helpman and Krugman, 1985; for a recent survey, see Markusen and Markus, 2001).

[2]A notable exception is a very recent paper by Falkinger (2002), who stresses informational capacity constraints on the individual level and the competition for attention as determinants of the total number of logos in the economy. Related aspects are analyzed in Grossman and Shapiro (1988) and Grossmann (2001), among others.

offered in these markets. Varieties are horizontally differentiated and imperfectly substitutable.

Each network has a *head department* which consists of *managers* who create and position a trademark. This formalizes a key role of managers, namely to enhance perceived quality and thus the demand for their firm's products. Managerial skills in the population are heterogenous. Moreover, for a given size of a firm's head department, it is the *average* skill level of managers employed by a firm which determines the valuation of its trademark by consumers.

There are two sectors in the economy, a traditional sector with a constant returns to scale technology and a modern sector which consists of networks. Networks can freely enter the economy, but have to incur sunk cost for wages paid to managers in their head department. Conditional on the implied consumers' valuation of a firm's trademark, each firm then (i) chooses its set of market locations in which to operate, (ii) decides how many varieties to supply in each of these markets, and (iii) are engaged in product market (Cournot) competition. This process is formalized as a non-cooperative extensive-form game with simultaneous decisions at any of these three stages.

Selling varieties in a particular market (location) has set up costs for firms, e.g. for establishing a branch, for marketing activities, and costs associated with red tape. These set up cost determine the number of markets in which a single firm offers products and thus its range. Quite plausibly, I find that networks with a higher-valued trademark typically choose to be active in more markets (i.e. are more "globalized") than firms with a less successful logo, offer at least as much varieties in any market they are active and have higher profits.

In fact, it is shown that the valuation of trademarks by consumers and thus profits of firms are typically asymmetric in equilibrium, despite complete symmetry of potential entrants in the economy. This result is shown to be a direct implication of the heterogeneity of managerial skills. Moreover, because perfect competition in the labor market drives up managerial wages until net profits of all firms are zero, the resulting distribution of profits among networks in turn determines the distribution of managerial wages.

The theoretical innovation of this paper is to combine the approach of Sutton (1991, 1998) to endogenize sunk costs for demand-enhancing activities with the recent literature on endogenous sorting of heterogenous workers into firms (Kremer, 1993; Saint-Paul, 2001; Kremer and Maskin, 2002). By hypothesizing that managerial skills are related to product demand and thus profits of firms, this combination provides a novel theory about the determinants of the number of firms and the distribution of profits in the economy under free entry.

The existing literature on these issues can be subdivided into two broad classes. In the first class of models, symmetric firms simultaneously enter a market as long as profits cover some exogenous entry costs, leading to symmetric equilibria.[3] The second class of models hypothesizes some initial asymmetry among firms, e.g. sequential entry or some heterogeneity in demand or cost functions. In contrast, this paper allows for endogenous sunk investments after entry of firms into the economy. Unlike in Sutton (1991, 1998), this is done by taking into account both an aggregate constraint and heterogeneity of the invested factor.

The present analysis is also related to Athey and Schmutzler (2001), who analyze the evolution of market power of asymmetric firms which can incur profit-enhancing investments. However, the main point of the present paper is considerably different. Athey and Schmutzler (2001) identify conditions for increasing market dominance with a fixed number of ex ante asymmetric firms in a partial equilibrium context. In contrast, this paper identifies conditions for an ex post asymmetric distribution of profits with an endogenous number of firms by taking a general equilibrium perspective.

The paper is organized as follows. In section 2, the model is presented. Section 3 derives the subgame perfect equilibrium of the extensive-form game in the modern sector, conditional on the distribution of consumers' valuation of trademarks. Section 4 analyzes the general equilibrium by considering free entry of firms and labor market competition for managerial skills. The last section concludes.

[3]For instance, see the well-known "love of variety" models of monopolistic comptition (Spence, 1976; Dixit and Stiglitz, 1977), the "ideal variety" approach (Lancaster, 1979; Helpman, 1981) or "transportation cost" models (Salop, 1979).

2 The Model

There is a unit mass of consumers/workers, who differ in their "managerial skill" level h (for creating and positioning a trademark) which is distributed over the interval $H = [h^0, h^1]$, where $h^1 > h^0 \geq 0$. Its measure is denoted by μ and has full support. However, with respect to all non-managerial tasks, labor is homogenous. The labor market is perfectly competitive. Consumers are uniformly distributed at "markets", indexed m, which are represented as points in the unit interval $[0, 1]$. They are immobile in the sense that they can consume only in the market where they are located.

There are two sectors in the economy. On the one hand, there is a traditional sector producing a homogenous consumption good with a constant-returns to scale technology, using labor as only input. This good is chosen as numeraire and labor productivity is normalized to one. Thus, the wage rate for each unit of non-managerial labor is equal to one as well. On the other hand, there is a modern sector, producing a horizontally differentiated good. For simplicity, marginal costs of producing varieties are assumed to equal zero. However, as explained in more detail later, there are endogenous sunk cost which firms have to incur to ensure positive demand for their product(s).

There is free entry into the economy and the (endogenously determined) number of firms in the modern economy is denoted by T. Define $\mathcal{T} = \{1, ..., T\}$ as the set of firms (trademarks) in the economy. A firm is said to be "active" in a market m, if it offers at least one variety to consumers at m. Let M_i be the set of markets in which firm $i \in \mathcal{T}$ is active. Let $n_i(m)$, K_m and N_m denote the number of products supplied by firm i in market m, the total number of available varieties in market m, and the number of firms which are active in market m, respectively. Moreover, define the sets $I_m = \{1, ..., n_i(m)\}$, $\mathcal{K}_m = \{1, ..., K_m\}$ and $\mathcal{N}_m = \{1, ..., N_m\}$.

The inverse demand function in market m for variety k is given by

$$p_k(m) = A_k(m) - \beta x_k(m) - \gamma \sum_{l \in \mathcal{K}_m \setminus k} x_l(m), \ \ \beta > \gamma, \tag{1}$$

where $A_k(m)$ is an index for the (perceived) quality and $x_k(m)$ denotes the quantity of product $k \in \mathcal{K}_m$ in market m, respectively.[4]

Creating and positioning a trademark involve the creation of customer services attached to the products, the invention of marketing strategies, improvement of delivery channels, product design activities, and other related tasks which take place *prior* to product market competition. These tasks are summarized under the term *managerial activities* (Bresnahan, 1999). Managers form the head department of a firm and are hired from the interval H. The head department of each firm consists of an exogenous mass s of workers.[5] Let H_i denote the subset of H which contains the skill levels of the mass s employed in the head department of firm $i \in \mathcal{T}$. Moreover, assume that the average managerial skill level $\bar{h}_i$ of each firm $i \in \mathcal{T}$ determines the perceived quality of *all* products $k \in I_m$ supplied in *any* market $m \in M_i$ it is active. Formally, for all $i \in \mathcal{T}$ one can write

$$A_k(m) = A_i(m) \text{ for all } k \in I_m \tag{2}$$

with

$$A_i(m) = \begin{cases} \alpha(\bar{h}_i) \equiv \alpha_i \text{ if } m \in M_i, \\ 0 \text{ if } m \notin M_i, \end{cases} \tag{3}$$

where $\alpha(\cdot)$ is assumed to be a twice differentiable and strictly monotonic increasing function with $\alpha(h_0) = 0$. For later use, define the vectors $\boldsymbol{\alpha} = (\alpha_1, ..., \alpha_T)$ and $\mathbf{A}(m) = (A_1(m), ..., A_T(m))$, $m \in [0, 1]$. Because typically $N_m \leq T$ will hold (since each firm turns out to be active in only a subset of markets), $\mathbf{A}(m)$ contains $T - N_m$ zeros.

Remark 1. Functions like $\alpha(\cdot)$ (given a fixed mass of workers) have been taken as *production functions* of homogenous consumption goods by Sattinger (1980) and Saint-Paul (2001). Saint-Paul (2001) allows for the case in which $\alpha''(\cdot) > 0$, which

[4] That is, consumers have identical, quasi-linear preferences which are represented by the utility function (suppressing m) $U = \sum_{k \in \mathcal{K}} \left(A_k x_k - (\beta/2)x_k^2\right) - \gamma \sum_k \sum_{l<k} x_k x_l + Y$, where Y is the quantity of the numeriare commodity.

[5] In fact, the crucial assumption in the model is that the valuation of a trademark by consumers is not necessarily increasing in the number of workers employed in a firm's head department, all other things equal. For instance, this can be justified by internal coordination problems which imply that a network's head department has an optimal size. For simplicity, this size is taken as exogenous here.

he justifies by positive spillover effects among workers within a firm.[6] In contrast, in the present paper $\alpha(\cdot)$ is associated with the perceived quality of a firm's varieties, rather than output. As shown below, product market competition generally leads to *profit functions* which are non-concave as function of the average managerial skill $\bar{h}_i$ of a network, even if $\alpha(\cdot)$ were a (strictly) concave function.

Denote q_i as the number (and thus the share) of markets in the economy in which firm i is active. For simplicity, let the (non-managerial) labor requirement for firm i to enter markets only depend on q_i, denoted by $Q(q_i)$. That is, set up costs for entering a single market are independent of its location. Neither does the location of a firm's head department play any role. Examples of set up costs to enter market locations include labor services for opening up a branch, introducing a marketing campaign at a location and costs associated with red tape. Assume that $Q(\cdot)$ is a strictly monotonic increasing and strictly convex function with $Q(0) = 0$. A strictly convex shape of $Q(\cdot)$ may be justified by logistic problems of networks of coordinating and governing single branches, i.e. by diseconomies of scale.[7]

For each firm $i \in \mathcal{T}$, to introduce $n_i(m)$ varieties in *any* single market $m \in M_i$ requires an amount $C(n_i(m))$ of non-managerial labor,[8] where $C(\cdot)$ is a strictly increasing and convex function with $C(1) > 0$. Note that, like the function $\alpha(\cdot)$, both $Q(\cdot)$ and $C(\cdot)$ are identical for all potential entrants, implying a completely symmetric situation *ex ante*. For later use, define the vector $\mathbf{n}(m) = (n_1(m), ..., n_T(m))$ of product numbers supplied by firms $i \in \mathcal{T}$ in market m which, again, contains $T - N_m$ zeros. (Note that $n_i(m) = 0$ if $m \notin M_i$.)

To summarize the model so far, a trademark is defined as a common asset for a firm in any market it operates, according to (2) and (3). Each firm $i \in \mathcal{T}$ is a network which is characterized by (i) its trademark (i.e. its α_i), (ii) the range of

[6] Models with non-concave production functions and and exogenous firm sizes have also been analyzed by Kremer (1993) and Kremer and Maskin (2002).

[7] See Remark 7 for an alternative specification in which $Q(\cdot)$ is linear.

[8] For instance, $C(\cdot)$ includes administrative and marketing costs at single markets. However, there are no economies of scope in introducing varieties, neither within nor across markets. This allows to compare my framework with existing models of multi-product firms (see Remarks 2 and 7 below), but is admittedly not entirely realistic.

markets it operates (i.e. its q_i or M_i, respectively), and (iii) the number of products supplied in each market (i.e. its $n_i(m)$ or I_m, respectively, $m \in M_i$).

The timing of events in the modern sector evolves according to the following five stages, with decisions at each stage made non-cooperatively and simultaneously by firms.

- At stage 0, firms freely enter the economy, i.e. the set $\mathcal{T}$ is determined.
- At stage 1, firms create and position a logo by hiring managers from the set H, i.e. each firm $i \in \mathcal{T}$ chooses a set H_i. Wage costs for managers are sunk at later stages.[9]
- At stage 2, firms decide at which markets to be active, i.e. each network $i \in \mathcal{T}$ chooses a set M_i. In turn, this determines the sets $\mathcal{N}_m$, i.e. the number of active firms in each market $m \in [0, 1]$.
- At stage 3, firms decide how many products to offer in each market they are active, i.e. each firm $i \in \mathcal{T}$ chooses $n_i(m)$, i.e. a set I_m for any $m \in M_i$.
- At stage 4, firms sell products under Cournot competition, i.e. each firm $i \in \mathcal{T}$ chooses quantities $x_k(m)$ for any $k \in I_m$ and for any $m \in M_i$.

Let's define an extensive-form game Ψ as consisting of stages 2-4, which is analyzed first.

Remark 2. Stages 3 and 4 follow the existing literature on *multi-product* firms (e.g. Anderson and de Palma, 1992; Ottaviano and Thisse, 1999). However, in this literature entry at only one market is considered. In the model developed here, stage 2 substitutes the usual entry stage in this literature. It thus adds two dimensions (or endogenous variables, respectively) by which firms are characterized (before product market competition). That is, besides their range of products, each firm i is characterized by its range of markets q_i and an index α_i, which indicates the perceived quality of its products.

[9] For a general account of the theory of endogenous sunk costs, see Sutton (1991, 1998).

3 Equilibrium Analysis of Game Ψ

In this section, the subgame perfect equilibrium of the extensive-form game Ψ is derived by backwards induction. Note that the decisions of a firm $i \in \mathcal{T}$ at stages 3 and 4, respectively, are taken at each single market $m \in M_i$, i.e. provided that $i \in \mathcal{N}_m$. In order to save notation, the index m is thus suppressed in the analysis of these last two stages (i.e. in the following two subsections).

3.1 Cournot Competition (Stage 4)

Consider the decision problem of firms at stage 4. The revenue of firm i in this final subgame of Ψ (in any market it is active) is given by $R_i = \sum_{k \in I} p_k x_k$ (which equals its profit at stage 4 due to the absence of production costs). Thus, taking the output of each variety by its rival firms as given, each firm $i \in \mathcal{N}$ solves

$$\max_{x_k \geq 0, k \in I} \left\{ \sum_{k \in I} \left(A_i - \beta x_k - \gamma \sum_{l \in K \setminus k} x_l \right) x_k \right\}, \tag{4}$$

according to (1) and (2). The following first result is obtained.

Lemma 1. *In Cournot-Nash equilibrium in the final subgame of the extensive-form game Ψ each firm $i \in \mathcal{N}$ produces the same output level $\tilde{x}_k \equiv \tilde{x}_i$ for all $k \in I$ with*

$$\tilde{x}_i(\mathbf{n}, \mathbf{A}) = \frac{\Lambda_i}{\left(1 + \sum_{j \in \mathcal{N}} \Gamma_j\right)(2(\beta - \gamma) + \gamma n_i)} \tag{5}$$

and earns revenue

$$\tilde{R}_i(\mathbf{n}, \mathbf{A}) = n_i(\beta - \gamma + \gamma n_i)\tilde{x}_i(\mathbf{n}, \mathbf{A})^2, \tag{6}$$

where $\Gamma_j \equiv \frac{\gamma n_j}{2(\beta-\gamma)+\gamma n_j}$ *and* $\Lambda_i \equiv A_i \left(1 + \sum_{j \in \mathcal{N} \setminus i} \Gamma_j\right) - \sum_{j \in \mathcal{N} \setminus i} A_j \Gamma_j$.

Proof. See appendix. ■

Intuitively, due to the symmetry of the varieties in the demand function (1), a multi-product firm $i \in \mathcal{N}$ produces equal output levels $\tilde{x}_i$ for all n_i products it offers in the considered market.

For the sake of convenience, the n_is are treated as continuous variables on the domain $[1, \bar{n}]$, where $\bar{n} < \infty$ is some arbitrarily high constant. The following corollary will prove particularly helpful for the (comparative static) analysis of the next subsection, in which the firms' choice of their number of varieties is considered.

Corollary 1. *For all $i, j \in \mathcal{N}$, $i \neq j$, (i)* $\frac{\partial \tilde{R}_i(\mathbf{n},\mathbf{A})}{\partial n_i} > 0$ *and* $\frac{\partial^2 \tilde{R}_i(\mathbf{n},\mathbf{A})}{\partial n_i^2} < 0$*; (ii)* $\frac{\partial \tilde{R}_i(\mathbf{n},\mathbf{A})}{\partial A_i} > 0$ *and* $\frac{\partial \tilde{R}_i(\mathbf{n},\mathbf{A})}{\partial A_j} < 0$*; (iii)* $\frac{\partial^2 \tilde{R}_i(\mathbf{n},\mathbf{A})}{\partial n_i \partial A_i} > 0$ *and* $\frac{\partial^2 \tilde{R}_i(\mathbf{n},\mathbf{A})}{\partial n_i \partial A_j} < 0$*; (iv)* $\frac{\partial \tilde{R}_i(\mathbf{n},\mathbf{A})}{\partial n_j} < 0$*. (v) Moreover, let $(\hat{\mathbf{n}}, \hat{\mathbf{A}})$ be such that $(\hat{n}_i, \hat{A}_i) = (n_j, A_j)$ and $(\hat{n}_j, \hat{A}_j) = (n_i, A_i)$ for any pair $i, j \in \mathcal{N}$, $i \neq j$, and $(\hat{n}_z, \hat{A}_z) = (n_z, A_z)$ for all $z \in \mathcal{N} \backslash \{i, j\}$. Then $\tilde{R}_i(\hat{\mathbf{n}}, \hat{\mathbf{A}}) = \tilde{R}_j(\mathbf{n}, \mathbf{A})$ and $\tilde{R}_z(\hat{\mathbf{n}}, \hat{\mathbf{A}}) = \tilde{R}_z(\mathbf{n}, \mathbf{A})$ for all $z \in \mathcal{N} \backslash \{i, j\}$.*

Proof. See appendix. ■

Part (i) of Corollary 1 states that the revenue $\tilde{R}_i$ of each firm i at stage 4 is strictly increasing and strictly concave as function of its number of varieties n_i offered in the considered market. Part (ii) says that, from the perspective of a single firm i, $\tilde{R}_i$ increases with the valuation of its own trademark A_i, but decreases with the perceived quality of other firm's varieties A_j, $j \neq i$, holding the number of products supplied by each firm constant. According to part (iii), the incentive of firm i to introduce additional varieties increases with A_i, but decreases with A_j, $j \neq i$, all other things equal. Part (iv) means that a firm's revenue in the considered market declines if any other firm offers additional products. Part (v) will play a particular role for the comparative static analysis of the next subsection. It states that if two firms i and j would "exchange" their situations (i.e. exchange both the consumers' valuation of logos, A_i and A_j, and their product number, n_i and n_j), they would exchange their revenue, $\tilde{R}_i$ and $\tilde{R}_j$, as well, but would not affect revenues of any other firm than i and j.

3.2 Firms' Choice of Number of Products (Stage 3)

Note that in the limit $\gamma \to \beta$, i.e. varieties are perfect substitutes, the limiting revenue function of firm $i \in \mathcal{N}$ in the considered market is given by

$$\lim_{\gamma \to \beta} \tilde{R}_i(\mathbf{n}, \mathbf{A}) = \frac{\left(N A_i - \sum\limits_{j \in \mathcal{N} \backslash i} A_j \right)^2}{\gamma (1+N)^2}, \tag{7}$$

according to lemma 1. (Remember that N denotes the number of active firms in the considered market.) Obviously, due to the positive (wage) costs to introduce an additional variety, each firm does only supply one variety in each market in this limit case. From this one can also conclude that, if products are close - albeit imperfect - substitutes (i.e. $\beta - \gamma$ is close to zero), (some) firms may choose to offer only one variety (e.g. Sutton, 1998, ch. 2). However, such "fragmented" equilibria are rarely observed in reality.

Because in the case with imperfect substitutes it may be optimal for firms to introduce more than one variety into a market, conceptually one *has* to allow for a stage in which firms choose the number of products in any market they are active. The equilibrium actions at this stage 3 are derived next.

The profit maximization problem for each firm $i \in \mathcal{N}$ at stage 3 is given by

$$\max_{n_i \in [1,\bar{n}]} \left\{ \tilde{R}_i(\mathbf{n}, \mathbf{A}) - C(n_i) \right\}. \tag{8}$$

Denote the equilibrium number of varieties offered by firm $i \in \mathcal{N}$ by $\tilde{n}_i(\mathbf{A})$ and let $\tilde{\mathbf{n}}(\mathbf{A}) = (\tilde{n}_1(\mathbf{A}), ..., \tilde{n}_T(\mathbf{A}))$, with $\tilde{n}_i(\mathbf{A}) = 0$ if firm $i \in \mathcal{T}$ is not active in the considered market. To simplify things, I focus on the case with an interior solution of (8). According to the discussion above, this is ensured if varieties are not too close substitutes, i.e. if $\beta - \gamma$ is sufficiently large. In this case, $\tilde{\mathbf{n}}(\mathbf{A})$ is given by the following set of first-order conditions:[10]

$$\frac{\partial \tilde{R}_i(\tilde{\mathbf{n}}(\mathbf{A}), \mathbf{A})}{\partial n_i} = C'(\tilde{n}_i(\mathbf{A})), \; i \in \mathcal{N}. \tag{9}$$

The corresponding profit function $\pi_i(\mathbf{A})$ of each firm $i \in \mathcal{N}$ at stage 3 is given by

$$\pi_i(\mathbf{A}) = \tilde{R}_i(\tilde{\mathbf{n}}(\mathbf{A}), \mathbf{A}) - C(\tilde{n}_i(\mathbf{A})). \tag{10}$$

How does the equilibrium number of products $\tilde{n}_i$ of a firm i in the considered market depend on the consumers' valuation A_i of its own logo and the perceived quality of other firms' varieties A_j, $j \neq i$? Define the vector $\bar{\mathbf{A}}$ with the components

[10] Note that $\tilde{R}_i(\mathbf{n}, \mathbf{A}) - C(n_i)$ is strictly concave as function of n_i for all $i \in \mathcal{N}$, according to Corollary 1 (i) and the convexity of $C(\cdot)$. This does not only imply that "reaction functions" exist, but also ensures the existence of an equilibrium, according to e.g. Friedman (1977; Theorem 7.1). Note that for a single decision variable as it is the case here, this presumes that strategy sets of each player $i \in \mathcal{N}$ are compact subsets of $\Re$. This is why an upper bound $\bar{n} < \infty$ has to be introduced.

$A_i = \bar{A}$ for all $i \in \mathcal{N}$ and $A_i = 0$ for all $i \notin \mathcal{N}$. (This corresponds to the symmetric case with $\alpha_i = \bar{\alpha}$ for all $i \in \mathcal{T}$, according to (3).) Using the implicit function theorem implies

$$\frac{\partial \tilde{n}_i}{\partial \bar{A}} = -\frac{N \frac{\partial^2 \tilde{R}_i}{\partial n_i \partial \bar{A}}}{\frac{\partial^2 \tilde{R}_i}{\partial n_i^2} + \sum\limits_{j \in \mathcal{N} \setminus i} \frac{\partial^2 \tilde{R}_i}{\partial n_i \partial n_j} - C''(\cdot)}, \tag{11}$$

for all $i \in \mathcal{N}$, according to (9) with $\mathbf{A} = \bar{\mathbf{A}}$. It can be shown that $\partial^2 \tilde{R}_i / (\partial n_i \partial n_j) < 0$ for all $i, j \in \mathcal{N}$, $i \neq j$, in this symmetric case (i.e. n_i and n_j are strategic substitutes). Thus, parts (i) and (iii) of Corollary 1 imply $\partial \tilde{n}_i / \partial \bar{A} > 0$, according to (11). However, in the present context, symmetric equilibria are not particularly interesting since they typically do not prevail, as will be shown below. But comparative static analysis of asymmetric equilibria can be very messy and requires very strong assumptions if the implicit function theorem is applied (e.g. Samuelson, 1947; Takayama, 1985; Dixit, 1986).

Fortunately, in the present context there is an easy way to overcome this problem, since it allows to apply a result recently derived by Athey and Schmutzler (2001; Theorem 1), which relies on the analysis of supermodular games (Topkins, 1979). However, to do so requires to restrict attention to the case where the number of varieties are *strategic substitutes* among firms. It can be shown that n_i and n_j, $i, j \in \mathcal{N}$, $i \neq j$, are indeed strategic substitutes if $A_i - A_j$ is not too positive (for a given A_j). That is, in the following I focus on the case where firms are not too heterogenous.

Moreover, although one does not have to assume the existence of a unique equilibrium as often done in the literature, I exclusively focus on cases in which the set of equilibrium actions at stage 3 satisfy "conditional uniqueness" (Athey and Schmutzler, 2001). In the present context, this means the following.[11]

Definition 1. (Athey and Schmutzler, 2001). The set of equilibrium actions at

[11] A sufficient condition for conditional uniqueness of equilibria is $\left|\frac{\partial^2 \tilde{R}_i}{\partial n_i^2}\right| > \left|\frac{\partial^2 \tilde{R}_i}{\partial n_i \partial n_j}\right|$ for all $i, j \in \mathcal{N}$, $i \neq j$. Note that in the case $N > 2$, the latter inequality is much weaker than the usual "dominant-diagonal" conditions $\left|\frac{\partial^2 \tilde{R}_i}{\partial n_i^2}\right| > \sum\limits_{j \in \mathcal{N} \setminus i} \left|\frac{\partial^2 \tilde{R}_i}{\partial n_i \partial n_j}\right|$ for all $i \in \mathcal{N}$, which are often assumed in the context of non-cooperative games with globally concave objective functions. (The latter ensures uniqueness of equilibrium.)

stage 3 satisfies conditional uniqueness if for each $i, j \in \mathcal{N}$ and each $\mathbf{A}$, if $A_i \neq A_j$ and there exist two vectors of equilibrium actions $\tilde{\mathbf{n}}(\mathbf{A})$ and $\tilde{\mathbf{n}}^*(\mathbf{A})$ which fulfill $\tilde{n}_z(\mathbf{A}) = \tilde{n}_z^*(\mathbf{A})$ for all $z \in \mathcal{N} \backslash \{i, j\}$, then $\tilde{n}_i(\mathbf{A}) = \tilde{n}_i^*(\mathbf{A})$ and $\tilde{n}_j(\mathbf{A}) = \tilde{n}_j^*(\mathbf{A})$.

In sum, the following is maintained for the subsequent analysis (for all markets $m \in [0, 1]$).

Condition 1. For all $i, j \in \mathcal{N}$, $i \neq j$, $\frac{\partial^2 \tilde{R}_i(\mathbf{n}, \mathbf{A})}{\partial n_i \partial n_j} \leq 0$, and the set of equilibrium actions at stage 3 satisfies conditional uniqueness.

Lemma 2. *Under Condition 1, for all $i, j \in \mathcal{N}$, if $A_i > A_j$ then $\tilde{n}_i(\mathbf{A}) \geq \tilde{n}_j(\mathbf{A})$.*

Proof. The result is proven by applying Theorem 1 of Athey and Schmutzler (2001). Part (iii) of Corollary 1 implies that $\tilde{R}_i(\mathbf{n}, \mathbf{A}) - C(n_i)$ is *supermodular* as function of both (n_i, A_i) and $(n_i, -A_j)$, $j \neq i$, respectively. Using this fact together with both part (v) of Corollary 1 and Condition 1 confirms the result. ■

Remark 3. If the n_is are treated as continuous variables (with $n_i \geq 1$), then Lemma 2 implies both $\partial \tilde{n}_i / \partial A_i \geq 0$ and $\partial \tilde{n}_i / \partial A_j \leq 0$ for all $i, j \in \mathcal{N}$, $i \neq j$.[12] However, comparative static analysis with supermodular functions does not require differentiability.[13] Thus, Lemma 2 also holds if one restricts the n_is to positive integers. Although this restriction does not pose problems at this stage, it turns out to be rather inconvenient later on.

The intuition of Lemma 2 can nicely be illustrated. Note that part (v) of Corollary 1 implies that exchanging the roles (or labels, respectively) of two firms does not affect profits of any other player. Athey and Schmutzler (2001) exploit this condition to generalize comparative static results of non-cooperative games which hold in a two-player case to a $N-$player game (under the additional assumptions that

[12] Also note that allowing for a cost function $C(n_i) = c(n_i, A_i)$ does not change the result of Lemma 2 as long as $\frac{\partial^2 \tilde{R}_i(\mathbf{n}, \mathbf{A})}{\partial n_i \partial A_i} \geq \frac{\partial^2 c(n_i, A_i)}{\partial n_i \partial A_i}$ for all $i \in \mathcal{N}$, i.e. as long as $\tilde{R}_i(\mathbf{n}, \mathbf{A}) - c(n_i, A_i)$ is still supermodular as function of both (n_i, A_i) and $(n_i, -A_j)$, $j \neq i$, respectively. For instance, $\frac{\partial^2 c(n_i, A_i)}{\partial n_i \partial A_i} < 0$ can be justified by lower marketing requirements of firm i to introduce an additional variety in the considered market, if trademark i is higher-valued.

[13] A real-valued and smooth function $f(x, y)$ is supermodular if and only if $f_{x,y} > 0$. See e.g. Athey and Schmutzler (2001, Definition 3) for a more general definition of supermodular functions.

the players' actions are strategic substitutes, profit functions are supermodular and conditional uniqueness is satisfied). Also note that in the case with only two firms, conditional uniqueness in Definition 1 is the same as uniqueness of equilibrium. Now consider the two-player case in Fig. 1.

<Figure 1>

With n_1 and n_2 being strategic substitutes, the reaction functions of both firms are negatively sloped.[14] Uniqueness of equilibrium obviously requires that the reaction function of firm 1 is steeper than that of firm 2. According to part (iii) of corollary 1, an increase in, say, A_1 shifts the reaction function of firm 1 rightward and that of firm 2 downward, implying that $\tilde{n}_1$ increases and $\tilde{n}_2$ decreases.[15]

How does the equilibrium revenue $\tilde{R}_i(\tilde{\mathbf{n}}(\mathbf{A}), \mathbf{A})$ of a firm $i \in \mathcal{N}$ change if a rival $j \neq i$ enters the considered market or raises A_j, respectively? Note that this has three effects. First, holding the number of products offered by firms constant, $\tilde{R}_i$ directly decreases with A_j, $j \neq i$, according to part (ii) of Corollary 1. Second, as Lemma 2 implies $\partial \tilde{n}_j / \partial A_j \geq 0$ for all $j \in \mathcal{N}$, $\tilde{R}_i$ also decreases if firm $j \neq i$ raises its number of products $\tilde{n}_j$, according to part (iv) of Corollary 1. Third, however, Lemma 2 also implies $\partial \tilde{n}_z / \partial A_j \leq 0$ for all $z \neq j$. According to this effect, $\tilde{R}_i$ increases if other firms $z \in \mathcal{N} \backslash \{i, j\}$ decrease their product numbers $\tilde{n}_z$ if A_j increases or firm j enters the considered market, respectively. (Of course, the third effect only exists if $N > 2$.) However, it is implausible that this counteracting effect is so strong to (weakly) dominate the first two effects. Hence, I only focus on the case where the following is fulfilled.

Condition 2. For all $i, j \in \mathcal{N}$, $i \neq j$, $\left|\frac{\partial \tilde{R}_i}{\partial A_j}\right| + \left|\frac{\partial \tilde{R}_i}{\partial n_j}\frac{\partial \tilde{n}_j}{\partial A_j}\right| > \sum_{z \in \mathcal{N} \backslash \{i,j\}} \frac{\partial \tilde{R}_i}{\partial n_z}\frac{\partial \tilde{n}_z}{\partial A_j}$.

[14] Fig. 1 is of course a standard picture for a unique Cournot-Nash equilibrium, observing the fact that $n_i \geq 1$ for all i in the context of multi-product firms. Note that the unique equilibrium is also "stable" if reaction functions are interpreted as describing dynamic behavior with alternate-period decisions.

[15] In contrast, if the reaction function of firm 2 would be steeper than that of firm 1, it is easy to check that for an interior equilibrium the opposite would hold. This is why (conditional) uniqueness has to be assumed.

Corollary 2. *Under Conditions 1 and 2, for all $i, j \in \mathcal{N}$, $i \neq j$, $\pi_i(\mathbf{A})$ is strictly increasing in A_i and strictly decreasing in A_j.*

Proof. See appendix. ■

Thus, if firm i has a higher-valued trademark than firm j (i.e. if $A_i > A_j$) then profits $\pi_i(\mathbf{A})$ of firm i in the considered market exceed profits $\pi_j(\mathbf{A})$ of firm j. The next subsection considers the equilibrium actions of the networks $i \in \mathcal{T}$ with respect to the choice of markets in which firms are active.

3.3 Firms' Choice of Markets (Stage 2)

The maximization problem of each firm $i \in \mathcal{T}$ at stage 2, i.e. the choice of the set of markets M_i in which network i chooses to be active, can be written as

$$\max_{M_i \subseteq [0,1]} \left\{ \int_{M_i} \pi_i(\mathbf{A}(m))dm - Q(q_i) \right\}, \tag{12}$$

given the sets M_j, $j \neq i$.[16] Denote $\tilde{M}_i(\boldsymbol{\alpha})$ as the equilibrium set of markets and $\tilde{q}_i(\boldsymbol{\alpha})$ as the corresponding share of markets in which firm $i \in \mathcal{T}$ is active. Note both are functions of $\boldsymbol{\alpha}$, according to (3). The corresponding profits $\Pi_i(\boldsymbol{\alpha})$ for firm i in equilibrium of the extensive-form game Ψ (called "gross profits" hereafter) then equal

$$\Pi_i(\boldsymbol{\alpha}) = \int_{\tilde{M}_i(\boldsymbol{\alpha})} \pi_i(\mathbf{A}(m))dm - Q(\tilde{q}_i(\boldsymbol{\alpha})). \tag{13}$$

Moreover, let $\bar{m}_i(\boldsymbol{\alpha})$ denote the market(s) in which equilibrium profits $\pi_i(\mathbf{A}(m))$ are highest among all markets $m \in \tilde{M}_i(\boldsymbol{\alpha})$ in which firm $i \in \mathcal{T}$ is active in equilibrium, i.e. $\bar{m}_i(\boldsymbol{\alpha}) \in \arg\max_{m \in \tilde{M}_i(\boldsymbol{\alpha})} \{\pi_i(\mathbf{A}(m))\}$. The equilibrium outcome at stage 2 can be characterized as follows.

Lemma 3. *In any set of equilibrium actions at stage 2, for all $i \in \mathcal{T}$ the condition*

$$\pi_i(\mathbf{A}(\bar{m}_i(\boldsymbol{\alpha}))) \geq Q'(\tilde{q}_i(\boldsymbol{\alpha})) \tag{14}$$

must hold.

Proof. Each network $i \in \mathcal{T}$ decides to operate in an additional market m if $\pi_i(\mathbf{A}(m)) \geq Q'(q_i)$ holds. Now suppose $\pi_i(\mathbf{A}(\bar{m}_i)) < Q'(\tilde{q}_i)$. As $\bar{m}_i$ is defined as the

[16] Note that $\pi_i(\mathbf{A}(m)) = 0$ if $m \notin M_i$.

market at which equilibrium profits for firm i are highest and $Q''(\cdot) > 0$, this is a contradiction. ∎

Lemma 3 simply states that the additional set up costs at the "last" market in which a network decides to be active must not exceed the highest profits the network attains across all markets it operates.

Corollary 3. *Under Conditions 1 and 2, for all $i, j \in \mathcal{T}$, $i \neq j$, $\tilde{q}_i(\boldsymbol{\alpha})$ is strictly increasing in α_i and strictly decreasing in α_j.*

Proof. Directly follows from Corollary 2 and Lemma 3. ∎

Thus, firms with a higher-valued trademark are more "globalized". Note that (14) implies an upper bound $\overline{\Pi}_i(\boldsymbol{\alpha})$ for gross profits of a firm $i \in \mathcal{T}$, which is given by[17]

$$\overline{\Pi}_i(\boldsymbol{\alpha}) = \tilde{q}_i(\boldsymbol{\alpha})\pi_i(\mathbf{A}(\bar{m}_i(\boldsymbol{\alpha}))) - Q(\tilde{q}_i(\boldsymbol{\alpha})), \tag{15}$$

according to (13).

Remark 4. If the number of firms T is high and ranges $\tilde{q}_i$ are wide, (14) implies

$$\pi_i(\mathbf{A}(m)) \approx Q'(\tilde{q}_i(\boldsymbol{\alpha})) \text{ for all } m \in M_i, \tag{16}$$

since markets are ex ante identical (also remember (3)).[18] Thus, $\overline{\Pi}_i(\boldsymbol{\alpha}) = \Pi_i(\boldsymbol{\alpha})$ approximately holds, where

$$\Pi_i(\boldsymbol{\alpha}) \approx \tilde{q}_i(\boldsymbol{\alpha})Q'(\tilde{q}_i(\boldsymbol{\alpha})) - Q(\tilde{q}_i(\boldsymbol{\alpha})), \tag{17}$$

according to (15) and (16).

Corollary 4. *Under Conditions 1 and 2, for all $i \in \mathcal{T}$, $i \neq j$, the upper bound $\overline{\Pi}_i(\boldsymbol{\alpha})$ for profits in the extensive-form game Ψ is strictly increasing in α_i and strictly decreasing in α_j.*

[17] Note that, with $Q(0) = 0$, $Q'(\cdot) > 0$ and $Q''(\cdot) > 0$ as assumed, we have $Q'(q) > Q(q)/q$. Using this and (14) implies $\overline{\Pi}_i(\mathbf{A}) > 0$.

[18] In contrast, consider the following example with $T = 3$, $\alpha_1 > \alpha_2 > \alpha_3$ and $\tilde{q}_1 = 1$, $\tilde{q}_2 = 2/3$ and $\tilde{q}_3 = 1/3$. Then, for instance, there may be an equilibrium where in a share 2/3 of markets firms 1 and 2 are active in the same market (but not firm 3) and in the remainder 1/3 of markets firms 1 and 3 are active (but not firm 2). Obviously, in these duopoly situations, firm 1 makes higher profits in markets in which it competes with firm 3 than in the markets in which it competes with firm 2. (But note that in this example at least firms 2 and 3 have similar profits everywhere they are active.)

Proof. See appendix. ■

According to Remark 4, the same result typically holds for gross profits $\Pi_i(\boldsymbol{\alpha})$. To summarize, the analysis so far has shown the following.

Proposition 1. (Equilibrium of game Ψ). *Under Conditions 1 and 2, for all $i, j \in \mathcal{T}$, if firm i has a higher-valued trademark than firm j (i.e. if $\alpha_i > \alpha_j$), then i is active in more markets (i.e. $\tilde{q}_i(\boldsymbol{\alpha}) > \tilde{q}_j(\boldsymbol{\alpha})$), offers at least as much products in each market it operates (i.e. $\tilde{n}_i(\mathbf{A}(m)) \geq \tilde{n}_j(\mathbf{A}(m))$* for all $m \in M_i$), *and typically earns higher gross profits* (i.e. $\Pi_i(\boldsymbol{\alpha}) > \Pi_j(\boldsymbol{\alpha})$) *than j.*

Proof. Directly follows from Lemma 2, Corollary 3 and Corollary 4, respectively. ■

Before proceeding with the general equilibrium analysis (i.e. stages 0 and 1), consider a very simple example, which serves two purposes. First, it demonstrates that typically each gross profit function $\Pi_i(\boldsymbol{\alpha})$ is increasing and at least locally strictly convex as function of α_i. (This will play an important role in the next section.) Second, it illustrates the derivation of an equilibrium of game Ψ.

Example 1. Let $\beta - \gamma$ be so low, i.e. varieties are very good substitutes, such that $\tilde{n}_i(\mathbf{A}(m)) = 1$ for all $m \in M_i$, $i \in \mathcal{T}$. In this case, the revenue function of firm i at stage 4 in any considered market becomes

$$\tilde{R}_i = \beta \left(\frac{A_i(2\beta - \gamma + \gamma N) - \gamma \sum_{j \in \mathcal{N} \setminus i} A_j}{(2\beta - \gamma + \gamma N)(2\beta - \gamma)} \right)^2, \tag{18}$$

according lemma 1, and profits at stage 3 equal $\pi_i = \tilde{R}_i - C(1)$. Thus, π_i is strictly convex as function of A_i. For simplicity, assume (16) and thus (17) exactly hold (i.e. replace "$\approx$" by "$=$"). Thus, applying the implicit function theorem to (16) yields $\partial \tilde{q}_i / \partial \alpha_i = \left(\partial \tilde{R}_i / \partial A_i \right) / Q''(\tilde{q}_i)$, using $A_i(m) = \alpha_i$ if $m \in M_i$, according (3). (Hence, $\partial \tilde{q}_i / \partial \alpha_i > 0$ and, similarly, $\partial \tilde{q}_i / \partial \alpha_j < 0$, which illustrates Corollary 3.) Using this result, (17) implies $\partial \Pi_i / \partial \alpha_i = \tilde{q}_i \left(\partial \tilde{R}_i / \partial A_i \right)$. Thus, $\partial^2 \Pi_i / \partial \alpha_i^2 > 0$, according to (18). Hence, in this example, gross profits $\Pi_i(\boldsymbol{\alpha})$ are not only strictly increasing but also *strictly convex* as function of α_i. For further illustration, now consider the symmetric case with $\alpha_i = \bar{\alpha}$ for all $i \in \mathcal{T}$ and specify $Q(q_i) = q_i^2 / \delta_Q$

and $C(n_i) = n_i/\delta_C$. Thus, for all $i \in \mathcal{N}$,

$$\pi_i = \frac{4\beta^3\bar{\alpha}}{(2\beta - \gamma + \gamma N)^2(2\beta - \gamma)^2} - \frac{1}{\delta_C} \equiv \hat{\pi}(N, \bar{\alpha}, \delta_C) \tag{19}$$

and $\hat{\pi}(N, \bar{\alpha}, \delta_C) = 2\tilde{q}_i/\delta_Q$ in equilibrium of the extensive-form game Ψ, according to (18) and (16), respectively. Thus, $\tilde{q}_i$ is identical for all $i \in \mathcal{T}$, denoted $\bar{q}$. Moreover, note that $N = \bar{q}T$ must hold, where T is given at this stage. Thus, $\bar{q}$ is implicitly given by $\hat{\pi}(\bar{q}T, \bar{\alpha}, \delta_C) - 2\bar{q}/\delta_Q = 0$. (For instance, this implies $\partial\bar{q}/\partial\delta_C > 0$ and $\partial\bar{q}/\partial\delta_Q > 0$.)

4 General Equilibrium

At stage 1, each firm $i \in \mathcal{T}$ maximizes gross profits $\Pi_i\left(\alpha\left(\bar{h}_i\right), \boldsymbol{\alpha}_{-i}\right)$, given $\boldsymbol{\alpha}_{-i} = (\alpha_1, ..., \alpha_{i-1}, \alpha_{i+1}, ..., \alpha_T)$, by choosing a subset $H_i \subset H$ which contains an exogenous mass s of types. (Remember $\alpha_i = \alpha\left(\bar{h}_i\right)$.) At stage 0, the equilibrium number of networks is determined under free entry into the economy. First, the conditions for a general equilibrium are defined.

Definition 2. An equilibrium consists of (i) a set of networks $\mathcal{T}$, (ii) a mapping η from $\mathcal{T} \times [0, s]$ to H, $(i, u) \to h_{iu}$ (assignment of managers into networks), and (iii) a mapping w from H to $\Re_+$ (wage schedule) such that (a) for all $i \in \mathcal{T}$,

$$\Pi_i(\boldsymbol{\alpha}) = \int_0^s w(h_{iu})du, \tag{20}$$

(b) for all mappings from $[0, s]$ to $H : u \to \hat{h}_u$,

$$\Pi_i\left(\alpha\left(\left[\int_0^s \hat{h}_u du\right]/s\right), \boldsymbol{\alpha}_{-i}\right) \leq s \leq \int_0^s w(\hat{h}_u)du, \tag{21}$$

and (c) labor is fully employed at wages $w(h) \geq 1$ for all $h \in H$.

Condition (a) must hold because there is free entry of firms into the economy. First, it implies that net profits are zero in equilibrium. Second, it means that the total wage bill for managers employed in the head department of a firm is equal to its gross profit. This reflects the assumptions that there are no exogenous costs for firms to enter the economy, but endogenous sunk costs for creating and positioning a trademark. Condition (b) means that no potential entrant in the modern sector can make positive profits in equilibrium, whether paying a higher or the same wage than in the traditional sector. Condition (c) says that all workers who are not employed as managers in the modern sector (with T networks, this is a mass of $1-sT$ workers), work in the traditional sector (which is characterized by a completely elastic labor demand curve). Moreover, as no one would work in the modern economy for a wage less than what is obtained in the traditional sector, all wages must be at least as high as unity.

Remark 5. Definition 2 is applied from Saint-Paul (2001) for the present model. In fact, the basic model of Saint-Paul (2001) can be viewed as a special case of the present framework, in which (i) there is no traditional sector, (ii) the extensive-form game Ψ of the present paper is absent, and (iii) $\Pi_i(\boldsymbol{\alpha}) = \alpha_i$ for all i (where α_i is output of a homogenous good in his model).

Proposition 2. (Managerial wage schedule). *For all $i \in \mathcal{T}$, the equilibrium wage schedule in the modern sector is such that*

$$w(h) \geq \frac{\Pi_i(\boldsymbol{\alpha})}{s} + \frac{\partial \Pi_i(\boldsymbol{\alpha})}{\partial \alpha_i} \frac{\alpha'(\bar{h}_i)}{s}(h - \bar{h}_i) \tag{22}$$

for all $h \in H$, whenever the right-hand side of (22) is not below unity, and with strict equality if $h \in H_i$.

Proof. See appendix. ■

Proposition 2 gives equilibrium wages which managers with skill level h obtain in firm i, conditional on the average skill level $\bar{h}_i$ in firm i and the number of logos T in the economy (or, more precisely, the distribution of the α_is). Note that, within each firm $i \in \mathcal{T}$ (i.e. for all $h \in H_i$), the wage schedule is linear. Moreover, managers with an average skill level $\bar{h}_i$ within firm i obtain the average gross profit per manager, i.e. $w(\bar{h}_i) = \Pi_i/s$, according to (22).[19]

[19] However, the overall wage schedule is not necessarily smooth but convex, as Proposition 2 implies that $w(\cdot)$ is the maximum of a set of linear functions (compare Saint-Paul, 2001, Corollary to Proposition 1). Thus, small differences in the individual skill levels may lead to large differences in wages across firms. Also note that the slope of the wage schedule for any type $h \in H_i$ (i.e. for any h employed in firm i) equals the marginal profit with respect to $\bar{h}_i$, divided by s; i.e. $w'(h) = (\partial \Pi_i(\boldsymbol{\alpha})/\partial \alpha_i)\, \alpha'(\bar{h}_i)/s$.

Remark 6. Although the function

$$f(\bar{h}_i) \equiv \Pi_i \left(\alpha \left(\bar{h}_i\right), \boldsymbol{\alpha}_{-i}\right) \tag{23}$$

is generally not concave in $\bar{h}_i$, due to the fixed mass s of managers for all $i \in \mathcal{T}$, one can proceed *as if* the Kuhn-Tucker theorem can be applied (see Saint-Paul, 2001, footnote 11). Moreover, since s is fixed, there are no strategic interactions among networks in determining α_i by hiring managers, even if $f(\bar{h}_i)$ would be strictly concave. That is, every firm tries to end up with a valuation of its trademark as high as possible.

Proposition 3. (Distribution of profits). *The equilibrium can be characterized by a partition of H into adjacent intervals $H_\rho \subset H$, indexed $\varphi \in \Phi$, such that the following holds.*

(i) Generally, the mapping η: $\mathcal{T} \times [0, s]$ to H is such that there exists $\varphi(i) \in \Phi$ with $H_i \subseteq H_{\rho(i)}$ for all $i \in \mathcal{T}$.

(ii) If $f(\bar{h}_i)$ is strictly convex everywhere, then the mapping η is such that the most profitable firm has hired the highest skilled mass s from H, the second-most profitable firm has hired the second-highest skilled mass s from H, and so on (full segregation equilibrium).

(iii) If $f(\bar{h}_i)$ is strictly concave everywhere, then $\bar{h}_i = \bar{h}$ and consequently $\alpha_i = \bar{\alpha}$ for all $i \in \mathcal{T}$ (symmetric equilibrium).

Proof. See appendix. ■

As an implication of Corollary 4, entry of an additional network in the economy (at stage 0) decreases gross profits of all other firms, all other things equal. Entry into the economy is free (e.g. there are no legal restrictions or formal barriers to entry), but firms have to incur sunk costs for establishing a head department (stage 1). These are endogenous in the model and specified as managerial wages, with a perfectly competitive labor market for heterogeneous managerial skills.[20] Firms enter the economy as long as they can hire sufficiently skilled managers to

[20] The general idea to endogenize sunk costs (affecting demand functions) follows Sutton (1991, 1998), who is interested in testable predictions regarding industry structures in a partial equilibrium context. In contrast, the present paper introduces an aggregate resource constraint (i.e. for managerial skills) for the factor which accounts for the sunk costs incurred by firms (at stage 1).

ensure non-negative gross profits, anticipating the situation in the subsequent non-cooperative game Ψ. An equilibrium is reached if the number of logos T (and the associated distribution of the α_is) is such that all firms make zero net profits with the wage schedule derived in Proposition 2. Note that, for this argument, it does not matter whether firms are identical. Higher gross profits of a firm just transmit into a higher average wage per manager in that firm.

In fact, part (i) of Proposition 3 says that the equilibrium distribution of profits is generally asymmetric, as each firm $i \in \mathcal{T}$ hires from just one subinterval $H_{\rho(i)} \subset H$ of the skill distribution. According to Proposition 1, this leads to a situation in which "globally" (i.e. in many markets) operating networks and "locally" operating firms coexist. Moreover, the former typically have higher profits and offer at least as much products under its logo than the latter.

A symmetric equilibrium, i.e. $\alpha_i = \bar{\alpha}$ for all $i \in \mathcal{T}$, occurs if profits $f(\bar{h}_i) = \Pi_i\left(\alpha\left(\bar{h}_i\right), \boldsymbol{\alpha}_{-i}\right)$ are strictly concave as function of $\bar{h}_i$ *everywhere* (part (iii) of Proposition 3). However, in the present context, this is highly implausible. As Example 1 has demonstrated, $\Pi_i\left(\alpha_i, \boldsymbol{\alpha}_{-i}\right)$ may be strictly convex as function of α_i everywhere, i.e. $\alpha(\cdot)$ would have to be a "sufficiently" concave function in this case for $f''(\bar{h}_i) < 0$ to hold everywhere. But in the present context, standard arguments to justify decreasing marginal returns to not apply.

Intuitively, if a marginally better average skill level in a head department has an increasing impact on profits (i.e. if $f(\bar{h}_i)$ is at least locally convex), this provides a force towards segregation in the sense that workers with similar skill levels are matched in a head department.[21] So generally, sorting of managers into head departments takes place in a way that networks hire from just one interval in the skill distribution (part (ii) of Proposition 3). This is depicted in Figure 2 (a)-(c), which shows three possible partitions of H into intervals from which single firms hire, for the case $T = 4$ with $\alpha_1 \geq \alpha_2 \geq \alpha_3 \geq \alpha_4$.

<Figure 2>

Moreover, the present analysis distinguishes between entry into the economy (stage 0) and entry into single markets (stage 2).

[21] Extensive discussions of an analogous argument (but in a different context) can be found in Kremer (1993), Saint-Paul (2001) and Kremer and Maskin (2002).

In panel (a), firms 1 and 4 exclusively hire from two intervals with a mass s of workers, i.e. $H_1 = H_{\varphi(1)}$ and $H_4 = H_{\varphi(4)}$, whereas firms 2 and 3 hire from the same interval, i.e. $H_{\varphi(2)} = H_{\varphi(3)}$. Moreover, note that in any case, no worker employed in the traditional sector can have higher managerial skills than any worker employed in the modern sector. Panel (b) depicts the extreme case of a full segregation equilibrium (part (ii) of Proposition 3), where the highest-skilled mass s in H form the most successful head department, the second-highest mass s are hired in the second-most profitable network, and so on (i.e. $H_i = H_{\varphi(i)}$ for all $i \in \mathcal{T}$ and $\bigcap_{i\in\mathcal{T}} H_{\varphi(i)} = \emptyset$). Another extreme case is when all networks hire from a single interval, which is depicted in panel (c). For instance, this necessarily holds in a symmetric equilibrium (part (iii) of Proposition 3).

Example 1 reconsidered. For further illustration, reconsider Example 1. Note that $\Pi_i = \bar{q}^2/\delta_Q$ with $Q(q_i) = q_i^2/\delta_Q$ and $\tilde{q}_i = \bar{q}$, according to (17). First, consider the *benchmark case* where the labor force is homogenous in *any* respect, i.e. $h_1 = h_0$ and thus $w(h) = 1$ for all h. Thus, sunk costs at stage 1 simply equal s. (Also note that $\alpha_i = \bar{\alpha} = \alpha(h_0)$ for all $i \in \mathcal{T}$ in this case.) Thus, $\Pi_i = s$ in general equilibrium, i.e. $\bar{q}^* = \sqrt{s\delta_Q}$ must hold, where superscripts (*) indicate general equilibrium levels. Since also $\hat{\pi}(N^*, \bar{\alpha}, \delta_C) = 2\bar{q}^*/\delta_Q$ must hold, in general equilibrium, the number of active firms N^* in each market is implicitly given by $\hat{\pi}(N^*, \bar{\alpha}, \delta_C) - 2\sqrt{s/\delta_Q} = 0$. (Thus, $\partial N^*/\partial\delta_C > 0$ and $\partial N^*/\partial\delta_Q > 0$.) Moreover, the equilibrium number of trademarks equals $T^* = N^*/\bar{q}^*$. However, things are more complicated with a non-degenerate skill distribution (i.e. $h_1 > h_0$). For instance, note that in the symmetric case with $\bar{h}_i = \bar{h}$ for all i, if T^* increases for some reason (e.g. because δ_C increases), then $\bar{h}$ must decline (see Fig. 2 (c)). In contrast, in a full segregation equilibrium entry of an additional network in the economy implies that managers have to be hired from the bottom of the skill distribution, leaving average managerial skills of all other firms unaffected (see Fig. 2 (b)).

Remark 7. As already pointed out in Remark 2, one innovation of this paper is to consider entry of firms at more than one market. In order to clarify some implications of this extension, this may be compared with standard frameworks. For instance, Ottaviano and Thisse (1999) consider entry into a single market only, which requires exogenous entry costs G. (Moreover, they assume constant costs $C'(\cdot) = F$ for each variety introduced by a firm.) One may thus ask how the results of the present analysis change if $Q(q) = Gq$, $G > 0$, (i.e. $Q''(\cdot) = 0$ rather than $Q''(\cdot) > 0$) is assumed. In this case all firms $i \in \mathcal{T}$ which enter the economy at stage 0, choose $\tilde{q}_i = 1$ (since markets are identical). Thus, one can write $\pi_i(\mathbf{A}(m)) = \bar{\pi}_i(\boldsymbol{\alpha})$ for all $m \in [0, 1]$. Hence, $\Pi_i(\boldsymbol{\alpha}) = \bar{\pi}_i(\boldsymbol{\alpha}) - G$, $i \in \mathcal{T}$. The remainder of the analysis would be unchanged. Also note that Ottaviano and Thisse (1999) consider a symmetric situation with $A_i = \alpha$ for all i in the considered market. In this case, one can write $\bar{\pi}_i(\boldsymbol{\alpha}) = \pi(T, \alpha)$ for all $i \in \mathcal{T}$, where $\partial\pi/\partial T < 0$. That is, in their model, the equilibrium number of firms T^* is determined by the (standard) condition $\pi(T^*, \alpha) = G$, whereas in the present analysis the equilibrium condition $\bar{\pi}_i(\boldsymbol{\alpha}) - G = w(\bar{h}_i)$ has to hold for all $i \in \mathcal{T}$, if $Q(q) = Gq$ would be assumed. In fact, this highlights the difference between the present analysis and the existing literature which assumes an ex ante symmetry of potential entrants (e.g. Spence, 1976; Dixit and Stiglitz, 1977; Salop, 1979; Lancaster, 1979; Helpman, 1981). However, this example also shows that the coexistence of global and local players requires the assumption $Q''(\cdot) > 0$ in the presented model.

5 Conclusion

In a multi-market model as developed here, one has to distinguish between entry of firms into the economy and entry into a single market. This paper has hypothesized that entry into the economy implies that firms have to incur sunk costs for establishing a head department which consists of managers. The key role of managers in the model is to create and position a trademark, which serves as a common asset by affecting the (perceived) quality of any product supplied of a firm in any market it is active. The average managerial skill within a firm ultimately determines its profits. Hypothesizing that managerial skills in the economy are heterogenous typically leads to segregation in the process of sorting (or assignment) of managers into firms (i.e. in the contest of firms for managerial skills). As an implication, despite complete symmetry of all potential entrants, the model endogenously generates global players (i.e. firms with a wide range) and locally operating rivals as well as a non-degenerate distribution of profits among firms. Moreover, it has been shown that global players not only have higher profits than their less successful rivals, but typically also offer more product varieties in any single market they operate.

On a more general level, this paper has made two points. First, if there is free entry into the economy and firms are allowed to make sunk investments (i.e. for demand-enhancing activities), then asymmetric equilibria typically arise whenever the invested factor is heterogenous. For instance, this general insight contributes to an understanding of an uneven distribution of both profits and market power, and thus is potentially relevant in a number of applications. Second, it has been demonstrated that such a framework leads to plausible and testable predictions regarding other differences among firms, which are associated with this uneven distribution of market power. To further exploit these general insights is left for future research.

Appendix

Proof of Lemma 1:[22] The first-order condition of the maximization problem (4) for firm $i \in \mathcal{N}$ is given by

$$A_i - 2\beta x_k - \gamma \sum_{l \in \mathcal{K} \setminus k} x_l - \gamma \sum_{l \in I \setminus k} x_l = 0. \tag{A.1}$$

Adding and subtracting $2\gamma x_k$ implies

$$A_i - 2(\beta - \gamma) x_k - \gamma X - \gamma \sum_{l \in I} x_l = 0, \tag{A.2}$$

where $X \equiv \sum_{l \in \mathcal{K}} x_l$. Thus, $x_k = x_i$ for all $k \in I$, which implies $\sum_{l \in I} x_l = n_i x_i$ and $X \equiv \sum_{z \in \mathcal{N}} n_z x_z$. Thus,

$$x_i = \frac{A_i - \gamma X}{2(\beta - \gamma) + \gamma n_i}. \tag{A.3}$$

Multiplying both sides by n_i and summing over all $i \in \mathcal{N}$ one obtains the total output level $\tilde{X} = \sum_{z \in \mathcal{N}} n_z \tilde{x}_z$ in Cournot-Nash equilibrium with

$$\tilde{X} = \frac{\sum_{z \in N} A_z \Gamma_z}{1 + \sum_{z \in \mathcal{N}} \Gamma_z}. \tag{A.4}$$

[22] For the special case $A_i = 1$ for all i, a similar proof can be found in Sutton (1998, p. 501f.).

(Remember $\Gamma_z = \frac{\gamma n_z}{2(\beta-\gamma)+\gamma n_z}$, $z \in \mathcal{N}$, from lemma 1.) Note that (A.4) implies

$$A_i - \gamma \tilde{X} = \frac{\Lambda_i}{\left(1 + \sum_{z \in \mathcal{N}} \Gamma_z\right)}, \tag{A.5}$$

where Λ_i is defined in lemma 1. Using (A.3) and (A.5) yields (5).

Moreover, note that (1) can be written as $p_k = A_i - (\beta-\gamma)x_k - \gamma X$ for all $k \in I$. Since $\tilde{x}_k = \tilde{x}_i$ for all $k \in I$, we have $\tilde{p}_k = \tilde{p}_i$ for all $k \in I$. Using (A.3) then yields equilibrium prices $\tilde{p}_i = (\beta - \gamma + \gamma n_i)\tilde{x}_i$. Finally, noting that $\tilde{R}_i = n_i \tilde{p}_i \tilde{x}_i$ confirms the result. □

Proof of Corollary 1: First, note that $1 + \sum_{z \in \mathcal{N}} \Gamma_z$ can be written as $1 + \Phi_{\backslash i} + \Gamma_i$, where $\Phi_{\backslash i} \equiv \sum_{z \in \mathcal{N} \backslash i} \Gamma_z$. Thus,

$$1 + \sum_{z \in \mathcal{N}} \Gamma_z = \frac{\left(1 + \Phi_{\backslash i}\right)\left(2(\beta-\gamma) + \gamma n_i\right) + \gamma n_i}{2(\beta-\gamma) + \gamma n_i} \tag{A.6}$$

by using $\Gamma_i = \frac{\gamma n_i}{2(\beta-\gamma)+\gamma n_i}$. Substituting (A.6) into (5) implies

$$\tilde{R}_i(\mathbf{n}, \mathbf{A}) = \frac{n_i(\beta - \gamma + \gamma n_i)\Lambda_i^2}{\left[\left(1 + \Phi_{\backslash i}\right)\left(2(\beta-\gamma) + \gamma n_i\right) + \gamma n_i\right]^2}, \tag{A.7}$$

according to (6). Taking the first and second partial derivative yields

$$\frac{\partial \tilde{R}_i}{\partial n_i} = \frac{(\beta-\gamma)\left[2(\beta-\gamma)\left(1+\Phi_{\backslash i}\right) + \gamma n_i\left(2 + 3\Phi_{\backslash i}\right)\right]\Lambda_i^2}{\left[\left(1 + \Phi_{\backslash i}\right)\left(2(\beta-\gamma) + \gamma n_i\right) + \gamma n_i\right]^3} \tag{A.8}$$

and

$$\begin{aligned} \frac{\partial^2 \tilde{R}}{\partial n_i^2} &= -\frac{\gamma(\beta-\gamma)\Lambda_i^2}{\left[\left(1 + \Phi_{\backslash i}\right)\left(2(\beta-\gamma) + \gamma n_i\right) + \gamma n_i\right]^4} \times \\ &\quad \left(8(\beta - \gamma + \gamma n_i) + \left[14(\beta-\gamma) + 22\gamma n_i\right]\Phi_{\backslash i} + \left[6(\beta-\gamma) + 9\gamma n_i\right]\Phi_{\backslash i}^2\right), \end{aligned} \tag{A.9}$$

respectively, after some straightforward but tedious manipulations. This confirms part (i). Moreover, note that $\frac{\partial \Lambda_i}{\partial A_i} > 0$ and $\frac{\partial \Lambda_i}{\partial A_j} < 0$ for all $i, j \in \mathcal{N}$, $i \neq j$. Using this confirms parts (ii) and (iii), according to (A.7) and (A.8), respectively. In order to prove part (iv), note that we can write

$$\Lambda_i = A_i \left(1 + \sum_{z \in \mathcal{N} \backslash \{i,j\}} \Gamma_z\right) - \sum_{z \in \mathcal{N} \backslash \{i,j\}} A_z \Gamma_z + (A_i - A_j)\Gamma_j, \tag{A.10}$$

which implies $\partial\Lambda_i/\partial n_j = (A_i - A_j)\partial\Gamma_j/\partial n_j$ if $i \neq j$, and

$$\Lambda_j = \Lambda_i - (A_i - A_j)\left(1 + \sum_{z\in\mathcal{N}}\Gamma_z\right), \tag{A.11}$$

respectively. Using these facts reveals after some tedious manipulations that, for $i \neq j$,

$$\frac{\partial\tilde{R}_i}{\partial n_j} = -\frac{4\gamma(\beta-\gamma)n_i\left(\beta-\gamma+\gamma n_i\right)\left(2(\beta-\gamma)+\gamma n_i\right)\Lambda_i\Lambda_j}{\left(2(\beta-\gamma)+\gamma n_j\right)^2\left[\left(1+\Phi_{\setminus i}\right)\left(2(\beta-\gamma)+\gamma n_i\right)+\gamma n_i\right]^3}, \tag{A.12}$$

which confirms part (iv). Finally, part (v) directly follows from (A.7) and (A.10). This concludes the proof. □

Proof of Corollary 2: Partially differentiating both sides of (10) with respect to A_i yields

$$\frac{\partial\pi_i}{\partial A_i} = \sum_{j\in\mathcal{N}\setminus i}\frac{\partial\tilde{R}_i}{\partial n_j}\frac{\partial\tilde{n}_j}{\partial A_i} + \frac{\partial\tilde{R}_i}{\partial A_i} + \left(\frac{\partial\tilde{R}_i}{\partial n_i} - C'(\cdot)\right)\frac{\partial\tilde{n}_i}{\partial A_i}. \tag{A.13}$$

Similarly, if $i \neq j$,

$$\frac{\partial\pi_i}{\partial A_j} = \frac{\partial\tilde{R}_i}{\partial A_j} + \frac{\partial\tilde{R}_i}{\partial n_j}\frac{\partial\tilde{n}_j}{\partial A_j} + \sum_{z\in\mathcal{N}\setminus\{i,j\}}\frac{\partial\tilde{R}_i}{\partial n_z}\frac{\partial\tilde{n}_z}{\partial A_j} + \left(\frac{\partial\tilde{R}_i}{\partial n_i} - C'(\cdot)\right)\frac{\partial\tilde{n}_i}{\partial A_j}. \tag{A.14}$$

Note that the terms in brackets in (A.13) and (A.14), respectively, vanish by the envelope theorem, according to (9). In order to prove $\partial\pi_i/\partial A_i > 0$, note that $\partial\tilde{R}_i/\partial A_i > 0$ for all $i \in \mathcal{N}$, according to part (ii) of Corollary 1. Finally $\partial\pi_i/\partial A_j < 0$, $i \neq j$, follows from Condition 2. This concludes the proof. □

Proof of Corollary 4: For all $i, j \in \mathcal{T}$,

$$\frac{\partial\overline{\Pi}_i}{\partial\alpha_j} = \tilde{q}_i\frac{\partial\pi_i}{\partial\alpha_j} + \left[\pi_i(\mathbf{A}(\bar{m}_i)) - Q'(\tilde{q}_i)\right]\frac{\partial\tilde{q}_i}{\partial\alpha_j}. \tag{A.15}$$

(Note that for all $i \in \mathcal{T}$, $A_i(m) = \alpha_i$ if $m \in M_i$, according to (3).) According to (14), the term in square brackets in (A.15) is non-negative. Hence, $\frac{\partial\overline{\Pi}_i(\alpha)}{\partial\alpha_j} > 0$ if $j = i$, and $\frac{\partial\overline{\Pi}_i(\alpha)}{\partial\alpha_j} < 0$ if $j \neq i$, according to Corollary 2 and Corollary 3. □

Proof of Proposition 2:[23] At stage 1, each firm $i \in \mathcal{T}$ solves

$$\max_{H_i\subset H}\left\{\Pi_i\left(\alpha\left(\left[\int_{H_i}\kappa(h)hd\mu\right]/s\right), \boldsymbol{\alpha}_{-i}\right) - \int_{H_i}\kappa(h)w(h)d\mu\right\} \tag{A.16}$$

[23] The proofs of Proposition 2 and 3 draw on Saint-Paul (2001).

s.t. $\kappa(h) \geq 0$ for all $h \in H$, and $\int_{H_i} \kappa(h) d\mu = s$, where $\kappa(h)$ is the density of each type h firm i wants to hire. Differentiating (A.16) with respect to $\kappa(h)$ and observing that, in equilibrium, the resulting expression must not exceed the Langrange multiplier on the second constraint, denoted λ_i, one obtains the first-order condition

$$\frac{\partial \Pi_i(\boldsymbol{\alpha})}{\partial \alpha_i} \alpha'(\bar{h}_i) \frac{h}{s} - w(h) \leq \lambda_i \tag{A.17}$$

for any type $h \in H$, with strict equality if $\kappa(h) > 0$. Integrating both sides of (A.17) over all types $h \in H_i$ yields

$$s\lambda_i = \frac{\partial \Pi_i(\boldsymbol{\alpha})}{\partial \alpha_i} \alpha'(\bar{h}_i) \bar{h}_i - \int_0^s w(h_{iu}) du, \tag{A.18}$$

where $\bar{h}_i = \left(\int_0^s h_{iu} du\right)/s$ has been used. Substituting (A.18) into (A.17) and observing the equilibrium condition (a) from Definition 2 yields (22). Finally, note that $w(h) \geq 1$, according to condition (c) of Definition 2. This concludes the proof. □

Proof of Proposition 3: Consider any two different firms $i, j \in \mathcal{T}$. Take any two types $h_i^- \in H_i$ and $h_j^+ \in H_j$ from both firms. Remember the definition of $f(\bar{h}_i)$ in (23), implying

$$f'(\bar{h}_i) = \frac{\partial \Pi_i(\boldsymbol{\alpha})}{\partial \alpha_i} \alpha'(\bar{h}_i). \tag{A.19}$$

Then (22) implies

$$sw(h_i^-) = f(\bar{h}_i) + f'(\bar{h}_i)(h_i^- - \bar{h}_i) \geq f(\bar{h}_j) + f'(\bar{h}_j)(h_i^- - \bar{h}_j) \tag{A.20}$$

and

$$sw(h_j^+) = f(\bar{h}_j) + f'(\bar{h}_j)(h_j^+ - \bar{h}_j) \geq f(\bar{h}_i) + f'(\bar{h}_i)(h_i^+ - \bar{h}_i), \tag{A.21}$$

respectively. Consequently,

$$\left[f'(\bar{h}_i) - f'(\bar{h}_j)\right]\left(h_i^- - h_j^+\right) \geq 0. \tag{A.22}$$

Similarly, one finds

$$sw(\bar{h}_i) = f(\bar{h}_i) \geq f(\bar{h}_j) + f'(\bar{h}_j)(\bar{h}_i - \bar{h}_j) \tag{A.23}$$

and

$$sw(\bar{h}_j) = f(\bar{h}_j) \geq f(\bar{h}_i) + f'(\bar{h}_i)(\bar{h}_j - \bar{h}_i), \tag{A.24}$$

respectively. Now suppose $\bar{h}_i > \bar{h}_j$, i.e. $\alpha_i > \alpha_j$. Thus,

$$f'(\bar{h}_i) \geq \frac{f(\bar{h}_i) - f(\bar{h}_j)}{\bar{h}_i - \bar{h}_j} \geq f'(\bar{h}_j), \tag{A.25}$$

according to (A.23) and (A.24).

Consider part (ii) of Proposition 3 first. Note that, if $f''(\cdot) > 0$ and $\bar{h}_i > \bar{h}_j$, then $f'(\bar{h}_i) > f'(\bar{h}_j)$. Thus, (A.22) implies $h_i^- \geq h_j^+$ for *any* $h_i^- \in H_i$ and $h_j^+ \in H_j$. Consider h_i^- as the least skilled type in firm i and h_j^+ as the most skilled type in firm j. Hence, any type hired in the more successful firm i is at least as skilled as any type in the less successful firm j (i.e. there is full segregation of types across i and j). It remains to be shown that $\bar{h}_i \neq \bar{h}_j$ for all $i, j \in \mathcal{T}$ if $f''(\cdot) > 0$. Suppose $\bar{h}_i = \bar{h}_j$, i.e. both firms i and j offer the same wage schedule, according to Proposition 2. If, say, firm i could increase profits by successfully replacing a small mass of type h by a small mass $\hat{h} > h$ it hires from firm j (!), $\bar{h}_i = \bar{h}_j$ cannot hold in equilibrium. Formally, $f'(\bar{h}_i)\hat{h}/s - w(\hat{h}) \leq f'(\bar{h}_i)h/s - w(h)$ must hold for $\bar{h}_i = \bar{h}_j$ to be an equilibrium, according to (A.16) (and observing (23)). Note that

$$f'(\bar{h}_i)h/s - w(h) = \left[f'(\bar{h}_i)\bar{h}_i - f(\bar{h}_i)\right]/s \text{ for all } h \in H_i, \tag{A.26}$$

according to Proposition 2. Also note that by the replacement of h by $\hat{h} > h$ the average skill level in firm i increases. However, if $f''(\cdot) > 0$, this means that the right-hand side of (A.26) increases, thus implying $f'(\bar{h}_i)\hat{h}/s - w(\hat{h}) > f'(\bar{h}_i)h/s - w(h)$. As this violates the equilibrium condition, part (ii) is confirmed.

To prove part (iii), suppose, say, $\bar{h}_i > \bar{h}_j$. Note that $f''(\cdot) < 0$ then implies $f'(\bar{h}_i) < f'(\bar{h}_j)$. However, from (A.17) also $f'(\bar{h}_i) \geq f'(\bar{h}_j)$ has to hold, which is impossible. Thus, $\bar{h}_i = \bar{h}_j$ for all $i, j \in \mathcal{T}$.

Part (i) can be proven by replicating the following argument in Saint-Paul (2001, p. 12). If $\bar{h}_i > \bar{h}_j$, then one can find $h_i^- \in H_i$ and $h_j^+ \in H_j$ with $h_i^- < h_j^+$ (i.e. the least skilled manager in the more successful firm i has lower skill than the most skilled manager in firm j) only if $f'(\bar{h}_i) \leq f'(\bar{h}_j)$, according to (A.22). But since $\bar{h}_i > \bar{h}_j$ also implies $f'(\bar{h}_i) \geq f'(\bar{h}_j)$, according to (A.25), $H_{\varphi(i)} = H_{\varphi(j)}$ (i.e. one can find a partition of H into adjacent intervals such that firms i and j hire from the same interval) only if $f'(\bar{h}_i) = f'(\bar{h}_j)$, i.e. firms i and j offer the same wage schedule.

The same argument applies for any other firm z with $\bar{h}_i > \bar{h}_z > \bar{h}_j$. This concludes the proof. □

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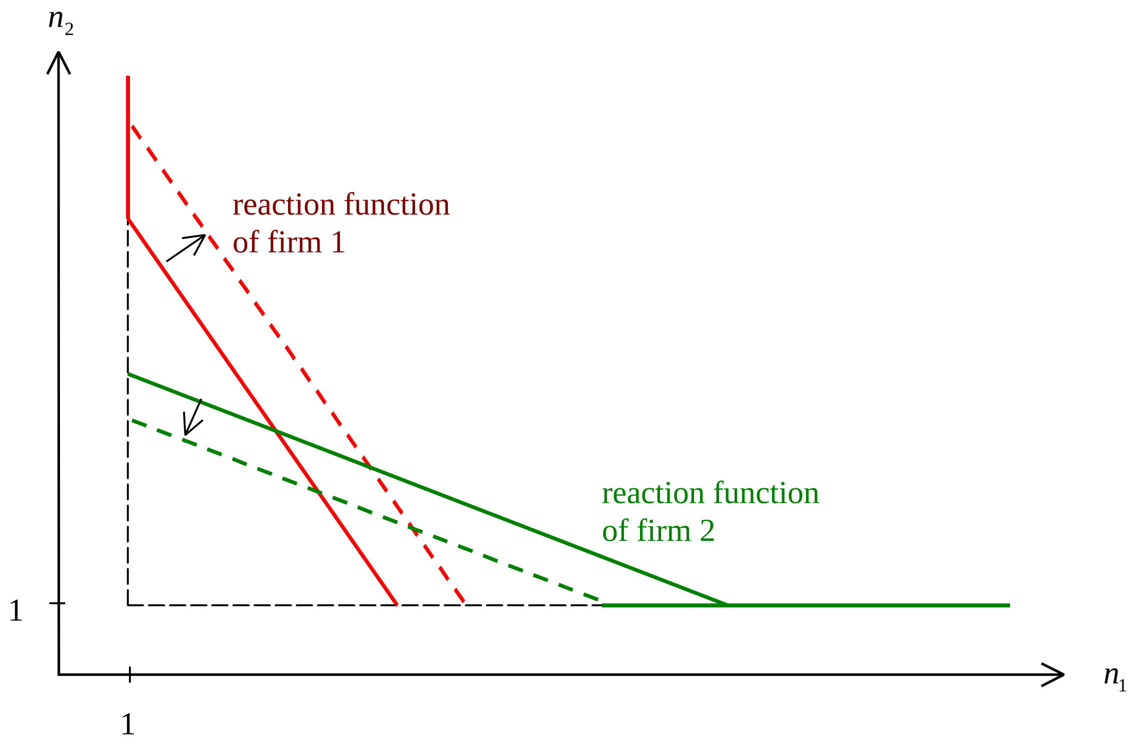

Figure 1: The impact of an increase in A_1 on $\tilde{n}_1$ and $\tilde{n}_2$.

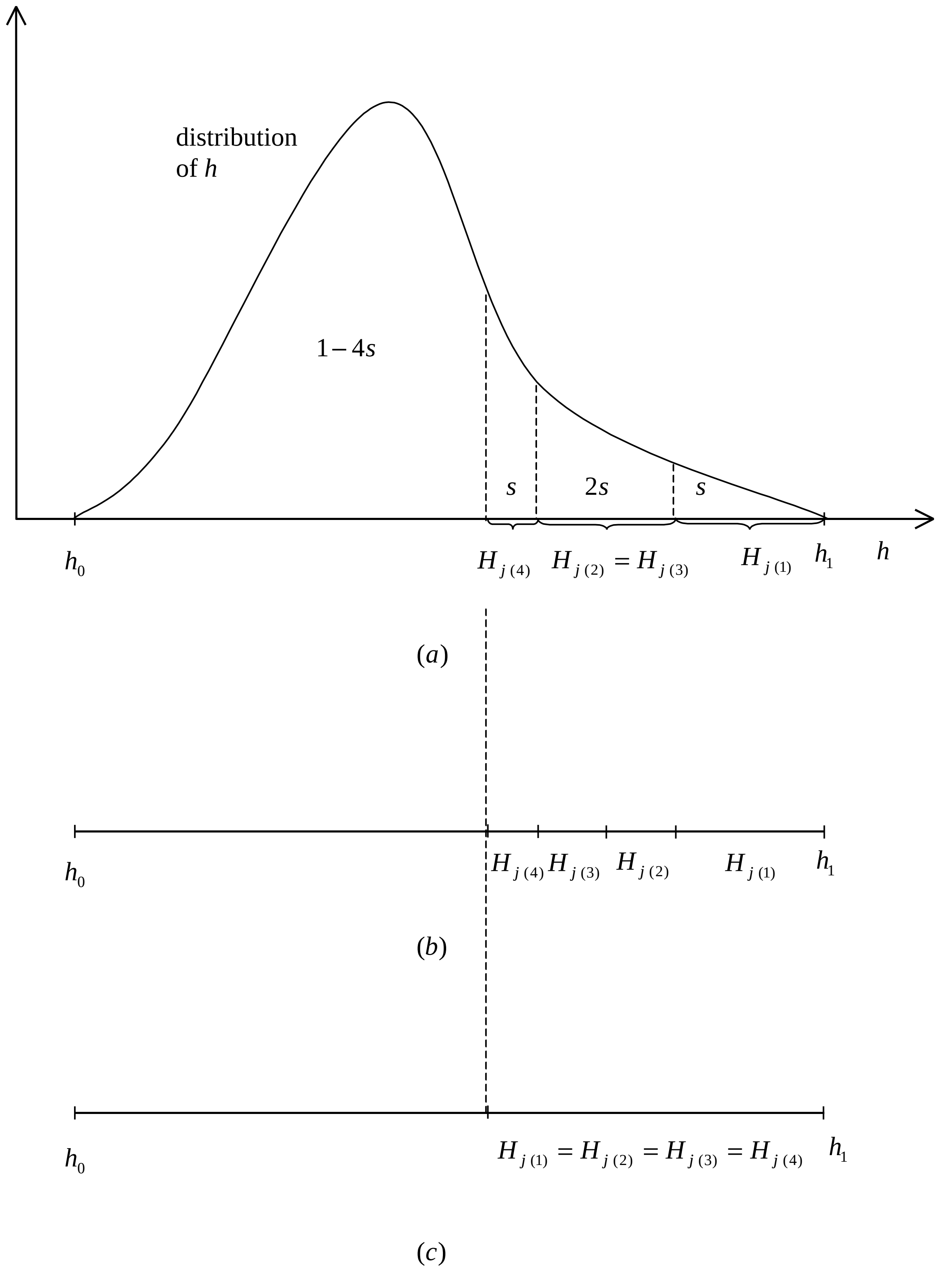

Figure 2: Three possible equilibrium partitions in the case *T*=4.