Communication Networks and Cooperation

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Abstract

I study a network based mechanism of norm enforcement in a community where agents play a repeated Prisoner's Dilemma with changing partners. Each agent chooses a number of close friends to whom he communicates his partners' actions. This enforces cooperation since an agent's close friends punish a defecting partner. In a world with noiseless communication, it is optimal for each agent to have close contacts to all other network members. Moreover, it is optimal to have a large network. If communication is noisy a lower number of close contacts is optimal. As the number of network members gets large norm enforcement fails.

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1 Introduction

This paper models a communication network in the setting of a repeated Prisoner's Dilemma with changing partners. I focus on two issues: how many contacts enabling communication should each network member have and what are the implications of a large network? In a world with noiseless communication, it is optimal for each agent

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to have close contacts to all other network members. Moreover, it is optimal to have a large network. With noisy communication it is not socially optimal to have close contacts to all other network members and, as the size of the network gets large, cooperation fails.

Network based mechanisms are important institutions for trust enforcement and cooperative behavior. The mechanisms work through communication and fear of punishment for misbehavior or anticipation of rewards for good behavior. In the parlance of sociology, a communication network builds up social capital.

> ...Social capital is defined by its function. It is not a single entity but a variety of different entities, with two elements in common: they all consist of some aspect of social structures, and they facilitate certain actions of actors-whether persons or corporate actors-within the structure...(Coleman [8])

An example are wholesale diamond markets (see Coleman [8]). During the bargaining over a sale, a trader will hand over to another trader a bag of stones for the latter to examine in private. There is no formal insurance whatsoever that the latter will not substitute inferior stones. A given diamond merchant community is ordinarily very close, both in the frequency of interaction and in ethnic and family ties. Coleman mentiones the wholesale diamond market in New York City, an essentially closed Jewish community. Ethnic communities such as the Dominicans in New York and the Cubans in Miami use network based systems as well to sustain informal credit channels (see Portes and Sensenbrenner [21]). The well known rotating credit associations among Asian immigrants, for example the Chinese on Java, rely also on trust enforcement through network mechanisms (see Granovetter [13]).

The number of close contacts network members have among each other[1] is an important input to foster cooperation. A dense network, that is, a network where each network member has many close ties seems beneficial since these ties are an effective threat to enforce cooperation. Boissevain [6], however, studies the structure of relations inhabitants of Malta have. He shows with an example of two inhabitants that people generally do not maintain close ties to all members in their network. Hence there must

[1] In the language of sociology this would be the closure of a network, see Coleman [8].

be some cost in having ties to all network members. In opposite to the benefits, it is less clear what the costs of a dense network are. Most of the literature on networks interprets these costs as time costs since individuals have to spend time to maintain links.[2] Observations suggest that there are other costs from a dense network. For example, an exacerbation of the obligations within a network may lower the benefits of a network. Portes and Sensenbrenner display interesting examples like faulty assaults or constraints on freedom. I shall model the costs of a dense network as costs originating from noisy communication.

Another relevant dimension is the overall number of participants in the network. Granovetter [13] notes the success of the overseas Chinese on Java, which is based on rotating credit associations. The Javanese failed in establishing such a mechanism. Being immigrants, the Chinese are a small community in relation to the Javanese: in a Javanese town dubbed "Modjokuto" they numbered 1.800 out of a total of approximately 18.000. Granovetter offers the following explanation. Successful Javanese face demands for a piece of the cake achieved from an unlimited number of other Javanese (relatives, kins, etc.). The Chinese immigrants did not suffer from such excessive claims, since their immigrant status simplified the process of "decoupling" from relatives and kins. This paper suggests that there is an additional detrimental effect of large numbers, namely the effect of *gossip*: large communities are plagued by gossip. This aggravates cooperative behavior: if agents cannot distinguish true messages from gossip then the required punishments might not take place.

I address these issues using the insights of (repeated) game theory and model a communication network in the setting of a repeated Prisoner's Dilemma with changing partners. The basic framework is that of a repeated game with changing partners à la Kandori [17]. Before the repeated interaction starts, a subset of all agents form a network and each network member chooses a number of other network members - close contacts or friends - to whom he communicates the events in each of his per-period interactions. If communication is frictionless harshest punishments are achieved by choosing as many contacts as possible. Moreover, it is optimal to have as many network members as possible. If communication is noisy, it is no longer socially optimal to have a maximum number of contacts possible: noisy communication might lead to sanctions with positive probability although there was no misbehavior. The same mechanism

[2] e.g. Bala and Goyal [5].

which supports cooperation through communication in the first place is detrimental to the community if communication is noisy. However, private incentives to maintain close contacts differ from social incentives and an overinvestment in relations occurs. In a further step, individuals receive not only messages from their close friends, but also pure noisy messages - *gossip* - from other network members. As the number of network members gets large, cooperation cannot be sustained anymore. Intuitively, if the number of network members gets large, agents cannot distinguish between gossip and truth. If network formation were costly no network would form! This suggests also why in many communities we do *not* observe communication networks but other, more centralized institutions, such as courts which rely not only on communication as a means of investigation.

There are some papers using repeated game theory to analyze trust in social structures.[3] Milgrom, North and Weingast [19] model the role of courts in enforcing cooperation in a repeated game setting similar to the one in this paper. In their paper, agents may inform a court about a partner's misbehavior and they also may obtain information about a partner's previous behavior from the court. Hence information flows through a central institution whereas here information flows are decentralized. Greif's [15] remarkable work on the Maghribi trader's coalition studies an efficiency wage based mechanism which the Maghribi used to enforce honest behavior from their agents. The wage which has to be paid to enforce honest behavior is decreasing in the probability of future hiring. Hence a merchant prefers to hire an honest agent. This mechanism also rests on information flow among the Maghribis. In contrast to the present work, Greif does not model the details of the communication network.

There is also a growing literature on strategic noncooperative network formation which focusses on individual incentives to form links, e.g. Bala and Goyal [5] or Jackson and Wolinsky [16].[4] Bala and Goyal interpret their model as a model of information flow. An agent's benefits and costs in a network in their paper are given directly by the *number of agents* he is linked to. The authors show that specific network architectures, e.g. a star or a ring, emerge as a Nash equilibrium in a strategic game of link formation. I do not study the pattern of a network but examine the number

[3]See Gibbons [12] for an overview.

[4]See Aumann and Myerson [4] and van den Nouweland [20] for approaches using cooperative game theory.

of links or contacts agent must have to sustain cooperation. Benefits *and cost* for a contact are determined from the repeated interaction that follows the stage of contact formation. The aspect of networks with noise or frictions is missing in most of the literature mentioned (section 5 in Bala and Goyal [5] is a notable exception). Another paper which models strategic incentives for link formation network is Boorman's [7] analysis on the impact of communication networks on job search.

Kranton and Minehart [18] also analyze patterns of strategic network formation. Network benefits are given by an economic environment of trade among buyers and sellers. Buyers must form links to sellers before they can buy one unit of a good. After the stage of link formation an efficient trading mechanism allocates the goods among the buyers. Kranton and Minehart show that the efficient network structure, i.e. a network structure which maximize overall surplus, is always an equilibrium. In contrast, I show that if communication is noisy, private and social incentives for network formation differ.

The paper is organized as follows. Section 2 sets up the model. Conditions for a network equilibrium without noise are derived in section 3. Section 4 contains the result on the optimal number of contacts with noise, while section 5 presents the result on cooperation failure when the number of network members gets large. Section 6 discusses how the results of the model are affected if certain assumptions are changed and section 7 concludes.

2 The Model

I shall first describe the situation *without* communication network. There are $i = 1, ..., I > 2$ agents, where I is even. The number $i \in \{1, ..., I\}$ is also called the *identity* of agent i. Time is discrete, $t = 1, 2, ...$, and runs from one to infinity. In each period, all agents get matched pairwise. If a match forms, the two agents play the following stage game.

	C	D
C	a, a	b, c
D	c, b	$0, 0$

with $c > a > 0 > b$.

Each agent only observes his private history but does not observe the events in other matches. For each agent, a strategy is a function from his private history to his action set. Each agent maximizes his average discounted payoff. Let $\delta \in (0,1)$ denote the discount factor for all agents. This setting is equivalent to those studied by Kandori [17] or Ellison [10]. Whereas Kandori and Ellison sustain cooperation through punishments relying on contagion effects coming back to hit a noncooperator, I shall consider a more direct punishment mechanism.[5] This more direct punishment mechanism comes from a communication network.

The Game with Communication Network

Suppose that $N \leq I$ agents have decided to set up a **communication network**. The following assumptions hold.

(N1) all agents inside the network know and recognize the identities of other network members. Let $\mathcal{N}$ denote the set of network members.

(N2) in period 0 - *before* the repeated interaction starts - all network members choose simultaneously $L_i \leq N-1$ contacts or close friends. Let $\mathcal{L}_i$ denote the set of contacts of agent i. These contacts will sometimes be called *outgoing contacts*. We say that i has a contact to j if $j \in \mathcal{L}_i$. Let $\overline{\mathcal{L}}_i$ be the set of agents who have a contact to i, that is $\overline{\mathcal{L}}_i = \{j \in \mathcal{N}, j \neq i | i \in \mathcal{L}_j\}$. Let $\overline{L}_i$ denote the number of network members who have a contact to i, that is, $\overline{L}_i = \#\overline{\mathcal{L}}_i$. These contacts will sometimes be called *incoming contacts*. A network $\mathcal{L}$ is defined as the pattern of contacts agents have among each other. Each network $\mathcal{L}$ implies, for each network member i, a number of contacts i has to other agents, L_i, and a number of contacts other agents have to i, $\overline{L}_i$. Let $L_N(\mathcal{L}) \in \mathbb{R}^{N\times 2}, L_N = \{(L_1, \overline{L}_1), ..., (L_N, \overline{L}_N)\}$ be the matrix of these numbers. For the rest of the paper only these numbers, not the exact pattern of contacts of agents, are relevant and I write L_N. [6] Agents who are not network members cannot choose contacts.

(N3) Relations among agents are not symmetric, that is $j \in \mathcal{L}_i \nRightarrow i \in \mathcal{L}_j$ for all $i, j \in \mathcal{N}$. Hence, for agents $i \in \mathcal{N}$ and $j \in \mathcal{N}$, i has a contact to j *and* j has a

[5] I shall explain the relation of this paper to their papers in more detail below.

[6] Hence two networks $\mathcal{L}$ and $\mathcal{L}'$ are equivalent if $L_N(\mathcal{L}) = L_N(\mathcal{L}')$

contact to i if and only if $j \in \mathcal{L}_i$ and $i \in \mathcal{L}_j$. Moreover, relations among agents are not transitive, that is, $i \in \mathcal{L}_j \wedge z \in \mathcal{L}_i \nRightarrow z \in \mathcal{L}_j$ for all $i, j, z \in \mathcal{N}$.

(N4) The repeated interaction itself is as follows: in any period $t = 1, 2, ...$, agents get matched pairwise. All agents can get matched both with agents in the network and with agents outside the network. If two network members i and j meet they observe the number of contacts L_i and L_j of their partner and then choose their action, C or D.

(N5) after network members i and j have played the stage game, both i and j inform their respective contacts (L_i contacts for i and L_j contacts for j) of their partners' actions in the match. Hence the information of i given to his L_i contacts contains an element of the set $\{C, D\}$ and the identity (i.e. a number $i \in \{1, ..., I\}$) of his partner. This information about i is available to the L_j contacts of j at the beginning of the next period.

Assumptions (N1)-(N5) define a *game with communication network*.

Equilibrium Strategies in the Repeated Game:

In the repeated game with network formation, strategies for a network member i are now functions from private histories *and* from the information i receives from the $\overline{L}_i$ agents who chose to have contacts to i in period 0. For the periods $t = 1, 2, ...$, that is, for the infinitely repeated interaction following any choice of a communication network $\mathcal{L}$, I consider equilibria involving the following strategies:

(OUTSIDE) agents outside the network choose D in each period.

(COOPERATE) agents inside the network choose C if they have the information that their partner has always chosen C. If there is no information since the partner is not a network member an agent chooses D.

(PUNISH) agents inside the network choose D if they have the information that their partner has chosen D in any previous period. An agent who has chosen D meeting an agent who plays C chooses D forever after.

(COMMUNI) if a network member i deviated in a match with network member j, then j informs all his L_j contacts about this. That is, he sends the message D and

i's identity to all his L_j contacts. Agent i sends the message C and the identity of j. From the next period on, those L_j contacts punish i forever after (assumption (PUNISH)). If both i and j chose C, then both agents send the message C to their respective contacts.

(NO CONTAG) Once punishment for an agent i takes place, i sends the message C to all his L_i contacts for the rest of the game.

To understand (NO CONTAG), suppose that network member i did not cooperate when playing with network member j and that j informs all his L_j contacts about this. Assumption (NO CONTAG) says that noncooperator i does not trigger punishments on his punishers by informing his L_i contacts. Hence there is no contagion in punishments. The assumption assures that punishing is always a best response in the equilibrium described for the punishers. If assumption (NO CONTAG) does not hold, an agent who is required to punish another agent might fear further punishment and chooses C when he is supposed to choose D. As in Kandori [17] one could circumvent the problem and fix stage game payoffs such that punishing *is* a best response (i.e. b has to be sufficiently small). Then, all the qualitative results of the paper continue to hold even if (NO CONTAG) is not imposed. I chose to impose assumption (NO CONTAG) since, in contrast to Kandori's analysis I do not want to focus on contagion effects. I discuss the implications of relaxing (NO CONTAG) for the results of this paper in more detail in section 6.

The other assumptions on equilibrium strategies are imposed since they seem natural in a communication network. Network members inform their close contacts about any network member choosing D. This noncooperator is then punished by the close friends of the agent who was cheated. The strategies resemble in a simple way the communication network mechanisms used by ethnic communities as mentioned in the introduction.

Let σ_n denote the strategy profile for network member described above while σ_{-n} denotes the strategy profile for all agents not in the network. The strategy profile for all agents is denoted by σ. The payoff $V_i^n(\sigma)$ for a network member, is

$$V_i^n(\sigma) = \frac{N-1}{I-1}a \tag{1}$$

The average expected payoff $V_i(\sigma)$ for an agent not in the network is $V_i(\sigma) = 0$.

I take a network with N members as given and ask wether network members are willing to participate. A network of size N is beneficial if and only if all network members are willing to participate. This participation constrained is required to hold for all histories of play. Hence, to have a network which is beneficial to all participants, $V_i^n(\sigma) \geq 0$ has to hold for all histories of play. Note that this participation constraint is satisfied for any network size N. If the payoff from both partners playing D is $d > 0, c > a > d > b$, then N has to be sufficiently large. If there is some exogenous cost for network formation, for example, if it is costly to maintain contacts, then $V_i^n(\sigma)$ has to exceed this cost as well.

3 Equilibrium and Choice of Contacts

In this section I analyze if and under what conditions the strategies specified in the previous section form an equilibrium indeed. Moreover, I determine the number of contacts agents choose in period 0.

I shall first show the optimality of (COMMUNI) and (NO CONTAG).

Lemma 1. *For any $\mathcal{L}$, information transmission as specified in* (COMMUNI) *and (*NO CONTAG*) is always optimal.*

Proof. (COMMUNI). Suppose that agents behave according to (NO CONTAG). Pick agent $i \in \mathcal{N}$. Suppose that in a match with i agent j plays C. If agent i informs his L_i contacts that j played D, they punish j forever after, which does not increase i's payoff. Suppose that j played D. If agent i does not inform his contacts about this, his payoff does not increase either (due to (NO CONTAG)) . Moreover, any information of i about some other agent $z \neq j$ would not increase i's payoff either.

(NO CONTAG). With a similar argument, a noncooperator j cannot increase his payoff sending the message D instead of the message C. □

Given this lemma, punishing a noncooperator is an equilibrium in the continuation game after any defection, as required in (PUNISH).

To have network members cooperate as required in (COOPERATE), the following incentive constraint has to hold for all $i \in \mathcal{N}$:

$$\frac{N-1}{I-1}a \geq \frac{N-1}{I-1}c + \frac{\delta}{1-\delta}\left(\frac{N-L_j-1}{I-1}a\right) \tag{2}$$

for all $j \in \mathcal{L}_j$.

The benefit of cooperation is given by the probability of meeting a network member, $(N-1))/(I-1)$ times the payoff of cooperation, a. The benefit from defection is the defection payoff c and the continuation payoff. The continuation payoff from defection is zero with probability $L_j/(I-1)$ and with probability $(I-N)/(I-1)$ and a with probability $(N-L_j-1)/(I-1)$. Due to Lemma 1 it is always a best response for j's L_j contacts to punish i.

Solving inequality (2) yields

$$L_j \geq L^\circ = \frac{(N-1)(1-\delta)(c-a)}{\delta a} \tag{3}$$

as the minimum number of contacts each network members needs to have in order to sustain cooperation.[7] Thus it is not necessary that all network members have contacts to *all* other network members. Behavior as specified in (OUTSIDE) is clearly optimal. Last note that $V_i^n(\sigma) \geq 0$ holds for all histories of play.

To determine the number of contacts agents choose in period 0, I focus on network members' equilibrium choices which induce the maximal threat on their partner.

> (EXTREMAL) network members choose a number of contacts such that their partners payoff is lowest in case of noncooperation.

Contact choices induce *extremal equilibria* in the sense of Abreu, [1] or [2]. Hence, it is optimal for each agent during the stage of contact formation that every agent chooses $L_i = N-1$. This number of links provides the harshest punishment for a noncooperator.

If the communication network $\mathcal{L}$ is organized such that it maximizes each network member's utility, it is also an optimal strategy that all network members have $L_i = N-1$ contacts.[8] This implies also that each network member is contacted by all other network members, $\overline{L}_i = N-1$. Hence network members' private incentives and social

[7] This number is actually $[L^\circ]+1$, where $[x]$ denotes the next largest integer to x.

[8] The network members might achieve this through some sort of bargaining.

incentives for the network as a whole are the same! Efficient communication network formation is always an equilibrium on the game of network in stage 0, given repeated interaction from period 1 on.

Considering network size, note that equilibrium payoffs are increasing in N so that agents prefer large networks. If $c > a/(1-\delta)$, the number of contacts required to sustain cooperation would strictly exceed $N-1$; cooperation based on a network mechanism fails in that case. The following Proposition summarizes these observations.

Proposition 1. *Under all the assumptions made above the following hold.*

(i) Cooperation is sustainable in the network if and only if

$$L_i \geq \frac{(N-1)(1-\delta)(c-a)}{\delta a}$$

for all $i \in \mathcal{N}$.

(ii) Suppose that contact choices induce extremal equilibria. For all $N^* \geq 2$, it is then optimal to form a network in which each network member chooses $L_i = N-1$ contacts. Then punishment is maximal and cooperation in the network can be sustained for $\delta \geq (c-a)/c$. The choice of $L_i = N-1$ is optimal for each individual network member and also optimal if the network maximizes each network member's utility: efficient communication network formation is always an equilibrium.

(iii) Since equilibrium payoffs for network members are strictly increasing in N, it is optimal to have all agents in the network, that is $N = I$ is optimal.

(iv) If $c > a/(1-\delta)$ it is not possible to sustain cooperation.

3.1 Cooperation without Institutions ?

In Kandori [17] and Ellison [10] cooperation is sustained by punishments relying on contagion effects coming back to hit a noncooperator. In such a contagion equilibrium, all agent initially cooperate. If an agent ever meets an opponent who defects, he defects from then on. Hence, playing D today will eventually lead all agents to play D. If these strategies are an equilibrium depends on how fast contagion spreads, which in turn depends on the number of agents and on the stage game payoffs. The main problem is that agents prefer to continue choosing C even after meeting an agent who plays D in order to slow down the spread of contagion. In particular, Kandori shows that, for any fixed number of agents, the contagion strategies are an equilibrium for discount factors close to 1 if stage game payoffs are such that the payoff to playing C against an agent playing D is sufficiently negative. Ellison extends Kandori's work and assumes that a publicly observable random variable is available. The public randomization allows to adjust the severity of the punishments. Punishments can be tailored such that agents fear a breakdown of cooperation, so they do no deviate first. Moreover, they do not fear the breakdown so much that they do not spread out the play of D.

In those papers cooperation is possible without any institutions. However, we do observe communication networks (see the introduction of this paper)! A reason for this could be that the direct punishments in this paper work for discount factors smaller than the discount factor required to sustain Kandori's or Ellison's mechanism.

Proposition 2. *The discount factor necessary to sustain cooperation through network formation is smaller than the discount factor necessary to sustain cooperation through contagion.*

Proof. Cooperation through contagion requires a larger discount factor than cooperation through network formation if a noncooperator's continuation payoff after defection is larger. With network formation, a noncooperator is punished immediately forever after a deviation. Hence, a noncooperator's continuation payoff after a deviation is zero.

In punishments relying on a contagion effects and it lasts at least $(I-2)/2$ periods until a noncooperator is punished with probability 1 in every period. Hence, in the first $(I-2)/2$ periods after a deviation there is always a strictly positive probability that a noncooperator is not punished. This implies that the infimum of a noncooperator's continuation payoff is strictly bounded away from zero. □

Hence this more effective way of punishment might be a reason why we observe institution such as communication networks or courts (see Milgrom, Weingast and North [19]).

4 Noisy Communication

In the previous section it was optimal for each agent to choose as many contacts as possible. This choice was optimal since contacts never became active in equilibrium. A large number of contacts is a very effective threat! However, in many networks, the same mechanism which supports cooperative actions may also be detrimental to the agents in the network. In particular, if communication is noisy, those contacts might not cooperate with me although I myself did cooperate. It may then be optimal if my partner has fewer than $N-1$ contacts. Moreover, it could be that I have to punish other network members even though they did cooperate. Hence, I prefer that $\overline{L}_i$, the number of other network members who have a contact to me, is not too high. I shall analyze the socially optimal choice of a communication network $\mathcal{L}$ when there is noise in the transmission of information.

4.1 Noise and Equilibrium Restrictions

I model the presence of noise as follows. Suppose that there is noise in the stage of the game where each agent informs his L_i contacts about the behavior of his partner j in a given period. Two things can happen: the partner j of agent i did deviate, but none of i's contacts received the message and all of i's contacts continue to believe that j did cooperate. Or, j did not deviate, but the L_i contacts of agent i did receive the message that agent j did deviate.

Formally, each message between two agents generates one of two signals at each period. The signal space is given by $X = \{\mathcal{C}, \mathcal{D}\}$ and is the same for all matches. The signal $\mathcal{C}$ is interpreted as a "good" signal, the signal $\mathcal{D}$ is interpreted as a "bad" signal. Let, for all matches between i and $j, i, j \in \mathcal{N}$,

$$\begin{aligned}
\alpha &= \Pr(L_i \text{ contacts receive signal } \mathcal{C} \mid j \text{ reports } C) \\
1-\alpha &= \Pr(L_i \text{ contacts receive signal } \mathcal{D} \mid j \text{ reports } C) \\
\beta &= \Pr(L_i \text{ contacts receive signal } \mathcal{C} \mid j \text{ reports } D) \\
1-\beta &= \Pr(L_i \text{ contacts receive signal } \mathcal{D} \mid j \text{ reports } D).
\end{aligned}$$

I assume $\beta < \alpha$. It is more likely that signal $\mathcal{C}$ results if C was reported than if D was chosen. All contacts of a network member receive the same signal. Each agent receives $\overline{L}_i$ signals and each agent can identify the network member to whose behavior a given signal is related. Moreover, after a match between i and j, it is common knowledge for j and all the network members in the set $\mathcal{L}_i$ which signal the agents in the set $\mathcal{L}_i$ observe. Similarly for i and the network members in the set $\mathcal{L}_j$.

The game is now as follows: assumptions (N1)-(N5) from section 2 hold. The only difference is that information transmission is noisy. For the repeated interaction in periods $t = 1, 2, ...$, for any network choice $\mathcal{L}$, I consider the following equilibria.

($\mathcal{E}1$) agents outside the network choose D in each period

($\mathcal{E}2$) agents inside the network start the repeated interaction by playing C. An agent inside the network continues to choose C if he receives the signal $\mathcal{C}$ about the previous behavior of his new partner. If an agent does not receive any signal about his new partner, he chooses D.

($\mathcal{E}3$) if an agent i receives a signal $\mathcal{D}$ about the previous behavior of an agent j, then agent j is punished by i. The punishment of i lasts forever after a deviation, that is, all agents who receive signal D about j punish j whenever they meet j.

($\mathcal{E}4$) information transmission: if in a match between i and j both agents choose C, then both agents send the message C to their respective contacts. If i deviates while j plays C then i sends the message C while j sends D. Once punishment for an agent i takes place, i sends the message C to all his L_i contacts for the rest of the game. Messages produce signals according to the signal technology described afore.

($\mathcal{E}5$) equilibrium strategies for all network members are symmetric, that is, all agents use the same strategies.

Requirement ($\mathcal{E}4$) contains the analogue to assumption (NO CONTAG) in section 2: punishments are not contagious. Moreover, I assume that (EXTREMAL) from the previous section holds: agents choose contacts which induce the most severe punishment on their partners. I also assume that all network members choose the same number of contacts, $L_i = L$ for all $i \in \mathcal{N}$. In the rest of this section I analyze the conditions for $(\mathcal{E}1) - (\mathcal{E}5)$

to be an equilibrium in the repeated game. I also characterize the communication network $\mathcal{L}$ which maximizes the network members' overall surplus and compare the result to the communication network $\mathcal{L}$ which arises from network members' private incentives under assumption (EXTREMAL).

4.2 Analysis of Equilibria with Noisy Communication

In contrast to the previous section, not only the number of contacts each agent i has is important for the analysis. Since each network member may have to punish other agents in each period with positive probability, it is also important, how many other agents have contacts to agent i. Recall that $\overline{L}_i$ denotes the number of agents in the network who have contacts to i, that is $\overline{L}_i := \{\# \cup_j \mathcal{L}_j | i \in \mathcal{L}_j\}$ where # denotes the cardinality of the set $\cup_j \mathcal{L}_j$. Let $p(\overline{L}_i)$ denote the probability that agent i receives a signal $\mathcal{C}$. Given that these other network members did actually cooperate, $p(\overline{L}_i)$ is a function of α. The exact expression for $p(\overline{L}_i)$ is complicated and depends on $\mathcal{L}$, that is, the exact pattern of contact choices in period 0. However, it clearly holds that $p'(\overline{L}_i) < 0$ and $0 < p(\overline{L}_i) < 1$ for $\overline{L}_i > 0$. That is, the more other agents chose agent i as a close friend, the lower the probability that agent i does *not* have to fulfill any punishment obligations.

Denote by V_i^+ the payoff from the proposed equilibrium strategy profile. It is given by

$$V_i^+ = (1-\delta)\frac{N-1}{I-1}a + \delta\alpha p(\overline{L}_i)V^+ + \delta(1-\alpha)p(\overline{L}_i)\frac{N-L_j-1}{I-1}a, \tag{4}$$

where L_j are the contacts each partner $j \in \mathcal{N}$ of i has. Using symmetry, $L_j = L$ for all $j \in \mathcal{N}$, this can be rewritten as

$$V_i^+ = \frac{[(1-\delta)(N-1) + \delta p(\overline{L}_i)(1-\alpha)(N-L-1)]a}{(I-1)(1-\delta\alpha p(L))}. \tag{5}$$

It is easy to show that this expression is strictly monotonically decreasing in L and in $\overline{L}_i$. The more network members have contacts to i, the more often i has to punish another network member. The larger the number of contacts i's partners $j \neq i$ have to *other* agents, the more often i is punished. In both cases, an increase in the respective number of contacts lowers i' s payoff. Note that V_i^+ does not depend on the number of contacts i has to other network members.

Moreover, the following incentive constraint has to hold to fulfill $(\mathcal{E}2)$:

$$V_i^+ \geq (1-\delta)\frac{N-1}{I-1}c + \delta\beta p(\overline{L}_i)V_i^+ + \delta(1-\beta)p(\overline{L}_i)\frac{N-L_j-1}{I-1}a. \quad (6)$$

Note that the probability $p(\overline{L}_i)$ is the same on both sides of that equation: $p(\overline{L}_i)$ denotes the probability that agent does *not* have to punish other agents he gets matched to and thus obtains a positive payoff from the interaction with them. Using the expression for V_i, we obtain that

$$L \geq \frac{(N-1)(c-a)(1-\delta\alpha p(\overline{L}_i))}{(\alpha-\beta)\delta a p(\overline{L}_i)} \quad (7)$$

has to hold to sustain cooperation, given the stage of contact formation in period 0. Note that L is increasing in $\overline{L}_i$. The more network members have contacts to i, the higher is his deviation payoff relative to the payoff from cooperation. Hence a larger number of contacts outgoing from network members, L, is required. Note that $(\mathcal{E}3), (\mathcal{E}4)$ and $(\mathcal{E}1)$ are satisfied by the same arguments as in section 3.

If the network aims to maximize overall surplus of its members, it maximizes in the game of contact formation in period 0 each network member's utility. Since all that matters are the numbers of ingoing and outgoing contacts L_N induced by a network $\mathcal{L}$, the network maximizes

$$\max_{L_N} \quad V_i^+ \qquad \text{for all } i \in \mathcal{N} \text{ subject to (7).}$$

We know that the objective function V_i^+ is strictly decreasing in $L_j, j \neq i$ and $\overline{L}_i$ for all i. On the other hand, the incentive constraint (7) has to hold. Suppose, for simplicity, that the network maximizes overall welfare of its members by choosing L_N such that $\overline{L}_i = L_j = L, j \neq i$. This means that each network member has the same number of incoming and outgoing contacts. This is the case, for example, if all network members have contacts to all other network members. Then, the incentive constraint reads as

$$\frac{ap(L)L}{(1-\delta\alpha p(L))} \geq \frac{(N-1)(c-a)}{(\alpha-\beta)\delta}. \quad (8)$$

If $L = 0$, the left-hand side of equation (8) is zero, hence there must be a choice of contacts with $L > 0$. On the other hand note that the left-hand side of the equation (8) is strictly increasing in L! Hence there exists a unique L^* with $0 < L^* < N-1$ such that this incentive constraint is satisfied indeed for any strictly decreasing function $p(L)$. Moreover, since each network member's utility is decreasing in L, it is optimal to choose L^* as the number of contacts agents have and agents receive.

Proposition 3. *Suppose that the network maximizes the surplus of its members by choosing* $\overline{L}_i = L_i = L$ *for all* i.

(i) The strategies $(\mathcal{E}1) - (\mathcal{E}5)$ *form an equilibrium in the infinitely repeated game in periods* $t = 1, 2, ...$ *if every agent has* L *contacts, where* L *satisfies*

$$\frac{ap(L)L(1-\delta)}{(1-\delta\alpha p(L))} \geq \frac{(N-1)(1-\delta)(c-a)}{(\alpha-\beta)\delta}$$

Punishment is induced by bad signals, $\mathcal{D}$, *providing incentives for cooperation which occurs on good signals,* $\mathcal{C}$.

(ii) Equilibrium payoffs are equal to

$$V_i^+ = \frac{[(1-\delta)(N-1) + \delta p(L)(1-\alpha)(N-L-1)]a}{(I-1)(1-\delta\alpha p(L))}. \tag{9}$$

for all $i \in \mathcal{N}$ *and are monotonically decreasing in* L.

(iii) The optimal number of contacts for each network member L^* *is given by the unique number* L^* *that solves*

$$\frac{ap(L^*)L^*(1-\delta)}{(1-\delta\alpha p(L^*))} = \frac{(N-1)(1-\delta)(c-a)}{(\alpha-\beta)\delta}. \tag{10}$$

The proposition states that it may well be optimal to restrict the number of contacts and close contacts each agent has. In particular,

$$L^* < N-1 \quad \Leftrightarrow \quad a < \frac{(c-a)(1-\delta\alpha p(N-1))}{p(N-1)(\alpha-\beta)\delta}.$$

In that case it is optimal to have fewer than $N-1$ contacts in the network.[9] As one easily checks, the following comparative statics results hold.

$$\frac{\partial L^*}{\partial a} < 0, \quad \frac{\partial L^*}{\partial \delta} < 0, \quad \frac{\partial L^*}{\partial c} > 0, \quad \frac{\partial L^*}{\partial N} > 0, \quad \frac{\partial L^*}{\partial \alpha} < 0, \quad \frac{\partial L^*}{\partial \beta} > 0 \tag{11}$$

The optimal number of contacts decreases as α increases (β decreases). The intuition for this would be that less noise in communication reduces the need for powerful punishments so less close contacts are necessary to sustain cooperation. On the other hand,

[9] In the language of graph theory, the optimal network is not connected, i.e. there is not a path between every pair of agents. Connectedness is a standard assumption in much of recent work on social learning and local interaction; see e.g. Anderlini and Ianni [3], Ellison [9] or Ellison and Fudenberg [11].

network size N increases, many contacts are needed: as N increases, a noncooperator finds more easily partners for future cooperation. This increases the incentives to deviate from the cooperative equilibrium. Then, more contacts are needed to induce more severe punishment. As the incentives for deviation, c, increases, the number of contacts must be higher as well.

The *private incentives* for network formation differ significantly from the social incentives. It is easy to see that, for each network member, a choice of $N-1$ contacts is again an equilibrium of the network formation game in period 0 if the choice of contacts is supposed to induce extremal equilibria. This holds since each network members payoff function V_i^+ does *not* depend on the number of contacts agent i has to other agents. The payoff of network member i is decreasing in the number of contacts other network members have to i and is also decreasing in the number of contacts other partner have. But the payoff of i does not depend on L_i.

5 Gossip

In the previous section, the distribution of the signal depended only on the outcome of any given match. In this section I add more noise to the situation and model network gossip. I assume that each agent is informed about the behavior of a friend's partner not only through a more or less informative signal which is generated from any given match. Rather, each agent receives a noisy message from each other network member. The interpretation is that even network member who did not observe an agent's behavior gossip about what that agent did.

Formally, for any given agent i, the partner matched to i receives now

- one signal about i's behavior which is informative according to the above defined probabilities α and β. It still holds that $\alpha > \beta$. Suppose for simplicity that $\alpha = 1$ and $\beta = 0$.
- and also signals from all the other $N-2$ agents in the network. Those agents did not observe what i did in the previous period. So, they gossip and transmit their gossip to the new partner of i: each of the $N-2$ signals can be $\mathcal{C}$ with probability γ and $\mathcal{D}$ with probability $1-\gamma$. The probability γ is not conditioned

on the action agent i and his partner took in the previous period. An agent does not know the source of the signal: he does not know if any of the $N-1$ signals is generated by gossip or by the last match of agent i. The signal technology is hence anonymous in the sense of Green [14].

Hence, each agent receives $N-1$ signals about the behavior of his new partner. Each agent then uses a simple rule to evaluate the $N-1$ signals and to condition his future actions on the signals.

- If a sufficiently large share $K \in (0,1)$ of the signals is $\mathcal{C}$, an agent cooperates with his new partner. Otherwise, he chooses action D.

Let $|\mathcal{C}|$ denote the number of signals with the value $\mathcal{C}$ an agent received about his new partner and define, for all $i, j \in \mathcal{N}$,

$$\begin{aligned} \widehat{\alpha} &= \Pr(L_i \text{ contacts receive at least } (N-1)\cdot K \text{ signals } \mathcal{C} \mid j \text{ reports } C) \\ \widehat{\beta} &= \Pr(L_i \text{ contacts receive at least } (N-1)\cdot K \text{ signals } \mathcal{C} \mid j \text{ reports } D). \end{aligned}$$

I assume $\widehat{\alpha} > \widehat{\beta}$. It is more likely that signal $\mathcal{C}$ results relatively more often if C was chosen than if D was chosen. All network members receive the same signals. I shall for further use rewrite these probabilities as

$$\widehat{\alpha} = \Pr(|\mathcal{C}|/(N-1) \geq K|C)$$

and

$$\widehat{\beta} = \Pr(|\mathcal{C}|/(N-1) \geq K|D).$$

Since these probabilities are the same for $i, j \in \mathcal{N}, j \neq i$, subscripts are omitted. Basically, we have now a situation similar to the one in section 4, but with "more" noise.

Given the probabilities $\widehat{\alpha}$ and $\widehat{\beta}$, one can repeat the exercise from the previous section and solve for the number of L contacts each agent has. I consider the same equilibria as in the previous section and impose the same assumptions on contact choice.

I denote with $q(\overline{L}_i)$ the probability that a network member does receive enough signals $\mathcal{C}$ so that he does not have to punish another network member.

I now analyze the possibilities for network cooperation as N gets large. Recall that the incentive constraint is given by

$$L \geq \frac{(N-1)(c-a)(1-\delta\widehat{\alpha}p(\overline{L}_i))}{(\widehat{\alpha}-\widehat{\beta})\delta a p(\overline{L}_i)} \tag{12}$$

for all $i \in \mathcal{N}$. This can be rewritten as

$$(\widehat{\alpha}-\widehat{\beta}) \geq \frac{(N-1)(c-a)(1-\delta\widehat{\alpha}q(\overline{L}_i))}{aq(\overline{L}_i)L\delta}. \tag{13}$$

for all $i \in \mathcal{N}$. I use this incentive constraint to show that cooperation fails as N gets large.

Proposition 4. *There exists $\overline{N}$ such that for all $N \geq \overline{N}$ cooperation cannot be sustained.*

Proof. Note that

$$\widehat{\alpha} = \Pr(|\mathcal{C}|/(N-1) \geq K|C) = \Pr(|\mathcal{C}|/ \geq K(N-1)-1|C).$$

This follows from the fact that in case of the partner's action being C, any new partner of any of the agents involved in the match obtains one signal $\mathcal{C}$ for sure (Recall that $\alpha = 1$). This lowers the critical bound of other signals an agent has to obtain to continue with cooperation.

On the other hand, since $\beta = 0$, we have

$$\widehat{\beta} = \Pr(|\mathcal{C}| \geq K(N-1)|D) = \Pr(|\mathcal{C}| \geq K(N-1)|D).$$

The argument is now that $\lim_{N\to\infty} |\widehat{\alpha}-\widehat{\beta}| = 0$ while the right hand side of (13) tends to infinity as $N \to \infty$. To see this, note that

$$\widehat{\beta} = 1 - \sum_{k=1}^{K(N-1)} \binom{K(N-1)}{k} \gamma^k (1-\gamma)^{K(N-1)-k}$$

and

$$\widehat{\alpha} = 1 - \sum_{k=1}^{K(N-1)-1} \binom{K(N-1)-1}{k} \gamma^k (1-\gamma)^{K(N-1)-1-k}.$$

Then, $\widehat{\alpha} - \widehat{\beta}$ is given by

$$\sum_{k=1}^{K(N-1)} \binom{K(N-1)}{k} \gamma^k (1-\gamma)^{K(N-1)-k} -$$

$$\sum_{k=1}^{K(N-1)-1} \binom{K(N-1)-1}{k} \gamma^k (1-\gamma)^{K(N-1)-1-k}.$$

Note that

$$\sum_{k=1}^{K(N-1)} \binom{K(N-1)}{k} \gamma^k (1-\gamma)^{K(N-1)-k} =$$

$$\sum_{k=1}^{K(N-1)-1} \binom{K(N-1)-1}{k} \gamma^k (1-\gamma)^{K(N-1)-1-k} + \gamma^{K(N-1)}.$$

Hence, $|\widehat{\alpha} - \widehat{\beta}| = \gamma^{K(N-1)}$ and $\lim_{N\to\infty} |\widehat{\alpha} - \widehat{\beta}| = 0$. So there exists $\varepsilon > 0$ such that $|\widehat{\alpha} - \widehat{\beta}| < \varepsilon$ for all $N \geq \overline{N}$. Moreover, as $N \to \infty$ the right hand side of (13), tends to infinity. This holds since all other expressions in the denominator of the expression on the right hand side of that equation cannot tend to infinity as well as $N \to \infty$. Moreover, no expression in the numerator can go to zero as $N \to \infty$. But since the left hand side of the equation cannot be larger than ε, there exists $\overline{N}$ such that for all $N \geq \overline{N}$ equation (13) does not hold.

It is straightforward to extend the Proposition for any $\alpha \in (0,1)$ and any $\beta \in (0,1), \alpha > \beta$.

□

If there is a cost for network formation, the network would not form if $N \geq \overline{N}$! The finding also suggests why in many communities we do *not* observe decentralized cooperation networks. If the number of community members gets large, other institutions such as courts might be needed (see Milgrom, North and Weingast [19]). Courts use a huge legal apparatus which does not rely on decentralized communication alone.

6 Discussion

I briefly discuss the robustness of these results with respect to assumption (NO CONTAG). This assumption prevents contagious punishments. Without this assumption, it is not clear if agents fulfill their punishment obligations. I shall demonstrate how the qualitative results of this paper hold even without assumption (NO CONTAG).

Consider first the analysis and results of section 3, the situation without noise. The relevant condition is that the payoff from punishing a noncooperator has to be higher than the payoff from playing C:

$$0 + \frac{\delta}{1-\delta}\left[\frac{I-N}{I-1}0 + \frac{L_j+1}{I-1}0 + \frac{N-L_j-2}{I-1}a\right] \geq \tag{14}$$
$$b + \frac{\delta}{1-\delta}\left[\frac{I-N}{I-1}0 + \frac{1}{I-1}0 + \frac{N-2}{I-1}a\right].$$

There are two possibilities to satisfy this constraint without making assumption (NO CONTAG). First, the payoff b from playing C when my partner plays D can be made small enough to provide the correct incentives. This is the same requirement as in Kandori [17] who needs this assumption to make his contagion mechanism work (see section 3.1). A second possibility would be the incentive constraint for L_j and derive an upper bound $\widehat{L}$ on the number of contacts. Then, we would have two restrictions on the number of contacts. First, as shown in section 3, the number of contacts has to be high enough to ensure cooperation:

$$L_j \geq L^{\circ} = \frac{(N-1)(1-\delta)(c-a)}{\delta a}. \tag{15}$$

Second, the number of contacts has to be less than $\widehat{L}$ to ensure network member's participation in a punishment stage of the game. Hence, $L^{\circ} < \widehat{L}$ would have to hold for such an equilibrium to exist. Suppose that this condition holds and that network members choose contacts which induce extremal equilibria (assumption (E6)). This numbers of contacts is then given by $\widehat{L}$. This is also a number which is optimal from a social point of view. Hence, private and social incentives overline as in section 3.

Consider now the model with noise. Recall that network members' payoffs are decreasing in the number of contacts their partners have. The number of contacts has to be high enough to ensure cooperation. On the other hand, punishment has to be ensured. Again, this can be achieved by making b sufficiently small. Or, the number of contacts has to be low enough to ensure network members' participation in the punishment but high enough to sustain cooperation in the first place. Clearly, since network members payoffs are decreasing in the number of contacts of their partners, it is socially optimal to choose the lower number of the two which is just enough to support cooperation. Private incentives would again go in the other direction and agents would choose too much contacts in the game of contact formation in stage 0. In other words: the basic results from section 4 continue to hold. This is also valid for the main result in section 5.

7 Conclusion

This paper models a communication network and shows how the optimal number of contacts in such a network is affected when communication is noisy. If communication is noisy, it is not optimal to have close contacts to all network members. Moreover, as the number of network members gets large, cooperation in the network fails. If network formation is costly, we cannot expect a network to form.

There are many other institutions sustaining cooperation in such situations, for example courts. It would be interesting to examine under which circumstances we observe decentralized institutions such as communication networks and under which circumstances we observe the existence of centralized institutions such as courts. Moreover, when would we expect a mechanism without any institution as the contagion mechanism proposed by Kandori and Ellison?

Another line of research could focus on communication networks in a setting where there is additional source of externalities among network members. While in my setting honest information transmission is always a best response, one can easily imagine a situation where an agents payoff strictly increases if an other agent gets punished. This could be the case if there is not only a double-sided moral hazard problem but also if there is the situation that a network member benefits from another network member being punished. Suppose, for example, that there is some competitive relation among some or all network members. Hence, there is a direct incentive to assault another network member even if that network member did cooperate. The network would then serve not only as a means to support cooperation but information flow is then also a means to hurt a potential competitor.

References

[1] D. Abreu. Extremal Equilibria of Oligopolistic Supergames. *Journal of Economic Theory*, (39):191–225, 1986.

[2] D. Abreu. Towards a Theory of Discounted repeated games. *Econometrica*, (56):383–396, 1988.

[3] L. Anderlini and A. Ianni. Path-dependence and Learning from Neighbors. *Games and Economic Behavior*, (13):141–178, 1996.

[4] R. Aumann and R. Myerson. *Endogenous formation of links between players and coalition. An application of the Shapley value,* in *The Shapley value* (A.Roth, Ed.),. Cambridge University Press, Cambridge, 1988.

[5] V. Bala and Goyal S. A noncooperative model of network formation. *Econometrica*, (68):1181–1229, 2000.

[6] J. Boissevain. *Friends of Friends. Networks, Manipulations and Coalitions.* Basil Blackwell, Oxford, 1974.

[7] S.A. Boorman. A combinatorial Model for Transmission of Job Information through Contact Networks. *Bell Journal of Economics*, (6):216–49, 1975.

[8] J. Coleman. Social Capital in the Creation of Human Capital. *American Journal of Sociology*, (94):S95–S120, 1988.

[9] G. Ellison. Learning, Local Interaction and Coordination. *Econometrica*, (61):1047–1072, 1993.

[10] G. Ellison. Cooperation in the Prisoner's Dilemma with Anonymous Random Matching. *Review of Economic Studies*, (61):567–588, 1994.

[11] G. Ellison and D. Fudenberg. Rules of Thumb for Social Learning. *Journal of Political Economy*, (101):612–644, 1993.

[12] R. Gibbons. *Trust in Social Structures: Hobbes and Coase Meet Repeated Games,* in *Trust in Society* (Karen Cook, Ed.). Russell Sage Foundation, New York, 2001.

[13] M. Granovetter. *The Economic Sociology of Firms and Enterprise,* in *The Economic Sociology of Immigration. Essays on Networks, Ethnicity, and Entrepreneurship* (Alejandro Portes, Ed.). Russell Sage Foundation, New York, 1995.

[14] E. Green. Non-cooperative price taking in large dynamic markets. *Journal of Economic Theory*, (22):155–182, 1980.

[15] A. Greif. Contract Enforceability and Economic Institutions in Early Trade: The Magribi Traders' Coalition. *American Economic Review*, (83):525–548, 1993.

[16] M. Jackson and A. Wolinsky. A Strategic Model of Economic and Social Networks. *Journal of Economic Theory*, (77):44–74, 1996.

[17] M. Kandori. Social Norms and Community Enforcement. *Review of Economic Studies*, (59):63–80, 1992.

[18] R. Kranton and D. Minehart. Buyer-Supplier Networks. *American Economic Review*, (91):485–508, 2001.

[19] P. Milgrom, D. North, and B. Weingast. The Role of Institutions in the Revival of Trade: The Law Merchant, Private Judges, and the Champagne Fairs. *Economics and Politics*, (1):1–23, 1990.

[20] A. van den Nouweland. *Games and Networks in Economic Situations*. PhD thesis, University of Tilburg, 1993.

[21] A. Portes and J. Sensenbrenner. Embeddedness and Immigration: Notes on the Social Determinants of Economic Actions. *American Journal of Sociology*, (98):1320–1350, 1993.