Can Carbon Tax Eat Opec's Rents?

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Abstract

We study the effects of carbon dioxide taxation on fossil-fuel markets where the supply is regulated by a resource-exporting cartel. To clarify some previous results, we show that the optimal tax set by a coalition of buyers is not a neutral Pigouvian tax but a combination of a tariff and Pigouvian tax. Because of the tariff element, the tax can shift more rents from the cartel than the pollution causes damage-related costs. Thus, the pollution problem accompanied by the coordination of carbon taxation can entail net benefits for the buyer side.

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1 Introduction

Fuel taxes are controversial. Some find them punitively high and argue that they should be reduced in response to increased fuel prices. Others argue that fuel taxes should be increased to reduce traffic congestion and various environmental externalities, including global warming which fossil-fuel burning possibly contributes to.[1] Politicians often like fuel taxes because they have good revenue-raising properties and since the threat of global warming provides a prominent efficiency reason for taxing the carbon content of fuels.[2] However, some recent studies provide support for the conclusion that the market power in the resource market may reduce the appeal of the carbon tax: Cartels like Opec may react strategically and increase the producer price to receive a major part of the tax revenues that would otherwise remain in oil importing countries ([18], [20], [19]).

Our objective is to show the contrary: market power in the sense of cartelized supply can increase the appeal of the carbon tax. The reason is that when the buyer side coordinates the emission taxation, the tax includes an import-tariff element shifting rents from the resource-exporting cartel. We even show that the tax must shift more rents than the pollution causes damage-related costs if the pollution problem is not too severe. The result means that the payoff for the importing countries can exceed the level realized in the absence of the pollution - in this sense, the pollution problem accompanied by carbon taxation can entail net benefits for the buyer side.

This result should not be surprising, given the literature on optimal tariffs in exhaustible resource markets. Karp [8] and Karp and Newbery [9] show that buyers with monopsonistic

[1]During the summer and fall 2000 the soaring fuel prices led to protests across Europe and to pressure on politicians to reduces fuel taxes. See [4] and [5] for arguments in favor and agains the tax reductions.

[2]In Britain, where the gasoline excise taxes are record high, the Labour government has defended high taxes on three grounds: pollution and emissions of greenhouse gases, traffic congestion, and revenue raising (see [11]).

power can use a dynamic optimum tariff to decrease the sellers' resource rent. Analogously, coordinating carbon taxation creates a strategic incentive to use the tax as a partial import tariff.[3] What is more surprising and contradicting the static optimal tariff arguments is that the tax includes an import-subsidy, not an import-tariff, component when the pollution problem is severe. These results lead us to conclude that the optimal carbon tax generally deviates from the neutral Pigouvian tax which would indicate only the damage cost of pollution: the tax exceeds or falls short of the Pigouvian level depending on the damage-related costs.

That the buyers' optimal carbon tax deviates from a neutral Pigouvian tax is in sharp contrast with a recent paper by Rubio and Escriche [12] who conclude that "the tax corrects only the market inefficiency caused by the stock externality, and has no effect on the cartel's monopoly power" (p. 298).[4] We explicitly consider the cartel's pre- and after-tax producer price path and show that the entire path must fall because of the taxation, provided that the pollution damage is not too severe. The result also contradicts the Wirl's [18] conclusion that the buyers' emission tax is purely Pigouvian and that the sellers' strategic behavior can lead to a loss of tax revenues due to an increased producer price. Wirl and Dockner [19] argue that the buyer side must have Leviathan motives (tax revenues are valued as such) to limit the cartel's possibilities of receiving tax revenues. In our view, this previous literature has overemphasized the strategic advantage on the seller side and has not analyzed the tariff components the carbon tax. In a similar vein, Tahvonen [15] and [16] finds non-Pigouvian

[3] Similar issues are discussed in a static context by [2] (pp. 266-278).

[4] Authors show the equivalence of the following two equilibria. In one, the buyers' tax and sellers' price are strategically chosen. In another, buyers' do not use a tax but internalize external pollution costs in their strategic demand decisions. Because both approaches involve the same degree of coordination on the buyer side, the two equilibria are equivalent. This does not imply that the tax is a neutral Pigouvian tax in any traditional sense – the coordination on the buyer side will lead to the deviations identified here.

motives in general settings, but he does not analyze the appropriation of the cartel's rents.

We follow the previous literature by basing on the Wirl's (1994) differential game where the buyers' agency coordinates the tax and the cartel sets the producer price. The dynamic approach is needed because the payoffs on both sides depend on the sellers' unextracted resource stock. We use the same equilibrium concept as the previous literature, the Markov Perfect Nash Equilibrium (MPNE), consider only stock pollutants, and include stock-dependent extraction costs (as in [20], [12], [15]). We deviate from the previous studies in that we compare the MPNE not only to the efficient equilibrium but also directly to the cartel equilibrium in the absence of the carbon tax. The latter equilibrium seems to be the relevant pre-tax case. We also explicitly characterize the time paths implied by the MPNE, which leads to a sharper comparison of the equilibria.

The next section sets up the model and solves for two benchmark equilibria which determine the neutral Pigouvian tax and the pre-tax cartel price. Section 3 characterizes the MPNE and compares it to the efficient equilibrium. Section 4 decomposes the MPNE tax into the Pigouvian and tariff elements. Similarly, the MPNE producer price is decomposed in order to clarify the reasons for rent shifting. The equations appearing in the text as well as the propositions are derived in Appendix.

2 The Model and Benchmark Equilibria

Consumers of the resource have a linear-quadratic utility function $aq - bq^2$ for private consumption, where a and b are parameters and q is the flow of the resource consumed. The consumer price is $p + \Psi$ where p is the producer price set by the resource-exporting cartel and Ψ is the emission tax set by the buyers' agency controlling pollution damage from resource consumption on the buyer side. The solution for the individual buyers' problem implies a demand function $q(p,\Psi) \equiv (a-p-\Psi)/2b$ ($q=0$ if $q(p,\Psi)<0$). The emission tax set by the buyers' agency is a solution to

$$V_1 = \underset{\Psi}{Max} \int_0^\infty \{aq - bq^2 - pq - dz^2\} e^{-\delta t} dt \tag{1}$$

$$s.t.\ \dot{z} = q,\ z(0) = z_0, q = q(p,\Psi), \tag{2}$$

where $\delta > 0$ is the discount rate, d is a damage parameter, and dz^2 is the buyers' pollution-damage flow from the pollutant stock z. Note that the tax revenues are reimbursed as lump-sum transfers to consumers, meaning that the after-tax surplus is independent on the tax bill.

The producer price set by the sellers' cartel is a solution to

$$V_2 = \underset{p}{Max} \int_0^\infty \{pq - q(c_1 - c_2 x)\} e^{-\delta t} dt \tag{3}$$

$$s.t.\ \dot{x} = -q,\ x(0) = x_0, q = q(p,\Psi), \tag{4}$$

where x is the sellers' stock of the nonrenewable resource. Extraction costs are given by $q(c_1 - c_2 x)$ where $c_1 > 0$, $c_2 > 0$ and $c_1 - c_2 x_0 > 0$. We assume throughout the paper that $c_1 > a$, meaning that the physical resource constraint $x \geq 0$ will not be binding. We thus focus on the economic exhaustion (cf. [6]), which is empirically more relevant than the physical exhaustion (see [14] for a discussion).

To show the deviation from the usual Pigouvian tax and the appropriation of the cartel's rents in the game between the buyers' agency and the cartel, we need two benchmark equilibria: the *efficient* and the *cartel* equilibrium. The former maximizes the total surplus from trade with the polluting resource. The latter is the cartel's optimum in the absence of the buyers' agency controlling pollution and the tax (i.e., in the absence of climate change treaty).[5] Consider first the efficient equilibrium and let φ be the resource rent which gives the seller's marginal valuation of the unextracted resource stock. The efficient equilibrium

[5] While Opec is not a textbook-like cartel, the price elasticity of crude oil is an important determinant of

is fully characterized by the following set of equations:

$$x_e(t) = (x_0 - x_e^\infty)e^{\alpha_e t} + x_e^\infty \tag{5}$$

$$q_e(t) = -\alpha_e(x_0 - x_e^\infty)e^{\alpha_e t} \tag{6}$$

$$\Psi_e(t) = \frac{2d}{\alpha_e - \delta}(x_0 - x_e^\infty)e^{\alpha_e t} + \frac{2d}{\delta}z_e^\infty \tag{7}$$

$$\varphi_e(t) = \frac{c_2\alpha_e}{\alpha_e - \delta}(x_0 - x_e^\infty)e^{\alpha_e t} \tag{8}$$

$$p_e(t) = 2(x_0 - x_e^\infty)(b\alpha_e - \frac{d}{\alpha_e - \delta})e^{\alpha_e t} + a - \frac{2d}{\delta}z_e^\infty \tag{9}$$

where $\alpha_e \equiv \frac{\delta}{2} - \frac{1}{2}[\delta^2 + \frac{2}{b}(2d + \delta c_2)]^{1/2} < 0$, $z_e^\infty = x_0 + z_0 - x_e^\infty$, and where the steady state resource stock is $x_e^\infty = \frac{2d(x_0+z_0)+\delta(c_1-a)}{c_2\delta+2d}$. Note that the steady state tax equals the present value of steady state marginal damages, $\Psi_e^\infty = \frac{2d}{\delta}z_e^\infty$, and that the steady state producer price is the choke price less the tax, $p_e^\infty = a - \Psi_e^\infty$. Throughout the paper, we consider interior steady state equilibria in the sense that $p_e^\infty > 0$. This characterization of the efficient equilibrium implies that

(i) the consumer price increases toward the choke price,

(ii) the resource use and resource rent decline toward zero,

(iii) and the producer price and the tax both increase.

The first property follows because there is no resource use in the long run. The value of the unextracted resource declines for two reasons. First, the sellers' unit cost of extraction increases as the stock is depleted. Second, the smaller is the unextracted stock of the resource the larger is the pollutant stock, which increases damage-related costs. For the first reason, sellers' resource rent declines, and for the latter reason, the emission tax increases.

An important implication of the efficient equilibrium is that the tax equals the present

the oil supply [5]. An oligopolistic structure for the oil market would lead to serious complications even in the absence of the pollution problem (see [14]). For this reason and for ease of comparison with the previous studies, we assume that the resource is supplied by a monopoly.

value of marginal damages at any point in time,

$$\Psi_e(t) = \int_t^{\infty} 2dz_e(\tau)e^{-\delta(\tau-t)}d\tau, \tag{10}$$

where $z_e(\tau) = x_0 + z_0 - x_e(\tau)$. This expression is our benchmark for a neutral Pigouvian tax. Any deviation from this principle implies the presence of non-Pigouvian motives.

Consider then the cartel equilibrium, i.e. the pure monopoly case, which is obtained by solving (3)-(4) in the absence of the tax (the consumer demand is $q(p,0)$). Although the cartel's price policy can be written as a state-dependent decision rule, it proves useful to express the equilibrium time paths as follows:

$$x_c(t) = (x_0 - x_c^{\infty})e^{\alpha_c t} + x_c^{\infty} \tag{11}$$

$$q_c(t) = -\alpha_c(x_0 - x_c^{\infty})e^{\alpha_c t} \tag{12}$$

$$\varphi_c(t) = (4b\alpha_c + c_2)(x_0 - x_c^{\infty})e^{\alpha_c t} \tag{13}$$

$$p_c(t) = 2b\alpha_c(x_0 - x_c^{\infty})e^{\alpha_c t} + a \tag{14}$$

where $\alpha_c \equiv \frac{\delta}{2} - \frac{1}{2}[\delta^2 + \frac{\delta c_2}{b}]^{1/2} < 0$ and $x_c^{\infty} = (c_1 - a)/c_2$. These imply that in the cartel equilibrium

(i) the resource use and resource rent decline to toward zero,

(ii) and the producer price increases toward the choke price.

Our benchmark for a cartel price will be

$$p_c(t) = p_s(t) + \frac{1}{2}\int_t^{\infty} q_c(\tau)c_2 e^{-\delta(\tau-t)}d\tau, \tag{15}$$

where $p_s(t) \equiv \frac{1}{2}(a + c_1 - c_2 x(t))$ is the price that a myopic or static monopoly would charge. The integral part indicates the shadow value of the resource stock: a marginal quantity of the stock sold today reduces future earnings by $q_c c_2$ through increased extraction costs, which is reflected in the current price. Any deviation from the principle expressed in (15) implies that the cartel's rents are altered by the tax used on the buyer side.

3 Markov Perfect Nash Equilibrium

The equilibrium of the game (1)-(4) can be solved after defining the sequence of moves and the strategy spaces for the players, i.e. for the buyers' agency and the cartel (see [1] and [3] for texts on differential games). We consider simultaneous moves and focus on stationary Markov strategies. Moreover, to ensure the existence of equilibrium independently of the stock level, we consider linear Markov strategies.[6] These assumptions together with the linear-quadratic structure of the game, imply that V_1 and V_2 defined in (1) and (3) take the following forms: $V_1(x) = A_1 + A_2x + \frac{1}{2}A_3x^2$ and $V_2(x) = B_1 + B_2x + \frac{1}{2}B_3x^2$, where the parameters are determined using standard procedures. The equilibrium (linear) Markov strategies are $\Psi(x) = A_2 + A_3x$ and $p(x) = \frac{1}{2}[B_2 - A_2 + a + c_1 + (B_3 - A_3 - c_2)x]$.

For ease of comparison with our benchmark equilibria, we express the time paths implied by the Markov Perfect Nash Equilibrium (MPNE) as follows:

$$x_n(t) = (x_0 - x_e^\infty)e^{\alpha_n t} + x_e^\infty \tag{16}$$

$$q_n(t) = -\alpha_n(x_0 - x_e^\infty)e^{\alpha_n t} \tag{17}$$

$$\Psi_n(t) = A_3(x_0 - x_e^\infty)e^{\alpha_n t} + \Psi_e^\infty \tag{18}$$

$$\varphi_n(t) = B_3(x_0 - x_e^\infty)e^{\alpha_n t} + \varphi_e^\infty \tag{19}$$

$$p_n(t) = \frac{1}{2}(B_3 - c_2 - A_3)(x_0 - x_e^\infty)e^{\alpha_n t} + p_e^\infty \tag{20}$$

where $\alpha_n = (A_3 + B_3 - c_2)/4b < 0$. Some of the properties of the MPNE will depend on the damage parameter d. We say that the damage is small if $d < \underline{d}$, intermediate if $\underline{d} < d < \overline{d}$, and large if $\overline{d} < d$, where $\underline{d} \equiv \frac{\delta}{2}[b\delta + \frac{c_2}{2} - (b^2\delta^2 + b\delta c_2)^{1/2}]$ and $\overline{d} \equiv \frac{\delta}{8}[-b\delta + 2c_2 + (b^2\delta^2 + 4b\delta c_2)^{1/2}]$.

The MPNE has the following characteristics:

[6] [18] and [19] consider nonlinear strategies in a similar game and find that that they are less efficient in a Pareto sense. While this is no reason to rule out nonlinear strategies, the linear strategies are know to be global: their domain of definition extends (or can be extended) to the entire state space [17].

(i) the MPNE approaches the efficient equilibrium;

(ii) the MPNE is more conservative than the efficient equilibrium;

(iii) if $d \leq \underline{d}$*, the MPNE is more conservative than the cartel equilibrium;*

(iv) for $d < \underline{d}$*,* $\Psi_n(t)$ *decreases and* $p_n(t)$ *increases over time;*

(v) for $\underline{d} < d < \overline{d}$*, both* $\Psi_n(t)$ *and* $p_n(t)$ *increase over time;*

(vi) for $\overline{d} < d$*,* $\Psi_n(t)$ *increases and* $p_n(t)$ *decreases over time.*

The first observation implies that the strategic behavior can alter only the dynamics of variables but not their steady state levels (compare (5)-(9) and (16)-(20)). The reason for the steady-state equivalence is that there is no resource use in the long run: in both cases the steady state net payoffs are the same implying the equivalence of trade-offs determining the steady state. The second observation follows from $\alpha_n > \alpha_e$ and it implies that for any given resource stock, the resource use in the MPNE is below the efficient resource use. Because the total amount of the resource used is the same in both cases, the time path for $q_e(t)$ must first exceed but later fall below the path for $q_n(t)$. The third observation is qualitatively similar to the second but the paths $q_n(t)$ and $q_c(t)$ need not intersect because the total MPNE resource use is lower. The fourth observation implies that the MPNE tax approaches the neutral Pigouvian tax Ψ_e^∞ from above when the damage parameter is small. From this, one might conjecture that the MPNE tax along the transition is above the neutral Pigouvian tax. In the sequel we show that this conjecture is correct when d is small. The last observation implies that the tax alone is responsible for choking off the demand, as opposed to the efficient case where both the tax and the producer price increase.

4 Decomposing the Tax and Producer Price

We can now address the following two questions: (1) is the tax Ψ_n in the MPNE a neutral Pigouvian tax in the sense of the efficient tax Ψ_e, and (2) does the tax Ψ_n appropriate part of the cartel's rents? To answer the first question, we rewrite the tax in the MPNE as

$$\Psi_n(t) = \int_t^\infty \{2dz_n(\tau) - q_n(\tau)p_n'(x_n(\tau))\}e^{-\delta(\tau-t)}d\tau, \tag{21}$$

where $z_n(\tau) = x_0 + z_0 - x_n(\tau)$. We then note, by properties (iv)-(v) of the MPNE, that the sellers' Markov strategy $p_n(x)$ is downward sloping when d is sufficiently small and, by property (vi), that $p_n(x)$ is upward sloping when d is sufficiently large. This together with expression (21) implies the following nonneutrality result.

Proposition 1 *The emission tax in the MPNE is generally not a neutral Pigouvian tax. The tax exceeds, equals, or falls short of the present value of marginal damages when* $d < \overline{d}$, $d = \overline{d}$, *or* $d > \overline{d}$, *respectively.*

The first sum in (21) is the present value of marginal damages and thus represents Pigouvian motives, as in the definition of the neutral Pigouvian tax (see (10)). The second sum in (21) is the tariff element, which would alone determine the shadow value of the resource stock for the buyer side if the damage were absent ($d = 0$). When $d = 0$ *the tax is a pure import tariff* shifting the sellers' rents by altering the timing of the resource extraction but not the total amount of the resource used. The tariff is initially high and gradually decreasing to zero, causing the extraction path be more conservative than the cartel path. The optimal tariff is determined by the trade-off between costs from delayed consumption and benefits from reduced producer price path. Another extreme is the case $d = \overline{d}$ where the rent-shifting motive is absent and *the tax is a pure Pigouvian tax* in the sense that it exactly equals the present value of marginal damages (the seller's decision rule is flat, $p_n'(x_n) = 0$).

Because damages are increasing, the tax is increasing over time. For a damage between the two extremes $d = 0$ and $d = \overline{d}$, the tax path is decreasing or increasing over time depending on whether the rent-shifting or Pigouvian motive dominates – these motives exactly cancel out each other when $d = \underline{d}$, which implies a constant tax path. For a severe damage ($\overline{d} < d$), the tax is increasing but less than the Pigouvian tax because it includes *an import-subsidy element* responding to the sellers' attempt to delay extraction through a high initial price. As opposed to the import-tariff case, it is now the seller side which has an incentive to delay the resource use.

To see the sellers' incentives more precisely consider next the MPNE producer price which we rewrite as

$$p_n(t) = p_s(t) + \frac{1}{2}\int_t^\infty \{c_2 - \Psi_n'(x_n(\tau))\} q_n(\tau) e^{-\delta(\tau - t)} d\tau, \tag{22}$$

where $p_s(t) \equiv \frac{1}{2}(a + c_1 - c_2 x(t) - \Psi_n(t))$ is again the price that a static monopoly would charge. As in our benchmark case (see eq. (15)) the producer price has the static-price and the user-cost elements, but now the latter also reflects how a marginal quantity of the stock sold today alters future earnings due to changes in the tax shifting the demand curve. Note that because Markov strategies are linear, this effect on the demand curve is pulling in one direction throughout the equilibrium path.

The producer price remains constant over time if the tax is purely Pigouvian ($d = \overline{d}$). The reason is that the increasing tax keeps the demand curve falling at a rate which exactly cancels out the changes in the static-price and user-cost elements. For a severe damage ($d > \overline{d}$), the demand curve falls faster than this benchmark rate, implying a declining producer price over time. Because the rapidly falling demand is a function of the extracted stock, the seller side has an incentive to delay the resource use, which explains the buyer's import-subsidy element in the tax discussed above. For a less severe damage ($d < \overline{d}$), the producer price is increasing over time, and the tax includes an import-tariff element.

Consider then the case where the buyers' import-tariff and Pigouvian motives exactly cancel out ($d = \underline{d}$). Because the tax remains constant, the sellers' price is determined by the static-price and traditional user-cost elements as in the benchmark case (eq. (15)). However, the entire MPNE price path must be lower than $p_c(t)$ because the MPNE is more conservative. This argument can be extended to *all* $d < \underline{d}$ as the buyers' rent-shifting motives keep the MPNE more conservative than the cartel equilibrium.

Proposition 2 *The MPNE producer price $p_n(t)$ falls short of the cartel price $p_c(t)$ for all t if $d \leq \underline{d}$ or if, for any given d, b is sufficiently large.*

Because the MPNE extraction path is more conservative than the cartel path, the falling price path implies that the sellers' payoff is unambiguously reduced. The latter part of the result can be explained by noting that a large b tends to make both price paths p_n and p_c flat − a sufficiently inelastic demand means that there is no role for intertemporal manipulation of the resource use. Because the long-run difference between the two prices is $p_n^\infty - p_c^\infty = \Psi_n^\infty > 0$, sufficiently flat price paths imply that the difference remains positive throughout the equilibrium.

Proposition 3 *For d sufficiently small, the buyers' MPNE payoff exceeds the level realized in the absence of the pollution problem and the tax.*

Note that the proposition compares two states of the world. In one, there is a pollution problem and the related coordination of the emission tax. In another, the pollution and hence the policy coordination are absent. The result is startling in that going from the latter world to the former introduces a costly pollution damage but still increases the payoff on the buyer side. The reason is that the optimally designed emission tax necessarily includes the import-tariff element which shifts more rents than the pollution causes damage. Of course, the increase in the payoff can also be achieved if the damage is absent and the buyer side controls the resource use by a pure import tariff.[7] Introducing the pollution problem together with the emission tax can reduce the payoff if the damage is significant, although the tariff element still reduces the buyers' costs compared to the neutral Pigouvian taxation.

5 Conclusions

The majority of anthropogenic emissions of carbon dioxide are caused by the combustion of fossil fuels [7]. While the prices of these fuels are to a large extent beyond the control of individual countries importing the fuels, there is no reason to believe that these prices cannot be affected by a coalition of importing countries coordinating their emission taxation. We argued that the carbon tax can be used to reduce the producer price of fossil fuels and thereby to shift resource rents from the resource-exporting countries. In fact, the optimal carbon tax for the coalition of buyers' who suffer from the pollution damage and face a cartelized supply is partly a tariff and, therefore, the tax is not purely Pigouvian. The rent-shifting result is in contrast with some recent studies elaborating this issue ([18], [20], [19], [12]) but entirely consistent with tariff retaliation arguments in trade theory ([21], ch 12), with [15] and [16], and with studies on optimal tariffs in exhaustible resource markets ([8], [9]).

An important restriction in the above literature as well as in this paper is the assumption of fully cartelized supply. One extension is the cartel-competitive fringe setup [10]. Another extension is an oligopolistic market where several producers extract their own stocks, serve

[7] However, the political costs of such a policy coordination are likely to be high in the absence of the pollution problem. Moreover, it is conceivable that for a given cost of coordination (e.g., the cost of setting up an agency coordinating actions), the cost may exceed the gains from the coordination when the pollution problem is absent − the cost may fall short of the gains in the presence of the pollution damage.

the same market and do not directly coordinate the market price. Only recently, [14] have characterized the Nash-Cournot equilibria in such a market without resting on the Nash open-loop equilibrium concept. The optimal design of the carbon tax under these market structures is left open for future research.

Appendix to Section 2

Equations (5)-(9). These can be derived by finding an equilibrium consumption trajectory under the assumption of price taking behavior on both sides of the resource market and the assumption the buyers' agency applies Pigouvian taxation to control the externality. The equilibrium conditions on the buyer side include: $\Psi = -\lambda$ and $\dot{\lambda} = 2dz + \lambda\delta$, where λ is the buyers' shadow value for the pollutant stock. The equilibrium conditions on the seller side include: $p - c_1 + c_2 x - \varphi = 0$ and $\dot{\varphi} = -qc_2 + \varphi\delta$, where φ is the sellers' shadow value of the unextracted stock (the resource rent). Using $p - c_1 + c_2 x - \varphi = 0$ and the demand equation yields $a - 2bq - c_1 + c_2 x - \varphi - \Psi = 0$. This together with $\dot{x} = \dot{\varphi} = \dot{\Psi} = 0$ and $z = x_0 + z_0 - x$ gives the steady state stock x_e^∞ which is positive by $c_1 - a > 0$. Using again $z = x_0 + z_0 - x$ we can eliminate the shadow prices Ψ and φ to write the interior equilibrium conditions as a system:

$$\begin{aligned} \dot{x} &= -q \\ \dot{q} &= [2d(x_0 + z_0 - x) - \delta(a - c_1) + 2\delta qb - \delta c_2 x]\frac{1}{2b}. \end{aligned}$$

A solution of $(\dot{x}, \dot{q})$ going to the steady state and thus satisfying transversality conditions ([13], p. 237, theorem 15) is given by (5)-(6). Using the solution (6) and $\dot{\varphi} = -qc_2 + \varphi\delta$ we obtain the solution for the resource rent, (8). Using the solution (5), equation $z = x_0 + z_0 - x$, and $\dot{\Psi} = -2dz + \Psi\delta$ yields the solution for the tax, (7). Finally, the demand equation gives the producer price, (9).■

Properties of the efficient equilibrium follow directly from (5)-(9).■

Equations (11)-(14). In an interior cartel equilibrium the sellers' value function satisfies $\delta V_2(x) = \max_p\{(p - c_1 + c_2 x - V_2'(x))(a - p)\frac{1}{2b}\}$, where the consumers' demand is $q(p, 0) = (a - p)\frac{1}{2b}$. By the linear-quadratic structure, the value function takes the form $V_2(x) = J_1 + J_2 x + \frac{1}{2}J_3 x^2$. This implies that the cartel's feedback rule is $p(x) = \frac{1}{2}[a + c_1 + J_2 + (J_3 - c_2)x]$.

Using these forms for $V_2(x)$ and $p(x)$ in $\delta V_2(x) = \max_p\{..\}$, collecting terms on both sides and equating coefficients, we obtain three equations determining the parameters J_1, J_2, and J_3:

$$\begin{aligned}
J_1 &= \frac{1}{8\delta b}(a - c_1 - J_2)^2 \\
J_2 &= \frac{(c_1 - a)(J_3 - c_2)}{4\delta b - J_3 + c_2} \\
J_3 &= 2\delta b + c_2 - 2(\delta^2 b^2 + \delta b c_2)^{1/2}.
\end{aligned}$$

By the demand equation and the feedback rule, $q = \frac{1}{4b}[a - J_2 - c_1 - (J_3 - c_2)x]$. Using this together with $V_2'(x_c^\infty) = 0$ in $\dot{x} = -q$ implies the steady state resource stock x_c^∞, and the solution (11) given in the text. The solution (11) determines the resource use (12) directly and the price (14) by the demand equation. The solution for the resource rent follows from $\varphi_c = V_2'(x)$ and $V_2'(x_c^\infty) = 0$.■

Properties of the cartel equilibrium follow directly from equations (11)-(14).■

Appendix to Section 3

Value functions. The players' value functions satisfy

$$\delta V_1(x) = \max_{\Psi}\{(a - p - V_1'(x))q(p, \Psi) - bq(p, \Psi)^2 - d(x_0 + z_0 - x)^2\} \tag{23}$$

$$\delta V_2(x) = \max_{p}\{(p - c_1 + c_2 x - V_2'(x))q(p, \Psi)\} \tag{24}$$

where $q(p, \Psi) \equiv (a - p - \Psi)/2b$ and $z = x_0 + z_0 - x$. By the linear-quadratic structure, the value functions associated with linear strategies take the forms $V_1(x) = A_1 + A_2 x + \frac{1}{2}A_3 x^2$ and $V_2(x) = B_1 + B_2 x + \frac{1}{2}B_3 x^2$, which imply that the linear feedback rules (Markov strategies) are given by $\Psi(x) = A_2 + A_3 x$ and $p(x) = \frac{1}{2}[B_2 - A_2 + a + c_1 + (B_3 - A_3 - c_2)x]$. Using these functional forms in (23)-(24), collecting terms on both sides, and equating coefficients gives the following six equations:

$$\delta A_1 = \frac{1}{16b}[(B_2+A_2)^2+(a-c_1)^2+2A_2(c_1-a)+2B_2(c_1-a)]-d(x_0+z_0)^2 \quad (25)$$

$$\delta A_2 = \frac{1}{8b}[(B_2+A_2+c_1-a)A_3+B_3(A_2+B_2+c_1-a)+c_2(a-c_1-A_2-B_2)] \quad (26)$$
$$+2d(x_0+z_0)$$

$$\delta\frac{1}{2}A_3 = \frac{1}{16b}[(B_3+A_3)^2+c_2(c_2-2(A_3+B_3))]-d \quad (27)$$

$$\delta B_1 = \frac{1}{8b}[(A_2+B_2)^2+a^2-2a(A_2+B_2+c_2)+2c_2(B_2+A_2+c_1)-{c_1}^2] \quad (28)$$

$$\delta B_2 = \frac{1}{4b}[(B_2+A_2+c_2-a)A_3+B_3(A_2+B_2+c_2-a)+c_2(a-A_2-B_2-c_2)] \quad (29)$$

$$\delta\frac{1}{2}B_3 = \frac{1}{8b}[(B_3+A_3)^2+c_2(c_2-2(A_3+B_3))] \quad (30)$$

Because (27) and (30) are nonlinear, the system has several solutions. The solution implying a stable steady state can be solved as follows. Equation (27) is quadratic in A_3 and equation (30) is quadratic in B_3. (i) Using the negative root of (27) we solve A_3 as a function of B_3, $A_3(B_3)$. (ii) Plugging $A_3(B_3)$ into the equation (30) gives two roots for B_3, of which the smaller one is chosen. A_3 and B_3 depend now only on the parameters of the model. (iii) Using these in (26) and (29) we can solve for A_2 and B_2. These solutions are unique since (26) and (29) are linear in A_2 and B_2. (iv) Given (A_2, A_3, B_2, B_3), A_1 and B_1 can be solved from equations (25) and (28). In this way, we obtain the parameters of the value functions. However, only A_3 and B_3 are needed in the sequel:

$$A_3 = \frac{4}{9}[b\delta+\frac{3}{4}c_2-3\frac{d}{\delta}-(b^2\delta^2+\frac{3}{2}b\delta c_2+3db)^{1/2}] \quad (31)$$

$$B_3 = 2A_3+\frac{4d}{\delta}.\blacksquare \quad (32)$$

Equations (16)-(20). Using the feedback rules $\Psi(x)=A_2+A_3x$ and $p(x)=\frac{1}{2}[B_2-A_2+a+c_1+(B_3-A_3-c_2)x]$ in the demand equation gives $q(x)=\frac{-1}{4b}[(A_2+B_2+c_1-a)+(A_3+B_3-c_2)x]$. This implies that the steady state resource stock is $x_n^\infty=-(A_2+B_2+c_1-a)/(A_3+B_3-c_2)$. The steady state stock also satisfies $B_2=-B_3x_n^\infty$ and $A_2=\frac{2dz_n^\infty}{\delta}-A_3x_n^\infty$, where the former follows by $V_2'(x_n^\infty) = 0$ and the latter by $\Psi(x_n^\infty) = \frac{2dz_n^\infty}{\delta}$. These three equations involving x_n^∞ can be used to verify that $x_n^\infty = x_e^\infty$. Using $q(x)$, the equation $\dot{x} = -q$, and the initial condition gives the time path for the resource stock (16). The rest of the equations in (16)-(20) follow by using the solution (16) together with $\Psi(x) = A_2 + A_3 x$, $\varphi(x) = B_2 + B_3 x$, and the feedback rule $p(x)$.■

Properties of the MPNE. Property (i) was shown in the text. Property (ii) follows from $\alpha_n > \alpha_e \Leftrightarrow \frac{1}{6b}[2b\delta - \frac{6d}{\delta} - (4b^2\delta^2 + 6b\delta c_2 + 12db)^{1/2}] > \frac{\delta}{2} - \frac{1}{2}[\delta^2 + \frac{2}{b}(2d + \delta c_2)]^{1/2}$. For property (iii) note that $\alpha_n \geq \alpha_c$ and $x_e^\infty \geq x_c^\infty$ when $d \leq \underline{d}$. Properties (iv)-(vi) can be seen from the slopes of the decision rules: $\Psi'(x) = A_3 > 0$ and $p'(x) = \frac{1}{2}(B_3 - A_3 - c_2) < 0$ if $d \leq \underline{d}$; $\Psi'(x) < 0$ and $p'(x) < 0$ if $\underline{d} < d < \overline{d}$; $\Psi'(x) < 0$ and $p'(x) > 0$ if $\overline{d} < d$. (the qualification $p_e^\infty > 0$ made in section 2 ensures that the tax does not exceed the choke price).■

Appendix to Section 4

Expression (21). Write $\mu \equiv V_1'(x)$ and note that the right hand side of (23) equals the buyers' maximized Hamiltonian evaluated at $(x, z) = (x, x_0 + z_0 - x)$ when the sellers' Markov strategy $p(x)$ is taken as a constraint. The co-state variable μ satisfies $\dot{\mu} = \delta\mu - 2dz + p'(x)q$. Intergrating this and using $\mu = \Psi$ yields (21).■

Expression (22). Write $\eta \equiv V_2'(x)$ and express the right hand side of (24) as the sellers' maximized Hamiltonian, H_2, evaluated at $(x, z) = (x, x_0 + z_0 - x)$ when the buyers' Markov strategy $\Psi(x)$ is taken as a constraint. The co-state η satisfies $\dot{\eta} = \delta\eta - c_2 q + \Psi'(x)(p - c_1 + c_2 x - \eta)\frac{1}{2b}$ or $\dot{\eta} = \delta\eta - c_2 q + \Psi'(x)q$, where the latter follows from $\partial H_2/\partial p = 0$ and the demand equation. Integrating the co-state equation yields $\eta(t) = \int_t^\infty q(\tau)[c_2 - \Psi'(x(\tau)]e^{-\delta(\tau - t)}d\tau$. The condition $\partial H_2/\partial p = 0$ implies $p = \frac{1}{2}(a + c_1 - c_2 x - \Psi + \eta)$. Using $\eta(t)$ and (21) in this expression for p gives the price equation (22).■

Proposition 2. The first part was shown in the text. For the latter part (large b), note that $lim_{b\rightarrow\infty}\alpha_c = lim_{b\rightarrow\infty}b\alpha_c = 0$. By (14), $lim_{b\rightarrow\infty}p_c(t) = a$ for any given t (x_c^∞ is independent of b). We then note that $lim_{b\to\infty}\alpha_n = 0$. This implies that $q_n(t)$ for any given t is close to zero when b is sufficiently large. Thus, $p_n + \Psi_n \approx a \Leftrightarrow p_n < a - \Psi_n$ for any given t. This shows that $p_n(t) < p_c(t)$ for large b, and completes the proof of Proposition 2.■

Proposition 3. Denote the value function for the buyer side in the MPNE by V_{1n} and in the cartel equilibrium by V_{1c}. For $d = 0$, these value functions satisfy

$$\begin{aligned} V_{1n} &= \int_0^\infty \{(a - p_n)q_n - bq_n^2\}e^{-\delta\tau}d\tau \\ &= \int_0^\infty (b\alpha_n^2 - \alpha_n A_3)(x_0 - x_n^\infty)^2 e^{(2\alpha_n - \delta)\tau}d\tau, \\ V_{1c} &= \int_0^\infty \{(a - p_c)q_c - bq_c^2\}e^{-\delta\tau}d\tau \\ &= \int_0^\infty b\alpha_c^2(x_0 - x_c^\infty)^2 e^{(2\alpha_c - \delta)\tau}d\tau. \end{aligned}$$

For $d = 0$, $x_n^\infty = x_c^\infty$ and $0 > \alpha_n > \alpha_c$. Then, for $d = 0$, $V_{1n} > V_{1c} \Leftarrow b\alpha_n^2 - \alpha_n A_3 > b\alpha_c^2$. Thus, we show first that $b\alpha_n^2 - \alpha_n A_3 > b\alpha_c^2$. To this end, note that $b\alpha_n^2 - \alpha_n A_3 = b\alpha_c^2 = 0$ when $c_2 = d = 0$. Note next that $\partial(b\alpha_n^2 - \alpha_n A_3)/\partial c_2 > \partial b\alpha_c^2/\partial c_2$, implying that $V_{1n} > V_{1c}$ holds for any given $c_2 > 0$ and $d = 0$. By the continuity of V_{1n} w.r.t. d, $V_{1n} > V_{1c}$ must hold when d is positive but not too large. This completes the proof of Proposition 3.■

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