A Bounded Influence Estimation and Outlier Detection for ARCH/GARCH Models With an Application to Foreign Exchange Rates

Jinliang Li

Finance and Insurance Group
Northeastern University
Boston, MA 02115-5000
Jin.li@neu.edu
(617) 373-4707

Chihwa Kao

Center for Policy Research and
Department of Economics
Syracuse University
Syracuse, NY 13244-1090
cdkao@maxwell.syr.edu
(315) 443-3233

January 15, 2002

A Bounded Influence Estimation and Outlier Detection for ARCH/GARCH
With an Application to Foreign Exchange Rates

Abstract

In this paper, we propose a bounded influence estimation (BIE) and outlier detection procedure for the ARCH and GARCH models of foreign exchange rates. It is known that due to outliers, exchange rates present heavy tails that are not explained by standard ARCH and GARCH models. The proposed BIE is robust against departure of the disturbances from normality, and its performance is compared with the MLE and semi-parametric models. A method of detection of additive outliers is developed. The economic events that lead to outliers in foreign exchange rates are identified and studied.

1. INTRODUCTION

The purpose of this paper is twofold. First, we propose a bounded influence estimator (BIE) for ARCH /GARCH models. Second, we propose an outlier detection procedure with an application to foreign exchange rates.

Knowledge of the distribution of exchange rates has important implications for theories of international finance and their applications. It is also of importance to varying issues related to foreign exchange. For instance, options pricing on foreign currencies relies on the right specification of the stochastic processes of exchange rates. In testing exchange market efficiency, the information of the statistical properties of exchange rate distribution is essential. Moreover, the volatility of exchange rates itself is a major risk component in international investing. Hence, clear understanding of the behavior and variance of exchange rates is important both for portfolio selection and for the evaluation of the performance of international asset portfolios.

Empirical evidence on the distribution of exchange rates, however, has been far from conclusive. While most previous studies have recognized that the rate of change in a foreign currency is not normally distributed, there is a lack of consensus on what type of distribution is most appropriate for describing the behavior of exchange rates. Examples of alternative statistical distributions, which have been commonly suggested in describing changes in exchange rates, include the symmetric stable Paretian, the Student t, the mixture of normal distributions, and the normal distribution with time-varying parameters (e.g., Friedman and Vandersteel, 1982; Booth and Glassman, 1987; and Tucker and Scott, 1987; Canova, 1993; and Lye et al., 1998). Nevertheless, none of these well-documented alternatives has gained general acceptance.

An alternative approach to the issues of exchange rates is the ARCH model (see Engle, 1982; Bollerslev et al., 1992, 1995, for a survey). This model is intuitively appealing because of the observed volatility clustering of exchange rates, i.e., periods of high volatility tend to follow periods of high volatility. Hsieh (1988, 1989a) and Baillie and Bollerslev (1989) applied the ARCH model to daily exchange-rate series. Diebold and Nerlove (1989) estimated the ARCH model for weekly spot-exchange rates. Recently, Andersen and Bollerslev (1998) captured the volatility persistence (ARCH) of intraday exchange rates[1]. Overall, the findings of ARCH in exchange rates are important. First, ARCH models are consistent with unconditional leptokurtosis in the changes of exchange rates (e.g., see Westerfield, 1977; Boothe and Glassman, 1987; Bollerslev et al., 1993, 1995). Second, ARCH models may prove to provide particularly helpful tools in future analyses and enhance the understanding of currency-option pricing with stochastic volatilities models (e.g., Hull and White, 1987; Melino and Turnbull, 1990)[2].

Among all assumptions of ARCH models, a very strong one is the conditional normal distribution of the disturbance. However, numerous studies (e.g., Hsieh, 1988, 1989; Baillie and Bollerslev, 1989; Andersen et al., 2001) showed that the distribution of the changes in exchange rates is, unconditionally as well as conditionally, far from being normal. In fact, leptokurtosis and skewness are frequently present. Hence, the normality assumption seems to be inadequate and often leads to spurious or inefficient inferences. This is mainly due to the fact that exchange rates are contaminated by some outliers or extreme values so that the conditional distribution looks heavy-tailed.

To account for heavy tails of the conditional distribution, student-t, among other alternatives, is often adopted instead of normal (e.g., Engle and Bollerslev, 1986; and Bollerslev, 1987). However, estimated residuals from GARCH models are still frequently observed with excess kurtosis, even when conditional student-t errors are allowed (Franses and Ghijsels, 1999). Alternatively, Hsieh

---

[1] Drost and Nijman (1993) and Drost and Werker (1996) provided some theoretic results regarding temporal aggregation of ARCH.

[2] An alternative approach is to study the implied standard deviation (ISDs) derived from currency and exchange rate options. Previous studies indicated that ISDs were either biased forecasts for future volatility, or less efficient in predicting than historical time-series (Lamoureux and Lastrapes, 1993; Canina and Figlewski, 1993; Jorion, 1995). Andersen et al. (2001) constructed model-free estimates of daily exchange rate volatility.

(1989b) and Nelson (1991) used the generalized error distribution (GED), which encompasses the normal, exponential, and uniform distributions. Nelson (1991), nevertheless, noted that the GED has only one parameter to control the shape of the conditional distribution, which may not be flexible enough in the presence of many outliers.

This paper proposes a BIE that is robust against departure from normality (of the conditional distribution) to describe the behavior of exchange rate changes. BIE limits the influence of any small subset of the data and is asymptotically normal (Krasker and Welsh, 1982). By construction, BIE provides a mechanism to detect over influential observations and limit their impact on the parameter estimation. In this paper, the BIE is used with ARCH and GARCH to identify additive outliers (AO) and other outliers caused by abnormal information arrivals that may be triggered by changes in domestic policies and international shocks. The identification of outliers allows us to analyze major economic and political factors that are not described by the statistical model but directly contribute to dramatic changes in exchange rates.

Balke and Fomby (1994) and Dijk et al. (1999) found that neglecting AOs can erroneously suggest misspecification or inadequate descriptive models for financial (especially ARCH) modeling. Franses and Ghijsels (1999) documented that neglected AOs substantially dampen the forecasting properties of GARCH models. The proposed BIE brings robustness to ARCH/GARCH models against outliers including AOs. Previous studies attempted to detect and remove an outlier, in order to obtain “true” estimators of ARCH/GARCH models. However, one concern arises that no observation can be regarded as an outlier with 100% assurance. Hence, an observation may be deleted by error. In addition, it is widely contended that an outlier may contain important information indeed, though its influence should not be assigned as high as its nominal magnitude suggests. The BIE identifies abnormal deviations from normality and downweights the influence of these observations accordingly. Hence, it achieves efficiency and robustness simultaneously. In this paper, the performance of the BIE will be compared with the maximum likelihood estimate (MLE), and a semiparametric estimator (SP) (Engle and Gonzalez-Rivera, 1991). Issues related to the assumption of the distribution such as non-normality, leptokurtosis, and outlying observations will also be addressed.

The remainder of this paper is organized as follows. Section 2 provides the background on the proposed BIE and places it in context with related work. Section 3 describes the BIE in details. Section 4 gives the estimation results from an application of the foreign exchange rates. Finally, Section 5 provides a summary.

2. The MODEL: ARCH/GARCH

Consider the ARCH model suggested by Geweke (1986),

$$y_t \mid \psi_{t-1} \sim N(0, \sigma_t^2) \qquad (1)$$
$$\log \sigma_t^2 = \alpha_1 + \alpha_2 \log y_{t-1}^2,$$

where y_t is the rate of change for the foreign exchange spot rate, ψ_{t-1} is the information set available at time t - 1 and σ_t^2 is the conditional variance. Note that the conditional variance σ_t^2 is positive for all values of α. Equation (1) is sometimes referred as the log-ARCH model. The log-likelihood function is

$$\ln L = -\sum_{t=1}^{T} \left(\log \sigma_t^2 + \frac{y_t^2}{\sigma_t^2}\right). \qquad (2)$$

The MLE either maximizes equation (2) or solves the following first order condition

$$\frac{\partial \ln l}{\partial \theta} = 0, \qquad (3)$$

where $\theta = (\alpha_1, \alpha_2)$. Note that if $\alpha_2 = 0$, the changes in exchange rates reduce to a random walk.

Hsieh (1989), Engle and Bollerslev (1986), and Baillie and Bollerslev (1989) have found that the MLE of ARCH is sensitive to distributional assumptions. One explanation is that the observations are contaminated by outliers and/or extreme values that make the conditional distribution look heavy tailed. Consequently, the outliers may not be helpful in predicting future variances, and the estimates in the variance function may be unduly influenced by a few extreme observations. These arguments strongly suggest the need of constructing robust-resistant ARCH parameter estimates and use these robust estimates to detect outliers.

Note that equation (1) can be written as

$$\log y_t^2 = \alpha_1 + \alpha_2 \log y_{t-1}^2 + v_t, \tag{4}$$

where $v_t = \log y_t^2 - \log\sigma_t^2$ are uncorrelated for t = 1,2,..., T. Thus equation (1) can be rewritten as an autoregressive model of order 1 (AR(1)) for $\log y_t^2$. Hence, the process $\log y_t^2$ has the same correlation structure as that of an AR(1) process with AR parameter α_2.

Pantula (1986) introduced the following generalized ARCH (GARCH(1, 1)) model that allows the conditional variance to depend not only on past residuals, but also on its own past realizations:

$$\begin{aligned} &y_t \mid \psi_{t-1} \sim N(0, \sigma_t^2) \\ &\log \sigma_t^2 = \alpha_1 + \alpha_2 \log y_{t-1}^2 + \alpha_3 \log \sigma_{t-1}^2. \end{aligned} \tag{5}$$

Equation (5) can also be written as

$$\log y_t^2 = \alpha_1 + (\alpha_2 + \alpha_3)\log y_{t-1}^2 + \alpha_3 v_{t-1} + v_t, \tag{6}$$

where $v_t = \log y_t^2 - \log\sigma_t^2$. This reveals that $\log y_t^2$ in equation (5) follows an autoregressive and moving average model (ARMA(1, 1)) with serially uncorrelated v_t.

Outlier detection is important for achieving the right statistical inference and has been an intriguing topic of numerous studies (see, e.g., Cook, Holschuh and Weisberg, 1982; Chen and Liu, 1993; Davies and Gather, 1993; Hadi and Simonoff, 1993; Ljung, 1993; Rocke and Woodruff, 1996; Penny, 1996). However, little attention has been given to outlying observations in the standard ARCH/GARCH. Among a few others, Dijk et al. (1999) and Franses and Ghijsels (1999) found that neglecting AOs the tests and estimates of ARCH effects. Jorion (1988) has a model that is very similar to the AO model. In his model, Jorion allows the mean of the exchange rate to follow a jump process, while the variance of the exchange rate follows an ARCH process. However, the present study considers the AO in the ARCH process to the variance but not the mean.

Now we take a more careful look at the outlying observations on ARCH models. Before assessing the effects of outliers on the ARCH models, we define what we mean by outliers in the time series models. Two major types of outliers have been defined by Fox (1972): One is called the additive effects outliers (AO) model; the other is referred as the innovation outlier (IO) model[3]. An IO represents an extraordinary shock at time t influencing y_t, y_{t+1}, ..., through the dynamic system described by equation (1).

In the IO model, occasional innovations have larger variance than the majority and therefore, can appear as outliers. In the AO model, on the other hand, the isolated outlier has an additive transient character that is unrelated to the time series model. Thus the AO is also called a gross error, since only the level of t^{th} observation is affected. In fact, IO-type outliers transmit their effect through to later observations; AO-type outliers do not. We also note that IO model will create a heavy-tailed distribution and ARCH model is heavy-tailed. ARCH model, therefore, are able to capture IOs by construction. Assume that the observations are generated from

$$z_t = x_t + e_t \qquad (7)$$

[3] Chen and Liu (1993) conducted a joint examination on the effects of IO, AO, and two other types of outlier, a level shift (LS) and a temporary change (TC), in time series.

where $x_t = \log y_t^2$ follows an AR(1) model in equation (4), and e_t is an independent sequence of variables, independent of the sequence of x_t. The variable e_t has distribution H, given by

$$H = (1 - \varepsilon)\delta_0 + \varepsilon G,$$

where δ_0 is the distribution that assigns probability 1 to the origin and G is an arbitrary distribution. Therefore, with probability $1 - \varepsilon$, the AR(1) process x_t itself is observed, and with probability ε the observation is the AR(1) process x_t plus an error with distribution G. Further insights into the effects of AOs to the ARCH model can be seen as follows: Let

$$z_t = x_t + e_t$$
$$x_t = \alpha_1 + \alpha_2 x_{t-1} + v_t$$
$$e_t \sim (1 - \varepsilon)\delta_0 + \varepsilon G.$$

Making the autoregressive transformation of z_t, we have that

$$z_t - \alpha_2 z_{t-1} = x_t - \alpha_2 x_{t-1} + e_t - \alpha_2 e_{t-1}. \tag{8}$$

Note that the sum of the two uncorrelated moving average (MA(1)) processes on the RHS of equation (8) is MA(1). Hence equation (8) represents an ARMA(1, 1) process. That is, the AR(1) model with AOs becomes an ARMA(1, 1) model in equation (8). In other words, the ARCH(1) model with AOs will become a GARCH(1, 1). Hence, GARCH (1, 1) model in equation (5) is able to capture AOs.

Looking at equations (6) and (8), it appears that AO hypothesis implies a testable restriction on the parameters of a GARCH(1,1) model. In particular, the AO hypothesis implies that from equation (8) the estimated AR parameter will be equal to the estimated MA parameter in a GARCH (1,1) model. This AO hypothesis will be tested in a later paper.

3. BOUNDED INFLUENCE ESTIMATION

The foregoing analysis shows that the MLE of the ARCH models may be sensitive to AO-type outliers. Consequently, detection of outlying observations implies that a robust estimation should be used. The motivation for BIE arises from studies such as Krasker and Welsh (1982), Kao and Dutkowsky (1989), Peracchi (1990a, 1990b, 1991), Naranjo and Hettmansperger (1994), Heritier and Ronchetti (1994), and Shen (1995).

The BIE proposed here is an iteratively reweighting technique where the weights decrease as some norms of the score function increases. The BIE for θ, denoted by $\hat{\theta}$, solves

$$\sum_{t=1}^{T} w(y_t,\hat{\theta})s(y_t,\hat{\theta}) = 0, \tag{9}$$

where w(.) is a nonnegative weight function, θ is a K by 1 vector of parameters to be estimated and s(.) is the score function such that

$$w(x, \theta) = \min\left\{1, \frac{bK^{1/2}}{\left[s^T(x,q)A^{-1}s(x,q)\right]^{1/2}}\right\}, \tag{10}$$

where

$$A = E[w^2(y, \theta)s(y, \theta)s^T(y, \theta)]. \tag{11}$$

The influence bound b is specified prior to estimation. Krasker and Welsch (1982) demonstrated that b has lower bound of unity.

The problem of selecting the optimal influence bound has not been conclusively resolved (see Samarov 1985; Powell, 1990). Suggested by Krasker et al. (1982), a criterion requires a predetermined level of asymptotic efficiency relative to the MLE at the "ideal" model. Hampel, Rousseeuw, Ronchetti, and Stahel (1986, p. 252) pointed out, however, that such an approach may lead to estimators with very low robustness. They suggested choosing the influence bound near 1. Carroll and Ruppert (1987) and Kao and Dutkowsky (1989), used these bounds ranged from 1.1 to 1.7 in their empirical studies. Peracchi (1990a) suggested that b is chosen so as to obtain an average weight of about 95%.

Equation (9) implies that the BIE falls within the class of weighted MLE. The BIE modifies the score function and finds the roots of the resulting likelihood functions. Equation (10) describes the choice of observation weights based on a Mahalanobis-type distance of $s(y_t, \theta)$ from the centroid of $\{s(y_t, \theta): t = 1,2, ..., T\}$. An observation is downweighted only if its influence exceeds the maximum allowable influence $bK^{1/2}$. Observations with influence below this bound receive a weight of unity. In this way the BIE compares with the MLE while, at the same time, the estimator protects against highly influential observations. From Equation (11) we see that A is a robust version of the second-moment matrix of $s(y, \theta)$.

The influence function (IF) of the BIE is

$$IF(y, \theta) = B^{-1} w(y, \theta)s(y, \theta), \tag{12}$$

where

$$B = -E\{\partial[w(y, \theta)s(y, \theta)]/\partial\theta\}. \tag{13}$$

Note that the influence function (IF) (see, e.g., Hampel, 1986, Peracchi, 1990b) measures the effect, on the asymptotic bias of an estimator, of an arbitrarily small contamination of the assumed statistical model.

The corresponding asymptotic covariance matrix of the BIE, denoted by V, is then

$$V = B^{-1} A \, B^{-1}. \tag{14}$$

Since the IF is a K x 1 vector, there is no natural ordering for influence. Obtaining a scalar in measuring of influence requires the application of appropriate norm for IF(y, θ). This norm maps the IF into R^1, combining the influence of a given observation over each parameter in θ to compute an overall measure of this influence. The Euclidean norm cannot be used here since it depends heavily upon the scaling of independent variables. A more suitable measure that is independent of particular parameterization is the self-standardized gross-error sensitivity (e.g., Krasker and Welsch, 1982),

$$\gamma = \max\{s^T(y,\theta)\ A^{-1}\ s(y,\theta)\}^{1/2}. \tag{15}$$

The γ in equation (15) measures the worst effect that a small amount of contamination by gross-error can have on the bias of the BIE. The construction of the weights in equation (15) implies that $\gamma < b$ for suitable choices of the influence bound. Therefore, the foregoing estimator achieves bounded influence. Bounding the gross-error sensitivity ensures robustness, with greater robustness produced by smaller bounds. The details of the computational algorithm can be found in Carroll et al. (1987), Kao et al. (1989), and Peracchi (1990a, 1990b).

Note that bounded influence weights, w(.), provide useful diagnostic information for outliers and influential observations, in particular, and identifying potential sources of model failure. Recently, several nonparametric and semiparametric estimators for the ARCH/GARCH have been discussed in the literature (e.g., Robinson, 1988; Pagan and Ullah, 1988; Diebold and Nason, 1990; Pagan and Schwert, 1990). Gallant el al. (1990) used a semi-nonparametric method where the conditional density is estimated with a polynomial expansion using ARCH as a leading term. Engle and Gonzalez-Rivera (1991) estimated the conditional distribution using a nonparametric penalized likelihood density estimation of Tapia and Thompson (1978). Weiss (1986) and Bollerslev and Woodridge (1988) proposed a quasi-maximum likelihood (QMLE) for ARCH and GARCH.

These estimators have certain robustness properties (such as consistency), but can be very inefficient, for they disregard entirely the information contained in the parametric assumptions. For example, Engle and Gonzalez-Rivera (1991) showed that the loss of efficiency of the QMLE could go up to 84% due to misspecification of the density. The BIE, on the other hand, provides a compromise between efficiency and robustness, since they take parametric assumptions into account.

4. DATA AND EMPIRICAL RESULTS

The data set consists of daily spot rates of foreign exchange rates (in terms of U.S. dollar)[4]. A ten-year sample of five major currencies are selected: the British Pound (BP), Canadian Dollar (CD), Deutsche Mark (DM), French Franc (FF) and Japanese Yen (JY). There are 2599 daily observations from January 2, 1991 to April 30, 2001. The analyzed series for each of the U.S. exchange rate is the first differences of the logarithms of the spot price of a specific currency in terms of U.S. dollar. Hence, the data represent the continuously compounded percentage rate of return for holding the particular currency one day. Table 1 reports the descriptive statistics of the data. The return skewness of each currency is negative except the British Pound. Meanwhile, British Pound, Canadian Dollar and Japanese Yen observe excess kurtosis, while Deutsche Mark and French Franc present less kurtosis.

4.1 The Estimation Results

Table 2 reports the MLE and BIE of the parameters of the ARCH(1) processes. A DUMINF subroutine of the IMSL libraries is used to compute the maximum likelihood estimators. The algorithm of computing BIE is written in FORTRAN and (9) is solved by subroutine DNEQNF in the IMSL libraries. For a given currency, the leading two rows display the parameter estimates of

[4] Federal Reserve Bank of Chicago provided the daily foreign exchange rate data.

ARCH(1) process. Standard errors appear in the parentheses. The MLE of α_1 and α_2 are significantly different from zero.

As mentioned earlier, there are some outliers in the daily exchange rate data that may not be representative of the true exchange rate process. Including these non-representative data may cause bias in the parameter estimation. To assess the effect of outlying observations on the parameter estimates, the ARCH(1) process is re-estimated with the BIE. Column three to column six in Table 2 report the estimates of the BIE for the ARCH(1) process. Different values of bounds are set (1.1 to 1.7) in the estimation. The smaller the bounds, the more the data were down-weighted. Table 2 shows that all the MLE and BIE parameter estimates are significant at the 1% level. However, the MLE estimates are very sensitive to the outliers. The BIE estimates are stable with respect to different bounds and obtained estimates of α_1 close to that of MLE. On the contrary, the estimates of α_2 of the five currencies increased about 22% to 300% for the BIE compared to the MLE. If the BIE represents the true parameter estimates of the population, then the ARCH effect of exchange rates has been underestimated by a substantial amount when the MLE is used as in most of previous studies.

The preceding results show that using BIE-ARCH(1) leads to significant difference with respect to ARCH(1). This is due to the fact that BIE is less sensitive than the MLE to local violations of the model assumptions. By construction, both BIE-GARCH(1,1) and GARCH(1,1) accommodate IO and AO-type outliers, which tend to produce close inferences. However, BIE-GARCH(1,1) is more robust than GARCH(1,1) since it also captures outliers other than IO and AO. Table 3 reports the MLE and BIE for the GARCH(1, 1) model. It shows that the BIE estimates are stable with respect to different bounds. The MLE of GARCH(1,1) tends to overestimate the constant component of volatility (α_1), and underestimate the ARCH effect of the exchange rates (α_2). Among the five currencies, only JY observed a bigger α_2 in MLE than in BIE. JY disassociated itself with the group again after it presented a significantly larger kurtosis (4.5088) in Table 1.

Table 3 also presents the estimation results of the various currencies using the semiparametric GARCH, which was proposed by Engle and Gonzalex-Rivera (1990) (see Engle and Gonzalez-Rivera for details on the computations). The semi-parametric and MLE estimates are shown to be close. It seems that using semi-parametric GARCH does not lead to significant difference with respect to MLE. Hence, EG's semi-parametric ARCH is not robust with respect to outliers, which is not surprising (see Huber, 1981, p. 6). For example, the sample mean is a nonparametric estimator of the population mean, but it is highly sensitive to outliers and therefore very non-robust.

To test the robustness of the models, we run the foregoing analysis on the two half-samples of British Pound. The first half-sample covers a period from January 2, 1991 to December 30, 1995, while the second covers from January 2, 1996 to April 2001. The finding is consistent with that of the full sample. Table 4 reports the robustness test. The BIE estimates are stable with respect to different bounds, for the log-ARCH and log-GARCH analyses, respectively. For both models in each half-sample, MLE is sensitive to outliers compared to BIE. In both log-ARCH and log-GARCH, MLE tends to underestimate the ARCH effect of the BP exchange rates substantially. The semi-parametric estimates are close to the MLE for the log-GARCH model, which is also sensitive to outliers.

4.2 Detection of AO

The BIE-GARCH(1, 1) identified two groups of abnormal data in the foreign exchange rates. The first group includes the "shocks" that cannot be explained by the ARCH(1) and GARCH(1, 1) process. As shown in Table 5 and Table 7, these are large fluctuations in foreign exchange associated with important political and economic events. The second group includes the AO-type outliers that are also captured by the GARCH(1, 1) process. The procedure of identifying these AO outliers is as follows. Using BIE we fitted the exchange rate data to the BIE-ARCH(1) and the BIE-GARCH(1, 1) process. We found some observations are down-weighted substantially for the ARCH(1) process but are either not down-weighted or down-weighted slightly for the GARCH(1, 1). This means that these observations do not fit the ARCH(1) process well but fit fairly well to the

GARCH(1, 1) process. Since the only difference between these two models is the inclusion of a moving average component in the GARCH(1, 1) process, these observations must be associated with the AO-type outliers. In this way, we identify the AO effects of economic and political changes that cause the jumps in exchange rate movements.

Table 5 reports the data points that were substantially down-weighted by the BIE for the British Pound for the purpose of demonstration. As shown in the table, most of the observations down-weighted in the BIE-GARCH(1, 1) process are also down-weighted in the BIE-ARCH(1) process. The AO-type outliers are listed in Table 5. The unexplained outliers in Table 5 may be due to the level-shift (LS) type outliers or structural change in Lastrapes (1989), Diebold and Pauly (1988), Chen and Tiao (1990), Lamoureux and Lastrapes (1990), and Chen and Liu (1993). Further work is needed for explaining ARCH or GARCH with LS-type outliers (e.g., Gourieroux and Monfort, 1990; Chu, 1991; McCulloch and Tsay; 1993).

4.3 Common Factor in Foreign Exchange Markets

BIE downweights the observations that exceed a given bound. The preceding analysis shows that BIE estimates are less sensitive to outliers, compared to the MLE and semi-parametric estimates of ARCH and GARCH. An intriguing question thus arises: What observations were down-weighted? Let's first look at the commonality in foreign exchange rates. Information related to the general economy and market leads to a common trend in returns, price volatility, and liquidity in equity markets, in both intraday and daily levels (e.g., see Karolyi and Stulz, 1996; Chordia et al., 2000; and Hasbrouck and Seppi, 2001). Similarly, changes of the U.S. dollar value and shocks to the global economy contribute to the widely observed commonality in foreign exchange rates. We conduct principle component analysis on the five currencies and report the results in Table 6. Panel A shows that the first component explains 56% of the variation of exchange returns of the five currencies, which suggests the presence of a strong common factor. The rest four components are negligible. If the five currencies were perfectly independent, each component should explain 20% of the variation. Panel B reports the principal component analysis on the weighted returns of the five currencies, using the weights from the BIE-GARCH with bound 1.5. After down-weighting, the first component explains 52% of the total variation, slightly less than that of Panel A. This suggests that, some down-weighted observations (outliers) contributed to the co-movement of currencies, which should be shocks related to the intrinsic value of U.S. dollar or the global economy. Panel C presents the principal component analysis on the five series of weights from the BIE-GARCH. The first component explains 47% of the total variation. This suggests that, the independently searched optimal weights of the five currencies share a common factor. We conduct the same test on the raw and weighted return volatilities (absolute returns) of the five currencies, and get similar and consistent results. Hence, a significant portion of the outliers were shocks related to the fundamental of U.S. currency and the global economy.

Table 7 documents some major events occurring on those dates identified in Table 5. The events displayed in Tables 7 reflect major policy changes and international turbulences. The findings indicate that these events led to abnormal jumps or fluctuations in the foreign exchange rates of the British Pound. Expectedly, these events were also likely to affect other currencies.

5. CONCLUSION

This paper proposes BIE to estimate the conditional heteroskedasticity of foreign exchange rates, which are robust against outliers. It extends the current literature on the distribution of exchange rate changes in a number of ways. First, the distribution was estimated with a robust parametric model BIE. The preceding results show that exchange rate changes estimated from the same set of data can differ significantly depending on the choice of the model and estimation technique. In particular, the ARCH(1) can differ significantly from BIE as a consequence of the presence of only a small fraction of extreme observations. This BIE estimation procedure offers an efficient mechanism to down-weight outlying observation and therefore, provides more accurate estimates for the parameters of the exchange rate changes distribution. Second, the proposed BIE-ARCH and BIE-GARCH are able to detect additive outliers by construction. Third, the down-weigting technique of BIE produces stable estimates with respect to varying bounds.

We found a clustering of outliers among currencies. That is, the causal events of outliers in one currency are permutational, which are likely to influence other currencies. Major political and economic events that caused jumps and abnormal fluctuations in exchange rates were identified. The effects of policy changes and international events on exchange rate movements were carefully analyzed. This analysis provides policy makers very valuable information on the sensitivity of exchange rate to policy shifts and economic events.

REFERENCES

Andersen, T.G., and Bollerslev, T. (1998), "DM-Dollar Volatility: Intraday Activity Patterns, Macroeconomic Announcements, and Longer-Run Dependencies," Journal of Finance 53, 219-265.

Andersen, T.G., Bollerslev, T., Diebold, F.X., and Labys, P. (2001), "The Distribution of Realized Exchange Rate Volatility," Journal of the American Statistical Association 96, 42-55.

Bailey, W., and Chung Y.P. (1995), "Exchange Rate Fluctuations, Political Risk, and Stock Returns: Some Evidence from an Emerging Market," Journal of Financial and Quantitative Analysis 30, 541-561.

Baillie, R. T., and Bollerslev, T. (1989), "The Message in Daily Exchange Rates: A Conditional-Variance Tale," Journal of Business and Economic Statistics 7, 297-305.

Balke, N.S., and Fomby, T.B. (1994), "Large Shocks, Small Shocks, and Economic Fluctuations: Outliers in Macroeconomic Time Series," Journal of Applied Econometrics 9, 181-200.

Bodurtha, J., and Courtadon, G. (1987), "Tests of an American Option Pricing Model on the Foreign Currency Options Market," Journal of Financial and Quantitative Analysis 22, 153-167.

Bollerslev, T. (1987), "A Conditionally Heteroskedastic Time Series Model for Speculative Prices and Rates of Return," Review of Economic and Statistics 69, 542-547.

Bollerslev, T. (1988), "On the Correlation Structure for the Generalized Autoregressive Conditional Heteroskedastic Process," Journal of Time Series Analysis 9, 121-131.

Bollerslev, T., and Engle, R.F. (1993), "Common Persistence in Conditional Variances," Econometrica 61, 167-186.

Bollerslev, T., Engle, R.F., and Nelson, D.B. (1995), "ARCH Models," The Handbook of Exonometrics vol. 4. ed. Engle, R.F., and McFadden, D.L., Elsevier, Amsterdam, 2959-3040.

Bollerslev, T. P., and Woodridge, J. M. (1988), "Quasi-maximum Likelihood Estimation of Dynamic Models with Time Varying Covariance," Massachusetts Institute of Technology Department of Economics Working Paper: 505.

Bollerslev, T., Chou, R. Y., and Kroner, K. (1992), "ARCH Modeling in Finance: A Review of the Theory and Empirical Evidence," Journal of Econometrics 52, 5-59.

Boothe, P., and Glassman, D. (1987), "The Statistical Distribution of Exchange Rates," Journal of International Economics 22, 297-319.

Canina, L., and Figlewski, S. (1993), "The Informational Content of Implied Volatility," Review of Financial Studies 6, 659-681.

Canova, F. (1993) "Modeling and Forecasting Exchange Rates with a Bayesian Time-Varying Coefficient Model," Journal of Economic Dynamics and Control 17, 223-261.

Carroll, R. J., and Ruppert, D. (1987), "Diagnostics and Robust Estimation when Transforming the Dependent Variable and the Response," Technometrics 29, 287-299.

Chen, C., and Liu, L. (1993), "Joint Estimation of Model Parameters and Outlier Effects in Time Series," Journal of the American Statistical Association 88, 284-297.

Chen, C., and Tiao, G. (1990), "Random Level-Shift Time Series Models, ARIMA Approximations, and Level-Shift Detections," Journal of Business and Economic Statistics 8, 83-97.

Chordia, T., Roll, R., and Subrahmanyam, A. (2000), "Commonality in Liquidity," Journal of Financial Economics 56, 3-28.

Chu, C. -J. (1991), "Tests for Instability of Variance Parameters: Corollaries of Recent Developments," Manuscript, Department of Economics, University of Southern California.

Cook, R.D., Holschuh, N., and Weisberg, S. (1982), "A Note on Alternative Outlier Model," Journal of the Royal Statistical Society 44, 370-376.

Crowder, M. J. (1976), "Maximum Likelihood Estimation for Dependent Observations," Journal of the Royal Statistical Society B 38, 45-53.

Davies, L., and Gather, U. (1993), "The Identification of Multiple Outliers," Journal of the American Statistical Association 88, 782-792.

Diebold, F. X., and Nerlove, M. (1989), "The Dynamics of Exchange Rate Volatility: A Multivariate Latent Factor ARCH Model," Journal of Applied Econometrics 4,1-22.

Diebold, F. X., and Nason, J. A. (1990), "Nonparametric Exchange Rate Prediction?" Journal of International Economics 28, 315-332.

Diebold, F., and Pauly, P. (1988), "Has the EMS Reduced Member-Country Exchange Rate Volatility," Empirical Economics 13, 81-102.

Dijk, D.V., Franses, P.H., and Lucas, A. (1999), "Testing for ARCH in the Presence of Additive Outliers," Journal of Applied Econometrics 14, 539-562.

Drost, F.C., and Nijman, T.E. (1993), "Temporal Aggregation of GARCH Processes," Econometrica 61, 909-927.

Drost, F.C., and Werker, B.J.M. (1996), "Closing the GARCH Gap: Continuous Time GARCH Modeling," Journal of Econometrics 74, 31-57.

Engle, R. F. (1982), "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of U. K. Inflation," Econometrica 50, 987-1008.

Engle, R. T., and Kraft, D. (1983), "Multiperiod Forecast Error Variances of Inflation Estimated from ARCH Models," in Applied Time Series Analysis of Economic Data, ed. A. Zellner, Bureau of Census, pp. 293-302.

Engle, R. T., and Bollerslev, T. (1986), "Modeling the persistence of Conditional Variances," Econometric Review 5, 1-50.

Engle, R. T., and Gonzalez-Rivera, G. (1991), "Semiparametric ARCH Models," Journal of Business and Economic Statistics 9, 345-360.

Fox, A. J. (1972), "Outliers in Time Series," Journal of the Royal Statistical Society, Series B, 34, 350-363.

Franses, P.H., and Ghijsels, H. (1999), "Additive Outliers, GARCH and Forecasting Volatility," International Journal of Forecasting 15, 1-9.

Friedman, D., and Vandersteel, (1982), "Short-Run Fluctuations in Foreign Exchanges Rates," Journal of International Economics 13, 171-186.

Gallant, A. R., Hsieh, D. A., and Tauchen, G. E. (1990), "On Fitting a Recalcitrant Series: The Pound/Dollar Exchange Rate, 1974-83," in Nonparametric and Semiparametric Methods in Econometrics and Statistics, ed. W. A. Barnett, J. Powell, and G. Tauchen.

Garman, M, and Kohlhagen, W. (1983), "Foreign Currency Option Values," Journal of International Money and Finance 2, 231-237.

Geweke, J. (1986), "Comment - Modeling Persistence of Conditional Variances," Econometric Review 5, 57-61.

Gourieroux, C., and Monfort, A., (1990), "Qualitative Threshold ARCH Models," CREST Manuscript.

Guttman, I., and Tiao, G. C. (1978), "Effect of Correlation on the Estimation of a Mean in the Presence of Spurious Observations," Canadian Journal of Statistics 6, 229-247.

Hadi, A.S. and Simonoff, J.S. (1993), "Procedures for the Identification of Multiple Outliers in Linear Models," Journal of the American Statistical Association 88, 1264-1272.

Hampel, F. R., Rousseeuw, P. J., Ronchetti, E. M., and Stahel, W. A. (1986), Robust Statistics: The Approach Based on Influence, New York: John Wiley.

Hasbrouck, J., and Seppi, D.J. (2001), "Common Factors in Prices, order flows and Liquidity," Journal of Financial Economics 59, 383-411.

Heritier, S., and Ronchetti, E. (1994), "Robust Bounded-Influence Tests in General Parametric Models," Journal of the American Statistical Association 89, 897-904.

Hsieh, D. (1988), "The Statistical Properties of Daily Foreign Exchange Rates," Journal of International Economics 15, 130-145.

Hsieh, D. (1989a), "Modeling Heteroskedasticity in Daily Foreign-Exchange Rates," Journal of Business and Economic Statistics 7, 307-317.

Hsieh, D. (1989b), "Testing Nonlinear Dependence in Daily Foreign Exchange Rates," Journal of Business 62, 339-368.

Huber, P. (1981), Robust Statistics, Wiley, New York.

Hull, J., and White, A. (1987), "Hedging the Risks from Writing Foreign Currency Options," Journal of International Money and Finance 6, 131-152.

Jorion, P. (1988), "On Jump Processes in Foreign Exchange and Stock Market," Review of Financial Studies 1, 427-445.

Jorion, P. (1995), "Predicting Volatility in the Foreign Exchange Market," Journal of Finance 50, 507-528.

Kao, C., and Dutkowsky, D. H. (1989), "An Application of Nonlinear Bounded Influence Estimation to Aggregate Bank Borrowing From the Federal Reserve," Journal of the American Statistical Association 84, 700-709.

Karolyi, G.A., and Stulz, R.M. (1996), "Why Do Markets Move Together? An Investigation of U.S.-Japan Stock Return Comovements," Journal of Finance 51, 951-986.

Krasker, W. S., and Welsh, R. E. (1982), "Efficient Bounded-Influence Regression Estimation," Journal of the American Statistical Association 69, 595-604.

Lamoureux, C. G., and Lastrapes, W. D. (1990), "Persistence in Variance, Structural Change, and the GARCH Model," Journal of Business and Economic Statistics 8, 225-234.

Lamoureux, D., and Lastrapes, W. (1993), "Forecasting Stock-Return Variance: Toward an Understanding of Stochastic Implied Volatilities," Review of Financial Studies 6, 293-326.

Lastrapes, W. (1989), "Exchange Rate Volatility and U.S. Monetary Policy: An ARCH Application," Journal of Monetary, Credit, and Banking 21, 67-77.

Ljung, G.M. (1993), "On Outlier Detection in Time Series," Journal of the Royal Statistical Society 55, 559-567.

Lye, J.N., Martin, V.L., and Teo, L.E. (1998), "Parametric Distributional Flexibility and Conditional Variance Models with an Application to Hourly Exchange Rates," International Monetary Fund Working Paper: WP/98/29.

McCulloch, R.E., and Tsay, R.S. (1993), "Bayesian Inference and Prediction for Mean and Variance Shifts in Autoregressive Time Series," Journal of the American Statistical Association 88, 968-978.

Melino, A., and Turnbull, S. M. (1990), "Pricing Foreign Currency Options with Stochastic Volatility," Journal of Econometrics 45, 239-265.

Naranjo, J.D., and Hettmansperger, T.P. (1994), "Bounded Influence Rank Regression," Journal of the Royal Statistical Society Series B, Vol. 56, 209-220.

Nelson, D. (1991), "Conditional Heteroskedasticity in Asset Returns: A New Approach," Econometrica 59, 347-370.

Penny, K.I. (1996), "Appropriate Critical Values When Testing for a Single Multivariate Outlier by Using the Mahalanobis Distance," Applied Statistics 45, 73-81.

Pagan, A. R. and Ullah, A. (1988), "The Econometric Analysis of Models with Risk Terms," Journal of Applied Econometrics 3, 87-105.

Pagan, A., and Schwert, G. W. (1990), "Alternative Models for Conditional Stock Volatility," Journal of Econometrics 45, 267-290.

Pantula, S. G. (1986), "Comment: Modeling the Persistence of Conditional Variance," Econometric Review 5, 71-74.

Peracchi, F. (1990a), "Bounded-Influence Estimators for the Tobit Model, " Journal of Econometrics 44, 107-126.

Peracchi, F. (1990b), "Robust M-Estimators," Econometric Review 9, 1-30.

Peracchi, F. (1991), "Bounded-Influence Estimators for the SURE Model, " Journal of Econometrics 48, 119-134.

Powell, J. L. (1990), "Comment: Robust M-Estimators," Econometric Review 9, 31-33.

Rich, R.W., Raymond, J., and Butler, J. S. (1991), "Generalized Instrumental Variables Estimation of Autoregressive Conditional Heteroskedastic Models," Economics Letters 35, 179-185.

Robinson, P. M. (1988), "Semiparametric Econometrics: A Survey," Journal of Applied Econometrics 3, 35-51.

Rocke, D.M., and Woodruff, D.L. (1996), "Identification of Outliers in Multivariate Data," Journal of the American Statistical Association 91, 1047-1061.

Rogalski, R. J., and Vinso, J. D. (1978), "Empirical Properties of Foreign Exchange Rates," Journal of International Business Studies 9, 69-79.

Samarov, A. M. (1985), "Bounded-Influence Regression Via Local Minimax Mean Squared Error," Journal of the American Statistical Association 80, 1032-1040.

Shen, L.Z. (1995), "On Optimal B-Robust Influence Functions in Semiparametric Models," Annals of Statistics 23, 968-989.

Tapia, R. A., and Thomas, J. R. (1978), Nonparametric Probability Density Estimation, The Johns Hopkins University Press, Baltimore and London.

Tucker, A., and Scott, E. (1987), "A Study of Diffusion Processes for Foreign Exchange Rates," Journal of International Money and Finance 6, 465-478.

Weiss, A.A. (1986), "Asymptotic Theory for ARCH Models: Estimation and Testing," Econometric Theory 2, 107-131.

Westerfield, J. M. (1977), "An Examination of Foreign Exchange Risk under Fixed and Floating Rate Regimes," Journal of International Economics 7, 181-200.

Table 1: Statistics of the Data

The data set consists of five major currencies in term of U.S. dollar for a period from January 2, 1991 to April 30, 2001.

Currency	British Pound	Canadian Dollar	Deutsche Mark	French Franc	Japanese Yen
Mean Exchange Rate	0.7385	1.6083	1.7039	5.7754	115.25
Daily Returns (%)					
Mean	-0.0109	-0.0118	0.0151	0.0145	-0.0033
Standard Deviation	0.3054	0.5882	0.6771	0.6506	0.7362
Skewness	0.0393	-0.2445	-0.0717	-0.1230	-0.5495
Kurtosis	2.3975	2.8137	1.5998	1.5250	4.5088

Table 2: MLE and BIE for the log-ARCH process

$$y_t \mid y_{t-1} \sim N(0, \sigma_t^2)$$
$$\log \sigma_t^2 = \alpha_1 + \alpha_2 \log y_{t-1}^2$$

Currency	MLE	BIE(1.7)	BIE(1.5)	BIE(1.3)	BIE(1.1)
British Pound					
α_1	-9.415 (0.060)	-9.244 (0.068)	-9.306 (0.066)	-9.407 (0.642)	-9.660 (0.059)
$\alpha_2 \times 10^2$	7.322 (0.454)	12.836 (0.531)	12.985 (0.517)	12.985 (0.502)	12.682 (0.464)
Canadian Dollar					
α_1	-10.572 (0.088)	-10.460 (0.086)	-10.513 (0.084)	-10.615 (0.083)	-10.851 (0.077)
$\alpha_2 \times 10^2$	7.813 (0.660)	12.310 (0.624)	12.380 (0.616)	12.260 (0.606)	11.947 (0.563)
Deutsche Mark					
α_1	-9.593 (0.070)	-9.196 (0.070)	-9.258 (0.069)	-9.416 (0.067)	-9.706 (0.061)
$\alpha_2 \times 10^2$	3.480 (0.563)	10.714 (0.546)	10.740 (0.544)	10.639 (0.527)	10.076 (0.488)
French Franc					
α_1	-9.584 (0.077)	-9.269 (0.082)	-9.332 (0.081)	-9.503 (0.077)	-9.808 (0.069)
$\alpha_2 \times 10^2$	4.224 (0.619)	10.351 (0.660)	10.432 (0.650)	10.358 (0.620)	9.759 (0.566)
Japanese Yen					
α_1	-8.997 (0.059)	-9.393 (0.065)	-9.457 (0.062)	-9.529 (0.061)	-9.799 (0.056)
$\alpha_2 \times 10^2$	7.334 (0.525)	9.003 (0.550)	9.809 (0.518)	9.697 (0.510)	9.206 (0.468)

Note: Asymptotic standard errors are in parentheses below each coefficient. All the parameter estimates are significant at the 1% level.
BIE(1.5) is the BIE with bound to be 1.5.
Sample period is from January 2, 1991 to April 30, 2001.

Table 3: MLE, BIE and SP for the log-GARCH Process

Currency	MLE	BIE(1.7)	BIE(1.5)	BIE(1.3)	SP
British Pound					
α_1	-0.066	-0.106	-0.107	-0.112	-0.065
	(0.015)	(0.019)	(0.019)	(0.017)	(0.015)
α_2 x 10^2	1.958	2.611	2.643	3.269	2.088
	(0.168)	(0.196)	(0.192)	(0.179)	(0.169)
α_3	0.971	0.961	0.961	0.955	0.970
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)
Log-likelihood	-12181				-9890
Canadian Dollar					
α_1	-0.101	-0.022†	-0.025†	-0.058	-0.109
	(0.025)	(0.017)	(0.017)	(0.018)	(0.025)
α_2 x 10^2	3.019	3.322	3.336	3.477	3.047
	(0.196)	(0.196)	(0.193)	(0.192)	(0.194)
α_3	0.957	0.962	0.962	0.958	0.956
	(0.004)	(0.003)	(0.002)	(0.003)	(0.004)
Log-likelihood	-13891				-11572
Deutsche Mark					
α_1	-0.105	-0.092	-0.085	-0.087	-0.133
	(0.028)	(0.025)	(0.022)	(0.021)	(0.033)
α_2 x 10^2	2.080	2.398	2.365	2.365	2.415
	(0.261)	(0.256)	(0.236)	(0.231)	(0.288)
α_3	0.966	0.965	0.966	0.966	0.960
	(0.005)	(0.004)	(0.004)	(0.004)	(0.006)
Log-likelihood	-11758				-9426
French Franc					
α_1	-0.118	-0.109	-0.112	-0.128	-0.096
	(0.032)	(0.029)	(0.028)	(0.028)	(0.026)
α_2 x 10^2	1.914	2.803	2.846	2.911	1.715
	(0.250)	(0.286)	(0.279)	(0.260)	(0.214)
α_3	0.966	0.959	0.958	0.957	0.971
	(0.006)	(0.005)	(0.005)	(0.005)	(0.005)
Log-likelihood	-11855				-9527
Japanese Yen					
α_1	-0.108	-0.095	-0.090	-0.096	-0.095
	(0.021)	(0.019)	(0.018)	(0.017)	(0.019)
α_2 x 10^2	2.841	2.292	2.435	2.767	2.947
	(0.225)	(0.194)	(0.185)	(0.180)	(0.219)
α_3	0.956	0.965	0.965	0.961	0.956
	(0.004)	(0.003)	(0.003)	(0.003)	(0.004)
Log-likelihood	-11605				-9345

Note: Asymptotic standard errors are in the parentheses. All the parameter estimates are significant at the 1% level, except those marked with symbol † .
BIE(1.5) is the BIE with bound to be 1.5.
Sample period is from January 2, 1991 to April 30, 2001.

Table 4: Robustness Test on British Pound

This test runs the log-ARCH and log-GARCH analysis on two subperiods of British Pound, respectively.

Currency	MLE	BIE(1.7)	BIE(1.5)	BIE(1.3)	SP
Log- ARCH analysis on data period from January 2, 1991- December 30, 1995					
α_1	-9.091	-8.805	-8.874	-8.982	
	(0.085)	(0.092)	(0.091)	(0.089)	
α_2 x 10^2	7.619	14.102	14.159	14.072	
	(0.641)	(0.723)	(0.717)	(0.704)	
Log- ARCH analysis on data period from January 2, 1996- April 30, 2001					
α_1	-10.347	-10.050	-10.126	-10.532	
	(0.141)	(0.130)	(0.128)	(0.117)	
α_2 x 10^2	2.852*	8.906	8.809	7.661	
	(1.112)	(1.004)	(0.989)	(0.916)	
Log-GARCH analysis on data period from January 2, 1991- December 30, 1995					
α_1	-0.159	-0.041*	-0.018†	-0.020†	-0.123
	(0.041)	(0.017)	(0.015)	(0.015)	(0.032)
α_2 x 10^2	2.525	3.011	3.164	3.794	2.309
	(0.313)	(0.224)	(0.206)	(0.207)	(0.266)
α_3	0.955	0.963	0.964	0.958	0.961
	(0.007)	(0.003)	(0.003)	(0.003)	(0.005)
Log-likelihood	-5679				-4575
Log-ARCH analysis on data period from January 2, 1996- April 30, 2001					
α_1	-0.338*	-0.254	-0.289	-0.383	-0.396*
	(0.136)	(0.082)	(0.084)	(0.093)	(0.156)
α_2 x 10^2	1.869	3.130	3.316	3.711	2.016
	(0.414)	(0.395)	(0.391)	(0.396)	(0.434)
α_3	0.947	0.942	0.938	0.927	0.940
	(0.015)	(0.010)	(0.010)	(0.011)	(0.017)
Log-likelihood	-6507				-5311

Note: Asymptotic standard errors are in the parentheses. All the parameter estimates are significant at the 1% level, unless otherwise noted.

† : Not significant at the 5% level.

* : Significant at the 5% level.

BIE(1.5) is the BIE with bound to be 1.5.

Table 5: Selected Downweighted Cases from the BIE(1.5): The Case of British Pound

Date	BIE-ARCH Weights	BIE-GARCH Weights	Outliers Type	Exchange Rate Returns (%)
3/15/1991	0.05	0.25	AO	-1.357
3/18/1991	0.08	0.11		-2.182
4/19/1991	0.05	0.06		-2.980
4/30/1991	0.06	0.07		2.707
5/17/1991	0.09	0.17		-1.850
7/12/1991	0.06	0.06		2.671
8/16/1991	0.04	0.15		-1.552
8/21/1991	0.09	0.08		2.342
11/27/1991	0.04	0.03		-1.728
1/9/1992	0.00	0.06		-2.161
1/10/1992	0.08	0.09		-2.153
5/14/1992	0.09	1.00	AO	0.495
6/29/1992	0.03	0.38	AO	0.843
8/24/1992	0.09	0.09		2.385
9/11/1992	0.02	0.07		-2.693
9/16/1992	0.04	0.04		-3.286
9/18/1992	0.06	0.07		-2.530
9/29/1992	0.08	0.10		2.258
10/1/1992	0.07	0.08		-2.562
10/16/1992	0.07	0.07		-2.653
11/16/1992	0.09	0.14		-2.022
1/5/1993	0.05	0.04		2.889
1/29/1993	0.07	0.11		-2.065
2/1/1993	0.08	0.11		-2.143
2/16/1993	0.03	0.11		2.094
5/10/1993	0.04	0.08		-2.350
7/13/1993	0.05	0.50	AO	1.003
8/26/1994	0.01	0.03		-1.165
3/10/1995	0.09	0.05		-2.191
11/13/1995	0.08	0.06		-1.525
11/13/1996	0.09	0.82	AO	0.485
12/3/1996	0.04	0.02		-2.526
12/19/1996	0.04	0.17		-1.185
1/22/1998	0.07	0.15		1.429
6/16/1998	0.01	0.05		1.279
8/28/1998	0.08	0.06		1.902
2/19/1999	0.08	0.64	AO	-0.509
2/8/2000	0.05	0.31	AO	1.174
7/20/2000	0.09	0.09		1.012
9/22/2000	0.10	0.07		1.997
12/1/2000	0.04	0.27	AO	1.141
2/1/2001	0.06	0.29	AO	1.103

Additive outliers are detected if the BIE-ARCH weights are small ($\leq 10\%$) and BIE-GARCH weights are large ($\geq 25\%$).

Table 6: Common Factor Among Exchange Rates

Panel A: Principal Component Analysis on the Returns of the Five Currencies

Component	Eigenvalue	Difference	Proportion	Cumulative
1	2.8030	1.8011	0.5606	0.5606
2	1.0019	0.2296	0.2004	0.7610
3	0.7723	0.3785	0.1545	0.9154
4	0.3938	0.3648	0.0788	0.9942
5	0.0290		0.0058	1.0000

Panel B: Principal Component Analysis on the Weighted (BIE 1.5) Returns of the Five Currencies

Component	Eigenvalue	Difference	Proportion	Cumulative
1	2.6169	1.6112	0.5234	0.5234
2	1.0057	0.1742	0.2011	0.7245
3	0.8315	0.3363	0.1663	0.8908
4	0.4952	0.4445	0.0990	0.9899
5	0.0507		0.0101	1.0000

Panel C: Principal Component Analysis on the Weights (BIE 1.5) of the Five Currencies

Component	Eigenvalue	Difference	Proportion	Cumulative
1	2.3438	1.3607	0.4688	0.4688
2	0.9831	0.0792	0.1966	0.6654
3	0.9039	0.2386	0.1808	0.8462
4	0.6653	0.5614	0.1331	0.9792
5	0.1039		0.0208	1.0000

Table 7: Important Events Coincide with the Shocks found by the BIE: The Case of British Pound

Date	Events
3/15/1991 - 4/19/1991	U.S. dollar started a strong climb on world currency markets, in a confirmed recession and suspected rebounding economy.
4/30/1991	U.S. dollar tumbled world wide, driven lower by a 0.5 percent cut in U.S. interest rates.
5/17/1991	On 5/18, the Commerce Department reported that U.S. trade deficit narrowed impressively to $4.05 billion in March, the lowest level in nearly eight years.
7/12/1991	Central banks aggressively sold U.S. dollars.
8/16/1991 -8/21/1991	A three-day political coup emerged and failed in the Soviet Union.
1/9/1992 -1/10/1992	Pound fell below its effective floor within the European exchange rate mechanism (ERM), partly because of worries about the UK economy.
6/29/1992	A number of gloomy economic surveys and rumors of possible resignation of the prime minister in UK were released over the weekend.
9/11/1992 -10/1/1992	The European financial crisis emerged with European currencies dropped against DM. UK withdrew from the ERM. German fiscal policy was widely criticized.
10/16/1992	The Bank of England cut key lending rates by a full percentage point to 8%, the lowest rate in four years.
1/29/1993 -2/1/1993	After a 1% interest rate cut to 6% on 1/26, rumors of further rate cuts and government mis-cooperation in UK emerged over the week and was denied on 2/1.
8/26/1994	Second quarter US GDP was revised to 3.8%, prompting a recovery in the US bond market. Rumors emerged that Mr. George Soros was selling yen for dollars.
3/10/1995	An unexpectedly buoyant US employment market (a four year low unemployment rate of 5.4%) and progress in resolving Mexico's financial crisis were reported.
11/13/1995	US producer prices for October was released, indicating a stabilizing price inflation.
11/13/1996	A sharp fall in UK unemployment in October of 40,800 to 2,030,000 was released.
12/19/1996	A surprise (32.2%) drop in the US trade deficit to $7.99 billion in October was released. On 12/18, Japanese government forecast a growth rate of 1.9% in the next fiscal year, the lowest projection since World War II.
8/28/1998	Dollar fell against European currencies in wake of global turmoil.
2/19/1999	The UK government achieved a record cash surplus of pounds 12.4 billion in January.
7/20/2000	Annual growth rate of UK GDP rose to 3.1%, its highest since the first quarter of 1998.
9/22/2000	The world's leading industrialized countries launched a surprise assault on the currency markets to prop up the ailing euro.
2/1/2001	UK Inflation forecast fell below 1.5%.