Fixed vs. Random coefficients in an unbalanced data set.

Øystein Døhl and Jan Larsson

Abstract

In this study we examine two different methods for dealing with heterogeneity in an unbalanced panel. Focus is on comparing the fixed and random coefficient specification of the substitution scale elasticities and technical change and investigating the potential variation elasticities across the firms. The empirical results, based on data from Norwegian pulp industry plants indicate more efficient estimates with the model with fixed coefficient. However, we cannot in our study see any significant differences in our estimated results, i.e. elasticities. But if we do not take account of heterogeneity, the estimates may not be consistent.

Keywords: Panel data, Heterogeneity, Random coefficients, Fixed coefficients, Hypothesis testing, Paper and Pulp.

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Address: Øystein Døhl, Trondheim kommune, Munkegata 1, N-7004 Trondheim, Norway, e-mail: oystein.dohl@trondheim.kommune.no

Jan Larsson, Statistics Norway, Research department, P.O. Box 8131, N-0033 Oslo, Norway, e-mail: jan.larsson@ssb.no.

1. Introduction.

A common problem in econometrics is how to handle heterogeneity. The problem will appear in almost every microeconometric study. Heterogeneity can shortly be described as unobserved differences between the units studied. How serious the problem is, will affect our choice of how detailed the modelling of heterogeneity will be. On one extreme, we can assume the units studied are so different that we have to model them separately. If this is the case, then data often lay restrictions. The other extreme is when the units studied are so similar that we can estimate a common model for the data as a whole without taking the heterogeneity problem into consideration. In some models, not taking the heterogeneity into account, may not affect consistence of the estimates, but if we don't take the structure of the covariance matrix to the total error vector into account, we lose efficiency in the estimates. On the other hand, using a stochastic coefficient model may also cause problems. If the stochastic coefficients are correlated with the observable variables, a model with stochastic coefficients, which is assumed to be stochastic uncorrelated to the dependent variables, will give us biased estimates. This is not the case in a model with fixed coefficients.

How can we then treat heterogeneity? One possibility is to add a fixed coefficient to the parameters where we assume, the differences between the studied units might occur. Another option is to assume that the differences between the studied units will follow a probability distribution. If we have many observed units and/or wish to model heterogeneity in several dimensions, then the first possibility will demand a large amount of degrees of freedom. The other option will save a lot of parameters, but we may have chosen the wrong probability distribution.

The aim of this study is to compare the above mentioned methods, and investigate which application is best suited for our data set. In this study we consider a general framework for analysing the production process i.e. factor substitution, technical change from an unbalanced panel data. The paper discuss the importance of choosing correct specification, dealing with plant heterogeneity in econometric analysis of output supply and factor demand. A generalized Leontief profit function, suggested by Lau (1978), is applied on plant level panel data for Norwegian paper and pulp industry. In our study we will compare the estimated price elasticities, returns to scale and technical change between the different methods. The elasticities are important policy parameters. In our study, we show the importance of including heterogeneity, and that the fixed effect approach seems to be more efficient, than models with stochastic effects. However, the choice of model has a small effect the on the estimated elasticities.

The rest of the paper is organised as follows. In Section 2, we present the model and comments of the restrictions on the profit function, which follow from capital constrained profit maximisation. The econometric specifications with fixed and stochastic coefficients in selected profit function parameters are discussed. The equation system we work with, i.e. the output supply and factor demand equations, derived from Hotelling lemma, is formally a system of regression equations with fixed or stochastic coefficients. We also present the estimation procedure. In Section 3, we briefly present the data, and Section 4 we report and discuss our results.

2. The Model.

Heterogeneity between plants may occur in many dimensions. A lot of it depends on the different technologies. In this context, technology has a broad definition, and cover more than only different capital equipment. It could for example include differences in human capital and/or differences in management. For analysing the problem, we have chosen to model plants behaviour with a Generalized Leontief (GL) profit function.

2.1 Profit function.

We assume that the average plant only produce one output (Y) by a five-factor technology, with electricity (E), fossil fuel (F), labour (L), intermediate material (M) as variable inputs and capital (K) as a quasi-fixed input. We also assume all plants to maximise profit, and to be price takers in both the output and the inputs markets. The plants profit maximising problem can be defined as:

$$
\begin{aligned}
&\pi(p,v,K,\tau) = Max\{py - vx\}\big|_{K,\tau} \\
&y \geq 0 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (2.1) \\
&x_i \geq 0 \\
&(y,x,K) \in Z
\end{aligned}
$$

where π is profit,

y is output,

x a vector of inputs,

p is output price,

v is a vector of input prices,

K is capital and considered as a quasi-fixed input factor,

τ is a time parameter and assumed to measure technical development, here represented by calendar years.

Z is the production possibilities set.

The problem the plants are facing is to find the corresponding volume of production and the volume of each factor input that will maximise the short-run profit, given capital and time. Necessary conditions for duality between production space and the short-run profit function are:

a) Non-negative real-value function for all *p* and *v* <<0.
b) Homogeneous of degree 1 in *p* and *v*.
c) Convex and continuously in *p* and *v*.
d) Non-decreasing in *p*, and non-increasing in *v*.

The real profit function is unknown, but locally it can be approximated by a second order taylor expansion. Several flexible function forms have been proposed, e. g. translog (Christensen, Jorgenson and Lau, 1971), normalised quadratic (Lau, 1978). We have though chosen the Generalized Leontief (GL) profit function (Diewert, 1973). Defined as:

$$\begin{aligned}\pi = {} & \beta_0 + \delta_p p + \sum_i^m \sum_j^m \beta_{ij} \sqrt{v_i}\sqrt{v_j} + \sum_i^m \delta_i \sqrt{p}\sqrt{v_i} \\ & + \beta_\tau \tau + \beta_K K + \beta_{KK} K^2 + \beta_{\tau\tau}\tau^2 + 2\beta_{K\tau} K\tau \\ & + \delta_K pK + \delta_\tau p\tau + \sum_i^m \beta_{Ki} v_i K + \sum_i^m \beta_{\tau i} v_i \tau\end{aligned} \tag{2.2}$$

If the profit function fulfils the above stated conditions, then according to Hotellings lemma, the produced output is given by:

$$\frac{\partial \pi}{\partial p} = y = \delta_p + \frac{1}{2}\frac{\sum_{i=1}^{m} \delta_i \sqrt{v_i}}{\sqrt{p}} + \delta_\tau \tau + \delta_K K \tag{2.3}$$

The demand for the variable input factors is then:

$$-\frac{\partial \pi}{\partial v_i} = x_{\mathrm{i}} = \beta_{ii} + \frac{\sum_{j \neq i}^{m} \beta_{ij} \sqrt{v_j}}{\sqrt{v_i}} + \frac{1}{2} \delta_i \frac{\sqrt{p}}{\sqrt{v_i}} + \beta_{\tau i} \tau + \beta_{Ki} K \ \ , \forall\ i \tag{2.4}$$

The output price elasticity can been derived from supply function (2.3):

$$El_p y \equiv \frac{\partial y}{\partial p} \frac{p}{y} = -\frac{1}{4} \frac{\sum_{i=1}^{m} \delta_i \sqrt{v_i}}{p\sqrt{p}} \frac{p}{y} = -\frac{1}{4} \frac{\sum_{i=1}^{m} \delta_i \sqrt{v_i}}{y\sqrt{p}} \tag{2.5}$$

Similar, the input price elasticity can be derived from the demand function (2.4):

$$El_{v_i} x_i \equiv \frac{\partial x_i}{\partial v_i} \frac{v_i}{x_i} = -\frac{2\sum_{j \neq i}^{m} \beta_{ij} \sqrt{v_j} + \delta_i \sqrt{p}}{4 v_i \sqrt{v_i}} \frac{v_i}{x_i} = -\frac{2\sum_{j \neq i}^{m} \beta_{ij} \sqrt{v_j} + \delta_i \sqrt{p}}{4 x_i \sqrt{v_i}} \tag{2.6}$$

We can also derive several cross price elasticities, as between output supply and factor prices, which by symmetry is equal to the elasticities between factor demand and output price except the sign

$$El_{v_i} y \equiv \frac{\partial y}{\partial v_i} \frac{v_i}{y} = -El_p x_i \equiv -\frac{\partial x_i}{\partial p} \frac{p}{x_i} = \frac{\delta_i}{4\sqrt{v_i}\sqrt{p}} \frac{v_i}{y} = \frac{\delta_i \sqrt{v_i}}{4y\sqrt{p}} \tag{2.7}$$

and cross price elasticities between factor demand

$$El_{v_j} x_i \equiv \frac{\partial x_i}{\partial v_j} \frac{v_j}{x_i} = -\frac{\beta_{ij}}{2\sqrt{v_i}\sqrt{v_j}} \frac{v_j}{x_i} = -\frac{\beta_{ij}\sqrt{v_j}}{2x_i\sqrt{v_i}} \tag{2.8}$$

The effect of output and variable factor demand on a change in capital can be calculated as

$$El_K y \equiv \frac{\partial y}{\partial K} \frac{K}{y} = \frac{\delta_K K}{y} \tag{2.9}$$

$$El_K x_i \equiv \frac{\partial x_i}{\partial K} \frac{K}{x_i} = -\frac{\beta_{Ki} K}{x_i} \tag{2.10}$$

The time trend is supposed to be a proxy for technical development. Assuming the technical development is constant over time, we can approximate the annual technical change effect on output supply and factor demand as

$$\varepsilon_{\tau} y = \frac{\partial y}{\partial \tau} \frac{1}{y} = \frac{\delta_{\tau}}{y} \tag{2.11}$$

and

$$\varepsilon_{\tau} x_i = \frac{\partial x_i}{\partial \tau} \frac{1}{x_i} = - \frac{\beta_{\tau i}}{x_i} \; . \tag{2.12}$$

2.2 Econometric specification.

Our analysis is made on an unbalanced data set. We assume that the selection rules for unbalanced panels are negligible, i. e. we assume that there exists no relations between the plants exits and entrances and any of the endogenous variables in the model (see Verbeek and Nijman, 1996). We choose to organise the plants after observation years, so that every subgroup is composed of a balanced data set. Subscript *(ir)t* describes plant *i* observed in *r* years in the period *t*. N_r is the number of plants observed in r years. In a balanced data set, t often runs from the first to the last observation year. when we organise the observation in this way, it is important to notify that the subscript *t*, can indicate different years. Or to put it in other words, the subscript *t* is the number of observation for plant *i*.

The supply and demand functions from (2.3) and (2.4) can be written in compact form as:

$$y_{(ir)t} = x_{(in)t}\,\beta + \alpha + \mu_{(ir)t}, \qquad i=1,....,N_r, \;\; r=1,....,R \text{ and } t=1,....,r \tag{2.13}$$

where we assume:

$x_{(ir)t}$ and $\mu_{(ir)t}$ are stochastic independent and (2.14)

$\mu_{(ir)t} \sim \text{IID}(0_{G,1}, \Omega^{\mu})$ (2.15)

IID signifies independently, identical normally distribution, and where

$$\Omega^{\mu} = \begin{bmatrix} \sigma_{11}^{\mu} & \cdots & \sigma_{1G}^{\mu} \\ \vdots & \ddots & \vdots \\ \sigma_{G1}^{\mu} & \cdots & \sigma_{GG}^{\mu} \end{bmatrix} \tag{2.16}$$

Writing the covariance matrix for the genuine disturbance term in this way makes it possible for the disturbances in the different equations to be correlated for the same plant at a given point of time. However within each equation the error components are homoskedastic.

Following (2.3) and (2.4) and include the inputs in our model, we can write the vectors and matrices in (2.13) as:

$$y = \begin{bmatrix} Y \\ -X_E \\ -X_F \\ -X_L \\ -X_M \end{bmatrix}, \quad \mu = \begin{bmatrix} \mu_Y \\ \mu_E \\ \mu_F \\ \mu_L \\ \mu_M \end{bmatrix}, \quad \alpha = \begin{bmatrix} \delta_p \\ \beta_{EE} \\ \beta_{FF} \\ \beta_{LL} \\ \beta_{MM} \end{bmatrix} \tag{2.17}$$

$$\beta = \begin{bmatrix} \beta_{EF} \\ \beta_{EL} \\ \beta_{EM} \\ \beta_{FL} \\ \beta_{FM} \\ \beta_{LM} \\ \delta_E \\ \delta_F \\ \delta_L \\ \delta_M \\ \beta_{\tau E} \\ \beta_{\tau F} \\ \beta_{\tau L} \\ \beta_{\tau M} \\ \beta_{KE} \\ \beta_{KF} \\ \beta_{KL} \\ \beta_{KM} \\ \delta_\tau \\ \delta_k \end{bmatrix}, \quad x' = \begin{bmatrix} 0 & \frac{\sqrt{v_F}}{\sqrt{v_E}} & \frac{\sqrt{v_E}}{\sqrt{v_F}} & 0 & 0 \\ 0 & \frac{\sqrt{v_L}}{\sqrt{v_E}} & 0 & \frac{\sqrt{v_E}}{\sqrt{v_L}} & 0 \\ 0 & \frac{\sqrt{v_M}}{\sqrt{v_E}} & 0 & 0 & \frac{\sqrt{v_E}}{\sqrt{v_M}} \\ 0 & 0 & \frac{\sqrt{v_L}}{\sqrt{v_F}} & \frac{\sqrt{v_F}}{\sqrt{v_L}} & 0 \\ 0 & 0 & \frac{\sqrt{v_M}}{\sqrt{v_F}} & 0 & \frac{\sqrt{v_F}}{\sqrt{v_M}} \\ 0 & 0 & 0 & \frac{\sqrt{v_M}}{\sqrt{v_L}} & \frac{\sqrt{v_L}}{\sqrt{v_M}} \\ \frac{1}{2}\frac{\sqrt{v_E}}{\sqrt{p}} & \frac{1}{2}\frac{\sqrt{p}}{\sqrt{v_E}} & 0 & 0 & 0 \\ \frac{1}{2}\frac{\sqrt{v_F}}{\sqrt{p}} & 0 & \frac{1}{2}\frac{\sqrt{p}}{\sqrt{v_F}} & 0 & 0 \\ \frac{1}{2}\frac{\sqrt{v_L}}{\sqrt{p}} & 0 & 0 & \frac{1}{2}\frac{\sqrt{p}}{\sqrt{v_L}} & 0 \\ \frac{1}{2}\frac{\sqrt{v_M}}{\sqrt{p}} & 0 & 0 & 0 & \frac{1}{2}\frac{\sqrt{p}}{\sqrt{v_M}} \\ 0 & t & 0 & 0 & 0 \\ 0 & 0 & t & 0 & 0 \\ 0 & 0 & 0 & t & 0 \\ 0 & 0 & 0 & 0 & t \\ 0 & K & 0 & 0 & 0 \\ 0 & 0 & K & 0 & 0 \\ 0 & 0 & 0 & K & 0 \\ 0 & 0 & 0 & 0 & K \\ t & 0 & 0 & 0 & 0 \\ K & 0 & 0 & 0 & 0 \end{bmatrix}$$

There are r (r=1,....,R) x matrices, and y and μ vectors for each plant. Ω^μ defined in (2.16) is a symmetric matrix that can be written as:

$$\Omega^\mu = \begin{bmatrix} \sigma^\mu_{YY} & \sigma^\mu_{YE} & \sigma^\mu_{YF} & \sigma^\mu_{YL} & \sigma^\mu_{YM} \\ \sigma^\mu_{EY} & \sigma^\mu_{EE} & \sigma^\mu_{EF} & \sigma^\mu_{EL} & \sigma^\mu_{EM} \\ \sigma^\mu_{FY} & \sigma^\mu_{FE} & \sigma^\mu_{FF} & \sigma^\mu_{FL} & \sigma^\mu_{FM} \\ \sigma^\mu_{LY} & \sigma^\mu_{LE} & \sigma^\mu_{LF} & \sigma^\mu_{LL} & \sigma^\mu_{LM} \\ \sigma^\mu_{MY} & \sigma^\mu_{ME} & \sigma^\mu_{MF} & \sigma^\mu_{ML} & \sigma^\mu_{MM} \end{bmatrix} \qquad (2.18)$$

Equation (2.13)-(2.15) can then be written in a compact form for the r observations of plant i.

$$y_{(ir)} = x_{(ir)}\beta + \alpha_{(ir)} + \mu_{(ir)} \tag{2.19}$$

where

$$y_{(ir)} = \begin{bmatrix} y_{(ir)1} \\ \vdots \\ y_{(ir)r} \end{bmatrix}, \; x_{(ir)} = \begin{bmatrix} x_{(ir)1} \\ \vdots \\ x_{(ir)r} \end{bmatrix} \text{ and } \mu_{(ir)} = \begin{bmatrix} \mu_{(ir)1} \\ \vdots \\ \mu_{(ir)r} \end{bmatrix}$$

and

$$\alpha_{(ir)} = \lfloor e_{(r)} \otimes \alpha \rfloor \tag{2.20}$$

where $\alpha_{(ir)}$ is a (Grx1) vector and $e_{(r)}$ is a (rx1) unit vector. In this case $\alpha_{(ir)}$ do not indicate any plant specific characterisation, because all elements are equal for all units. But below we shall se how we can modify (2.18) by incorporating heterogeneity.

From (2.14), (2.15) and (2.19) we have

$$\mu_{(ir)} \sim \text{IID}(0_{G,1}, \Omega^{\mu}_{(ir)}), \tag{2.21}$$

where

$$\Omega^{\mu}_{(ir)} = I_r \otimes \Omega^{\mu} = \begin{bmatrix} \Omega^{\mu} & 0 & \cdots & 0 \\ 0 & \Omega^{\mu} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Omega^{\mu} \end{bmatrix} \tag{2.22}$$

Since I_r is a *rxr* identity matrix and Ω^{μ} is a matrix of dimension (*G*x*G*), the $\Omega^{\mu}_{(ir)}$ have the dimension (*GR*x*GR*).

In our data set there are 45 plants observed over 22 years (1972-1993). It is not obvious, even though it is in our case, that there are firms observed over the whole period. But the way the data is organised, it is important to treat heterogeneity in the right way. Let's take a simple example. Suppose we have two firms, call them i=1,2, which only exist in one year (r=1). But the first plant exists in the beginning of the period, and the second in the end. These two plants will be grouped in same subgroup, but the heterogeneity between these plants will probably be significant, whit respect to technology. But if the heterogeneity only depends on the cause of existence in time, these general differences will be explained by the trend parameter δ_τ.

In this study, we will examine three different assumptions with respect to heterogeneity. In model A described in part 2.2.1, we assume absence of plant specific heterogeneity. Then we assume fixed heterogeneity coefficients in model B, described in part 2.2.2. In the last model C, described in part 2.2.3, we assume stochastic coefficients. We assume that heterogeneity can be modelled by plant specific coefficients in the parameters, δ_p, β_{EE}, β_{FF}, β_{LL}, and β_{MM}. This means that the intercepts in equation (2.3) and (2.4) are plant specific, whereas the slope coefficients are assumed to be plant invariant throughout.

2.2.1 Homogeneous plants

First, we will describe the case with homogeneous plants, i.e. we assume all plants in the examined industrial branch to be equal regarding technology, in the sense that, the model will not be improved by adding heterogeneity conditions in estimation. This is the reference model, which will be modified later, and will be referred to as model A.

We will estimate the model described in (2.19)-(2.20), given the vectors in (2.17), by maximum likelihood method. The density function for the disturbance term $\mu_{(ir)}$, which is same for all plants, is with the assumption of identical independent distribution given by

$$f_{(ir)}=(2\pi)^{-\frac{Gr}{2}}\left|\Omega^{\mu}_{(ir)}\right|^{-\frac{1}{2}}\exp\left[-\frac{1}{2}\mu'_{(ir)}{\Omega^{\mu}_{(ir)}}^{-1}\mu_{(ir)}\right] \qquad (2.23)$$

Taking the logarithm of (2.17) and adding (2.13), we will get the log-likelihood function for plant (ir). The coefficient $\alpha^{H}_{(ir)}$ has the superscript H, indicating a homogeneous model.

$$\ln(L_{(ir)}) = -\frac{Gr}{2}\ln(2\pi)-\frac{1}{2}\ln\left|\Omega^{\mu}_{(ir)}\right| \; -\frac{1}{2}\left[y_{(ir)}-x_{(ir)}\beta-\alpha^{H}_{(ir)}\right]'{\Omega^{\mu}_{(ir)}}^{-1}\left[y_{(ir)}-x_{(ir)}\beta-\alpha^{H}_{(ir)}\right] \quad (2.24)$$

Because the observations from the plants are assumed to be independent, the conditional log-likelihood function for all plants, will be the sum of (2.24) over all plants:

$$
\begin{aligned}
L = \sum_{r=1}^{R}\sum_{i=1}^{N_r} lnL_{(ir)} &= -\frac{G\sum_{r=1}^{R} rN_r}{2}\ln(2\pi) - \frac{1}{2}\sum_{r=1}^{R}\sum_{i=1}^{N_r}\ln\left|\Omega_{(ir)}^{\mu}\right| \\
&- \frac{1}{2}\sum_{r=1}^{R}\sum_{i=1}^{N_r}\left[y_{(ir)} - x_{(ir)}\beta - \alpha_{(ir)}^{H}\right]' \Omega_{(ir)}^{\mu\,-1}\left[y_{(ir)} - x_{(ir)}\beta - \alpha_{(ir)}^{H}\right]
\end{aligned}
\tag{2.25}
$$

where $\Omega_{(ir)}^{\mu}$ is the covariance matrix given by (2.22). If we set

$$
Q\left(\beta,\alpha,\Omega_{(ir)}^{\mu}\right) = \sum_{r=1}^{R}\sum_{i=1}^{N_r}\left[y_{(ir)} - x_{(ir)}\beta - \alpha_{(ir)}^{H}\right]' \Omega_{(ir)}^{\mu\,-1}\left[y_{(ir)} - x_{(ir)}\beta - \alpha_{(ir)}^{H}\right], \tag{2.26}
$$

we can simplify (2.24) as

$$
\mathrm{L} = -\frac{Gn}{2}\ln(2\pi) - \frac{1}{2}\sum_{r=1}^{R}\sum_{i=1}^{N_r}\ln\left|\Omega_{(ir)}^{\mu}\right| - \frac{1}{2}Q\left(\beta,\alpha,\Omega_{(ir)}^{\mu}\right). \tag{2.27}
$$

L will be maximised with respect to the unknown coefficient matrices β and α, and the parameters that characterise the covariance matrix $\Omega_{(ir)}^{\mu}$ (for all *ir*). Direct maximisation of L will be complicated, because we can not write the estimators in closed form. However, we can solve the first order conditions and the solution conditions by an iterative method. These simplified solution conditions are similar for all here stated estimation problems. We confine ourselves to describe this method in section 2.2.3 where we use stochastic parameters.

2.2.2 Fixed coefficients

In model B, we assume all coefficients to be fixed, but δ_p, β_{EE}, β_{FF}, β_{LL}, and β_{MM} will vary between the plants. This is modelled by utilising dummy variables of the plants.

Then we have to modify (2.19)

$$
y_{(ir)} = x_{(ir)}\beta + \alpha_{(ir)}^{F} + \mu_{(ir)} \tag{2.19'}
$$

For (2.20) to take care of the plant specific heterogeneity, we rewrite it to

$$\alpha^F_{(ir)} = e_r \otimes \tilde{\alpha}_{(rp)}, \qquad (2.20')$$

where $\tilde{\alpha}_{(ir)} = \left(\delta^F_{p(ir)}, \beta^F_{EE(ir)}, \beta^F_{FF(ir)}, \beta^F_{LL(ir)}, \beta^F_{MM(ir)}\right)'$ e_r is a *rx1* unit vector, then $\alpha^F_{(ir)}$ becomes a Grx1 vectorThe superscript F indicates that this is a fixed coefficient model.

We can best see the differences between (2.20) and (2.20'), if we aggregate all $N_r\ \alpha_{ir}$ into one matrix, then (2.20) can be rewritten as

$$\alpha^H_{N_r} = e_{N_r} \otimes \alpha^H_{(ir)}, \qquad (2.20b)$$

where e_{N_r} is a (rN×1) unit vector, $\alpha^H_{(ir)}$ is a ($GrNr$×1) vector where all rN_r elements is α. (2.20') can we rewrite as

$$\alpha^F_{N_r} = I_{N_r} \otimes \alpha^F_{(ir)}, \qquad (2.20'b)$$

where I_{N_r} is a (rN_r×rN_r) identity matrix, $\alpha^F_{N_r}$ is a (GrN_r×GN_r) matrix.

We solve this problem exactly as in the case with homogenous plants, except that we in this model have N_r intercepts to solve.

2.2.3 Random coefficients.

In model C we assume that the parameters δ_p, β_{EE}, β_{FF}, β_{LL}, and β_{MM}, vary stochastically between plants according to a multivariate normal distribution, while the remaining parameters are fixed and equal over plants. The random parameters can be decomposed by

$$\alpha^{S}_{(ir)} = \kappa + \varepsilon_{(ir)}, \tag{2.28}$$

where $\alpha^{S}_{(ir)}$ is a $(G\times 1)$ vector, which is supposed to catch the plant specific effects. The superscript *s* denotes stochastic coefficient model. κ is a $(G\times 1)$ vector with the common expected value for all plants. $\varepsilon_{(ir)}$ is a $(G\times 1)$ plant specific vector. In our case (2.28) can be written as

$$\alpha^{S}_{(ir)} \equiv \begin{bmatrix} \alpha_{\delta_p\ (ir)} \\ \alpha_{\beta_{EE}\ (ir)} \\ \alpha_{\beta_{FF}\ (ir)} \\ \alpha_{\beta_{LL}\ (ir)} \\ \alpha_{\beta_{MM}\ (ir)} \end{bmatrix} = \begin{bmatrix} \kappa_{\delta_p} \\ \kappa_{\beta_{EE}} \\ \kappa_{\beta_{FF}} \\ \kappa_{\beta_{LL}} \\ \kappa_{\beta_{MM}} \end{bmatrix} + \begin{bmatrix} \varepsilon_{\delta_p\ (ir)} \\ \varepsilon_{\beta_{EE}\ (ir)} \\ \varepsilon_{\beta_{FF}\ (ir)} \\ \varepsilon_{\beta_{LL}\ (ir)} \\ \varepsilon_{\beta_{MM}\ (ir)} \end{bmatrix} \tag{2.29}$$

We assume

$$\varepsilon_{(ir)} \sim \mathrm{IID}(0_{G,1}, \Omega^{\alpha}), \tag{2.30}$$

In our case (where G=5), Ω^{α} is

$$\Omega^{\alpha} = \begin{bmatrix} \sigma^{\alpha}_{YY} & \sigma^{\alpha}_{YE} & \sigma^{\alpha}_{YF} & \sigma^{\alpha}_{YL} & \sigma^{\alpha}_{YM} \\ \sigma^{\alpha}_{EY} & \sigma^{\alpha}_{EE} & \sigma^{\alpha}_{EF} & \sigma^{\alpha}_{EL} & \sigma^{\alpha}_{EM} \\ \sigma^{\alpha}_{FY} & \sigma^{\alpha}_{FE} & \sigma^{\alpha}_{FF} & \sigma^{\alpha}_{FL} & \sigma^{\alpha}_{FM} \\ \sigma^{\alpha}_{LY} & \sigma^{\alpha}_{LE} & \sigma^{\alpha}_{LF} & \sigma^{\alpha}_{LL} & \sigma^{\alpha}_{LM} \\ \sigma^{\alpha}_{MY} & \sigma^{\alpha}_{ME} & \sigma^{\alpha}_{MF} & \sigma^{\alpha}_{ML} & \sigma^{\alpha}_{MM} \end{bmatrix} \tag{2.31}$$

The matrix is symmetric so the subscript *ij=ji*. Since the off-diagonal elements are free, we allow for correlation between the plant specific effects in the different equations. But within the equations the error terms are homoskedastic. We can now rewrite (2.19), by substitute in (2.28)

$$y_{(ir)t} = x_{(ir)t}\beta_{(ir)} + \kappa + \eta_{(ir)t}, \tag{2.32}$$

where

$$\eta_{(ir)t} = \varepsilon_{(ir)} + \mu_{(ir)t.}$$

We assume $x_{(ir)t}$, $\varepsilon_{(ir)}$ and $\mu_{(ir)t}$ to be stochastically independent. We can now rewrite (2.19) in a compact form

$$y_{(ir)} = x_{(ir)}\beta_{(ir)} + \kappa_{(ir)} + \eta_{(ir)}, \qquad (2.19'')$$

where $y_{(ir)}$, $x_{(ir)}$ and $\beta_{(ir)}$ are the same vectors/matrices as in (2.19). The vector $\kappa_{(ir)}$ is given by

$$\kappa_{(ir)} = e_r \otimes \kappa \qquad (2.33)$$

where e_r is a $(r\times 1)$ unit vector, and where κ, as explained above, is common to all plants. Furthermore

$$\eta_{(ir)} \sim \mathrm{IID}(0_{Gr,1}, \Omega_{(ir)}) \qquad (2.34)$$

where the covariance matrix is given by

$$\Omega_{(ir)} = (e_r e_r') \otimes \Omega^{\alpha} + I_r \otimes \Omega^{\mu} \qquad (2.35)$$

$(e_r e'_r)$ is a $(r\times r)$ matrix containing only ones. Since Ω^{α} is a $(G\times G)$ matrix, $(e_r e_r') \otimes \Omega^{\alpha}$ is a matrix with dimension $Gr\times Gr$.

$$(e_r e_r') \otimes \Omega^{\alpha} = \begin{bmatrix} \Omega^{\alpha} & \Omega^{\alpha} & \cdots & \Omega^{\alpha} \\ \Omega^{\alpha} & \Omega^{\alpha} & \cdots & \Omega^{\alpha} \\ \vdots & \vdots & \ddots & \vdots \\ \Omega^{\alpha} & \Omega^{\alpha} & \cdots & \Omega^{\alpha} \end{bmatrix} \qquad (2.36)$$

I_r is an identity matrix with r elements on the diagonal. $I_r \otimes \Omega^{\mu}$ is the Kroenecker product between I_r and Ω^{μ}, which gives us the block-diagonal $(Gr\times Gr)$ matrix

$$\mathrm{I_r}\otimes\Omega^{\mu} = \begin{bmatrix} \Omega^{\mu} & 0 & \cdots & 0 \\ 0 & \Omega^{\mu} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Omega^{\mu} \end{bmatrix}. \tag{2.37}$$

Thus we have

$$\Omega_{(\mathrm{ir})} = \begin{bmatrix} \Omega^{\alpha}+\Omega^{\mu} & \Omega^{\alpha} & \cdots & \Omega^{\alpha} \\ \Omega^{\alpha} & \Omega^{\alpha}+\Omega^{\mu} & \cdots & \Omega^{\alpha} \\ \vdots & \vdots & \ddots & \vdots \\ \Omega^{\alpha} & \Omega^{\alpha} & \cdots & \Omega^{\alpha}+\Omega^{\mu} \end{bmatrix}. \tag{2.38}$$

We define $(e_r e'_r)=rA_r$ and $I_r=A_r+B_r$, where A_r is a $(r\times r)$ matrix, with all terms equal to $\frac{1}{r}$, and B_r is then a (r×r) matrix where the elements on the diagonal equal $1-\frac{1}{r}$.

$$A_r = \begin{bmatrix} \frac{1}{r} & 0 & . & 0 \\ 0 & \frac{1}{r} & . & . \\ . & . & . & . \\ 0 & . & 0 & \frac{1}{r} \end{bmatrix} \tag{2.39}$$

$$B_r = \begin{bmatrix} 1-\frac{1}{r} & 0 & \dots & 0 \\ 0 & 1-\frac{1}{r} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1-\frac{1}{r} \end{bmatrix} \tag{2.40}$$

Then we can write (2.35) as

$$\Omega_{(ir)} = A_r \otimes \left(\Omega^{\mu} + r\Omega^{\alpha}\right) + B_r \otimes \Omega^{\mu} \tag{2.41}$$

The inverse of $\Omega_{(\mathrm{ir})}$ given in (2.41), can the be written as

$$\Omega_{(ir)}^{-1} = A_r \otimes \left(\Omega^{\mu} + r\Omega^{\alpha}\right)^{-1} + B_r \otimes \Omega^{\mu^{-1}} \tag{2.42}$$

To estimate the equation (2.19") we use maximum likelihood. The density function for the compound error term $\eta_{(ir)}$, with conditioned on $x_{(ir)}$ for plant (*ir*) is given by

$$f_{(ir)}=(2\pi)^{-\frac{Gr}{2}}\left|\Omega_{(ir)}\right|^{-\frac{1}{2}}\exp\left[-\frac{1}{2}\eta'_{(ir)}\Omega^{-1}_{(ir)}\eta_{(ir)}\right] \tag{2.43}$$

By taking the logarithm of (3.43) and add (2.19"), the log-likelihood function for plant (*ir*) is given by

$$\ln(L_{(ir)}) = -\frac{Gr}{2}\ln(2\pi)-\frac{1}{2}\ln\left|\Omega_{(ir)}\right|-\frac{1}{2}\left[y_{(ir)}-x_{(ir)}\beta-\kappa_{(ir)}\right]'\Omega^{-1}_{(ir)}\left[y_{(ir)}-x_{(ir)}\beta-\kappa_{(ir)}\right] \tag{2.44}$$

(2.44) can also be written as

$$lnL_{(ir)}=-\frac{Gr}{2}\ln(2\pi)-\frac{1}{2}\ln\left|\Omega_{(ir)}\right|-\frac{1}{2}Q_{(ir)}\left(\beta,\kappa,\Omega^{\mu},\Omega^{\beta}\right) \tag{2.45}$$

where the quadratic form is given by

$$Q_{(ir)}\left(\beta,\kappa,\Omega^{\mu},\Omega^{\beta}\right)=\left[y_{(ir)}-x_{(ir)}\beta-\kappa_{(ir)}\right]'\Omega^{-1}_{(ir)}\left[y_{(ir)}-x_{(ir)}\beta-\kappa_{(ir)}\right] \tag{2.46}$$

Because the observations from the plants are independent, the conditioned log-likelihood function is the sum of (2.45) for all plants.

$$\ln L = \sum_{r=1}^{R}\sum_{i=1}^{N_r} lnL_{(ir)} =$$

$$-\frac{G\sum_{r=1}^{R}rN_r}{2}\ln(2\pi)-\frac{1}{2}\sum_{r=1}^{R}\sum_{i=1}^{N_r}\ln\left|\Omega_{(ir)}\right|-\frac{1}{2}\sum_{r=1}^{R}\sum_{i=1}^{N_r}\left[y_{(ir)}-x_{(ir)}\beta-\kappa_{(ir)}\right]'\Omega^{-1}_{(ir)}\left[y_{(ir)}-x_{(ir)}\beta-\kappa_{(ir)}\right] \tag{2.47}$$

where $\Omega_{(ip)}$ is given by (2.40). (2.47) can then be written as

$$\ln L = \sum_{r=1}^{R}\sum_{i=1}^{N_r} lnL_{(ir)} = -\frac{Gn}{2}\ln(2\pi)-\frac{1}{2}\sum_{r=1}^{R}\sum_{i=1}^{N_r}\ln\left|\Omega_{(ir)}\right|-\frac{1}{2}Q\left(\beta,\kappa,\Omega^{\mu},\Omega^{\beta}\right) \tag{2.48}$$

where

$$Q\left(\beta,\kappa,\Omega^{\mu},\Omega^{\beta}\right)=\sum_{r=1}^{R}\sum_{i=1}^{N_r}\left[y_{(ir)}-x_{(ir)}\beta-\kappa_{(ir)}\right]'\Omega^{-1}_{(ir)}\left[y_{(ir)}-x_{(ir)}\beta-\kappa_{(ir)}\right] \tag{2.49}$$

L is maximised with respect to the unknown variance components in $\Omega_{(ir)}$, the coefficient vector β and the vector κ . Direct maximisation of L is a complicated procedure, and we cannot write the estimators in a closed form. It is convenient to divide the maximisation problem into two parts.

1. Maximise (2.49) with respect to β and κ, given Ω^μ and Ω^α.
2. Maximise (2.49) with respect to Ω^μ and Ω^α, given β and κ.

The solution to the first part of the problem is similar to the more common Generalised Least Square (GLS) method. With small manipulations, following to Biørn (2000b), the solution of the GLS estimators can be written as

$$\hat{\beta}_{GLS} = \hat{\beta}\left(\Omega^\mu, \Omega^\alpha\right) =$$

$$\begin{bmatrix} \sum_{r=1}^{R}\sum_{i=1}^{N_r} \tilde{X}'_{(ir)}\left[A_r \otimes \left(\Omega^\mu + r\Omega^\alpha\right)^{-1}\right]\tilde{X}_{(ir)} \\ + \sum_{r=1}^{R}\sum_{i=1}^{N_r} \tilde{X}'_{(ir)}\left(B_r \otimes \Omega^{\mu^{-1}}\right)\tilde{X}_{(ir)} \end{bmatrix}^{-1} \times \begin{bmatrix} \sum_{r=1}^{R}\sum_{i=1}^{N_r} \tilde{X}'_{(ir)}\left[A_r \otimes \left(\Omega^\mu + r\Omega^\alpha\right)^{-1}\right]\tilde{y}_{(ir)} \\ + \sum_{r=1}^{R}\sum_{i=1}^{N_r} \tilde{X}'_{(ir)}\left(B_r \otimes \Omega^{\mu^{-1}}\right)\tilde{y}_{(ir)} \end{bmatrix} \qquad (2.50)$$

where A_r and B_r is defined as (2.41) and (2.42) and where

$$\tilde{X}_{(ir)} = \begin{bmatrix} x_{(i1)} \\ \vdots \\ x_{(ir)} \end{bmatrix} \text{ where } x_{(ir)t} = \begin{bmatrix} x_{(ir)1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & x_{(ir)G} \end{bmatrix}$$

and

$$\tilde{y}_{(ir)} = \begin{bmatrix} y_{(i1)} \\ \vdots \\ y_{(ir)} \end{bmatrix} \text{ where } y_{(ir)t} = \begin{bmatrix} y_{(ir)1} \\ \vdots \\ y_{(ir)G} \end{bmatrix} \qquad (2.51)$$

The plant specific mean for equation g can be written as

$$\bar{y}_{(ir)g} = \frac{1}{r}\sum_{j=1}^{r} y_{(ir)gj} \qquad (2.51)$$

and

$$\bar{x}_{(ir)g} = \frac{1}{r}\sum_{j=1}^{r} x_{(ir)gj} \quad \text{g=1,...,G} \tag{2.52}$$

We can write the weighted global mean as

$$\bar{y} = \left[\sum_{r=1}^{R}\sum_{i=1}^{r} r\theta_{B(ir)}\right]^{-1} \times \left[\sum_{r=1}^{R}\sum_{i=1}^{r}\left(r\theta_{B(ir)}\right)\bar{y}_{(ir)}\right] \tag{2.53}$$

and

$$\bar{x} = \left[\sum_{r=1}^{R}\sum_{i=1}^{r} r\theta_{B(ir)}\right]^{-1} \times \left[\sum_{r=1}^{R}\sum_{i=1}^{r}\left(r\theta_{B(ir)}\right)\bar{x}_{(ir)}\right] \tag{2.54}$$

where $\theta_{B(ir)}$ is a plant specific weight given by

$$\theta_{B(ir)} = \left(\Omega^{\mu} + r\Omega^{\alpha}\right)^{-1}\Omega^{\mu} \tag{2.55}$$

The GLS estimator for the constants can be written as

$$\hat{\kappa}_{GLS} = \hat{\kappa}\left(\Omega^{\mu}, \Omega^{\alpha}\right) = \bar{y} - \bar{x}\hat{\beta}_{GLS} \tag{2.56}$$

The first equality in (2.56) indicates that the estimators of κ and β, both are functions of Ω^{α} and Ω^{μ}.

Though the data set totally is unbalanced, in the way we have organised the data, every subset is a balanced data set. Every subset *r* contains all firms, which is observed for *r* years. Organising data this way, we can use Kronecker product in our calculations.

We now have to solve the second part of our problem. But before we continue we have to introduce some matrix operators, and to declare some properties of scalar functions. This is presented without any proofs. The definitions of the matrix operators are from Berck and Sydsæter (1991)

First, a *vec*-operator is defined as the transforms of an $m{\times}n$ matrix into an $m\,n \times 1$ vector.

$$\text{If } A = [a_1, a_2, ..., a_n]_{m\times n} \text{ then}$$

$$vec(A) = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}_{(mn\times 1)} \tag{2.57}$$

Secondly the trace of a matrix is defined as the sum of the terms of the diagonal elements:

$$tr(A) = \sum_{i=1}^{n} a_{ii} \tag{2.58}$$

We also know that the trace of the product of quadratic matrices is independent of the order of the matrices (see for ex. Lutkepohl, 1996, p 41).

$$\text{tr(ABCD) = tr(CDAB).} \tag{2.59}$$

Further, according to Lütkepohl, (1996 p 42), if we have four matrices $A\ (m\times n)$, $B\ (n\times p)$, $C(p\times q)$, and $D(q\times m)$ then

$$\begin{aligned} tr(ABCD) &= vec(D')'(C'\otimes A)vec(B) \\ &= vec(A')'(D'\otimes B)vec(C) \\ &= vec(B')'(A'\otimes C)vec(D) \\ &= vec(C')'(B'\otimes D)vec(A) \end{aligned} \tag{2.60}$$

According to Biørn (2000), if we have two quadratic matrices C_{mm} and D_{mm}, and let $A_T=(e_Te_T')/T$ and $B=I_T-(e_Te_T')/T$, then the determinant of $[A_T\otimes C)+(B_T\otimes D)]$ is equal to

$$|A_T\otimes C)+(B_T\otimes D)| = |C|\ |D|^{T-1} \tag{2.61}$$

(For proof se Magnus, 1982 pp 271-272.)

We have also the following matrix derivation rules from to Magnus and Neudecke (1988).

If A is a quadratic non-singular (m×m) matrix then

$$\frac{\partial |A|}{\partial A} = |A|(A')^{-1} \tag{2.62}$$

and

$$\frac{\partial \ln|A|}{\partial A} = (A')^{-1} \tag{2.63}$$

If B and C are two non-singular quadratic (m×m) matrices, then the (m×m) matrix CB^{-1} has following characteristic

$$\frac{\partial tr(CB^{-1})}{\partial B} = (B^{-1}CB^{-1})' \tag{2.63}$$

Using (2.57) on the $H_{(ip)}$ error matrix given by

$$\mathrm{H}_{(ir)} = \begin{bmatrix} \eta_{1(ir)1} & \cdots & \eta_{1(ir)r} \\ \vdots & \ddots & \vdots \\ \eta_{G(ir)1} & \cdots & \eta_{G(ir)r} \end{bmatrix}, \tag{2.64}$$

gives

$$(vec(H_{(ir)}))' = \left[\eta_{1(ir)1} \quad \eta_{2(ir)1} \quad \cdots \quad \eta_{G(ir)1} \quad \cdots \quad \cdots \quad \eta_{G(ir)r}\right], \tag{2.65}$$

where $(vec(H_{(ir)}))$ is a (Gr×1) vector.

According to Biørn (2000b), can we write (2.47) as

$$\begin{aligned} &Q_{(ir)}\left(\beta,\kappa,\Omega^{\mu},\Omega^{\beta}\right) = \tilde{\eta}'_{(ir)}\Omega^{-1}_{(ir)}\tilde{\eta}_{(ir)} = \\ &\tilde{\eta}'_{(ir)}\left(A_r \otimes \left(\Omega^{\mu} + r\Omega^{\alpha}\right)^{-1}\right)\tilde{\eta}_{(ir)} + \tilde{\eta}'_{(ir)}\left(B_r \otimes {\Omega^{\mu}}^{-1}\right)\tilde{\eta}_{(ir)} = \\ &vec\left(\mathrm{H}_{(ir)}\right)'\left(A_r \otimes \left(\Omega^{\mu} + r\Omega^{\alpha}\right)^{-1}\right)vec\left(\mathrm{H}_{(ir)}\right) + vec\left(\mathrm{H}_{(ir)}\right)'\left(B_r \otimes {\Omega^{\mu}}^{-1}\right)vec\left(\mathrm{H}_{(ir)}\right) \end{aligned} \tag{2.66}$$

where we in the second equality have used (2.42), and where the error term $\tilde{\eta}_{(ir)}$ is defined as in (2.32). Taking the trace of (2.66) we have

$$Q_{(ir)} = tr(Q_{(ir)}) = tr\left[\mathrm{H}_{(ir)}'\left(\Omega^{\mu} + r\Omega^{\alpha}\right)^{-1}\mathrm{H}_{(ir)}A_r\right] + tr\left[\mathrm{H}_{(ir)}'{\Omega^{\mu}}^{-1}\mathrm{H}_{(ir)}B_r\right]$$

$$= tr\left[\mathrm{H}_{(ir)}A_r\mathrm{H}_{(ir)}{}'\left(\Omega^{\mu} + r\Omega^{\alpha}\right)^{-1}\right] + tr\left[\mathrm{H}_{(ir)}B_r\mathrm{H}_{(ir)}{}'\Omega^{\mu^{-1}}\right] \qquad (2.67)$$

where A'=$H_{(ir)}$, B=$\left(\Omega^{\mu} + r\Omega^{\alpha}\right)^{-1}$, C= $H_{(ir)}$ og D=A_r and A_r'=A_r. In the second part, we use the same rule, but A'=$H_{(ir)}$, B= $\Omega^{\mu^{-1}}$, C= $H_{(ir)}$ and D=B_r and B_r'=B_r.

If we derivate (2.67) with respect on Ω^{μ} and Ω^{μ} respectively using the defined derivation rules (2.62) and (2.63) we get

$$\frac{\partial Q_{(ir)}}{\partial\Omega^{\mu}} = -\left(\Omega^{\mu} + r\Omega^{\alpha}\right)^{-1}\mathrm{H}_{(ir)}A_r\mathrm{H}_{(ir)}{}'\left(\Omega^{\mu} + r\Omega^{\alpha}\right)^{-1} + \Omega^{\mu^{-1}}\mathrm{H}_{(ir)}B_r\mathrm{H}_{(ir)}{}'\Omega^{\mu^{-1}} \qquad (2.68)$$

$$\frac{\partial Q_{(ir)}}{\partial\Omega^{\alpha}} = -r\left(\Omega^{\mu} + r\Omega^{\alpha}\right)^{-1}\mathrm{H}_{(ir)}A_r\mathrm{H}_{(ir)}{}'\left(\Omega^{\mu} + r\Omega^{\alpha}\right)^{-1} \qquad (2.69)$$

Using (2.61) we can rewrite (2.42) as

$$\left|\Omega_{(ir)}\right| = \left|A_r \otimes \left(\Omega^{\mu} + r\Omega^{\alpha}\right) + B_r \otimes \Omega^{\mu}\right| = \left|\Omega^{\mu} + r\Omega^{\alpha}\right|\left|\Omega^{\mu}\right|^{r-1} \qquad (2.70)$$

The derivate of the logarithm of (2.70) with respect to Ω^{μ} and Ω^{α} can be written as

$$\frac{\partial \ln\left|\Omega_{(ir)}\right|}{\partial\Omega^{\mu}} = \frac{\partial \ln\left|\Omega^{\mu} + r\Omega^{\alpha}\right|}{\partial\Omega^{\mu}} + (r-1)\frac{\partial \ln\left|\Omega^{\mu}\right|}{\partial\Omega^{\mu}} = \left(\Omega^{\mu} + r\Omega^{\alpha}\right)^{-1} + (r-1)\Omega^{\mu^{-1}} \qquad (2.71)$$

$$\frac{\partial \ln\left|\Omega_{(ir)}\right|}{\partial\Omega^{\alpha}} = \frac{\partial \ln\left|\Omega^{\mu} + r\Omega^{\alpha}\right|}{\partial\Omega^{\alpha}} = r\left(\Omega^{\mu} + r\Omega^{\alpha}\right)^{-1} \qquad (2.72)$$

The first order conditions of (2.44) with respect to Ω^{μ} and Ω^{α} can, by using (2.68), (2.69) (2.71) and (2.72), be written as

$$\frac{\partial L_{(ir)}}{\partial \Omega^{\mu}} =$$

$$-\frac{1}{2}\left[\left(\Omega^{\mu}+r\Omega^{\alpha}\right)^{-1}+(r-1)\Omega^{\mu^{-1}}-\left(\Omega^{\mu}+r\Omega^{\alpha}\right)^{-1}\mathrm{H}_{(ir)}A_{r}\mathrm{H}_{(ir)}{'}\left(\Omega^{\mu}+r\Omega^{\alpha}\right)^{-1}+\Omega^{\mu^{-1}}\mathrm{H}_{(ir)}B_{r}\mathrm{H}_{(ir)}{'}\Omega^{\mu^{-1}}\right] \tag*{(2.74}$$

$$\frac{\partial L_{(ir)}}{\partial \Omega^{\alpha}} = -\frac{1}{2}\left[r\left(\Omega^{\mu}+r\Omega^{\alpha}\right)^{-1}-r\left(\Omega^{\mu}+r\Omega^{\alpha}\right)^{-1}\mathrm{H}_{(ir)}A_{r}\mathrm{H}_{(ir)}{'}\left(\Omega^{\mu}+r\Omega^{\alpha}\right)^{-1}\right] \tag{2.73}$$

After putting in the first order conditions for maximisation of (2.46) $\frac{\partial L}{\partial \Omega^{\mu}} = \frac{\partial L}{\partial \Omega^{\alpha}} = 0_{G,G}$, where we have summarised the conditions in (2.72 and 2.73), we have a equation system with the solutions

$$\hat{\Omega}^{\mu} + r\,\hat{\Omega}^{\alpha} = \Omega^{\mu}(\beta,\kappa) + r\Omega^{\alpha}(\beta,\kappa) = \frac{1}{N_{r}r}\sum_{r=1}^{\sqrt{P}}\sum_{i=1}^{r}\mathrm{H}_{(ir)}A_{r}\mathrm{H}_{(ir)}^{'} \tag{2.74}$$

$$\hat{\Omega}^{\mu} = \hat{\hat{\Omega}}^{\mu}(\beta,\kappa) = \frac{1}{N_{r}r(r-1)}\sum_{r=1}^{R}\sum_{i=1}^{r}\mathrm{H}_{(ir)}B_{r}\mathrm{H}_{(ir)}^{'} \tag{2.75}$$

By inserting (2.75) into (2.74) and some manipulations we get

$$\hat{\Omega}^{\alpha} = \hat{\Omega}^{\alpha}(\beta,\kappa) = \frac{1}{rN_{r}r}\left[\sum_{r=1}^{R}\sum_{i=1}^{r}\mathrm{H}_{(ir)}A_{r}\mathrm{H}_{(ir)}^{'} - \sum_{r=1}^{R}\sum_{i=1}^{r}\mathrm{H}_{(ir)}B_{r}\mathrm{H}_{(ir)}^{'}\right] \tag{2.76}$$

In (2.75) and (2.76) we have described the solution to the two subproblems 1 and 2. Maximisations will be carried out by a iteration method where the two problems are solve simultaneously. We have used the PROC MIXED program module in SAS/STAT software (SAS, 1992), to solve this. The program is described in Appendix A.

It has to be a certain amount of observations units for the stochastic model to be parameter saving. In our case where we have five stochastic coefficients, we have to estimate 15 unknown components in Ω^{α} . This must be compared to the amount of parameters we have to estimate if we use the model with fixed coefficients. In our case this is 5×the amount of plants. This means that as long as we have more that three observation units, it is parameter saving to use a stochastic model.

3. Data

We are using an unbalanced data set of plants belonging to the Norwegian Pulp industries covering the period 1972-1993. Our primary source is the Manufacturing Statistic database of Statistic Norway, complemented with data from National Account. Every plant is observed at minimum four years and maximum 22 years. The total amounts of plants are 45, and the total amount of observations is 676. So at an average a plant is observed in 15 years. 16 of the plants are observed the whole period, and represent more than half the data set.

In our profit function, we use four variable inputs, Labour (L), Electricity (E), Fossil fuel (F) and Intermediate inputs (M), and one homogeneous output (Y). In addition we use capital (K) as a quasi fixed input. τ is a calendar time trend, which is included to proxy the technological level of the plant. The variables are defined as following:

Labour. Employment (L) is measured in man-hours per year, price of labour (v_l) as total compensation to labour divided by hour worked.

Capital. The Capital stock (K) is the most difficult factor to obtain. The manufacturing statistics have only one measure of capital, the insurance value of capital. One weakness with insurance value is that firms have had different insurance policies over time due to costs. Instead, the perpetual inventory method has been chosen, accumulating the historical investments from an initial bench-market value. To calculate a benchmark value for capital we have used the mean gross profit, i.e. the income minus the variable costs for the establishment, as a proxy variable for the user cost of capital. The capital has then been calculated as the gross profit divided by the user price of capital, defined as investment price index multiplied by the sum of the internal rent and the depreciation rate. The internal rate is set to equal the average interest rent for five years Governmental bounds. Capital is divided in two categories a) buildings and b) machinery and equipment. The index *n* for plants is dropped for convenience.

$$k_t = (1-\delta_t)^{-1} k_{t-1} + i_t \qquad (3.1)$$

$$k_0 = (\Sigma^{T}_{t=0} (p_t y_t - \Sigma_{j=l,m,e} (w_{jt}\, x_{jt}))/(T+1))/p_{kt} \qquad t= (0,..T) \qquad (3.2)$$

$$p_{knt} = p_{invt}(r_t^{*}+\delta_{vt}) \qquad (3.3)$$

$$p_{invt} = (p_{bt}\,\overline{k}_{bt} + p_{mt}\,\overline{k}_{mt}) / (\overline{k}_{bt} + \overline{k}_{mt}) \qquad (3.4)$$

$$r_t^* = (1-r_t)/(1-i_t), \qquad (3.5)$$

$$\delta_t = (\delta_b\,\overline{k}_{bt} + \delta_m\,\overline{k}_{mt}) / (\overline{k}_{bt} + \overline{k}_{mt}) \qquad (3.6)$$

where
p_{invt} = weighted price index for investment goods.
p_{bt} = price index for building investments.
p_{mt} = price index for investments in machinery and equipment's.
k_{bt} = insurance value for buildings.
k_{mt} = insurance value for machinery and equipment's.
r_t^* = the real rate of the long run interest rate.
r_t = the nominal long-term interest rate.
i_t = inflation.
δ_b = the depreciation rate for buildings.
δ_m = the depreciation rate for machinery and equipment.

The depreciation rate for buildings is assumed to be 4 per cent (50 years geometric depreciation). For machinery and equipment the depreciation rate is 8 per cent (25 years). These depreciation rates are used in National Accounts, Norway (see Todsen, 1997). The nominal long-term interest rate is the average interest rate from 5 years Government bonds (source: Banc of Norway)

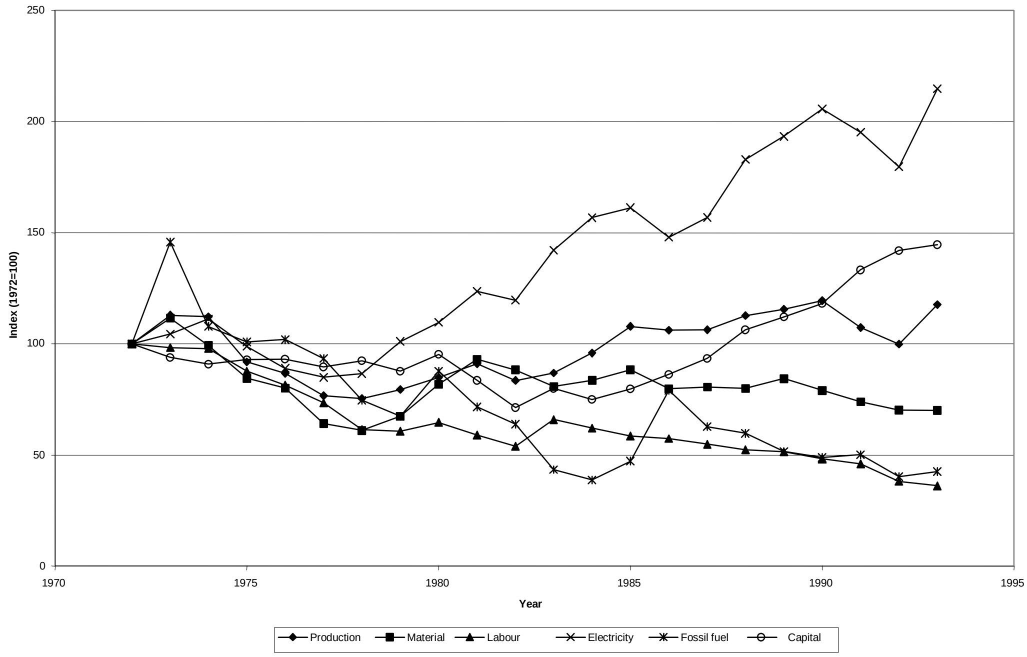
Electricity and fuels. Electricity (E) and fuels (F) inputs are measured in MWh consumed and prices (v_E, v_F) are obtained by dividing the value of input purchased by the quantity in kWh.

Intermediate inputs. (M) is measured in fixed prices (1990). The price (v_M) is an index collected from the National Accounts and is same for all plants.

Output. Output (Y) is measured as produced quantity in tonnes. The output price (p) is calculated as the value of the total production divided by the total produced volume, for each plant and year.

In Figure 3.1, we can se the development of the production and factor use for the whole Pulp industry over the period 1972-1993. There we can see, that the use of labour, fossil fuel and material have declined during the period, but electricity is more intensive used. The use of capital has nearly had the same development as production. This can partly be explained by a shift in technology. The structure of the industry branch has gone from minor plants producing mechanical pulp to major and more capital and energy intensive plants, producing termomechanical and chemical pulp. In the year 1972 there was 42 plants which produced at average 43 400 tonnes pulp. In the year 1993, 17 plants produced at an average 126 900 tonnes pulp.

Figure 3.1 Production and factor use in the Pulp industry. 1972-1993



4. Results

In our estimation of the models described in section 2, we have used the PROC MIXED procedure in SAS/STAT (SAS, 1999). First we will examine which model is best suited given our data set. Then we present different elasticity measures across the models and discuss their economic interpretations.

4.1 Random or fixed effect, what is best suited?

How can we decide what model is best suited given our data set? The model with fixed effects is very costly regarding to lost degrees of freedom, and a model with random effects may have better efficiency. However, as we stated in section 2.2.3, the plant specific effects are assumed to be distributed independently of the right hand side variables. This can lead to inconsistent estimation of the stochastic effects (Hausman and Taylor, 1981).

In table 4.1 we report the estimated elements of the parameter vector β (se equation 2.17). In the model with fixed effects we estimate 245 parameters, with random effects 45 parameters, but in the model with homogeneous coefficients we only estimates 25 parameters. It seems, like for most of the estimated parameters, that the results are similar for all models. We cannot tell, from the estimates, if omission or incorporating heterogeneity in the models gives us inefficient estimates of the coefficients. Most of the coefficients are also significant at the 5 % significance level according to the standard errors.

In table 4.2 we present different goodness of fit measures. Both Akaike's (Akaike, 1987) and Schwarz Bayesian's (Schwarz, 1978) information criteria support the model with fixed plant specific effects, but the model with no heterogeneity effects seems to have the worst performance. In the table we also presents the log-likelihood value, from these values we can calculate the log-likelihood ratio. But according to Biørn et. al. (1998), these ratios are not asymptotic χ^2-distributed, because the coefficients are on the boundary of the parameter space. More about inference in mixed linear models are discussed in Khuri et. al. (1998).

Table 4.1 Coefficient estimates and standard deviation

	Model A Homogeneous coeffisients[1]		Model B Fixed effects[2]		Model C Stochastic effects[3]	
Coefficients	Estimate	Standard deviation.	Estimate	Standard deviation.	Estimate	Standard deviation.
δ_P	-20.239*	5.156	-29.914*	9.615	4.866	9.170
δ_L	14.765*	1.202	12.040*	1.195	11.185*	1.118
δ_E	-5.399	3.159	-3.567	2.865	-4.566	2.855
δ_F	-13.360*	2.426	-4.162*	1.950	-4.938*	1.930
δ_M	14.765*	1.202	12.040*	1.195	11.185*	1.118
β_{LL}	16.944*	4.090	3.062	8.645	16.982*	6.187
β_{LE}	-24.786*	5.330	-20.584*	4.753	-20.326*	4.729
β_{LF}	-39.054*	4.961	-16.499*	3.308	-17.308*	3.367
β_{LM}	-24.787*	2.060	-12.279*	1.787	-11.931*	1.733
β_{EE}	26.187	17.720	95.183*	20.251	42.956*	19.459
β_{EF}	50.869*	11.214	14.627	8.037	14.864	8.134
β_{EM}	14.251*	4.074	3.545	3.198	4.808	3.235
β_{FF}	28.484	18.581	18.570	14.455	-27.052	19.89
β_{FM}	15.720*	3.815	8.460*	2.452	8.004*	2.498
β_{MM}	141.090*	20.873	190.040*	26.355	62.141*	29.495
β_{KL}	0.112*	0.008	-0.034*	0.0148	0.003	0.012
β_{TL}	0.743*	0.294	1.227*	0.275	0.947*	0.273
β_{KE}	-0.390*	0.020	-0.106*	0.028	-0.146*	0.026
β_{TE}	-4.110*	0.709	-5.748*	0.518	-5.519*	0.530
β_{KF}	-0.147*	0.021	0.123*	0.020	0.120*	0.019
β_{TF}	1.090	0.779	0.336	0.377	0.353	0.385
β_{KM}	-0.428*	0.020	0.073*	0.022	0.051*	0.022
β_{TM}	0.791	0.722	-1.268*	0.434	-1.129*	0.444
δ_K	0.266*	0.009	0.052*	0.013	0.073*	0.012
δ_T	0.440	0.317	1.309*	0.249	1.260*	0.249

[1] All coefficients are assumed to be homogeneous.

[2] Average of the fixed plant specific parameters δ_p, β_{LL}, β_{EE}, β_{FF} and β_{MM}.

[3] Expected value of the random parameters in the stochastic model $\delta_{p(ir)}$, $\beta_{LL(ir)}$, $\beta_{EE(ir)}$, $\beta_{FF(ir)}$ and $\beta_{MM(ir)}$.

* Significant at the 5 % level.

To test if a model with random heterogeneity coefficients is more suited than a model with fixed coefficients, we use the Hausman-test, as described in Green (1993). The test is based on hypothesis that, if the condition of stochastic independence described in section 2.2 holds, both the models with random and fixed effects will be consistent, but the model with fixed effects will be inefficient. It means that under the null hypotheses there will not be significant differences in the estimated β-vector between the stochastic and fixed models. Another property of the Hausman-test, is that the covariance between the efficient estimator and the difference between the efficient and inefficient estimator has to be zero.

$$\begin{aligned}
&\text{Cov}\left[(\hat{b}-\hat{\beta}),\hat{\beta}\right]=\frac{1}{2}\left[Var\left(\left(\hat{b}-\hat{\beta}\right)+\hat{\beta}\right)-Var\left(\hat{b}-\hat{\beta}\right)-Var\left(\hat{\beta}\right)\right]=\\
&\frac{1}{2}\left[Var\left(\hat{b}\right)-\left[Var\left(\hat{b}\right)+Var\left(\hat{\beta}\right)-2Cov\left(\hat{b},\hat{\beta}\right)\right]-Var\left(\hat{\beta}\right)\right]=\\
&Cov\left(\hat{b},\hat{\beta}\right)-Var\left(\hat{\beta}\right)=0\Rightarrow Cov\left(\hat{b},\hat{\beta}\right)=Var\left(\hat{\beta}\right)
\end{aligned}\tag{4.1}$$

We also know

$$Var\left(\hat{b}-\hat{\beta}\right)=Var\left(\hat{b}\right)+Var\left(\hat{\beta}\right)-2Cov\left(\hat{b},\hat{\beta}\right)\tag{4.2}$$

By substituting (4.1) into (4.2) we have

$$Var\left(\hat{b}-\hat{\beta}\right)=Var\left(\hat{b}\right)-Var\left(\hat{\beta}\right)=\Sigma\tag{4.3}$$

where Σ now is a matrix. The χ^2 - test based on the Wald criterion (Wald, 1943) is

$$W=X^{2}\left[K\right]=\left[\hat{b}-\hat{\beta}\right]'\hat{\Sigma}^{-1}\left[\hat{b}-\hat{\beta}\right]\tag{4.4}$$

Under the null hypotheses W is asymptotic χ^2 distributed with K degrees of freedom, where K is the number of additional parameters in the less restricted model.

The alternative hypotheses, is that the fixed coefficient model is consistent but not the stochastic coefficient model. Then the condition of stochastic independence fails, and then (4.1) also fails, because $\hat{\beta}$ no longer is consistent. If $\text{Cov}\left[(\hat{b}-\hat{\beta}),\hat{\beta}\right]$ deviates significant from zero, we have to reject the null hypotheses, that the stochastic model is more efficient than the fixed model. The cause is that under the alternative hypotheses, the fixed coefficient model gives consistent estimates, whereas this is not the case for the stochastic coefficient model. Then the condition for the Hausman test also fails. This applies even if $Var(b)<Var(\beta)$. In table 4.2 we can see the Hausman test falls out in favour to Model B with fixed effects Model C with stochastic effects. The χ^2-value with 19 degrees of freedoms is 192.2, which are significant at all reasonable levels of significance. But we have to be careful when we interpret the test, when we use maximum likelihood estimates, because if the conditions of

independence is violated the conditions for Hausman test will fall. We then risk testing inconsistent parameters of the stochastic effect model against the consistent parameters of the fixed effect model.

Table 4.2 Goodness of fit

	Model A	Model B	Model C
Estimated parameters	25	245	40
-2*Log-Likelihood	36282.6	33511.6	34331.5
Akaike Information criterion	36362.6	34031.6	34441.5
Schwarz Bayesian criterion	36543.3	35205.8	34540.9
Hausman test			192.2

An adequate way to evaluate the different models is to examine the covariance matrices for the estimated models. In table 4.3 we report the estimated covariance matrix for the stochastic effect model, and table 4.4 displays the genuine error term for all of the models reported. Besides a few cells the variance are lowest for the Model B with fixed plant specific effects and highest for Model A with no heterogeneity. All estimated covariance matrices fulfils the condition of positive definiteness, because the leading principal minors to the covariance matrices are positive (see Sydsæter, 1990).

Table 4.3 The Covariance matrix for the stochastic effects in Model B

	$\delta_{(ir)}$	$\beta_{L(ir)}$	$\beta_{E(ir)}$	$\beta_{F(ir)}$	$\beta_{M(ir)}$
$\delta_{(ir)}$	84.0956				
$\beta_{L(ir)}$	27.7668	38.2768			
$\beta_{E(ir)}$	-68.452	4.4496	378.66		
$\beta_{F(ir)}$	-79.0469	-28.019	26.0038	395.77	
$\beta_{M(ir)}$	-98.5906	-78.4154	111.88	301.9	869.96

Table 4.4 The Covariance matrix of the genuine error terms

Model A

	u_Y	u_L	u_E	u_F	u_M
u_Y	1965.51				
u_L	-46.2292	11241			
u_E	223.06	-1557.93	13133		
u_F	-1269.62	3008.56	8792.08	10950	
u_M	634.84	-3685.64	-1849.17	-3719.25	2182.76

Model B

	u_Y	u_L	u_E	u_F	u_M
u_Y	1326.63				
u_L	-347.85	4610.69			
u_E	-5.4321	-217.05	2282.51		
u_F	-557.12	1864.44	1290.74	2858.65	
u_M	459.73	-1626.06	-436.96	-1351.83	992.31

Model C

	u_X	u_L	u_E	u_F	u_M
u_X	1432.6				
u_L	-384.32	4946.19			
u_E	-10.0681	-231.34	2442.77		
u_F	-615.25	1993.67	1368.09	3048.19	
u_M	496.58	-1746.95	-468.08	-1432.11	1058.62

4.2 Economic interpretation of the results

In this section we want analyse the results from an economic theory point of view. This is done by reporting the estimated elasticities and compares the results from the different models. The elasticities presented here are the own price elasticities output and variable inputs, cross price elasticities, and output and variable input elasticities with respect to capital and time. All elasticities are defined in section 2.1, and calculated for an average plant in the year 1990.

All the estimated own price elasticities have the expected sign. But the level of the elasticities in Model A with homogeneous coefficients seems to differ. This may indicate that of homogeneity between the plants are too rigid. None of the variable input factors have price elasticity over unity.

Only one of the cross price elasticities is over unity, and that is between fossil fuel and electricity in model A. Corresponding elasticities for model B and C are 0.52 and 0.55. These results are similar to earlier studies, se for example Biørn et. al. (1998) and Døhl (2001).

Table 4.5 Own- and cross price elasticities for an average plant in year 1990.

Model A

	Output	Labour	Electricity	Fossil fuel	Material
Output	0.177	0.249	-0.026	-0.063	0.021
Labour	-0.003	-0.051	0.020	0.028	0.005
Electricity	0.003	0.213	-0.083	-0.123	-0.010
Fossil fuel	0.009	1.233	-0.514	-0.288	-0.042
Material	-0.036	0.903	-0.166	-0.161	-0.124

Model B

	Output	Labour	Electricity	Fossil fuel	Material
Output	0.181	0.203	-0.017	-0.020	0.017
Labour	-0.002	-0.029	0.017	0.012	0.003
Electricity	0.002	0.177	-0.141	-0.035	-0.003
Fossil fuel	0.003	0.521	-0.148	-0.122	-0.023
Material	-0.030	0.447	-0.041	-0.087	-0.470

Model C

	Output	Labour	Electricity	Fossil fuel	Material
Output	0.156	0.189	-0.022	-0.023	0.016
Labour	-0.002	-0.030	0.017	0.012	0.003
Electricity	0.003	0.174	-0.138	-0.036	-0.003
Fossil fuel	0.003	0.546	-0.150	-0.159	-0.021
Material	-0.028	0.435	-0.056	-0.082	-0.155

We have also estimated the effects on supply and demand caused by a change in the capital stock, reported in table 4.6. Here model A also gives different estimates compared to the other models, both regarding signs and level on the estimates. But there is no difference between the other two models. Except in one occasion, the effect on labour of a change in capital is positive in model B with fixed effects, but negative in the stochastic model. However, the effects are very close to zero. The highest effects are found in the demand for fuel. One per cent changes in the capital stock leads to a half per cent reduction on the fuel demand. At the same time the effect for electricity is positive, but not to the same degree. It indicates that the capital investments are energy saving.

Technical development is reported by a linear time trend. We can approximately calculate the annual technical change as $\varepsilon_{x\tau} = \frac{\partial x}{\partial \tau}\frac{1}{x}$. These technical changes, interpreted as a trend effect, are also reported in table 4.6. We can clearly see that the technical change has not been neutral. The sign of the effect differs between the input factors. In this case the homogeneous model does not differ except for, intermediate materials. Nota bene, in contrary to the other input factors, the demand for electricity grows over time.

Table 4.6 The effect on output and factor demand by a change in capital and trends.

	Model A	Model B	Model C
	Homogenous coefficients	Fixed effects	Stochastic effects
El(YK)	0.375	0.976	0.443
El(LK)	-0.123	0.038	-0.003
El(EK)	0.461	0.126	0.173
El(FK)	0.563	-0.472	-0.458
El(MK)	0.566	-0.097	-0.067
$\varepsilon_{L\tau}$	-0.0038	-0.0063	-0.0048
$\varepsilon_{E\tau}$	0.0226	0.0316	0.0303
$\varepsilon_{F\tau}$	-0.0194	-0.0060	-0.0062
$\varepsilon_{M\tau}$	-0.0049	0.0078	0.0069

5. Conclusions

In this study we have shown that it is very important to pay attention to heterogeneity in econometric modelling of panel data. If you don't take this into account, the parameter estimates can be inconsistent. According to our results, the question of modelling heterogeneity with fixed or stochastic coefficients may effect the efficiency of the estimates. However, further tests have to be done to obtain a definite conclusion. Though, our conclusion on this study is, that the choice of how to model heterogeneity does not effect our estimates on elasticities.

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Appendix A :Program PROC MIXED SAS/STAT

```
options PS=55 LS=80 NOCENTER nodate;
title ' GL profit function';
title2 'Demand and Supply functions with homogeneous functions';
* Estimering av modell A1;
proc mixed data=data.s34GL method=ml asycov mmeqsol maxiter=1000 ;
class bed aar;
where lig ne 6;
model profit = gaxx gaxl gaxe gaxf gaxm gall gale galf galm
                gaee gaef gaem gaff gafm gamm
                galk galt gaek gaet gafk gaft gamk gamt gaxk gaxt/
noint s;
repeated / type=un sub=bed(aar) r;

estimate 'gaxx' gaxx 1;
estimate 'gaxl' gaxl 1;
estimate 'gaxe' gaxe 1;
estimate 'gaxf' gaxf 1;
estimate 'gaxm' gaxl 1;
estimate 'gall' gall 1;
estimate 'gale' gale 1;
estimate 'galf' galf 1;
estimate 'galm' galm 1;
estimate 'gaee' gaee 1;
estimate 'gaef' gaef 1;
estimate 'gaem' gaem 1;
estimate 'gaff' gaff 1;
estimate 'gafm' gafm 1;
estimate 'gamm' gamm 1;
estimate 'galk' galk 1;
estimate 'galt' galt 1;
estimate 'gaek' gaek 1;
estimate 'gaet' gaet 1;
estimate 'gafk' gafk 1;
estimate 'gaft' gaft 1;
```

```
estimate 'gamk' gamk 1;
estimate 'gamt' gamt 1;
estimate 'gaxk' gaxk 1;
estimate 'gaxt' gaxt 1;
ods output Estimates=taba1;
ods output MMEQSOL=mmeqsa1;
ods output AsyCov=asya1;
ods output SolutionF=fixa1;
ods output listing exclude all;
run ;


* Estimering av modell A2;
title2 'Demand and Supply functions with fixed effects';

proc mixed data=data.s34GL method=ml asycov mmeqsol maxiter=1000 ;
class bed aar;
where lig ne 6;
model profit = gaxx(bed) gaxl gaxe gaxf gaxm gall(bed) gale galf
galm
                gaee(bed) gaef gaem gaff(bed) gafm gamm(bed)
                galk galt gaek gaet gafk gaft gamk gamt gaxk gaxt/
noint s;
repeated / type=un sub=bed(aar) r;

estimate 'gaxx' gaxx(bed) 1;
estimate 'gaxl' gaxl 1;
estimate 'gaxe' gaxe 1;
estimate 'gaxf' gaxf 1;
estimate 'gaxm' gaxl 1;
estimate 'gall' gall(bed) 1;
estimate 'gale' gale 1;
estimate 'galf' galf 1;
estimate 'galm' galm 1;
estimate 'gaee' gaee(bed) 1;
estimate 'gaef' gaef 1;
estimate 'gaem' gaem 1;
```

```
estimate 'gaff' gaff(bed) 1;
estimate 'gafm' gafm 1;
estimate 'gamm' gamm(bed) 1;
estimate 'galk' galk 1;
estimate 'galt' galt 1;
estimate 'gaek' gaek 1;
estimate 'gaet' gaet 1;
estimate 'gafk' gafk 1;
estimate 'gaft' gaft 1;
estimate 'gamk' gamk 1;
estimate 'gamt' gamt 1;
estimate 'gaxk' gaxk 1;
estimate 'gaxt' gaxt 1;


ods output Estimates=taba2;
ods output MMEQSOL=mmeqsa2;
ods output AsyCov=asya2;
ods output SolutionF=fixa2;
ods output listing exclude all;

run ;



* Estimering av modell A3;
title2 'Demand and Supply functions with random effects';
proc mixed data=data.s34GL method=ml asycov mmeqsol maxiter=1000 ;
class bed aar;
where lig ne 6;
model profit = gaxx gaxl gaxe gaxf gaxm gall gale galf galm
                gaee gaef gaem gaff gafm gamm
                galk galt gaek gaet gafk gaft gamk gamt gaxk gaxt/
noint s;
random gaxx gall gaee gaff gamm/type=un sub=bed s;
repeated / type=un sub=bed(aar) r;
```

```
estimate 'gaxx' gaxx 1;
estimate 'gaxl' gaxl 1;
estimate 'gaxe' gaxe 1;
estimate 'gaxf' gaxf 1;
estimate 'gaxm' gaxl 1;
estimate 'gall' gall 1;
estimate 'gale' gale 1;
estimate 'galf' galf 1;
estimate 'galm' galm 1;
estimate 'gaee' gaee 1;
estimate 'gaef' gaef 1;
estimate 'gaem' gaem 1;
estimate 'gaff' gaff 1;
estimate 'gafm' gafm 1;
estimate 'gamm' gamm 1;
estimate 'galk' galk 1;
estimate 'galt' galt 1;
estimate 'gaek' gaek 1;
estimate 'gaet' gaet 1;
estimate 'gafk' gafk 1;
estimate 'gaft' gaft 1;
estimate 'gamk' gamk 1;
estimate 'gamt' gamt 1;
estimate 'gaxk' gaxk 1;
estimate 'gaxt' gaxt 1;


ods output Estimates=taba3;
ods output MMEQSOL=mmeqsa3;
ods output AsyCov=asya3;
ods output SolutionF=fixa3;
ods output SulotionR=rana3;
ods output listing exclude all;

run ;
```