On the Simultaneity of Components of the Transaction Process

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Abstract

Highly preliminary version. In this paper a new estimation concept is proposed for the intraday transaction process in liquid markets, which is characterized by discrete price changes and an irregular, stochastic occurrence of transactions. It extends the work of Rydberg and Shephard (1998) and Russell and Engle (1998) by suggesting a different decomposition of the joint process of price changes and time between transactions, which includes a parsimonious modelling of the time between transactions component and allows to analyse the size and the sign component of the process of price changes separately.

Unlike previous approaches our model is based on an explicit formulation of the joint transaction process. Similiar to a VARMAX specification the model enables us to distinguish between systematic effects and innovations for the inter-trade durations and the price process.Therefore it is rich enough to discriminate between true structural dependence of the dependent variables and pure statistical correlations due to common innovations.

The empirical results obtained on a sample of intraday trading of the Bund future suggest that inter-trade durations and the size of price changes are indeed simultaneous. This is not the case for the process of the sign of price changes and the time between transactions. The latter were found to be independent.

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1 Introduction

This paper proposes a new specification to model the intraday trading process observed on typical liquid financial markets. The challenge involved in this task lies in the nature of the observed data. Two main features have to be accounted for. First, transactions occur irregularly with a random time between transactions, quite contrary to daily observations or intraday aggregates. Second, price changes observed take on only few distinct values. For instance, in the sample of Bund future trading at the DTB which is used in this study, 97% of all observations fall within three categories, no price change, one basis point up or down. These two characteristics invalidate standard time series procedures for the analysis of transaction data, as they render estimation results virtually uninterpretable.

There are two approaches discussed in the literature which both take the microstructure properties of transaction data into account and allow for a flexible time series structure to capture the salient features of the data. The approach suggested by Rydberg and Shephard (1998) decomposes the price changes d_t observed at time t into three components. An indicator for the occurrence of a price change, an indicator for the sign of the price change and the size of the price change itself. This decomposition allows to apply dynamic extensions to count data models in the form of generalised linear models (GLM) as they are surveyed e.g. in Nelder and Wedderburn (1972) or Cameron and Trivedi (1998). The dynamic extensions are mainly based on the work of Zeger and Qaqish (1988) and Shephard (1995), which include nonlinear functions of past observations as explanatory variables within the generalised linear modeling framework. A feature of the modeling strategy pursued by Rydberg and Shephard worth emphasizing is the discrete nature of time. They consider each second of the trading day and characterize it by the above mentioned random variables. The second approach proposed by Russell and Engle (1998) pursues a somewhat different modeling strategy by specifying a dynamic model for the evolution of price changes in the form of a generalized Markov Chain also employing the generalised linear modeling framework to include a more flexible dynamic, than just a plain mapping of the transition from a price change of one category to a price change of another category by a Markov chain. Their approach is to combine this model with an ACD model, proposed by Engle (2000)[1] and Engle and Russell (1998), to obtain a joint model of price changes and time between transactions in a continuous time framework.

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[1] The original working paper circulated as Engle (1996).

Other approaches, which consider also multivariate models of the joint transaction process, do not necessarily concentrate on the salient features of the data, i.e. the discreteness of price changes and the irregular occurrence of transactions. These include e.g. Hasbrouck (1991) who in his work on price changes and quote revisions plainly applies standard VAR techniques, which can be justified in some sense, as he concentrates on the effects of explanatory variables for the conditional mean function. This work was extended by Dufour and Engle (1999). They build on the setup by Hasbrouck in a way which allows the parameters of the process of price changes to depend on the trade intensity modelled by an ACD specification. Ghysels and Jasiak (1997) and Grammig and Wellner (1999) modify their estimators of volatility to account for the irregular spacing of observations. However, they apply a standard GARCH approach going back to the work of Drost and Nijman (1993) which allows the scaling of a modified GARCH model to account for a change in the aggregation interval. None of these approaches quoted above explicitly models the joint distribution of transaction times and price changes.

The model proposed in this paper combines the approaches of Rydberg and Shephard and Russell and Engle with a flexible dynamic for quantal response models proposed by Gerhard (2000). The decomposition framework of Rydberg and Shephard is modified to allow for a joint model of the size of price changes and the time between transactions within a continuous time framework. The advantage of the continuos time framework is the parsimony of the transaction frequency model. The particular dynamic for the discrete price changes on the other hand allows to build a particularly flexible joint model, which is much like a VARMAX model but accounts for the discrete nature of price changes. Contrary to previous approaches our model is parametrically rich enough to discriminate between true structural dependencies among the dependent variables and simple correlations arising from common shocks (e.g. news events).

In order to show the new approach at work we present estimates for the BUND future trading at the DTB, Frankfurt. The main result is that the size of price changes and the time between transactions are indeed simultaneous variables. The direction of price changes and the time between transactions on the other hand can be considered as independent processes. An analysis of the nonlinear impulse response function reveals that an impulse on the time between transaction equation has an influence on the contemporaneous size of price changes, but the shock dies out almost instantly.

The structure of the paper is as follows: In Section 2 alternative decompositions of the transaction process are discussed. Section 3 introduces the new model for the the joint process of inter-trade durations and price changes. The data set is described in section 4. Estimation results and an analysis of the impulse response function is provided in section 5. Section 6 concludes.

2 On the decomposition of the price process

2.1 A Generalized Markov Chain

Engle and Russell (1998) in their work on ACD models propose to decompose the transaction process y_t into price changes d_t and transaction intensity, i.e. time between transactions τ_t

$$y_t := \begin{bmatrix} d_t & \tau_t \end{bmatrix}', \tag{2.1}$$

where t denotes the t-th transaction. The joint distribution of y_t can be written as the product of the conditional distribution of the price changes $f_{d|\tau}$ and the marginal distribution of the time between transactions f_τ without any further implications as

$$f_y(y_t|\bar{d}_{t-1}, \bar{\tau}_{t-1}) = f_{d|\tau}(d_t|\bar{d}_{t-1}, \bar{\tau}_t) \cdot f_\tau(\tau_t|\bar{\tau}_{t-1}, \bar{d}_{t-1}), \tag{2.2}$$

where the bar on $\bar{d}_{t-1}$ and $\bar{\tau}_{t-1}$ indicates a vector of historical values of the corresponding variable up to transaction $t-1$. The conditional distribution $f_{d|\tau}$ can be modelled by autoregressive conditional multinomial (ACM) model which generalizes a standard Markov chain on the process of price changes and allows for a more complex dynamics compared to the latter .

The marginal distribution of the duration f_τ is modeled using the well-known ACD model. Thus Russell and Engle (1998) specify their joint model as a special competing risks model in which the hazard rate is split into intensities of price changes of size v_i or more concisely, the hazard rate $\lambda^{ACM}(\tau_t|I_{t-1})$ is a J-dimensional vector, assuming that d_t can take on J discrete values. The information set available up to the time where transaction $t-1$ occurs is denoted by I_{t-1}. The probability that discrete price change of size v_i and a trade occurs after a spell of length s given there was no trade until s is given by:

$$\lambda_{[i]}^{ACM}(s|I_{t-1}) = \lim_{c \to 0} \frac{\Pr\left[s \leq \tau_t < s + c, d_t = v_i \,|\tau_t \geq s, I_{t-1}\right]}{c}, \qquad (2.3)$$
$$i = 1, \ldots, J.$$

The joint probability in (2.3) is factored by Russell and Engle into a product of the conditional density to observe a certain price change and the standard ACD hazard rate λ^{ACD}, as it is implied by a regular ACD model,

$$\lambda^{ACM}(s|I_{t-1}) = \lambda^{ACD}(s|\bar{d}_{t-1}, \bar{\tau}_t)\mu_t^{ACM}(\bar{d}_{t-1}, \bar{\tau}_t). \qquad (2.4)$$

The conditional probability to observe a particular price change μ_t^{ACM} is derived from a modified GLM based on a multinomial Logit with an ARMA dynamics and some additional terms included to account for the time between transactions.

This approach has considerable advantages, in particular, the time between transactions specification in continuous time allows to build a parsimonious model of the trade intensity. The ACM model on the other hand imposes a quite complicated structure on the dynamics of price changes, since the transitions from one price change to another price change are modelled explicitly.

2.2 A Decomposition Model in Discrete Time

A alternative model suggested by Rydberg and Shephard (1998) is based on the decomposition of the process of price changes. Their approach rests on a discrete time framework, where now the index t for the observation denotes clock time measured in seconds since the trading start. Thus one observation is available for each second of the trading day. To discriminate between the price change d_t in a continuous time framework where d_t is only observed if a trade occurs and the price change in the discrete time model, random variables are marked by a circle in the latter case. So d_t° stands for the price change in the discrete time context.

The decomposition of Rydberg and Shephard heavily utilizes the dynamic framework for GLMs proposed by Shephard (1995), to capture the dynamics of the three components, price activity a_t°, direction of price changes s_t° and size of price changes z_t°. The price activity, a_t°, is a variable taking on a value of one, if a transaction was observed at time

t and a price change has taken place and is zero otherwise. The price change itself is decomposed into its direction s_t° taking on values $+1$ and -1 and its size z_t°, which is integer valued. As already mentioned the discrete time framework leads to an observation in each second having the form of a product

$$d_t^\circ = a_t^\circ s_t^\circ z_t^\circ \tag{2.5}$$

Using appropriate definitions of the three components, the distribution of d_t° can be formulated without loss of generality based on the conditional distribution of the components as

$$\Pr\left[d_t^\circ = v_i \left| I_{t-1}\right.\right] =$$

$$\begin{cases} \Pr\left[a_t^\circ = 0 \left| I_{t-1}\right.\right] & \text{if} \quad v_i = 0 \\ \begin{array}{l} \Pr\left[z_t^\circ = v_i \left| I_{t-1}, a_t^\circ = 1, s_t^\circ = 1\right.\right] \\ \cdot\Pr\left[s_t^\circ = 1 \left| I_{t-1}, a_t^\circ = 1\right.\right] \Pr\left[a_t^\circ = 1 \left| I_{t-1}\right.\right] \end{array} & \text{if} \quad v_i > 0 \\ \begin{array}{l} \Pr\left[z_t^\circ = -v_i \left| I_{t-1}, a_t^\circ = 1, s_t^\circ = -1\right.\right] \\ \cdot\Pr\left[s_t^\circ = -1 \left| I_{t-1}, a_t^\circ = 1\right.\right] \Pr\left[a_t^\circ = 1 \left| I_{t-1}\right.\right] \end{array} & \text{if} \quad v_i < 0 \end{cases} \tag{2.6}$$

Here d_t°, the price change in time period t, takes on J different values of size $v_i (i = 1, \ldots, J)$ Note that this specification does not discriminate between the occurrence of a trade without a price change and the event that no trade occurs at time t. Both events are captured by $a_t^\circ = 0$ and thereby $d_t^\circ = 0$.

Price activity a_t° is modeled as a GLM based on a logistic distribution including ARMA(p,q) type dynamics. The model of the trade direction s_t° is quite similar to this specification. Actually, it is just an autologistic model as the empirical results show that the direction of price changes seems to be driven mainly by the bid-ask bounce. The size of price changes z_t° is at least potentially a count variable. However, price changes larger than one are observed only rarely in liquid markets. To account for the nature of z_t° as a count variable Rydberg and Shephard employ again a GLM based on a negative binomial (NegBin) distribution with an ARMA dynamics to model the size of price changes minus one, $z_t^\circ - 1$.

The modeling framework proposed by Rydberg and Shephard has the considerable advantage that the decomposition of the price process allows to build flexible models for each component and allows to consider and interpret each component in turn. On the other hand, the individual processes can be combined via (2.5) or a similar definition of a composed random variable of interest, to obtain empirical evidence on the trading process as a whole.

The main problem associated with the discrete time framework chosen by Rydberg and Shephard is the identity of the observation index t with the clock time. The problem becomes apparent if one considers a very active trading phase, e.g. trading after the opening of an exchange, and a very slow trading phase, e.g. during lunch time. In order to proceed with a simple example without invalidating the argument in a more complex setting assume further, that the true DGP is such that the current time between transactions depends via an ACD(1,0) model only on the preceding time between transactions and the time of day, governing the intraday seasonals. In the log-ACD context the difference between the slow and fast trading could be captured by an additive component, e.g. a trigonometric seasonal, in the mean function. For the model in discrete time, however, a time varying coefficients model would be necessary to capture the seasonality effects, because in a fast trading phase lags of the dependent variable with a lower order should have the largest weight, in a slow trading phase, lags with a higher order should have a greater weight. A model for this kind of seasonally varying coefficients has been suggested in the GARCH context by Bollerslev and Ghysels (1996) in the form of a periodic GARCH model. Their work is also motivated by an intraday analysis on the basis of equally spaced observations. They, however, use time aggregates which of course suffer from the same effect, if the aggregation interval is chosen to be sufficiently small. A modification of the given GLM models in the sense of Bollerslev and Ghysels would however complicate matters considerably, if it is possible at all. These effects might be the reason for the large number of lags Rydberg and Shephard (1998) need to model the price activity a_t° in a satisfactory fashion. This is also the reason why a continuous time framework employing an ACD type dynamics is used in this work.

A comparison of the solutions of Russell and Engle and Rydberg and Shephard does not really yield a clear cut picture. The advantage of Russell and Engle (1998) is the use of an ACD model to obtain a parsimonious duration model in continuous time, which allows e.g. a straightforward inclusion of seasonalities. The advantage of Rydberg and Shephard's approach is the stringent inclusion of dynamics and the decomposition of the transaction process into components of interest, e.g. the size of price changes. Both models, however, suffer from the drawback that the simultaneity of individual components of the transaction process cannot be assessed from the estimation results. In particular it is possible to identify whether observed correlations between the single components of the transaction process are due to structural relations or due to common innovations (news events).

2.3 A Decomposition Model in Continuous Time

Our bivariate model of trade frequency and price changes introduced below is based on the standard decomposition in terms of the marginal distribution for the time between transactions and the distribution of the price change variable conditional on the time between transactions. Time between transactions is treated as a continuous random variable such that it can be modeled as an ACD type model. The response probabilities of a given price change conditional on the time between transactions can be decomposed in various ways. One way of decomposing the price change variable d_t is to define it solely for the event when trade occurs. In this case it can simply be factored into the size of price changes z_t and the sign of price changes s_t. This deviates substantially from the price change d_t° employed by Rydberg and Shephard which is observed at every period (second) of the trading day. Here again, the sign takes on values $s_t \in \{-1, 0, 1\}$ and the size $z_t \in \{0, 1, 2, \ldots\}$. The definition of the decomposition is thus

$$d_t = z_t \cdot s_t. \tag{2.7}$$

Contrary to Rydberg and Shephard there is no particular component which accounts for the occurrence of a price change, like a_t° in (2.5), this is captured in this model by the random variable z_t. Finally, the occurrence of a transaction is explicitly modelled by the time between transactions τ_t random variable. Trade occurrence is a component of this model and not only price changes.

The two processes of size z_t and sign s_t as defined above cannot be independent since the size of the price change is informative for the direction of the price change with respect to the zero price change event, i.e.

$$\Pr\left[s_t = 0 \,|z_t = \text{abs}\left[v_i\right] = 0, \tau_t, I_{t-1}\right] = 1 \tag{2.8}$$

$$\text{and} \qquad \Pr\left[s_t = 0 \,|z_t = \text{abs}\left[v_i\right] > 0, \tau_t, I_{t-1}\right] = 0. \tag{2.9}$$

However, it can be assumed that the size of price changes beyond the zero event carries no additional information for the sign of price changes, or more concisely

$$\begin{aligned}\Pr\left[s_t = \operatorname{sgn}\left[v_i\right] \left| z_t = \operatorname{abs}\left[v_i\right] > 0, \tau_t, I_{t-1}\right.\right] = \\ \Pr\left[s_t = \operatorname{sgn}\left[v_i\right] \left| z_t > 0, \tau_t, I_{t-1}\right.\right]. \quad (2.10)\end{aligned}$$

To model the conditional probabilities (2.10) in practice, there are two ways to proceed, either one models a random variable $s^{\dagger}_{t_k} \in \{-1, 1\}$, $k = 1, \ldots, K$, which is only observed, if there actually is a price change, i.e. $z_{t_k} > 0$. The latter defines the indices t_k which perform a thinning of the original process indexed by $t = 1, \ldots, T$, i.e. the indices are defined such that $t_k \in \{t = 1, \ldots, T : z_t > 0\}$. This in turn implies that the random variable $s^{\dagger}_t$ can be assumed to be independent of the size of price changes z_t, i.e.

$$\Pr\left[s^{\dagger}_{t_k} = \operatorname{sgn}\left[v_i\right] \left| z_t > 0, \tau_t, I_{t-1}\right.\right] = \Pr\left[s^{\dagger}_{t_k} = \operatorname{sgn}\left[v_i\right] \left| \tau_t, I_{t-1}\right.\right]. \quad (2.11)$$

Then again, the price change is decomposed as in (2.7) but the sign of the price change s_t is defined as

$$s_t = \begin{cases} 0 & \text{if } z_t = 0 \\ s^{\dagger}_{t_k} & \text{if } z_t > 0. \end{cases} \quad (2.12)$$

A dynamic model for the thinned process $s^{\dagger}_{t_k}$ can thus be employed to evaluate the conditional probabilities required to obtain the distribution of the price change d_t, this dynamic model could e.g. be a GLM or dynamic ordered probit model.

The thinning of the available observations yielding only those $K \leq T$ observations with a price change unequal zero, however, raises the problem that in a dynamic specification, where the information set I_t is made up mainly by past observations of the endogenous variable, the information available is rather limited. Lags of the dependent variable are easily included in the model, i.e. $\bar{s}^{\dagger}_{t_{k-1}} \in I_{t-1}$. Yet, the inclusion of zero price change observations raises substantial problems, i.e. some observations of d_t are not in the information set $\bar{d}_s \notin I_{t-1}$ if $d_s = 0$, $s \leq t - 1$. If one is concerned by this loss of information, one might well choose another alternative to model the process s_t to obtain the necessary conditional probabilities $\Pr\left[s_t = \operatorname{sgn}\left[v_i\right] \left| z_t > 0, \tau_t, I_{t-1}\right.\right]$. This involves the specification of the conditional probabilities via a model of the sign of price changes s_t

unconditional on the size of price changes. The conditional probability of the sign of the price change is thus defined for the observations showing a price change as

$$\Pr\left[s_{t_k}^{\dagger} = \operatorname{sgn}[v_i] \left| z_t > 0, \tau_t, I_{t-1}\right.\right] := \frac{\Pr\left[s_t = \operatorname{sgn}[v_i] \left|\tau_t, I_{t-1}\right.\right]}{1 - \Pr\left[s_t = 0 \left|\tau_t, I_{t-1}\right.\right]} \quad \text{if} \quad t = t_k. \tag{2.13}$$

By the description of the decomposition of the price process in (2.7) it was shown that the distribution of the process of price changes d_t can indeed be recovered from models of the conditional probabilities of the size of price changes $\Pr\left[z_t = \operatorname{abs}[v_i] \left|\tau_t, I_{t-1}\right.\right]$ and the sign of price changes $\Pr\left[s_t = \operatorname{sgn}[v_i] \left|\tau_t, I_{t-1}\right.\right]$.

The advantage of this decomposition is that it allows to examine the size and the sign process independently of each other. Supposedly, the sign process should be heavily influenced by the well-documented bid-ask bounce. The size of price changes or rather the absolute price change on the other hand is of particular interest for the analysis of volatility. For an extended exposition of the role of absolute price changes in the economic analysis of risk, see e.g. Granger and Ding (1995). They discuss the use of the conditional distribution of absolute price changes, particularly the use of various functions thereof, where z_t^2 as the variance under the mean zero assumption is just the major example.

The assumption needed to establish this decomposition is that the sign process is independent from the size of price changes beyond the fact whether a price change occurs or not, see (2.11). If these processes were not independent, it would imply that either positive or negative price changes would come along with particularly large or small price changes. If one would reject the assumption made and strive to assess this as a hypothesis, it would well be possible. In order to model the bivariate distribution of size and sign of price changes a bivariate ordered probit would be needed. In this context however, both processes are modelled independently of each other but as joint models with the duration process.

3 A Model of the Joint Transaction Process

In the preceding section a decomposition of the price change process d_t into a process of the size and the sign of price changes was proposed. This decomposition is used and two alternative joint models with dependent variables are defined as

$$y_t^{(s,\tau)} := \begin{bmatrix} s_t & \log \tau_t \end{bmatrix} \text{ and} \tag{3.1}$$

$$y_t^{(z,\tau)} := \begin{bmatrix} z_t & \log \tau_t \end{bmatrix}. \tag{3.2}$$

The model structure of the joint model involving the size of price changes $y^{(z,\tau)}$ and the model involving the sign of price changes $y^{(s,\tau)}$ is almost identical. Thus, for the sake of brevity, only the model for $y^{(z,\tau)}$ is described in depth, the derivation of the model for $y^{(s,\tau)}$ follows directly. The few minor differences will be pointed out along the model description.

The joint density $f_{z,\tau}$ of the bivariate process $y^{(z,\tau)}$ can be factored into the conditional density of the size of price changes $f_{z|\tau}$ and the marginal density of time between transactions f_τ to obtain

$$f_{z,\tau}(z_t, \tau_t | I_{t-1}) = f_{z|\tau}(z_t | \tau_t, I_{t-1}) f_\tau(\tau_t | I_{t-1}). \tag{3.3}$$

The conditional probability function of the discrete random variable z_t and its relationship to the process of price changes d_t was already discussed in the preceding subsection. See e.g. the discussion of (2.13). Thus, it can be used here as a building block for the joint model, i.e.

$$f_{z|\tau} = \Pr\left[z_t = \text{abs}\left[v_j\right] | \tau_t, I_{t-1}\right]. \tag{3.4}$$

To model $f_{z|\tau}$ and the analogon $f_{s|\tau}$ a latent dynamic model in the form of an ordered probit will be employed since it captures the discreteness of the dependent variable and the serial dependency of observations. The ordered probit with a simple dynamics was employed before by Hausman, Lo, and MacKinlay (1992) to analyse discrete price changes. In this study we apply an ordered probit augmented by an ARMA dynamics as proposed by Gerhard (2000). Besides the more general dynamics in the latent dependent variable and in the error term the model has the additional advantage, that weakly exogenous variables can easily be included either as static regressors or in an infinite lag structure. Furthermore, the latent dynamic resembles very closely the log ACD model employed to capture the dynamics of the time between transactions component of the trading process.

The marginal distribution f_τ will be derived from a log-ACD model which has a very similar structure to the model of the size of price changes, as it also builds on the state space representation of an ARMA model possibly including exogenous regressors.

The combination of both components, the duration and the price change, will allow to asses the potential simultaneity of the variables.

For notational convenience the latent model of the size of price changes takes the following form of a state space model. This formulation is particularly convenient to apply the filtering method suggested by Gerhard (2000) which relies on the notion of generalized residuals proposed by Gourieroux, Monfort, and Trognon (1987).

$$z_t^* = H_z' \xi_{z,t} + w_{z,t}\gamma_z + \kappa_z \log \tau_t \tag{3.5}$$

$$\xi_{z,t} = F_z \xi_{z,t-1} + e_1(x_{z,t}\beta_z + \epsilon_{z,t}^*). \tag{3.6}$$

This model includes weakly exogenous regressors either in $w_{z,t}$ as static components or with an infinite lag structure if they are included as $x_{z,t}$. The ARMA dynamics are included as usually in a state space form via H_z and F_z, see e.g. Hamilton (1994). The vector e_1 has a one in the first element and zeros elsewhere. The time between transactions τ_t can be included in this specification in spite of a potential simultaneity, as this is explicitly accounted for by the simultaneous model. The dependent variable z_t^* is not observed directly, but through an observation rule, which is parameterized as usually in ordered probits via thresholds $\mu_{z,j}$, $j = 1, \ldots, J_z - 1$. Here, three categories are used, $J_z = 3$. The observation equation maps the latent variable z_t^* into three observable categories:

$$z_t = \begin{cases} 0 & \text{, if} \quad z_t^* \leq \mu_{z,1} \\ 1 & \text{, if} \quad \mu_{z,1} < z_t^* \leq \mu_{z,2} \\ 2 & \text{, if} \quad \mu_{z,2} < z_t^* \end{cases} . \tag{3.7}$$

Price changes of an absolute value larger than two are recoded as two. This is necessary for estimation purposes, as the thresholds of the larger categories would be rendered virtually unidentified. Note that less than 0.5% of the data set are affected by this recoding and that the largest observed price change is five ticks. Thus, the effect of the recoding is limited. See also the discussion of the recoding effect and possible remedies in Pohlmeier and Gerhard (2000).

The time between transactions variable included in (3.5) is of particular interest, as it allows to assess the consequences of a slow or a fast order execution has on the size of price changes. Contrary to lags of the duration variable which capture only the general state of the market, this variable characterizes the particular transaction under consideration.

The contemporaneous time between transactions τ_t is however potentially simultaneous with the size of price changes z_t. The latter would render a standard maximum likelihood estimation of (3.5) and (3.6) inconsistent due to a simultaneous equation bias. Thus a nonlinear simultaneous equation system is estimated with the duration as a second component. The model of time between transactions, is based on a modified log ACD specification

$$\log \tau_t = H'_\tau \xi_{\tau,t} + w_{\tau,t}\gamma_\tau \tag{3.8}$$

$$\xi_{\tau,t} = F_\tau \xi_{\tau,t-1} + e_1(x_{\tau,t}\beta_\tau + \epsilon_{\tau,t}). \tag{3.9}$$

Both error terms are collected in a $\epsilon_t{}' := \begin{bmatrix} \epsilon^*_{z,t} & \epsilon_{\tau,t} \end{bmatrix}$. For the error terms the assumption of joint normality is introduced:

$$\epsilon_t \sim N(0, \Sigma_z) \tag{3.10}$$

$$\text{with } \Sigma_z = \begin{bmatrix} 1 & \rho_z \\ \rho_z & 1 \end{bmatrix}$$

Given that ϵ_t is a function of $y^{*\prime}_t := \begin{bmatrix} z^*_t & \log \tau_t \end{bmatrix}$, the bivariate Gaussian distribution translates into the needed distribution of the dependent variables given the Jacobian $J_\epsilon(z^*_t, \log \tau_t)$ using ϕ_Σ as the density of the bivariate Gaussian distribution with covariance Σ

$$f(z^*_t, \log \tau_t | \bar{x}_t, \bar{w}_t) = \phi_\Sigma(\epsilon_t) \cdot |J_\epsilon(z^*_t, \log \tau_t)|. \tag{3.11}$$

The Jacobian takes on a particularly simple form given that the derivative of $\epsilon_{\tau,t}$ with respect to z^*_t is zero because not the dependent variable itself is excluded from the duration equation (3.8) Thus the latent linear system is fully recursive and completely identified with the determinant of the Jacobian being equal to one. Obviously, the parameters of the equations for the size of the price changes (sign of the price changes) are only identified up to a scale due to the discrete nature of the observable dependent variable.

The dynamic systems (3.5),(3.6), and (3.8), (3.9), respectively along with the observation rule (3.7) are estimated by Maximum Likelihood. Given the normality assumption on the error terms (3.10) the log likelihood for the discrete absolute price changes and the inter-trade durations is of the form:

$$\log \mathcal{L} = \sum_{t=1}^{T} \sum_{j=1}^{J} 1\left[z_t = v_j\right] \cdot \log \int_{\mu_{z,j-1}}^{\mu_{z,j}} \phi_\Sigma(z^*_t, \log \tau_t) d(z^*_t), \tag{3.12}$$

with $v_j \in \{0, 1, 2\}$ and $\mu_{z,0} \to -\infty$ and $\mu_{z,J} \to +\infty$. The likelihood can be maximized by applying Gerhard's (2000) filtering algorithm adapted to the multivariate context.

The model introduced above can be directly extended to the case where τ_t is replaced by a multivariate process including not only the time between transactions but also variables like the volume associated with a particular trade. The only limitation for the type of included variables is that they should be observed on a scale which allows to use a multivariate state space model along the lines of the duration model.

4 Data

The sample contains transaction data of the DTB Bund future trading. The Bund future was at the sampling time one of the most actively traded future contracts in Europe. The Bund future is a notional 6% German government bond of DEM 250.000 face value which matures in 8.5 to 10.5 years at contract expiration. There are four contract maturities per year, March, June, September and December. Prices are denoted in basis points of face value. In the sampling interval one tick was equivalent to a contract value of DEM 25. The Bund future was traded at the LIFFE, London, as well. The main difference between the Bund future trading at either exchange is that the DTB relies on an electronic trading system, whereas the LIFFE builds on floor trading. A thorough comparison of both trading places and a closer description of the data set is found in Franke and Hess (2000).

This study uses a sample of five trading days between 1/23/95 and 1/27/95 comprising 12 490 observations of the Bund future contract expiring in March 1995.

5 Empirical evidence on Bund future trading

5.1 Estimation results

Based on the sample of the BUND future trading we estimate the two joint processes for $y^{(s,\tau)}$ and $y^{(z,\tau)}$ by maximum likelihood using the assumption that the sign and the absolute size of transaction price changes are independent apart from the occurrence of a price change. Given this assumption the observation of a price change implies trivially that the sign of the price change is either plus one or minus one, but the observation of exact size of the price change, e.g. equal to one tick or equal to two ticks, does not influence the probability to observe a price change upwards or downwards.

Several questions of economic relevance are tied to the analysis of these processes. The first one, was already raised in the context of the liquidity analysis, which is limited in this work to the analysis of time between transactions. It boils down to the question whether the risk of the asset under consideration due to the variability of the market price is linked to the liquidity of the asset. Several distinct cases can be thought of. First, liquidity and volatility are independent processes, governed by different determinants and thus need to be considered as two distinct risk factors. Second, liquidity and volatility might jointly depend on certain determinants, where a high liquidity, i.e. low time between transactions, implies a high volatility per transaction. This would more or less characterize the ideal relationship since investors have indeed the opportunity to adapt their positions to a changed market price. The third possibility is quite similar to the second one, in the respect that liquidity and volatility still depend jointly on certain determinants, yet a low liquidity implies a high volatility per transaction. This might be caused by the fact that the price formation is dependent on the observation of past trades and thus the adjustment of the market price to the changing value of the asset is occurring quite erratic. The latter would however raise substantial problems for an investor, as the last price which is observable might not be a reliable approximation of the underlying value of the asset and it might not be possible for him to change his portfolio position in the desired way after substantial price changes. These different hypotheses are linked to the analysis of the process of the size of price changes and the time between transactions $y^{(z,\tau)}$.

The second question relates to the dynamic implications the joint model has for the individual components of the process. Gerhard (2000) shows that the process of time between transactions reveals a serial dependency which is quite different from the pattern observed for the size of price changes. The question under consideration is, how an innovation in the liquidity equation, (i.e. the time between transactions) influences the future trade frequency and the contemporaneous and future size of price changes. In particular it seems valuable to get some quantitative assessment on how long an unexpected shock on the time between transactions prevails in the market, especially in the volatility and the liquidity component. The answer to this question will involve the impulse response function implied by the estimations of the joint process $y^{(z,\tau)}$.

For the case of futures trading simultaneity of the sign of price changes and time between transactions is rather questionable since this is only plausible if there is some systematic discrimination between buyer and seller initiated trades, as e.g. short sales constraints. This would imply that an unexpectedly high waiting time between transactions causes a higher probability for a price change upwards. If ρ is negative, the reverse argument holds and a positive duration shock implies a higher probability for a price change downwards. This issue will be examined just for the sake of completeness.

The empirical evidence reported in Table 1 of the Appendix is quite plausible under this light. The results given in panel A on the simultaneous process of the direction of price changes and the time between transactions $y^{(s,\tau)}$ show that neither the contemporaneous nor the past time between transactions is found to be significant. The latter is also the case in the univariate model. More important however is the fact that ρ is found insignificant and thereby a simultaneity between the duration and the price equation has to be rejected. As a first important result, it can be concluded, that innovations on the duration process τ_t have no impact on the direction of price changes s_t.

The intensity of the trading process is often considered weakly exogenous for the process of price changes, see e.g. Russell and Engle (1998). The results on the sign of price changes reported in specification A of Table 1 provided some evidence in this respect. However, weak exogeneity does not necessarily hold for the process of absolute price changes z_t and the time between transactions τ_t.

In the context of absolute price changes simultaneity is much more reasonable. For a positive ρ a positive shock on the duration equation (i.e. waiting time between transactions is lager than expected) implies an increasing probability to observe a price change. Thus an increase in the contemporaneous unexpected component of time between transactions increases the conditional volatility. This nicely corresponds to the reasoning of Easley and O'Hara (1992) where it is the exogenous but unobservable event of information accrual having simultaneously an impact on both, the process of absolute price changes and the intensity of the trading process.

The empirical evidence reported in table 1 supports the hypothesis of simultaneity. The correlation ρ of the error terms has a significantly positive sign, i.e. a shock on the duration equation implies actually a higher probability to observe a price change (see specification B). The severity of the simultaneous equation bias can be assessed from the erroneously significantly positive coefficient on τ_t reported in panel C. If simultaneity is taken into account the coefficient on τ_t is insignificantly negative as in Panel B or significantly negative as in panel D. In terms of the BIC we conclude that specification B is actually the favourable one. Specification E, which excludes explicitly the contemporaneous time between transactions and the correlation term suffers from a heavy loss in fit, compared to specifications B-D.

The empirical evidence provided above gives a clear answer to the question of the relationship between the size of price changes and the time between transactions. The two processes are not independent and the result is such that a large innovation on the time between transaction equation implies a large innovation on the size of price changes and thus an increased probability to observe larger price changes. Note that the contemporaneous τ_t has an insignificant impact in regression B. Yet, some care has to be taken in the interpretation of this result, because past time between transactions enters the specification with a negative sign.

5.2 Analysis of the Impulse Response function

While the concept of impulse response analysis in linear models is well-known and straightforward, the treatment of impulse responses in nonlinear models is far from being trivial. Some advances to the methodology are provided by Gallant, Rossi, and Tauchen (1993), Koop, Pesaran, and Potter (1996), and Gourieroux and Jasiak (1999). Intuitively, the question is how does the dynamic respond to a shock on the system. The dynamic system is typically characterized by the conditional expectation of the dependent variable, e.g. $\mathrm{E}\left[y_{t+s}^{(z,\tau)} \middle| \epsilon_{t+s}, \epsilon_{t+s-1}, \ldots, \epsilon_t, \bar{\epsilon}_{t-1} \right]$, given the history of innovations driving the system. This history of innovations can be seen to contain three distinct parts. First, the history of the system $\bar{\epsilon}_{t-1}$ until it is hit by the shock ϵ_t which is to be assessed (second). Third, the innovations driving the system until it reaches the state which is to be examined in the context of the impulse response $\epsilon_{t+1}, \epsilon_{t+2}, \ldots, \epsilon_{t+s}$.

A generic formulation of impulse responses serves as a starting point, where the impulse response is defined as the difference between two forecasts based on the same information set but deviating in the impulse ϵ^* inflicted on the system

$$\begin{aligned}\Gamma_{G,s} := \mathrm{E}\left[y^{(z,\tau)}_{t+s} \middle| \, \epsilon_{t+s}, \epsilon_{t+s-1}, \ldots, \epsilon_t, \bar{\epsilon}_{t-1} \right] \\ - \mathrm{E}\left[y^{(z,\tau)}_{t+s} \middle| \, \epsilon_{t+s}, \epsilon_{t+s-1}, \ldots, \epsilon_t = \epsilon^*, \bar{\epsilon}_{t-1} \right]\end{aligned} \tag{5.1}$$

The effect of the history dependence and the dependence on the innovations driving the system between the shock and the target period can be removed by considering the innovations stochastic and taking the appropriate expectation of (5.1) to obtain

$$\Gamma_s = \mathrm{E}_\epsilon[\Gamma_{\mathrm{G,s}}] = \mathrm{E}[\mathrm{y}^{(z,\tau)}_{\mathrm{t+s}}] - \mathrm{E}\left[\mathrm{y}^{(z,\tau)}_{\mathrm{t+s}} \middle| \, \epsilon_{\mathrm{t}} = \epsilon^* \right], \tag{5.2}$$

where ϵ^* is a particular impulse on the system.

The conditional expectation $\mathrm{E}\left[y^{(z,\tau)}_{t+s} \middle| \, \epsilon^* \right]$ will be estimated by Monte Carlo methods. To obtain the above conditional expectation a set of N replications for the conditional expectation of $y^{(z,\tau)}_{t+s}$ using draws $\zeta_{j,i}$ and $\bar{\zeta}_i$, $i = 1, \ldots, N$, from the distribution of ϵ_t

$$\begin{aligned} y^{(z,\tau)}_{t+s,i} = \\ \mathrm{E}\left[y^{(z,\tau)}_{t+s} \middle| \, \epsilon_{t+s} = \zeta_{1,i}, \epsilon_{t+s-1} = \zeta_{2,i}, \ldots, \epsilon_{t+1} = \zeta_{s,i}, \epsilon_t = \epsilon^*, \bar{\epsilon}_{t-1} = \bar{\zeta}_i \right].\end{aligned} \tag{5.3}$$

The history of the system $\bar{\epsilon}_{t-1}$ is assumed to consist of 30 periods in the practical application. The conditional expectation can thus be estimated from the ordinary sample mean of the above given observations as

$$\hat{\mathrm{E}}\left[y^{(z,\tau)}_{t+s} \middle| \, \epsilon_t = \epsilon^* \right] = \frac{1}{N} \sum_{i=1}^{N} y^{(z,\tau)}_{t+s,i}. \tag{5.4}$$

The procedure outlined by (5.3) and (5.4) can be repeated for a total of M different values of ϵ^* thus obtaining a whole range of sample paths $\hat{\mathrm{E}}\left[y^{(z,\tau)}_{t+s} \middle| \, \epsilon_t = \epsilon^* \right]$, $s = 1, \ldots, S$. The impulse ϵ^* is also drawn from the distribution of ϵ_t, so that the distribution of sample paths gives a representative picture of the system's dynamics.

To sum up, the individual steps carried out to analyse the impulse responses are

1. Historical and future realizations of shocks are drawn according to the distributional assumptions of ϵ_t^* $N = 1000$ times.

2. The conditional expectation $\hat{\mathrm{E}}\left[y_{t+s}^{(z,\tau)} \middle| \epsilon_t = \epsilon^* \right]$ is evaluated from the estimates of specification B in table 1 as the sample mean of the replications.

3. Steps 1 and 2 are carried out for a sample of $M = 100$ draws of the shock ϵ_* drawn from the distribution of ϵ_t^*.

4. The distribution of conditional expectations $\hat{\mathrm{E}}\left[y_{t+s}^{(z,\tau)} \middle| \epsilon_t = \epsilon_i^* \right]$, $i = 1, \ldots, 100$, is depicted for the forecast horizons $s = 0, \ldots, 10$.

Practically, the shock is inflected on the time between transactions equation τ_t at $s = 0$, via the orthogonal decomposition

$$\epsilon_{z,t}^* = \rho_z \epsilon_{\tau,t} + \nu_{z,t} \tag{5.5}$$

the simultaneous effect on the size of price changes is computed. The regressors not relevant to the dynamic are set to zero in this case. Note that the impulse responses given in figures 1 and 2 are not given as the difference denoted in (5.2), but depict only the conditional expectation $\hat{\mathrm{E}}\left[y_{t+s}^{(z,\tau)} \middle| \epsilon_t = \epsilon^* \right]$. Thus, the impulse response of the observable size of price changes z_t does not return to zero, as s increases but to the unconditional expectation $\mathrm{E}\left[y_{t+s}^{(z,\tau)} \right]$. For the other variables, the latent z_t^* and the time between transactions τ_t, this is zero, due to the omission of static regressors.

A clear cut picture of the dynamics of the system can be gained from an inspection of the impulse response function depicted in Figure 1. The two panels on the left give the conditional expectation of one particular impulse $\hat{\mathrm{E}}\left[y_{t+s}^{(z,\tau)} \middle| \epsilon_t = \epsilon^* \right]$. The two middle panels describe the distribution of the observable conditional expectations $\hat{\mathrm{E}}\left[y_{t+s}^{(z,\tau)} \middle| \epsilon_t = \epsilon_i^* \right]$, $i = 1, \ldots, M$. The right hand panel gives the distribution of the usually unobservable conditional expectation of the latent size of price changes $\hat{\mathrm{E}}\left[z_{t+s}^* \middle| \epsilon_t = \epsilon_i^* \right]$, $i = 1, \ldots, M$. The estimated values for the dynamics of $y_t^{(z,\tau)}$ imply a very quick dampening of the impulse inflicted on the time between transaction equation. The positive correlation ρ in conjunction with the significant negative influence of past values of the latent variable z_t^* on the duration process leads to a dampening of the impulse. This is intuitively plausible since it implies that trades, which occur unexpectedly quick, have no significant influence on future price changes, the influence is limited to the present transaction.

To demonstrate that it is indeed the positive correlation ρ, which has a substantial influence on the dynamic properties of the process, a little experiment is carried out in figure 2. The sign of the estimated correlation is exchanged so that all the parameters are identical, on which 1 and 2 are based, but the correlation which is changed to $\rho_z = -0.6521$. The effect is quite dramatic. The shock inflicted on the duration equation lasts for another two observations after the event.

6 Conclusion

In this paper a new estimation concept is proposed for the intraday transaction process. It extends the work of Rydberg and Shephard (1998) and Russell and Engle (1998) by suggesting a different decomposition of the joint process of price changes and time between transactions, which includes a parsimonious modelling of the time between transactions component and allows to analyse the size and the sign component of the process of price changes separately. The fact that the models of the time between transaction process and the components of the process of price changes are of a standard time series form allows a particularly easy and convenient combination of the two processes to obtain a specification which is very similar to a VARMAX specification. Yet, the model proposed here accounts explicitly for the fact that the price change variable takes on only few distinct values in a liquid financial market.

The empirical results obtained on a sample of intraday trading of the Bund future suggest that the process of time between transactions and the size of price changes are indeed simultaneous. This is not the case for the process of the sign of price changes and the time between transactions. The latter were found to be independent.

The new nonlinear simultaneous equation system proposed in this paper is quite easily extended to include other continuous-scaled variables such as the volume associated with a particular transaction. To include this variable in the specification, the standard state space model would be extended to a multivariate model. Estimation would proceed either by integrating over a trivariate normal distribution or by conditional maximum likelihood methods.

Other extensions include the use of this model to determine optimal intraday aggregation levels for the assessment of intraday risk. By explicitly accounting for the time between transactions, which could be used as a measure of liquidity and the size of price changes, the two most important risk dimensions are included.

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A Appendix

Table 1: Simultaneous estimation of the Log ACD model for the time between transactions and the sign of price changes reported in column A, and simultaneous estimation of the Log ACD model for the time between transactions and the size of price changes reported column B-E. Trigonometric expansions of intraday seasonals $s(\delta, t, 4)$ are included as additional regressors. Estimates are based on the sample 5 of trading days for the BUND future including 12490 trades. The mean log likelihood (log. lik.) and the Bayes Information Criterion (BIC) are also provided for each regression. White standard errors of the coefficients are given in parentheses.

Specification	A		B		C		D		E	
Dependent variable					Time between transactions					
equation one	τ_t		τ_t		τ_t		τ_t		τ_t	
					Parameters of the dynamic					
AR(1)	0.9708	(0.0122)	0.9592	(0.0130)	0.9587	(0.0164)	0.9674	(0.0119)	0.9660	(0.0133)
MA(1)	0.8906	(0.0220)	0.8450	(0.0295)	0.8776	(0.0250)	0.8863	(0.0216)	0.8824	(0.0237)
					Static regressors $w_{\tau,t}$					
$\mathrm{E}\left[z^*_{t-1} \middle\| \cdot\right]$			-0.4225	(0.1539)	-0.3180	(0.3187)			-0.5053	(0.3918)
δ_c	0.7639	(0.0678)	0.6096	(0.0823)	0.6787	(0.0819)	0.7523	(0.0572)	0.7496	(0.0595)
δ_t	-0.0044	(0.0504)	-0.0072	(0.0463)	-0.0059	(0.0476)	-0.0045	(0.0503)	-0.0059	(0.0514)
$\delta_{s,1}$	0.1315	(0.1300)	0.1026	(0.1106)	0.1163	(0.1155)	0.1291	(0.1249)	0.1276	(0.1283)
$\delta_{s,2}$	-0.0191	(0.0975)	-0.0297	(0.0793)	-0.0273	(0.0838)	-0.0253	(0.0930)	-0.0259	(0.0965)
$\delta_{s,3}$	-0.0271	(0.0863)	-0.0203	(0.0698)	-0.0205	(0.0741)	-0.0236	(0.0826)	-0.0220	(0.0862)
$\delta_{s,4}$	0.1064	(0.0794)	0.0793	(0.0654)	0.0887	(0.0696)	0.1046	(0.0761)	0.0994	(0.0791)
$\delta_{c,1}$	-0.1775	(0.0888)	-0.1377	(0.0687)	-0.1494	(0.0735)	-0.1726	(0.0825)	-0.1640	(0.0866)
$\delta_{c,2}$	0.0644	(0.0819)	0.0470	(0.0626)	0.0535	(0.0672)	0.0562	(0.0757)	0.0572	(0.0793)
$\delta_{c,3}$	-0.0168	(0.0770)	-0.0167	(0.0603)	-0.0139	(0.0645)	-0.0158	(0.0719)	-0.0106	(0.0751)
$\delta_{c,4}$	-0.0213	(0.0731)	0.0100	(0.0588)	0.0017	(0.0627)	0.0018	(0.0695)	-0.0058	(0.0727)

Specification	A		B		C		D		E	
Dependent variable	Sign of price changes		Size of price changes							
equation two	s_t		z_t		z_t		z_t		z_t	
					Thresholds					
μ_1	-1.0290	(0.0541)	0.1077	(0.1527)	0.5690	(0.0644)	0.0723	(0.0694)	0.3842	(0.0360)
μ_2	0.8863	(0.0526)	1.7528	(0.1571)	2.2094	(0.0829)	1.7188	(0.0856)	1.9801	(0.0642)
					Parameters of the dynamic					
AR(1)			0.2332	(0.0284)	0.2521	(0.0285)	0.2374	(0.0285)	0.2109	(0.0288)
MA(1)	0.3268	(0.0257)								
					Contemporaneous duration					
τ_t	-0.0623	(0.1014)	-0.2245	(0.1825)	0.3859	(0.0447)	-0.2208	(0.0448)		
					Static regressors $w_{\tau,t}$					
τ_{t-1}	-0.0219	(0.0684)	-0.1253	(0.0477)	-0.1166	(0.0429)	-0.1634	(0.0468)		
δ_t			0.0062	(0.0471)	0.0136	(0.0445)	0.0066	(0.0459)	0.0099	(0.0427)
$\delta_{s,1}$			-0.0019	(0.1085)	-0.0721	(0.1006)	0.0054	(0.1048)	-0.0437	(0.0966)
$\delta_{s,2}$			0.0380	(0.0735)	0.0580	(0.0692)	0.0382	(0.0724)	0.0467	(0.0668)
$\delta_{s,3}$			-0.0116	(0.0633)	0.0008	(0.0595)	-0.0129	(0.0626)	-0.0025	(0.0577)
$\delta_{s,4}$			0.0172	(0.0610)	-0.0348	(0.0555)	0.0226	(0.0583)	-0.0140	(0.0535)
$\delta_{c,1}$			-0.0502	(0.0657)	0.0497	(0.0552)	-0.0589	(0.0582)	0.0095	(0.0526)
$\delta_{c,2}$			-0.0141	(0.0551)	-0.0521	(0.0508)	-0.0130	(0.0537)	-0.0348	(0.0493)
$\delta_{c,3}$			-0.0160	(0.0529)	-0.0035	(0.0495)	-0.0152	(0.0525)	-0.0093	(0.0481)
$\delta_{c,4}$			0.0058	(0.0517)	0.0021	(0.0486)	0.0066	(0.0513)	0.0055	(0.0472)
					Correlation					
ρ	-0.0614	(0.1120)	0.6521	(0.1859)			0.5921	(0.1559)		
log. lik.	-0.5324		-0.4665		-0.4679		-0.4671		-0.4751	
BIC	-6618		-5862		-5879		-5865		-5948	

Figure 1: Impulse responses for the estimation of the simultaneous Log ACD model for the time between transactions and the sign of price changes based on the estimates of specification B of Table 1. The coefficients of intraday seasonals are set to zero.

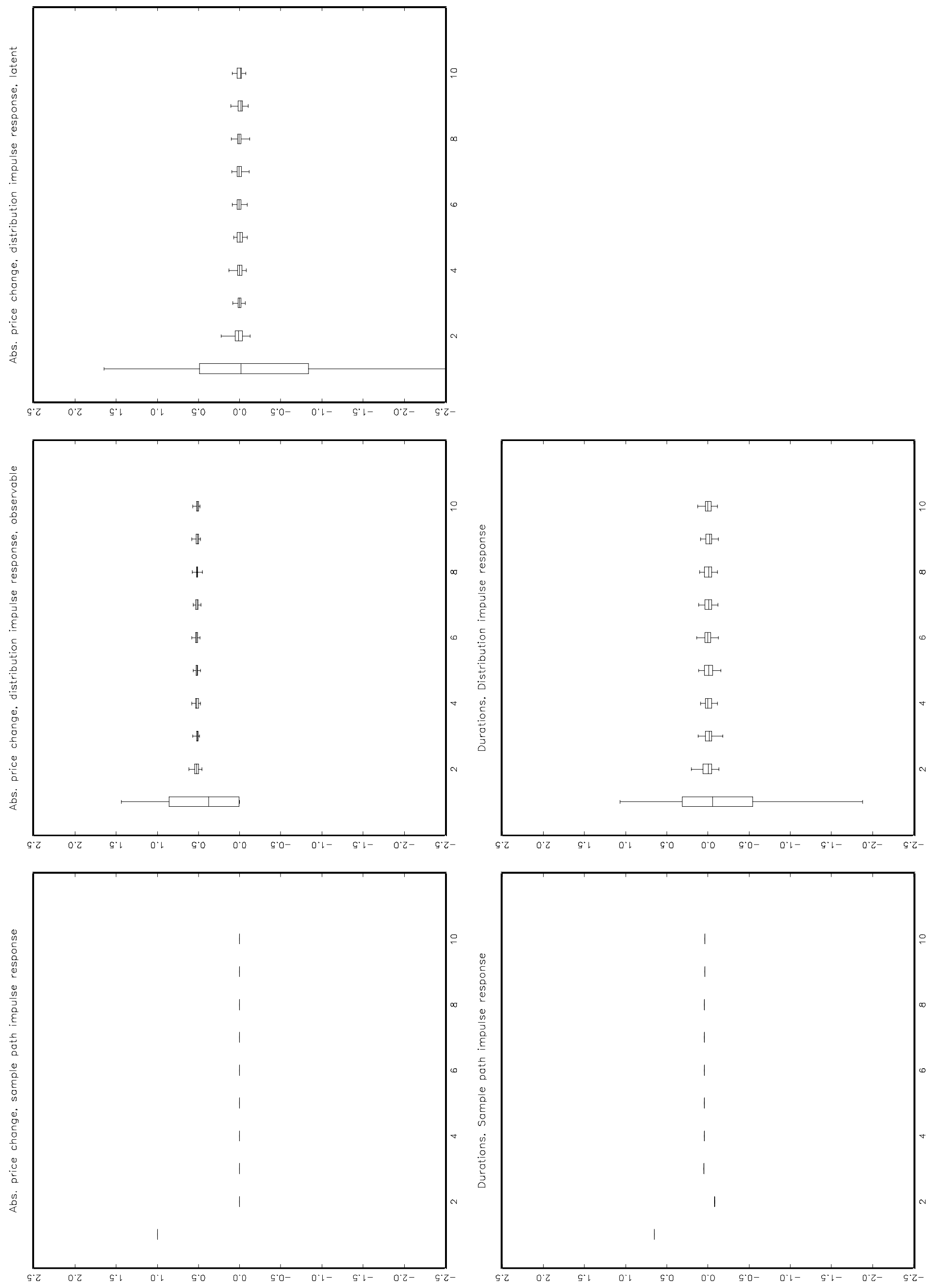

Figure 2: Impulse responses for the estimation of the simultaneous Log ACD model for the time between transactions and the sign of price changes based on the estimates of specification B of Table 1. The coefficients of intraday seasonals are set to zero. Compared to the indicated estimation, the sign of the correlation is switched, i.e. $\rho_z = -0.6521$.