Estimating Input Demand Elasticities for Australia

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Abstract

The view that recent changes in the distribution of income primarily reflect the influence of technical change favouring the use of skilled workers is well accepted among labour economists. Less established is how this skill biased technical change works its way into economic decision making. Capital embodied technical change has been argued to be one source. Thus, ‘modern’ capital equipment is thought to be more skilled labour ‘biased’, that is, relatively more complementary with skills. In order to investigate this claim, we need to establish the degree of substitutability between various types of labour and capital. Eleven years (1986–1996) of wage and capital rental data were collected for 11 sectors of the Australian economy. Using a translog cost function, own- and cross-partial price elasticities of factor demand are estimated. A full information estimation (SUR) procedure is used in estimating a system of factor cost shares. Elasticity values for each of the three types of inputs (skill, unskilled and capital) are found to offer broad support for the complementarity hypothesis, both at the economy wide and sectoral levels.

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1 Introduction

Much has been written about the role technology has played in the changing demand patterns for labour across the industrialised world (see, for example Berman, Bound and Machin 1998, Gaston 1998, and Bound & Johnson 1992). Over the last two decades in particular, a consensus has emerged attributing the observed rise in the share of employment and the wage bill accruing to skilled workers to a bias in technological change. Such a bias, it is thought, is the reason behind an increasing

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demand for skilled workers and, by extension, behind their increasingly favourable position in the distribution of income. Thus, new forms of technology (for instance, computers) may have resulted in a fundamental change in the way inputs are combined within the production process. In particular, it has been suggested that advances in technology have made capital and skills increasingly complementary (eg Goldin and Katz 1998)

The mix of factor inputs is a complex process determined simultaneously with output decisions within a technological environment. In examining how a firm transforms inputs into outputs and how technology plays a role in that transformation, it is important to use a correctly specified underlying production process. Accurate parameter estimates plays a crucial part in specification. In particular, it is important that input substitution elasticities be correctly estimated.

Numerous studies exist that estimate elasticity parameters under a variety of production technology assumptions (see Hamermesh 1993). Our purpose is to re-estimate these parameters for Australia to better understand the role of changing technology on input demand. It is important to estimate these parameters in an environment as free of *a priori* assumptions regarding technology as possible, such as is provided by flexible functional forms.

Production (and cost) functions must be sufficiently flexible to accommodate the data, yet yield economically credible results. The theory of production with several inputs allows us to infer how changes in the wage rate of one group of workers affect the demand for labour in other groups, or for other inputs. A flexible form requires no prior assumptions about the substitution relationship between factors of production and allows it to differ for each pair of inputs. The purpose of this work is to estimate these pairwise elasticities for a three input production function.

The paper proceeds as follows. Section 2 provides a brief review of some of the literature estimating input demand elasticities. Section 3 outlines the transcendental logarithmic (translog) production function, the flexible form on which this work is based. Section 4 presents results and section 5 concludes.

2 Literature review

To our knowledge, there are no recent estimates of partial input demand elasticities for Australia available. Higgs, Parham and Parmenter (1981) estimated substitution rates between labour using different occupations within a CRESH framework. They found values ranging from 0.10 to 1.05 between various types of labour for the period 1968/69 to 1973/74. Turnovsky and Donnelly (1984) estimated elasticities for Iron and Steel in Australia using data from 1959 to 1979, applying a translog approach. They found a substitution elasticity between capital and blue collar workers of 0.62

while that between white collar workers and capital was 1.24. The substitution elasticity between the two types of labour was -0.04.

Moving to the broader international literature, there have been more attempts to measure input substitution parameters. The methodologies vary, as do the functional forms, with flexible forms among the most frequently used.

The translog form is especially popular, particularly among labour economists, as it provides greater flexibility than some traditional alternatives[1] and, under certain conditions, yields economically plausible results. Hamermesh (1993) provides a good review of many of these studies, summarised in Table 1. Overall, the majority of studies focus on the manufacturing sector due to the greater availability of data.

Table 1 Estimates of substitution — selected studies[a,b]

Using translog functional form

Study	*Description*		σ_{bk}	σ_{wk}	σ_{bw}	η_{bb}	η_{ww}
Berndt & Christensen ('74)	Manufacturing, 1929-68		2.92	-1.94	5.51	-2.10	-2.59
Clark & Freeman ('77)	Manufacturing, 1950-76		2.10	-1.98	0.91	-0.58	-0.22
Berndt & White ('78)	Manufacturing, 1947-71		0.91	1.09	3.7	-1.23	-0.72
Grant ('79)	SMSAs, 1970		0.47	0.08	0.62	-0.32	-0.18
Freeman & Medoff ('82)	Manufac. 1972,	Union	0.94	0.53	-0.02	-0.24	-0.12
		Nonunion	0.90	1.02	0.76	-0.43	-0.61
Bergstrom & Panas ('92)	Manufacturing, 1963-80		0.15	-0.01	1.20	-0.43	-0.47

[a] Taken from Hamermesh (1993), p.110. All refer to large industry aggregate studies. [b] σ_{ij} refers to the partial elasticity of substitution and η_{ii} to the constant output demand elasticity. The subscripts b, w, k refer to blue collar workers, white collar workers and capital respectively.

As shown in the table, most studies support the relative complementarity hypothesis between capital and skilled labour. That is, unskilled workers are shown to be more highly substitutable for capital than skilled workers. In some instances, skilled workers and capital are absolute complements (when cross elasticities are negative). That is, the demand for skilled labour increases when the price of capital falls. However, the size and sign of the elasticities vary considerably, meaning that the choice of elasticity parameters to be used in modelling work is far from a straightforward one.

The flexible form, and the translog form in particular, have been criticised. This stems from its unstable nature, as well as the claim that it generates elasticity estimates

[1] In particular, the translog does not constrain partial cross elasticities to positive values only, as do the CES or Cobb-Douglas functions.

which are not plausible (Tishler and Lipovetsky 1997). Accordingly, recent studies have attempted to estimate parameters from the nesting of more stable forms, such as CES, or to adapt these forms to improve their flexibility. The goal is to arrive at a functional form which conforms with economic theory, and yields consistent, plausible results.

However, if parameter estimates can be shown to conform to certain conditions, such as regularity and concavity in input prices, the translog can be used to approximate known cost and production functions with satisfactory results. In this paper, we present an estimation procedure based on a standard translog flexible form model.

3 Estimation procedure

We estimate partial elasticities using a methodology similar to Binswanger (1974), Berndt and Christensen (1973), and Anderson and Thursby (1986). We start with the assumption that, for the overall production function *F[capital(k), skilled labour(s), unskilled labour(u)]*, there exists an input function which is weakly separable from all other inputs and exhibits a constant returns to scale technology (CRTS). Any technical change affecting *k*, *s* or *u* is assumed to be Hicks-neutral.

Every production function has a minimum cost function as its dual which relates factor prices to the cost of output. The cost function contains all the information about the production process which the production function contains. Starting with the traditional translog cost function for the three factor inputs:

$$\ln C = \alpha_0 + \alpha_y \ln Y + \alpha_k \ln P_k + \alpha_s \ln P_s + \alpha_u \ln P_u + ½ \;_{kk}(\ln P_k)^2$$

$$+ \;_{ks}\ln P_k \ln P_s + \;_{ku}\ln P_k \ln P_u + ½ \;_{ss}(\ln P_s)^2 + \;_{su}\ln P_s \ln P_u + ½ \;_{uu}(\ln P_u)^2$$

$$+ \;_{yk}\ln Y \ln P_k + \;_{ys}\ln Y \ln P_s + \;_{ys}\ln Y \ln P_u + ½ \;_{yy}(\ln Y)^2 \qquad (1)$$

Assuming CRTS we get:

$$\ln C = \alpha_0 + \alpha_k \ln P_k + \alpha_s \ln P_s + \alpha_u \ln P_u + ½ \;_{kk}(\ln P_k)^2 + \;_{ks}\ln P_k \ln P_s$$

$$+ \;_{ku}\ln P_k \ln P_u + ½ \;_{ss}(\ln P_s)^2 + \;_{su}\ln P_s \ln P_u + ½ \;_{uu}(\ln P_u)^2 \qquad (2)$$

Differentiating with respect to each input and applying Shephard's lemma yields a series of input demand functions stated as:

$$\ln C/ \; P_k = \alpha_k + \;_{kk}\ln P_k + \;_{ks}\ln P_s + \;_{ku}\ln P_u \qquad (3)$$

$$\ln C/ \; P_s = \alpha_s + \;_{sk}\ln P_k + \;_{ss}\ln P_s + \;_{su}\ln P_u \qquad (4)$$

$$\ln C/ \; P_u = \alpha_u + \;_{uk}\ln P_k + \;_{us}\ln P_s + \;_{uu}\ln P_u \qquad (5)$$

For all translog functions, symmetry (equality of the cross derivatives) applies, thus ${}_{ji} = {}_{ij}$.

If monotonicity holds (ie lnC/ P_i >0), and assuming CRTS, then the input demand variables can be viewed as cost shares (S_i). If they are cost shares, they must sum to one. This implies the following linear homogeneity restrictions on the parameters, which we impose during estimation:

$$\alpha_k + \alpha_s + \alpha_u = 1$$

$${}_{kk} + {}_{ks} + {}_{ku} = 0$$

$${}_{sk} + {}_{ss} + {}_{su} = 0$$

$${}_{uk} + {}_{us} + {}_{uu} = 0$$

We know that the translog function is not globally stable. However, it can be shown to be stable within a region. Stability conditions for the demand for factors of production can be shown by concavity in input prices, at least within the relevant range. This requires the matrix of partial elasticities of substitution to be negative semidefinite, which is the case here.

Finally, regularity requires that the (*n-1*) partial elasticities of substitution between any one factor and the others cannot all be negative. Thus, for the three factors here, we need to observe the following condition to ensure the function is stable over the relevant range:

$S_k \; {}_{sk} + S_u \; {}_{su} > 0$, and $S_s \; {}_{sk} + S_u \; {}_{ku} > 0$, and $S_s \; {}_{su} + S_k \; {}_{ku} > 0$

This condition is found to hold.

Once the system of equations (3) – (5) has been estimated, Allen Elasticities of Substitution (AES) can be derived from the ${}_{ij}$ coefficients in the usual way:

$${}_{ij} = ({}_{ij}/S_iS_j) + 1 \quad \text{and} \quad {}_{ii} = ({}_{ii} + S_i^2 - S_i) / S_i^2 \qquad (6)$$

and the constant output demand elasticities are calculated as $\eta_{ii} = S_i\sigma_{ii}$.

The system is estimated using cross sectional/time series ABS data. The pooled series covered 11 of the 12 sectors of the Australian economy for the period 1986–1996.[2] The input price data is represented by the log of hourly wages for skilled (*lnws*) and unskilled labour (*lnwu*) and the log of rates of return (*lnror*)

[2] The sectors are mining, manufacturing, electricity gas and water, construction, transport and storage, wholesale and retail trade, communication, finance, personal and business services, public administration, community services and personal/recreational services. Details of data set construction are available from the authors on request.

for each sector.[3] Symmetry and linear homogeneity are imposed on the system. Cost shares are represented by the share of value added accruing to a particular factor (see appendix).

4 Results

Given that the cost shares must add up to one, we can drop one equation and estimate the remaining two. Doing so as a system avoids possible endogeneity and improves efficiency. Estimating input demands through cost shares is influenced by an industry's cost structure. Given industry heterogeneity in this area (see appendix), estimating each industry separately would yield the most efficient and unbiased estimates of input demands functions. However, there is insufficient data to do this. Thus, we pool the data but account for the heterogeneity by estimating the equations in a SUR framework. This approach yields one 'average' estimate for each input demand coefficient. These estimates are presented in Table 2 along with the relevant test statistics.

Table 2 Translog model outcomes

Model	*Coefficient*	*Standard error*	*Z (P>|z|)*
Capital share equation			
lnws	-0.146	0.028	-5.158 (0.00)
lnwu	0.077	0.031	2.510 (0.01)
lnror	0.069	0.039	1.752 (0.08)
R-squared	*0.074*	*Chi-squared*	*30.550 (0.00)*
Skilled labour share eq.			
lnws	0.076	0.082	0.929 (0.35)
lnwu	0.069	0.077	0.897 (0.37)
lnror	-0.146	0.028	-5.158 (0.00)
R-squared	*0.1513*	*Chi-squared*	*27.195 (0.00)*
Breusch-Pagan test	*47.441 (0.00)*		

As shown in the table, the Breusch-Pagan test is significant, supporting the validity of the SUR approach. While the overall chi-squared shows a significant goodness of fit statistic, the R-squared values do not support this.[4] In addition, not all coefficients in the skilled labour share equation are significant.

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[3] There is a vast literature on the determination of 'skilled' employment. We have used occupational classifications, grouping the first three categories (managers and administrators, professionals, and para-professionals) as 'skilled' and the remaining classifications (trade persons, clerks, salespersons, plant and machinery operators and labourers) as 'unskilled'. Data details available from the authors upon request.

[4] Although care should be taken with interpreting goodness of fit measures when estimating a system of equations.

Table 3 shows the AESs obtained from the parameter estimates in table 2. Estimates are within the range reported in most studies. All elasticity estimates are significant, with the exception of skilled labour's own partial substitution elasticity. In addition, all, again with the exception of skill's own, are shown to maintain their sign within the confidence intervals calculated on the basis of parameter values.[5] As mentioned earlier, the system is found to be stable and regularity to hold.

While the results in tables 2 and 3 would appear to give contradictory evidence, namely in the significance of the unskilled-skilled relationship, it is the significance of the elasticity measure which should take precedence. This is because is that price movements alone do not necessarily capture the association between two factors of production. When cost shares are taken into account the consistency and significance of the results improves. Thus, this supplementary information improves the estimates' ability to predict the relationship between the variables.

The relationship between capital and skilled labour would appear to be one of absolute complementarity from the results given in table 3. That is, demand for skilled labour reacts *positively* to a decrease in the price of capital, which can be regarded as confirmation of the 'embodied technology' hypothesis. While most studies have shown capital and skill to be relative complements (with respect to unskilled labour), only a few have found evidence of absolute complementarity.[6] This lack of evidence may be due to the datasets being used. Most studies have relied on manufacturing data to derive elasticity estimates. We use a much broader cross-section which is dominated by the service sector. Indeed, 9 of the 11 sectors studied are service sectors (see footnote 1). When the two non-service sectors are dropped from the analysis, we obtain virtually the same results (not shown).[7] Thus, it may be argued that capital-skill complementarity is more pronounced in the services sectors than in manufacturing or mining. One possible explanation is that services sectors are increasingly skill-intensive, so that the installation of new machinery and equipment is likely to require a ratcheting up of the production process' skill content. In secondary or primary industries, on the other hand, this capital deepening may be aimed at economising on all types of labour.

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[5] Standard errors are computed as follows: $SE(\sigma_{ij}) = SE(\gamma_{ij})/\alpha_i\alpha_j$ and $SE(\eta_{ii}) = SE(\gamma_{ii})/\alpha_i$. See Binswanger (1974).

[6] For instance, in Table 1, Bergstrom and Panas (1992) and Clark and Freeman (1977).

[7] The value for σ_{ks} has the largest deviation from the whole dataset results, with the value for the services sector being only -0.375.

Table 3 Allen partial elasticities of factor substitution

Standard errors in parentheses

	AES	*95% Confidence Interval*[a]	
	σ_{ij}^{b}	*Low*	*High*
Capital-Skilled	-0.667(0.323)*	-1.30057	-0.0337
Capital-Unskilled	1.515(0.205)*	1.112843	1.917538
Skilled-Unskilled	1.789(0.879)*	0.042379	3.6817
Capital-Capital	-1.134(0.468)*	-1.65776	-0.61031
Skilled-Skilled	-1.922(1.47)	-5.02085	1.176825
Unskilled-Unskilled	-2.552(0.486)*	-1.14081	-3.96291

[a] Confidence intervals and standard errors based on parameter values. [b] * represents significance at the 5% level.

Economy wide results show the partial substitution elasticity between capital and unskilled labour to be at the high end of what is often used. However, its relationship to capital, *vis a vis* skilled labour is consistent with what is reported in the literature. Substitution between the two types of labour is slightly higher than that found in Higgs et al. (1981).

Table 4 Demand elasticities of substitution

Standard errors in parentheses

		95% Confidence Interval[a]	
	η_{ii}^{b}	*Low*	*High*
Capital-Capital	-0.435 (0.102)*	-0.5130	-0.2345
Skilled-Skilled	-0.437 (0.360)	-0.4121	0.2681
Unskilled-Unskilled	-0.989 (0.169)*	-0.8708	-1.1445

[a] Confidence intervals and standard errors based on parameter values. [b] * represents significance at the 5% level. All values conform with Allen's stability conditions.

Table 4 presents the (constant output) own price demand elasticities. Again, the skilled labour value has a confidence interval that includes both positive and negative values, implying instability in the measure. Overall, however the values are broadly consistent with those reported in table 1.

As stated above, industry structure is likely to play a role in the determination of input demand. While there is insufficient data to estimate each sector individually, we can use dummy variables to try and capture these heterogeneity effects. The following reports estimations incorporating industry dummies.

All the summary statistics from the model including industry dummies outperform the previous model's results. Both R-squared values are high, and all the coefficients are significant (table 5). However, as shown in table 6, the resulting elasticities of substitution are quite different from those previously reported. While the coefficient on the capital-skill relationship is still negative (table 5), the resulting elasticity of substitution is not (table 6). Further, the size of the cross elasticities calculated using industry dummies ranges between a half and a quarter of the value obtained without them (table 3).

Table 5 Translog model results with industry dummies[a]

Model	*Coefficient*	*Standard error*	*Z (P>|z|)*
Capital share equation			
lnws	-0.062	0.007	-8.782 (0.00)
lnwu	-0.066	0.009	-7.205 (0.00)
lnror	0.127	0.009	13.575 (0.00)
R-squared	*0.985*	*Chi-squared*	*7904.3 (0.00)*
Skilled labour share eq.			
lnws	0.113	0.019	5.894 (0.00)
lnwu	-0.051	0.020	-2.546 (0.00)
lnror	-0.062	0.007	-8.782 (0.00)
R-squared	*0.984*	*Chi-squared*	*7651.7 (0.00)*
Breusch-Pagan test	*23.682 (0.00)*		

[a] Dummy coefficients not reported. All coefficients, with the exception of Electricity, gas and water for the skilled labour equation, are significant.

Table 6 Allen partial elasticities of factor substitution with industry dummies

Standard errors in parentheses

	AES	*95% Confidence Interval*[a]	
	σ_{ij}^{b}	*Low*	*High*
Capital-Skilled	0.311 (0.078)*	0.1567	0.4644
Capital-Unskilled	0.530 (0.065)*	0.4026	0.6582
Skilled-Unskilled	0.476 (0.205)*	0.0734	0.8795
Capital-Capital	-0.799 (0.073)*	-0.9441	-0.6558
Skilled-Skilled	-1.192 (0.307)*	-1.7943	-0.5914
Unskilled-Unskilled	-0.785 (0.121)*	-0.4132	-1.1585

[a] Confidence intervals and standard errors based on parameter values. [b] * represents significance at the 5% level.

The own price demand elasticities are, again, about half the value of those obtained without industry dummies (table 7). All elasticity parameters pertaining to skilled labour are now significant. Another positive result for this group is that the confidence intervals show that the sign of the value is maintained for each elasticity.

While they differ from the economy wide results, results from the industry model confirm the complementarity hypothesis, in that skilled labour is less easily substituted for capital, compared to unskilled labour.

Table 7 Demand elasticities of substitution with industry dummies

Standard errors in parentheses

		95% Confidence Interval[a]	
	η_{ii}[b]	*Low*	*High*
Capital-Capital	-0.286(0.026)*	-0.3295	-0.2358
Skilled-Skilled	-0.298(0.076)*	-0.4484	-0.1478
Unskilled-Unskilled	-0.308(0.042)*	-0.2629	-0.4549

[a] Confidence intervals and standard errors based on parameter values. [b] * represents significance at the 5% level.

5 Conclusions

The paper set out to estimate elasticity parameters of a three-input demand function system. This is to enhance our understanding of embodied technology's effect on skilled labour demand. We arrive at two differing sets of estimates.

The first set using pooled time series/cross sectional industry data, dominated by services sectors, yielded estimates which are in line with what is often used and reported in the literature. An unexpected deviation is the finding that capital and skilled labour are absolute, as opposed to relative, complements. We attribute this result to the influence of services, where the link between skill and capital is probably stronger than in the rest of the economy.

The estimation procedure showed that the system is stable, the translog cost function well behaved in the relevant space, and most of the estimates of elasticities associated with skilled labour are significant.

In order to capture industry diversity across the data set and improve the quality of the skilled labour coefficient estimates, we included industry dummies in our approach. This second model showed much improved goodness of fit measures, but elasticities are very small. In many cases they are outside the bounds of those often applied in studies reported in the literature. However, complementarity between capital and skilled labour, albeit now relative, is confirmed. This provides further support for the hypothesis that embodied technology represents one avenue of skill biased technical change.

Appendix

Table A.1 Cost shares by sector (%)

	Skilled labour	*Unskilled labour*	*Capital*
Mining	7.6	19.4	73.0
Manufacturing	18.1	53.8	28.1
Electricity, Gas & Water	12.4	22.3	65.3
Construction	14.7	67.7	17.6
Wholesale/retail	23.6	53.3	23.1
Transport and storage	7.9	32.4	59.7
Communication	12.0	33.9	54.1
Finance, personal & business	33.0	33.2	33.8
Public administration	34.4	43.2	22.4
Community service	60.5	24.3	15.2
Recreation and personal	26.2	43.2	30.6
Economy wide average	***22.8***	***38.8***	***38.4***

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