Capital Asset Pricing Model, Bear, Usual and Bull Market Conditions and Beta Instability: A Value-At-Risk Approach

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Abstract: It has long been investigated in the finance literature that whether or not beta responds asymmetrically to good and bad news as measured by large positive and negative returns respectively. In this paper we define three market scenarios, namely, bad, usual and good, conditional on the quantiles of the market returns distribution. We investigate the asymmetric response of beta to these market conditions by modeling the mean and the volatility of CAPM as nonlinear threshold models with three regimes. We use daily returns on 30 Dow Jones Industrial Stocks for the period January 1991 to December 1999, and S&P 500 returns as a proxy for the market portfolio in this study. We find that for 21 stocks, beta is higher when the market is bearish than that when the market is bullish, while for other 9 stocks the reverse is true. However, the Dow Jones portfolio betas corresponding to poor, usual and good market conditions were found to be 1.071, 1.030, and 1.023 respectively, while ignoring the asymmetric effects, the portfolio beta was 1.032. The results are in accordance with the widely held view that the portfolio beta increases (decreases) when the market is bearish (bullish). Further, estimates of risk premiums in the cross-sectional beta-return relationship indicate that the risk premium is positive and highly significant for the usual-market-beta, while those corresponding to extreme market conditions are statistically insignificant. The strength of the relationship appears to be improved compared to that of a single factor beta-return model. These findings, we believe, have implications for portfolio diversification, performance measurement and risk management, among others.

Key Words: Large Returns, Value-at-Risk, Quantile Estimation, Threshold Model, Beta-Instability

1. Introduction

Since the seminal paper by Markowitz (1959), the capital asset pricing model (CAPM) has become an important tool in finance for assessment of cost of capital, portfolio performance, portfolio diversification, valuing investments, and choosing portfolio strategy, among others. Building on Markowitz's work, Sharpe (1964) and Black (1972) developed various version of the CAPM that can be empirically tested. The last half-century has witnessed a proliferation of empirical studies testing the validity of CAPM and the stability of beta. The beta, which is a measure of systematic risk, generally defined in terms of variances of and covariance between the stock and market returns.

Following the suggestion made by Levy (1974) to compute separate betas for bull and bear markets, Fabozzi and Francis (1977) were the first to formally estimate and test the stability of betas over the bull and bear markets. They have defined bull and bear markets in the following three ways: (i) these markets were delineated in accordance with the dates published in the investment textbook; (ii) positive market return is defined as up (bull) market, while the negative market return is defined as down (bear) market; and (iii) substantial up and down markets were defined as bull and bear markets respectively, as measured by absolute market returns larger than half of return standard deviation of entire sample. Using monthly returns on NYSE stocks and S&P500, and simple econometric tools, no evidence was found to support the hypothesis that the stock market has an asymmetric effect on beta. Extending this study by defining bull and bear markets using a threshold model, Kim and Zumwalt[1] (1979) found no evidence to support the beta instability, but concluded that investors would like to receive a positive premium for accepting downside risk, while a negative premium was associated with the up-market beta. They suggested that downside risk as measured by the beta corresponding to bear market may be an appropriate measure of portfolio risk than the conventional single beta; see also Chen[2] (1982) for using an improved statistical technique to overcome some econometric problems encountered by previous studies. Currently, there is emerging a literature on analyzing bull and bear markets, relating

[1] See also Pettengill, Sundaram and Mathur (1995).

them to duration dependence and macroeconomic variables. See, for example, Maheu and McCurdy (2000), Pagan and Sossounov (2000) and Lunde and Timmermann (2000) for details.

In this paper we use the methodology that is presently popular in measuring value-at-risk (VaR) to define various market conditions and investigate their effects on beta. We estimate the lower and upper quantile measures of the market returns distribution and define bear, usual and bull market conditions. Given the popularity and the importance of VaR in finance and risk management (see Jorian (1997) and Duffie and Pan (1997), our study is the first, we believe, to incorporate the VaR measures into the CAPM in order to investigate the stability of beta. The daily returns on thirty Dow Jones Industrial stocks and S&P 500 Index for the period 01 January 1991 to 31 December 1999 were used in this study. Various market conditions were allowed to affect the conditional mean and the volatility of stock returns in the CAPM through nonlinear threshold regime switching models with three regimes. We will estimate three betas for each stock corresponding to bear, usual and bull market conditions, and test whether these market conditions have systematic asymmetric effects on beta. Moreover, we test for the significance of cross-sectional beta-return relationship using the three sets of beta-estimates.

The paper by Fabozzi and Francis (1978) was the first to model the beta in CAPM as a random coefficient. Following this paper several studies have tested for randomness of beta in the early literature. However, since the introduction of ARCH/GARCH-type processes by Engle (1982) and others, testing for and modeling of time varying volatility (variance/covariance) of stock market returns, and hence the time varying beta, have been given considerable attention in the literature. See, for example, Bollerslev, Engle, and Wooldridge (1988), which was the first study to appear in modeling the beta in terms of time varying variance/covariance[3]. Recently, several studies have investigated the effect of good and bad news, as measured by positive and negative returns, ie., leverage effects, on beta

[2] See also Campbell and Hentschel (1992).

[3] See, for example, the survey paper Bolleslev, Engle and Nelson (1994) for details. Further, see Ng (1991) application of multivariate GARCH approach.

of CAPM; see, for example, Braun, Nelson, and Sunier (1995) (hereafter known as BNS), and Cho and Engle (2000) and the references therein.

Contrary to how good and bad news to stock market were defined in the literature, we use the quantile estimation methods to define three market conditions, namely, bear, usual and bull. We consider two methods, a GARCH process with *t*-distribution and the extreme value theory[4], to estimate the lower and upper quantiles of the market returns distribution at probabilities p and (1-p) with p fixed at 0.01, 0.025 and 0.05 levels. Having estimated the quantiles, then we define the market portfolio returns conditional on bear, usual or bull market environment, which is discussed in the next section in some details. As has been discussed earlier, non-linear threshold models with three regimes were utilized to capture the asymmetric response of beta and the time varying stock return volatility to various market conditions. Clearly, this model is considerably easier to estimate than the Markov regime-switching model as there is no need to estimate the various state probabilities, as they are predetermined in our study.

We find that the beta is higher for 21 stocks in the bear market than that in the bull market, and that the reverse is noted for other 9 stocks. The Dow Jones portfolio beta under the three market conditions was estimated and found that it is high in the bear market, compared to those of the other two market conditions. Further, comparing with the beta for the usual market condition, we notice that the beta in the bull market is only marginally lower. These results are in accordance with the widely held view that the portfolio beta increases (decreases) when the market is bearish (bullish).

BNS have investigated the variability of beta using exponential GARCH (EGARCH) models allowing market volatility, portfolio specific volatility and beta to respond asymmetrically to positive and negative markets and portfolio returns; that is, leverage effects. Using monthly data they found strong evidence of conditional heteroskedasticity in both market and non-market components of returns, and weaker evidence of time-varying beta. Cho and Engle (2000), on the other hand, have used the same

[4] See Longin (1996), Rachev and Mittnik (2000) and the references therein for theory and applications to financial time series..

models as BNS but daily data series and found strong evidence of time-varying beta in the CAPM of 25 blue chip stocks.

Hyung (2000) has used a Markov-regime switching model to investigate the CAPM, the instability of beta in particular. In this study, allowance is made for the possibility that the risk measure beta is drawn from two different states, namely the high-risk state and the low-risk state. The main finding is that the CAPM is stable in the low-risk state, while it is violated in the high-risk state. Given the popularity of VaR as a measure of downside risk (see Kuberston, 1998) and the quantile estimation, we believe that our definition of various market conditions and the nonlinear threshold-CAPM with time varying volatility would yield some interesting results.

We now discuss empirical testing of return-beta relationship, which appears to be a controversial issue. Several studies have tested this relationship over the years, and many found no significant relationship between the average stock return and the beta. Various explanations were given in the literature for this lack of empirical evidence, some relating to the selected market portfolio being inefficient and others to instability of beta; see, for example, Fama and MacBeth (1973), Fama and French (1992), Black (1993), Chan and Lakonishok (1993) and Roll and Ross (1994). We would use three betas corresponding to various market conditions studied in this paper to test the return-beta relationship, although only 30 stocks were studied in this paper.

. The paper is organized as follows: the next section briefly outlines the methodology used in estimating the quantiles of the S&P 500 returns distribution; defines the bear, normal, and bull market conditions; and specifies the regime switching threshold model with three regimes and time varying error variances. Section 3 briefly describes the data series used in this study. Section 4 reports and analyzes the results and Section 5 concludes the paper.

2. Various Stock Market Conditions and Threshold CAPM

In this section we first consider the market model with time varying volatility and define various market conditions that, we believe, have asymmetric effects on beta and stock return volatility. To capture the asymmetric effects of various market conditions on beta, we use three state regime switching threshold models with predetermined threshold parameters. Further, in order to define the conditional market portfolio returns given that various stock market conditions prevail, we utilize the quantiles, which were estimated using a GARCH process with *t*-distribution and the extreme value theory; see section 3 for details.

2.1 The Model

We consider the market model

$$R_t = \alpha + \beta R_{mt} + u_t, \quad u_t \sim N(0, h_t) \qquad (1)$$

$$h_t = \gamma_o + \sum_{i=1}^{p} \gamma_i h_{t-i} + \sum_{\tau=1}^{q} \delta_j u_{t-j}^2 \qquad (2)$$

where R_t is the stock return; R_{mt} is the market return at *t*, defined as $\ell n(p_t) - \ell n(p_{t-1})$, p_t is the price at time *t*, the error variance h_t follows a GARCH (p,q) process, and β is commonly known as beta – the systematic risk. Clearly, the model (1) is a simple linear model, which will not capture the asymmetric effects, if any, on beta of various market conditions. In the light of the discussion in the introduction, we define, low, usual and high market returns using quantile, estimation of which will be discussed in detail in the next section. For the purpose of introducing the threshold models, we assume that the *p* and (1-*p*) per cent quantile estimates of the market returns distribution are R_{mL} and R_{mH} respectively.

Now, we define the following indicator functions:

$$I_{Lt} = \begin{cases} 1 & \text{if } R_{mt} < R_{mL} \\ 0 & \text{otherwise} \end{cases}$$

$$I_{Ct} = \begin{cases} 1 & \text{if } R_{mL} \le R_{mt} \le R_{mH} \\ 0 & \text{otherwise} \end{cases}$$

and $$I_{Ht} = \begin{cases} 1 & \text{if } R_{mt} > R_{mH} \\ 0 & \text{otherwise} \end{cases}.$$

The following threshold model will be able to capture the asymmetric effects:

$$R_t = \alpha + \beta_L I_{Lt} R_{mt} + \beta_C I_{Ct} R_{mt} + \beta_H I_{Ht} R_{mt} + u_t . \quad (3)$$

Since there are a number of volatility models studied in the literature which were found to be capturing various features of stock returns, we consider the following volatility equations:

i *The ARCH/GARCH Model*:

$$h_t = \gamma_0 + \sum_{i=1}^{p} \gamma_i h_{t-i} + \sum_{j=1}^{q} \delta_j u_{t-j}^2 + \theta_L I_{Lt-1} R_{mt-1}^2 + \theta_C I_{Ct-1} R_{mt-1}^2 + \theta_H I_{Ht-1} R_{mt-1}^2 . \quad (4)$$

ii *The Threshold ARCH Model*:

The Threshold ARCH (TARCH) model was introduced by Zakoian (1990) and Gloster, Jaganathan, and Runkel (1993). In this model good news $(u_t < 0)$ and bad news $\left(u_t > 0\right)$ have distinct effects on the conditional variance, where u_t is the error term in the market model (3). Extending (4) to capture these effects, the new model is given as follows:

$$h_t = \gamma_0 + \sum_{i=1}^{p} \gamma_i h_{t-i} + \sum_{j=1}^{q} \delta_i u_{t-j}^2 + \theta_L I_{Lt-1} R_{mt-1}^2 + \theta_C I_{Ct-1} R_{mt-1}^2 + \theta_H I_{Ht-1} R_{mt-1}^2 + \sum_{k=1}^{q} \theta_k u_{t-j}^2 I_{ut-j} \quad (5)$$

where I_{ut} is an indicator function defined as

$$I_{ut} = \begin{cases} 1 & if \quad u_t < 0 \\ 0 & \text{otherwise} \end{cases}.$$

If θ_k is not zero for at least one *k*, then the news impact on the stock return volatility is asymmetric.

iii *The Exponential GARCH Model*

The exponential GARCH (EGARCH) model was first proposed by Nelson (1991). The model specification is given as

$$\log(h_t) = \gamma_0 + \sum_{j=1}^{p} \gamma_j \log(h_{t-j}) + \sum_{i=1}^{q} \left(\mu_i \left| \frac{u_{t-i}}{\sqrt{h_{t-i}}} \right| + \eta_i \frac{u_{t-i}}{\sqrt{h_{t-i}}} \right) + \theta_L I_{Lt-1} R_{mt-1}^2 + \theta_C I_{Ct-1} R_{mt-1}^2 + \theta_H I_{Ht-1} R_{mt-1}^2 . \quad (6)$$

The main advantage of this model is that the parameters are not restricted to be nonnegative, while those of the ARCH/GARCH model are. The news impact is asymmetric if $\eta_i \neq 0$ at least for one *i*. That is, the leverage effect is present.

iv *The Component ARCH Model*

The component ARCH (CARCH) model allows mean reversion to a time varying parameter q_t (say), while the conventional ARCH/GARCH models allow mean reversion to a constant parameter γ_0 as in (4). For exposition, consider the following simple model:

$$h_t = \bar{\gamma}_0 + \gamma(h_{t-1} - \bar{\gamma}_0) + \delta(u_{t-1}^2 - \bar{\gamma}_0) . \quad (7)$$

Clearly, this process is mean reverting to $\bar{\gamma}_0$ which is a constant long-term volatility for all *t*. Allowing the mean reversion to a time varying parameter, we can specify the CARCH process as follows:

$$h_t - q_t = \delta(u_{t-1}^2 - q_{t-1}) + \gamma(h_{t-1} - q_{t-1}) + \theta_L I_{Lt-1} R_{mt-1}^2 + \theta_C I_{Ct-1} R_{mt-1}^2 + \theta_H I_{Ht-1} R_{mt-1}^2 \quad (8)$$

where

$$q_t = \gamma_0 + \rho(q_{t-1} - \gamma_0) + \phi(u_{t-1}^2 - h_{t-1}) . \quad (9)$$

The CARCH model consists of three-volatility-components, namely, the time varying long run volatility q_t, the transitory component (8) and the permanent component (9). The endogenous variables, the condition market returns, can be allowed to affect either (8) or (9) or both. However, after some experiments, we have assumed that these variables affect only the transitory component.

v. *The Asymmetric Component ARCH Model*

The asymmetric component model is analogous to the threshold ARCH model. Introducing asymmetric effects in the transitory equation (8), the model is given as

$$(h_t - q_t) = \delta(u_{t-1}^2 - q_{t-1}) + \theta(u_{t-1}^2 - q_{t-1})I_{ut-1} + \gamma(h_{t-1} - q_{t-1}) + \theta_L I_{Lt-1}R_{mt-1}^2 + \theta_C I_{Ct-1}R_{mt-1}^2 + \theta_H I_{Ht-1}R_{mt-1}^2 \quad (10)$$

where the indicator function I_{ut-1} is defined in (11). If $\theta > 0$, then the transitory leverage effects in the conditional variance is significant.

The model (3) and the volatility models will be estimated to capture the following effects: asymmetric effects of various market conditions on the mean and the volatility of stock returns. Our interest is to estimate the CAPM model (3) and test whether or not all three β_L, β_C, and β_H are equal. Further, we test the significance and equality of parameters θ_L, θ_C, θ_H and significance of θ in the volatility models. It is of interest to estimate the nonlinear CAPM (3) with volatility equations (4) – (9) and select the best fit using the AIC criterion. Having estimated the betas of all thirty stocks across various market conditions, we then estimate the portfolio's betas, with portfolio being defined as equally weighted average of thirty stocks, commonly known as the Dow Jones Industrial average (DJIA).

2.2 The Beta-Return Relationship

Extending the single-factor cross-sectional beta-return relationship to include all three betas estimated using the models given in the previous section, we will provide some evidence on the significance of its relationship. The three-beta model is given as,

$$\overline{R}_i = \alpha + RP_L\beta_{Li} + RP_C\beta_{Ci} + RP_H\beta_{Hi} + w_i \qquad \text{for i = 1, 2, …, 30.} \qquad (11)$$

where $\overline{R}_i$ is the average return of stock i, β_{Li}, β_{Ci} and β_{Hi} are known betas of stock i corresponding to bear, usual and bull market conditions, and RP_L , RP_C and RP_H are unknown risk premiums need to be estimated. According to Kim and Zumwalt[5] (1979) and others, $RP_L > 0$ as investors would like to receive a positive premium for accepting downside risk, and $RP_H < 0$ as investors willing to pay a positive premium in the up-market.

3. Quantile Estimation Methods

We use GARCH with *t*-distribution and extreme value theory to estimate the quantiles of the market return distribution.

3.1. GARCH t-distribution

We assume that the stock returns follow a GARCH process with *t*-distribution.

$$R_t = \phi_0 + \sum_{i=1}^{n} \phi_i R_{t-i} + \varepsilon_t, \quad \varepsilon_t \sim g(\theta) \tag{12}$$

$$\sigma_t^2 = \sigma_0^2 + \sum_{t=1}^{p} \gamma_i \sigma_{t-i}^2 + \sum_{j=1}^{q} \delta_j \varepsilon_{t-j}^2, \tag{13}$$

where $g(\theta)$ is a student-*t* distribution with *κ* degrees of freedom given as

$$g(\varepsilon_t, \theta) = \Gamma\left(\frac{\upsilon+1}{2}\right)\Gamma(\upsilon/2)^{-1}(\upsilon-2)^{-\frac{1}{2}}\left(1+\frac{\varepsilon_t(\theta)}{\upsilon-2}\right)^{-(\upsilon+1)/2} \tag{14}$$

where $\Gamma(\cdot)$ is the gamma function. The aim is to first estimate the above three models (12), (13), and (14) simultaneously, assuming ν is unknown. Suppose ν is estimated as $\hat{\upsilon}$, and its closet integer value is denoted by $[\hat{k}]$. The *p* and (1-*p*) quantiles of return distributions are $(\hat{R}_t - t_{[\hat{\upsilon}],p}\hat{\sigma}_{t+1})$ and $(\hat{R}_t + t_{[\hat{\upsilon}],(1-p)}\hat{\sigma}_{t+1})$ respectively.

3.2.Extreme Value Theory

It has been shown in the literature that the form of the asymptotic distribution of the extreme returns is independent of the process generating returns and only the distribution parameters' values depend on it; see, for example, Longin (1996) for details. Following the recent literature on the extreme value theory and its extensive applications, we use Frechet extreme distribution to measure quantiles of stock market returns, as this family includes stable distributions and student *t*-distributions, and has been found very popular. The probability density function (pdf) of the generalized limiting distribution can be obtained as

[5] There is a huge literature on testing for the significance of beta-return relationship. See, for example, Chan and LaKonishok (1993),

$$f(x)=\begin{cases}(1+kx)^{1/k-1}\exp[-(1+kx)^{1/k}] & \text{if} \quad k\neq 0\\ \exp[x-\exp(x)] & \text{if} \quad k=0\end{cases} \tag{15}$$

where $-\infty < x < \infty$ for $k = 0$, $x < 1/k$ for $k < 0$, and $x > -1/k$ for $k > 0$.

$$f(x)=\begin{cases}\frac{1}{a}\left[1+\frac{k(x-b)}{a}\right]^{1/k-1}\exp\left\{-\left(1+\frac{k(x-b)}{a}\right)\right\}^{1/k} & \text{if } k\neq 0\\ \frac{1}{a}\exp\left\{\frac{x-b}{a}-\exp\left[\frac{x-b}{a}\right]\right\} & \text{if } k=0\end{cases}.$$

where k, a and b are the shape, scale and location parameters of the distribution.

3.2.1. Empirical Estimation of the Extreme Value Distribution

The parameter in (15) using non-parametric methods and those in (16) using parametric methods are estimated separately for the lower tail and upper tail of the distribution. To briefly discuss these methods, let us assume that there are *T* observations in the return series and they are divided into *g* non-overlapping sub-periods each with *m* observations, and hence *T* = *mg*. The divided return series are given as

$$\{R_1,\ldots,R_m \mid R_{m+1},\ldots,R_{2m} \mid R_{2m+1},\ldots,R_{3m} \mid \ldots \mid R_{(g-1)m+1},\ldots,R_{mg}\}.$$

The observed return series can be denoted as

$$R_{mi+j} \text{ where } i=0,1,\ldots,g-1 \text{ and } i\leq j\leq m.$$

Let us define the minimum and maximum of $\{R_{(i-1)m+j}\}$ for $i\leq j\leq m$ as

$$\min_{i\leq j\leq m}\{R_{(i-1)m+j}\}, i=1,\ldots,g$$

and

$$\max_{i\leq j\leq m}\{R_{(i-1)m+j}\}, i=1,\ldots,g$$

respectively. For a given sub-period length *m*, the sets of sub-period minima $\{R_{m,i}\}$ and maxima $\{R_{m,i}\}$ are the two data series that would be used to estimate the k_m, a_m and b_m parameters of the lower and

Black (1993) and Fama and French (1992).

upper tails of the extreme value distributions respectively. Now, we briefly outline the maximum likelihood procedure and nonparametric procedure to estimate the parameters.

3.2.2. Parametric Approach: The Maximum Likelihood Method.

Assuming that the sub-period minima $\{R_{m,i}\}$ follow a generalized extreme value distribution given in (16) with $x = (R_{m,i} - b_m) / a_m$, the pdf of $\{R_{m,i}\}$ can be written as

$$f(R_{m,i}) = \begin{cases} \dfrac{1}{a_m}\left[1 + \dfrac{k_m(R^{'}_{m,i} - b_m)}{a_m}\right]^{1/k_m - 1} \exp\left\{-\left(1 + \dfrac{k_m(R^{'}_{m,i} - b_m)}{a_m}\right)\right\}^{1/k_m} & \text{if } k_m \neq 0 \\ \dfrac{1}{a_m}\exp\left\{\dfrac{R^{'}_{m,i} - b_m}{a_m} - \exp\left[\dfrac{R^{'}_{m,i} - b_m}{a_m}\right]\right\} & \text{if } k_m = 0 \end{cases}. \qquad (16)$$

Where $\{R^{'}_{m,i}\}$ the ordered statistics of sequence $\{R_{m,i}\}$. The *m* subscript is to signify that the parameter estimate depends on the choice of *m*. The likelihood function of the subperiod minima to estimate the k_m, a_m and b_m parameters is given as

$$l(R_{m,1}, \ldots, R_{m,g} \mid k_m, a_m b_m) = \prod_{i=1}^{g} f(R_{m,i}).$$

Having estimated the shape, location and the scale parameters, the *p* per cent quantile can be estimated from the following equation:

$$p = 1 - \exp\left[-\left(1 + \frac{k_m(R_{pL} - b_m}{a_m}\right)\right]^{1/k_m}.$$

and (1-*p*) per cent upper quantile can also be computed from the above equation, replacing the parameter estimates with maximum of sub-period returns.

3.2.3. The Nonparametric Approach

The shape parameter *k* in (15) can be estimated using nonparametric methods, namely, Hill's (1975) and Pickards' (1975). We use Hill's (1975) method. The reason is that it was shown that Hill's (1975) method is applicable only to thin distribution such as Frechet distribution and more efficient than the Pickard's estimator. Further, Hill (1975) estimator is asymptotically normal with mean zero and variance k^2. The Hill (1975) estimator of the lower tail index k in (15) is defined as, for a positive integer *q*,

$$k_h^L(q) = \frac{1}{q}\sum_{i=1}^{q}\{\ell n[-R_{(i)}] - \ell n[-R_{(i+q)}]\}$$

when *q* in $k_h(\cdot)$ indicate that the estimates of $k_h(\cdot)$ depends on *q*. Note that the Hill (1975) estimator is applied directly to the returns $\{r_t\}_{t=1}^{T}$, but ordered series given as

$$R_{(1)} \leq \ldots \leq R_{(T)},$$

and the Hill estimator of the upper tail index k is defined as

$$k_h^U(q) = \frac{1}{q}\sum_{i=1}^{q}\{\ell n R_{(T-i)} - \ell n R_{(T-q)}\}.$$

The p per cent quantile of (15) is compute by solving for R_{pL} from the following equation:

$$p = 1 - \exp\left[-(1 + k_h^L R_{pL})\right]^{1/k_h^L}.$$

The (1-p) per cent quantile can similarly be computed replacing k_h^L with k_h^U.

4. Empirical Evidence

In this section, we describe the data series used in this study. We apply the procedures discussed in section 3 to estimate the quantiles of the S&P 500 returns distribution. Further, using the estimated quantiles of the market portfolio returns, we define the bear, usual and bull market conditions. The conditional capital asset pricing models were estimated for all thirty stocks. The beta estimates of CAPMs were used to study the beta-return cross-sectional relationship.

4.1. Data

We use daily price series on the thirty Dow Jones Industrial stocks from 01 January 1990 to 31 December 1999 consisting of 2529 observations. The S&P 500 Index was considered in this study as a proxy for the efficient market portfolio. The thirty stocks that constitute the market portfolio are heavily traded in the New York Stock Exchange. Stock returns were computed as $100 \times (\ell n(p_t) - \ell n(p_{t-1}))$. From some summary statistics of stocks and market returns (not reported in this paper to save space), we can say the following. Taking all 30 stocks together, the daily returns are in the range of -28 to 18 per cent, while the market portfolio, S&P 500, returns range only from –7 to 5 per cent with skewness –0.346 and kurtosis 8.233. Of 30 stocks, 11 are negatively skewed with skewness ranging from –0.002 to –1.426, while other 19 stocks were positively skewed, ranging from 0.009 to 0.464. Kurtosis of some stock returns was very high, generally ranging from 0.852 to 20.178. It is interesting to note that excluding just a one stock, PM, would reduce this range to 0.852 to 8.959. The series were collected from the CRIP database. See Granger and Silvapulle (2001) for details.

4.2 The Results and Discussion

In this section we estimate the quantiles of the distribution of the market index S&P 500 returns using the GARCH model and the extreme value theory at p = 0.01, 0.025 and 0.05. Using the lower and upper quantile estimates, the conditional bear, usual and bull market returns were defined via the indicator functions defined in section 2. The three-factor asset pricing model (3) with time varying volatility models (4) – (10) were estimated. The Akaike information criterion was used to select the best conditional mean and the volatility models. This also includes assessing the sensitivity of the results to various quntile estimates by defining the conditional market returns with various quantiles, obtained by GARH-t-distribution and extreme value theory.

We first explain the quantile estimation. The upper and lower tail indices *k* of (15) were estimated using the non-parametric method given in section 3.2, the results are given in Table 1. Choosing a number of values for *q*, we have examined the sensitivity of the Hill estimate of the shape parameter *k*. It appears that for *q* = 175 the estimates don't change noticeably. Further, we have used maximum likelihood methods to estimate the scale, location and shape parameters, a_m, b_m and k_m respectively, of the extreme value distributions, given in (16), fitted for both lower and upper tails, and the results are tabulated in Table 2. As has been discussed in Section 2.2, these estimates depend on *m*, and we have chosen three values of *m*, and corresponding quantile estimates are also given in Table 2. We chose the quantile estimates corresponding to *m* = 25, as *n* = 30 gives only 84 observations for nonlinear estimation of three parameters. Further, the standard errors are also very high for the estimates with *g* = 84. Table 3 reveals the GARCH model estimates with t-distribution. In Table 4 the lower and upper quantile estimates are given for *p* = 0.01, 0.025 and 0.05. After experimenting defining the conditional market returns using different quantiles and estimating CAPM, we reported the results of the models selected by the AIC. The quantiles of GARCH model with t-distribution appear to yield the best results for many stocks.

We begin our empirical analysis of CAPM by first estimating a single factor model given as

$$\begin{aligned} R_t &= \alpha + \beta R_{mt} + u_t, \quad u_t \sim N(0, h_t) \\ h_t &= \gamma_0 + \gamma_1 h_{t-1} + \delta_1 u_{t-1}^2 + \theta_1 R_{mt-1}^2 \end{aligned} \qquad (17)$$

Note that market returns were allowed to affect the conditional mean and the conditional variance equations. Further, we estimated the above mean equation with other forms of the volatility equation as given in (4) to (10).

To save space only the beta estimates corresponding to the model selected by AIC were reported in Table 5. The sample mean and the standard deviation of beta estimates, i.e., the DJ portfolio beta and the standard deviation, were calculated as 1.032 and 0.160 respectively.

As has been discussed in the introduction, several studies have argued that β depends on good news and bad news to market defined as positive and negative market returns respectively. Therefore, we have estimated the following model:

$$\begin{aligned} R_t &= \alpha + \beta_p I_{Pt} R_{mt} + \beta_n I_{nt} R_{mt} + u_t, \quad u_t \sim N(0, h_t) \\ h_t &= \gamma_0 + \gamma_1 h_{t-1} + \delta_1 u_{t-1}^2 + \gamma_{2p} I_{pt-1} R_{mt-1}^2 + \gamma_{2n} I_{nt-1} R_{mt-1}^2. \end{aligned} \tag{18}$$

where I_{pt} is an indicator function defined as *1* if $R_{mt} > 0$ and *0* otherwise, and I_{nt} is *1* if $R_{mt} < 0$ and *0* otherwise. The estimates of β_n and β_p in (18) are reported in Table 5 for all 30 stocks. There are some interesting differences among the beta estimates. For 16 stocks, beta estimates are higher for positive market returns than for negative returns while for other 14 stocks the reverse is true. The mean and the standard deviation of the portfolio beta were estimated as 1.039 and 0.186 respectively corresponding to positive market returns, and 1.031 and 0.230 respectively, for negative market returns. These results are in accordance with the widely held view that the portfolio beta is higher in the bear market than that in the bull market, although they differ only marginally.

Now, we estimate the conditional CAPM in which the betas pertinent to the bull and bear market conditions, and compare the results with those obtained for the models (17) and (18). We use the quantile estimates for the S&P 500 return series to define low, usual and high market returns as discussed in section 2, and estimate the model (4) with volatility models (5) to (9). The estimation was carried out for p = 0.01, 0.025 and 0.05. The model estimates chosen by the AIC were reported in Table 6. It is evident from the results that the volatility of stocks, except for ATT and GMM stocks, was significantly affected by the various market conditions. It is clear from these results that the beta estimates for 21 stocks in the bear market were higher than those in the bull market, while the reverse is noted for other 9 stocks. The portfolio beta is computed under various stock market scenarios. The mean and the standard deviation of portfolio beta are (i) 1.071 and 0.140 respectively when the market is bullish; (ii) 1.029 and 0.210 respectively when the market is usual; and (iii) 1.022 and 0.220 respectively when the market is bearish.

Further, the estimates of the risk premiums in the cross-sectional beta-return relationship given in (11) are presented in Table 7. Other estimates reported in this table are of single factor model in that the estimated beta of model (17) is the only explanatory variable, and of a two-factor model in that β_p and β_n in (18) are the two explanatory variables. The model estimates indicate that the risk premiums are positive and highly significant in the single- and the two-factor models. Contrary to what has been reported in the literature that the risk premiums in the down- and up-market are respectively positive and negative, the risk premium corresponding to up-market beta was found to be positive and significant, while that for down market is insignificant possibly due to a small number of stock studied in this paper. The estimate of the risk premium in the three-factor model (19), on the other hand, is significant only for the usual-market-beta, while other two betas corresponding to extreme market conditions are negative and statistically insignificant. The strength of the relationship however in this case appears to have improved compared to that of a single- and two-factor return-beta model.

5. Conclusion

In this paper we study three market scenarios, namely, bad, usual and good, conditional on quantiles of the S&P 500 returns distribution, and their effects on CAPM-beta. We first estimated the lower and upper quantiles using GARCH *t*-distribution and extreme value theory at p = 0.01, 0.025, and 0.05 levels, and these quantiles were used to define various market conditions. We investigate the asymmetric response of beta to these market scenarios by modeling the mean and the volatility of the capital asset pricing model as nonlinear regime switching threshold models with three regimes. We use daily returns on 30 Dow Jones Industrial Stocks for the period 1 January 1991 to 31 December 1999, and S&P 500 return series for the same period as a proxy for the market portfolio. We find that for 21 stocks, beta is higher when the market is bearish than that when the market is bullish, while for other 9 stocks the reverse is true. However, the estimated betas for the Dow Jones portfolio corresponding to poor, usual and good market conditions were found to be 1.071, 1.029, and 1.022 respectively. Ignoring the asymmetric effects, the portfolio beta was estimated as 1.032. The beta estimates of the threshold-CAPM are in accordance with the widely held view that the portfolio beta increases (decreases) when the market is bearish (bullish). Further, estimates of the risk premiums in the cross-sectional three-factor beta-return relationship indicate that the risk premium is positive and highly significant for the usual-market-beta, while other two betas corresponding to extreme market conditions are negative and statistically insignificant. The strength of the relationship however appears to have improved compared to that of a single factor return-beta model. The findings, we believe, have implications for portfolio diversification, performance measurement and risk management, among others.

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Table 1 Results of the Hill Estimator of the Shape Parameter k in the PDF (8) With Standard Errors Given in Parentheses				
Q	100	150	200	250
Minimum	-0.295 (0.025)	-0.299(0.020)	-0.294(0.023)	-0.285(0.0020)
Maximum	-0.321 (0.021)	-0.304(0.024)	-0.29(0.024)	-0.304(0.021)

Table 2 Maximum Likelihood Estimates of the Extreme Value Distribution for Daily Log Returns of S&P500 Return Series			
Length of Sub-period	**Scale a_m**	**Location b_m**	**Shape Param. K**
Minimal Returns:			
$m=20$, $g=126$	0.521(0.104)	-1.402(0.343)	-0.152(0.057)
$m=25$, $g=100$	0.601(0.172)	-1.632(0.470)	-0.189(0.090)
$m=30$, $g=84$	0.907(0.284)	-1.896(0.889)	-0.290(0.120)
Maximal Returns:			
$m=20$, $g=126$	0.599(0.194)	1.512(0.452)	-0.160(0.045)
$m=25$, $g=100$	0.720(0.218)	1.720(0.597)	-0.210(0.061)
$m=30$, $g=100$	0.991(0.320)	1.998(0.729)	-0.252(0.180)

Table 3

Estimates of Volatility Model for
Daily Return of S&P 500 Series

$$R_{mt} = 0.052 + u_t, \quad u_t \sim t-\text{distribution}$$

$$\underset{(2.189)}{h_t = 0.582} + \underset{(50.48)}{0.928\, h_{t-1}} + \underset{(7.201)}{0.128\, u_{t-1}^2}, \quad t =$$

The degrees of freedom of the t-distribution is estimated as 5.

Table 4 Estimates of Value-at-Risk and Upper Quantiles Of the Distribution of the S&P500 Stock Returns at the 1, 2.5, and 5 Percent Levels						
Methods	1	2.5	5	95	97.5	99
Hill Index	-2.500	-2.328	-2.012	2.028	2.412	2.625
Extreme Value Distrib..	-2.620	-2.128	-1.920	2.102	2.528	2.999
GARCH-t	-2.782	-2.318	-1.998	2.005	2.318	2.820

Table 5 Estimates of Beta in the conventional and the Conditional Capital Asset Pricing Models

1. $R_t = \alpha + \beta R_{mt} + u_t$,

2. $R_t = \alpha + \beta_n R_{mt}^{-} + \beta_\rho R_{mt}^{T} + u_t$,

3. $R_t = \alpha + \beta_L R_{mLt} + \beta_C R_{mCt} + \beta_H R_{mHt} + u_t$

Stocks	β	β_n	β_ρ	β_l	β_{us}	β_h	β_h
Dupont	0.998	0.939	1.060	1.015	1.000	0.947	1.006
Boeing	0.911	1.048	0.768	1.112	0.843	0.863	0.863
Caterpillar	0.941	0.896	1.021	0.998	0.947	0.898	0.997
Alcoa	0.836	0.925	0.733	1.039	0.795	0.841	0.781
Amex	1.260	1.261	1.256	1.235	1.242	1.504	1.344
ATT	0.908	0.902	0.914	0.955	0.903	0.813	0.813
CITIGRP	1.556	1.567	1.550	1.483	1.554	1.530	1.531
Coca Cola	1.098	1.029	1.208	1.143	1.105	1.085	1.149
Home Depot	1.297	1.279	1.327	1.275	1.360	1.245	1.245
GE	1.137	1.027	1.254	1.138	1.128	1.217	1.217
GM	1.004	0.977	1.030	1.043	0.997	0.966	0.929
Kodak	0.733	0.901	0.531	0.821	0.699	0.621	0.564
Exxon	0.722	0.700	0.748	0.723	0.736	0.603	0.674
Honeywell	0.946	1.017	0.884	1.144	0.911	0.985	0.936
HP	1.218	1.201	1.226	1.131	1.345	1.175	1.175
IBM	0.983	0.986	0.981	1.040	0.933	1.136	1.078
INTL Paper	0.845	0.871	0.815	1.161	0.792	0.885	0.826
JP Morgan	1.087	1.090	1.088	1.147	1.057	1.138	1.138
JJ	1.009	1.020	1.011	0.976	1.044	0.835	0.934
MCD	0.915	0.900	0.941	1.006	0.898	0.951	0.870
MERCK	1.090	1.107	1.045	1.144	1.091	0.999	1.058
MSFT	1.300	1.287	1.319	1.156	1.394	1.158	1.252
MMM	0.785	0.797	0.772	0.929	0.767	0.660	0.745
PM	0.994	1.032	0.959	1.081	0.926	0.970	0.999
PG	1.001	0.965	1.041	1.046	0.994	1.056	1.039
SBC	0.842	0.763	0.919	0.718	0.899	0.881	0.881
United Tec	0.879	0.943	0.811	0.980	0.834	0.924	0.938
Walmart	1.258	1.119	1.402	1.140	1.264	1.329	1.294
Disney	0.998	1.049	0.942	1.064	0.959	1.248	1.009
Intel	1.411	1.369	1.458	1.232	1.489	1.229	1.333
Market	1.006	1.018	0.997	1.039	1.004	0.992	0.990

Note: R_{mL} , R_{mC} , and R_{mH} are the conditional market returns in the lower-, center-, and the upper tails of the distribution. The quantiles were estimated at the 1, 2.5, and percent probability levels. The u_t is assumed to have time varying variance defined in (5-(9). For each stock, the AIC criterion was used to select the best model across volatility models for the variance process and quantiles. The reported beta estimates correspond to the best models.

Table 6

Estimates of Capital Asset Pricing Models with Time Varying Volatility Processes
Under Various Market Conditions: Bear, "Usual", and Bull estimated by Quantiles of Market Returns Distribution

Model: $R_t = \alpha_0 + \beta_L R_{mt} I_{Lt} + \beta_C R_{mg} I_{Ct} + \beta_H R_{mt} I_{Ht} + u_t$

Threshold GARCH: $h_t = h_o + \sum_{i=1}^{P} \gamma_i h_{t-i} + \sum_{j=1}^{q} \delta_j u_{t-j}^2 + \theta_L R_{mt-1}^2 I_{Lt-1} + \theta_C R_{mt-i}^2 I_{Ct-1} + \theta_H R_{mt-1}^2 I_{Ht-1}^2 + \theta u_{t-1}^2 I_t$

Parameters	Boeing (p = 0.05)	Home Depot (p=0.05)	GE (p=0.01)	Honeywell (p=0.025)	JP Morgan (p=0.05)	Merck (p=0.01)
α_o	0.012 (0.378)	0.070 (2.209)	0.024 (1.325)	0.022 (0.812)	-0.013 (0.502)	0.029 (1.122)
β_L	1.112 (21.43)	1.275 (20.46)	1.138 (19.56)	1.144 (15.96)	1.147 (19.64)	1.144 (12.84)
β_C	0.843 (15.26)	1.360 (26.17)	1.128 (44.82)	0.911 (23.82)	1.057 (24.58)	1.091 (35.05)
β_H	0.863 (10.84)	1.245 (15.96)	1.217 (16.03)	0.895 (12.73)	1.138 (16.79)	0.999 (9.720)
δ_1	0.017 (5.939)	0.157 (9.384)	0.036 (3.262)	0.206 (15.49)	0.033 (5.885)	0.001 (0.340)
γ_1	0.932 (210.5)	0.503 (9.082)	0.888 (43.14)	0.579 (21.20)	0.948 (127.9)	0.970 (125.807)
θ_L	0.068 (5.013)	0.291 (3.297)	0.031 (2.420)	0.309 (3.835)	0.068 (6.031)	0.920 (2.789)
θ_L	0.009 (0.366)	0.130 (1.309)	0.025 (2.272)	0.196 (3.587)	0.012 (0.669)	-0.011 (2.486)
θ_H	0.040 (1.875)	0.049 (0.934)	0.030 (1.825)	0.127 (2.318)	0.001 (0.005)	0.004 (0.430)
θ			0.034 (1.999)			0.035 (5.807)

Note: *t*-statistics are in parenthesis. Indicator functions except I_t are defined on page 16. I_t is one if $u_t < 0$, and zero otherwise.

Table 6 Continued									
Model: $R_t = \alpha_o + \beta_L R_{mt} I_{Lt} + \beta_C R_{mt} I_{Ct} + \beta_H R_{mt} I_{Ht} + u_t$									
EGARCH: $\log(h_t) = h_o + \sum_{i=1}^{p} \gamma_i \log(h_{t-i}) + \sum_{j=1}^{q} \left[\delta_j \mid \frac{u_{t-j}}{h_{t-j}} \mid + \mu_j \frac{u_{t-j}}{h_{t-j}} \right] + \theta_L R^2_{mt-1} I_{Lt-1} + \theta_C R^2_{mt-1} I_{Ct-1} + \theta_H R^2_{mt-1} I_{Ht-1}$									
Parameters	Alcoa (p=0.01)	ATT (p=0.05)	Coca Cola (p=0.01)	Kodak (p=0.01)	HP (p=0.05)	IBM (p=0.01)	Intl Paper (p=0.01)	Microsoft (p=0.025)	Intel (p=0.05)
α_o	-0.009 (0.318)	-0.008 (0.292)	0.027 (1.199)	0.002 (0.062)	0.018 (0.440)	0.020 (0.680)	0.016 (0.559)	0.097 (2.780)	0.053 (1.274)
β_L	1.039 (16.56)	0.995 (19.55)	1.143 (22.80)	0.821 (8.881)	1.131 (13.86)	1.040 (9.297)	1.161 (17.62)	1.156 (14.79)	1.232 (13.03)
β_C	0.795 (20.29)	0.903 (20.16)	1.106 (35.30)	0.699 (16.32)	1.345 (19.17)	0.933 (24.52)	0.792 (21.42)	1.394 (27.76)	1.489 (23.29)
β_H	0.841 (7.609)	0.813 (13.25)	1.085 (14.50)	0.621 (6.088)	1.175 (10.75)	1.136 (11.91)	0.885 (10.69)	1.158 (11.78)	1.229 (14.21)
h_o	-0.055 (7.810)	-0.070 (8.724)	-0.090 (6.879)	0.080 (2.482)	-0.027 (7.605)	-0.067 (10.16)	-0.029 (4.038)	-0.056 (2.782)	0.025 (1.210)
δ	0.076 (7.622)	0.095 (10.06)	0.108 (6.778)	0.306 (13.93)	0.037 (8.298)	0.098 (10.47)	0.037 (4.04 6)	0.184 (7.974)	0.099 (5.900)
γ	-0.008 (1.078)	0.005 (0.756)	-0.053 (5.301)	-0.023 (1.777)	-0.016 (4.039)	-0.044 (9.276)	-0.013 (1.849)	-0.053 (3.920)	-0.026 (2.815)
μ	0.900 (160.74)	0.972 (250.25)	0.910 (102.8)	0.659 (15.85)	0.980 (430.0)	0.987 (446.11)	0.993 (411.9)	0.906 (52.57)	0.930 (52.68)
θ_L	0.018 (3.620)	0.008 (1.989)	0.029 (3.427)	-0.012 (0.781)	0.004 (1.446)	0.015 (2.889)	0.004 (0.978)	0.011 (1.342)	0.013 (2.254)
θ_C	0.008 (2.214)	0.019 (1.942)	0.043 (4.312)	0.035 (2.092)	0.045 (6.935)	0.016 (6.103)	0.011 (3.314)	0.046 (2.697)	0.005 (0.314)
θ_H	-0.017 (2.302)	0.007 (1.047)	0.006 (0.431)	0.067 (4.400)	-0.005 (1.718)	-0.031 (4.315)	0.002 (0.319)	-0.024 (2.330)	-0.006 (0.763)

Table 6 Continued

Model: $R_t = \alpha_o + \beta_L R_{mt} I_{Lt} + \beta_C R_{mt} I_{Ct} + \beta_H R_{mt} I_{Ht} + u_t$

Transitory: $h_t = q_t + \gamma \mid u_{t-1}^2 - q_{t-1} \mid + \delta(h_{t-1} - q_{t-1}) + \theta_L R_{mt-1}^2 I_{Lt-1} + \theta_C R_{mt-1}^2 I_{Ct-1} + \theta_H R_{mt-1}^2 I_{Ht-1}$

Permanent: $q_t = h_o + \rho(q_{t-1} - h_o) + \phi(\Sigma_{t-1}^2 - h_{t-1})$

Parameters	Caterpillar (p=0.01)	Citigrp (p=0.01)	GM (p=0.01)	MCD (p=0.01)	PG (p=0.01)	SBC (p=0.05)	United Tech (p=0.025)	MKT (p=0.01)
α_o	0.015 (0.458)	0.044 (1.329)	0.027 (0.847)	0.024 (0.913)	0.028 (1.246)	0.009 (0.361)	0.034 (1.384)	0.002 (0.389)
β_L	0.998 (12.14)	1.483 (15.74)	1.043 (13.15)	1.006 (11.71)	1.046 (11.17)	0.718 (12.76)	0.980 (13.83)	1.039 (63.54)
β_C	0.947 (21.33)	1.554 (36.08)	0.997 (23.83)	0.898 (23.60)	0.994 (32.61)	0.899 (22.87)	0.834 (22.95)	1.004 (137.97)
β_H	0.898 (9.544)	1.530 (13.784)	0.966 (9.499)	0.951 (11.16)	1.056 (12.31)	0.881 (13.68)	0.924 (15.76)	0.992 (58.93)
h_o	2.941 (9.115)	2.421 (7.100)	2.894 (5.629)	1.125 (7.184)	0.983 (8.960)	1.863 (5.158)	0.810 (1.663)	0.041 (8.824)
q_o	0.955 (130.8)	0.960 (321.44)	0.995 (586.1)	0.978 (128.4)	0.931 (58.41)	0.979 (194.3)	0.995 (515.0)	0.919 (50.43)
μ	0.003 (2.770)	0.007 (3.984)	0.005 (1.529)	0.021 (3.629)	0.071 (5.713)	0.038 (5.032)	0.011 (2.931)	0.007 (0.564)
θ_L	0.004 (0.425)	0.117 (6.205)	0.003 (0.281)	0.004 (0.323)	0.047 (1.814)	0.064 (4.012)	0.029 (3.961)	0.006 (4.652)
θ_C	-0.007 (1.824)	0.001 (0.239)	0.005 (0.741)	0.024 (1.744)	0.046 (3.214)	-0.046 (2.500)	-0.009 (1.416)	0.005 (3.879)
θ_H	0.063 (4.297)	-0.063 (2.485)	0.028 (1.230)	`0.045 (1.964)	0.066 (2.193)	0.004 (0.256)	-0.016 (1.703)	0.002 (0.993)
γ	0.125 (8.212)	0.077 (4.260)	0.048 (3.742)	0.127 (6.455)	0.060 (2.584)	0.134 (4.957)	0.123 (6.147)	0.124 (5.667)
δ	0.350 (4.352)	0.714 (11.30)	0.838 (17.56)	-0.192 (2.138)	-0.543 (2.600)	0.066 (0.526)	0.541 (5.750)	0.433 (3.848)

Table 6 Continued								
Model: $R = \alpha_o + \beta_L R_{mt} I_{Lt} + \beta_C R_{mt} I_{Ct} + \beta_H R_{mt} I_{Ht} + u_t$								
Transitory: $h_t = q_t + \gamma(u_{t-1}^2 - q_{t-1}) + \delta(h_{t-1} - q_{t-1}) + \theta_L R_{mt-1}^2 I_{lt-1} + \theta_C R_{mt-1}^2 I_{Ct-1} + \theta_H R_{mt-1}^2 I_{Ht-1} + \theta(u_{t-1}^2 - q_{t-1})$								
Permanent: $q_t = h_o + \rho(\varepsilon_{t-1} - h_o) + \phi(u_{t-1}^2 - h_{t-1})$								
Parameters	Dupont (p=0.01)	Amex (p=0.05)	Exxon (p=0.01)	JJ (p=0.01)	MMM (p=0.01)	PM (p=0.025)	Walmart (p=0.01)	Disney (p=0.01)
α_o	0.009 (0.345)	0.007 (0.219)	0.010 (0.504)	0.037 (1.557)	0.014 (0.600)	0.008 (0.241)	0.029 (1.046)	0.016 (0.565)
β_L	1.015 (13.24)	1.235 (13.23)	0.723 (15.94)	0.976 (12.52)	0.929 (16.63)	1.081 (17.56)	1.140 (15.92)	1.064 (16.26)
β_C	1.000 (27.68)	1.242 (30.71)	0.736 (25.05)	1.044 (32.30)	0.767 (25.43)	0.926 (20.00)	1.264 (33.65)	0.959 (26.43)
β_H	0.947 (8.356)	1.504 (10.89)	0.603 (7.223)	0.835 (8.900)	0.660 (10.82)	0.970 (15.48)	1.329 (13.68)	1.248 (13.58)
h_o	0.937 (8.356)	1.841 (8.152)	0.525 (5.263)	1.370 (7.727)	1.894 (5.549)	3.062 (15.93)	1.968 (3.638)	3.100 (8.172)
q_o	0.995 (909.3)	0.952 (97.81)	0.974 (105.3)	0.967 (142.5)	0.995 (697.9)	0.937 (133.8)	0.994 (469.9)	0.985 (167.7)
μ	0.005 (2.458)	0.059 (4.868)	0.007 (0.881)	0.051 (5.387)	0.009 (3.509)	0.082 (7.153)	0.017 (3.610)	0.001 (0.251)
θ_L	0.033 (4.995)	0.084 (2.961)	0.051 (3.033)	0.038 (2.843)	0.008 (1.356)	0.098 (5.820)	0.026 (2.261)	0.037 (4.151)
θ_C	0.006 (1.508)	0.061 (2.670)	0.029 (2.556)	0.019 (2.165)	-0.011 (4.087)	0.027 (1.049)	0.005 (0.881)	-0.019 (5.125)
θ_H	0.008 (0.881)	0.062 (1.529)	0.012 (0.713)	-0.048 (3.012)	0.032 (4.799)	-0.065 (2.930)	-0.029 (1.981)	0.035 (2.744)
γ	0.105 (3.020)	0.047 (1.713)	0.125 (4.664)	0.049 (2.099)	0.105 (4.358)	-0.065 (3.324)	0.027 (1.774)	0.062 (3.175)
θ	-0.092 (2.188)	0.126 (3.452)	-0.072 (2.260)	0.085 (2.324)	-0.051 (-1.636)	0.196 (7.093)	0.083 (2.913)	0.075 (2.294
δ	0.256 (1.212)	-0.044 (0.305)	0.675 (9.586)	0.144 (0.860)	0.372 (3.130)	0.173 (1.020)	0.660 (7.892)	0.466 (4.065)

Table 7: Estimates of the Return-Beta Relationship	
Model 1:	$\overline{R}_i = \alpha_o + \beta_i RP + \varepsilon_i$ i = 1, …, 30 (total number of stocks)
Estimates:	$\hat{\overline{R}}_i = \underset{(0.0512)}{-0.077} + \underset{(5.673)}{0.136}\beta_i$, R^2 (adjusted) = 0.50
Model 2:	$\overline{R}_i = \alpha_o + RP^n \beta_i^n + RP^p \beta_i^p + \varepsilon_i$
Estimates:	$\hat{\overline{R}}_i = \underset{(2.569)}{-0.071} + \underset{(1.098)}{0.049}\beta_i^n + \underset{(2.435)}{0.083}\beta_i^p +$ R^2 (adjusted)= 0.52
Model 3:	$\overline{R}_i = \alpha_o + \beta_i^l RP^l + \beta_i^c RP^c + \beta_i^h RP^h + \varepsilon_i$
Estimates:	$\hat{R}_i = \underset{(1.960)}{-0.059} - \underset{(0.907)}{0.014}\beta_i^L + \underset{(4.908)}{0.183}\beta_i^C - \underset{(1.198)}{0.046}\beta_i^H$; R^2 (adjusted) = 0.63