A Simple Framework for Analyzing Bull and Bear Markets*

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Abstract

Bull and bear markets are a common way of describing cycles in equity prices. To fully describe such cycles one would need to know the data generating process (DGP) for equity prices. We begin with a definition of bull and bear markets and use an algorithm based on it to sort a given time series of equity prices into periods that can be designated as bull and bear markets. The rule to do this is then studied analytically and it is shown that bull and bear market characteristics depend upon the DGP for capital gains. By simulation methods we examine a number of DGP's that are known to fit the data quite well - random walks, GARCH models, and models with duration dependence. We find that a pure random walk provides as good an explanation of bull and bear markets as the more complex statistical models. In the final section of the paper we look at some asset pricing models that appear in the literature from the viewpoint of their success in producing bull and bear markets which resemble those in the data.

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1.Introduction

The behavior of asset prices has always been at the center of academic, media and business attention. “South Sea Island bubbles”, “Tulipmania”, ”irrational exuberance”, “financial melt-downs”, “bull and bear markets” are all terms that are familiar to us and which evoke strong reactions. Unlike the real economy which tends to change rather slowly, financial wealth can be created or destroyed very quickly, and it is no doubt this feature which accounts for the level of interest displayed in the meanderings of the Dow-Jones and NASDAQ indices, as well as foreign exchange rates and housing prices.

Modern finance has recognized the importance of accounting for asset price movements; correct pricing of a derivative written on the price of an asset requires some assumption about the data generating process (DGP) for that asset, while the market price of risk depends upon assumptions made about the nature of volatility in the class of assets being studied etc. Moreover, it is often recognized that any failure of these assumptions to be correct can lead to “event risk” that would seriously compromise many popular techniques of analysis such as Value at Risk (VaR).

Thus a study of the DGP's of asset prices is important. Since a DGP is a statistical concept it is natural to first try to infer it from an analysis of the data which are realizations from it. There is no doubt that this “data summary” step is an important one since it does narrow the class of potential DGP's a good deal. A huge literature has evolved in response to this demand e.g. see the surveys of Campbell, Lo and MacKinlay (1997) and Pagan (1996). We don't think that anyone would dispute that we have learned a great deal from this work about such characteristics as non-linearities in the conditional means and variances of returns, persistence of shocks into these same moments, volatility clustering etc. The DGP's that emerge from such investigations can then be utilized when making decisions that pertain to the asset prices.

One has to have some qualms about the use of DGP's established in this way. As one knows from the history of macroeconomics it is important to be able to interpret summaries of the data from a theoretical perspective in order to understand whether the statistical properties that have been documented are transitory features of the data or are somehow more fundamental. For example, it may be that the volatility seen in equity prices stems from volatility in the making of monetary policy, and hence it might disappear as monetary policy regimes change. Hence, we would prefer that one could interpret some of the observed characteristics of the data with models that are grounded in some economic behavior. In recognition of this fact we are now witnessing the emergence of models that emphasize economic features with the aim of providing explanations of the evolution of asset prices. These models are generally calibrated with a given set of parameters which have been chosen to replicate some feature of the data and are then used to explore the nature of the DGP pertaining to the price of the asset of interest. An example of such a development is the study of Gordon and St-Armour (2000) in which the evolution of the S&P500

index is accounted by allowing for stochastic risk aversion rather than the fixed degree that is the standard in models such as the C-CAPM. In a similar vein Campbell and Cochrane (1999a) provide another extension of the C-CAPM by allowing for a time-varying expected return based upon habit persistence in the standard model. This model works in a similar way to that of Gordon and St-Armour, in that the intertemporal marginal rate of substitution varies stochastically, although as a continuous rather than a discrete random variable. Finally, there is an emerging class of micro-simulation models, such as Lux and Marchesi (1998), at whose core are multiple agents classified as fundamentalists and chartists, with the fraction being determined by the relative profitability of playing each role.

One of the features that these models are designed to capture is “bull and bear markets” and there is no doubt that a satisfactory accounting for that phenomenon would be of great interest. Whether a bull market will end, and how high it can go is a perennial topic of conversation among financial strategists, and it has also been a topic of great interest and concern for monetary policy makers in the past few years. The depth of interest from the latter can be seen in Greenspan (1999) and Bernanke and Gertler (1999).

Given these developments it seems useful to enquire whether economic models of the DGP of asset prices are capable of producing bull and bear (B&B) markets. To date, despite a lot of references to these type of markets as a motivating factor in the construction of economic models, most attention has in fact been paid to the value of the equity premium and the volatility of capital gains. There is no doubt there is a relation between the latter two quantities and the nature of markets but it has rarely been given an explicit treatment. Perhaps this is not surprising, since one needs to define and measure bull and bear markets in order to be able to ask if a particular DGP for asset prices is capable of producing them.

There is of a course a large body of anecdotal evidence on the nature of bull and bear markets. An example would be the comment by Rothchild (1998) that U.S. bear markets come along about every 4.5 years. Niemira and Klein (1994, p.431) provide some widely used dates for expansions and contractions of the U.S. stock market which suggest a price cycle of around 40 months, a length that they observe was noticed at the end of the last century by Dow. Finally, Rhea (1932 p.37) reports Hamilton in 1921 as stating that bull markets in the preceding 25 years had lasted on average for 25 months while bear markets had persisted for 17.

In all of the evidence just cited it is not entirely clear what the definition of B&B markets used to extract these facts was. Consequently, it will be necessary to formulate a more precise definition of B&B markets so as to be able to replicate the stylized facts. Textbooks on financial investment sometimes give definitions such as that in Chauvet and Potter (2000, p90,fn 6),

> “In stock market terminology, bull (bear) market corresponds to periods of generally increasing (decreasing) market prices”,

although it has become apparent that recent usage in the financial press tends to qualify this by insisting on the rise (fall) of the market being greater (less) than 20%

in order to qualify for these names. In many ways the more general definition offered in the quotation above would seem to be closer to that used to describe contractions and expansions in the business cycle literature while the second, by emphasizing extreme movements, would be analogous to “booms” and “busts” in the real economy. We will adopt the first definition, although it will become clear that the analysis could equally have been done with the second and we actually do measure this to some extent.

The analogy with concepts in the business cycle literature which has just been invoked is one that continues throughout this paper. Turning points in the business cycle are widely known (at least in the U.S.) and are published by the NBER. A computer program set out in Bry and Boschan (1971) has often been used to automate the dating process, see King and Plosser (1994), Watson (1994) and Harding and Pagan (2001). Accordingly, in section 2 we adopt a variant of the approach set out in that program to perform the same task for stock markets. Once the turning points are established characteristics of the phases can be identified. This exercise is performed on monthly data for the equivalent of the S&P500 for the U.S. over the years 1835/1-1997/5. [1]

Section 3 gives a formal statement of the criterion selected to determine a turning point and section 4 canvasses some equivalent forms that are sometimes more useful for analysis. It emerges that the nature of bull and bear markets will depend upon the type of DGP which generates capital gains in the market. For example, if equity prices follow a random walk with normally distributed increments, all of the characteristics of bull and bear markets will depend solely upon the mean and volatility of capital gains. In section 5 we investigate how realistic such a DGP is by simulating data from it, replacing the unobserved mean and volatility with sample estimates, and then comparing the outcomes with the bull and bear market characteristics established in section 2. We find a good but not perfect fit. This leads us to consider other statistical models for capital gains. After some initial experimentation involving making capital gains exhibit volatility clustering we move on to consider the introduction of non-linear structure through hidden layer Markov chain models. Such models have become very popular in finance as a way of introducing non-linearities and we briefly consider how useful they really are in this regard.

Section 6 looks at the type of markets generated by economic models. We provide a general discussion about this and then focus in some detail upon the recent models of Gordon and St-Armour (2000) and Campbell and Cochrane (1999a). We also look at some work by Donaldson and Kamstra (1996) which explains the bull and bear markets of the 1920s and we consider some aspects of their explanation. Since economic models are generally of the calibrated variety they are capable of being simulated, whereupon the output from them may be passed through the dating algorithm to establish whether the bull and bear markets they imply match up with what has been observed. In general we find that all models have difficulty in producing realistic bull and bear markets and we use our framework to provide a simple explanation of why this is so. This strategy was also used in business cycle research by King and Plosser (1994), Watson (1994), Harding and Pagan (2001) and Hess and Iwata (1977). Finally, one advantage of the framework is that it facilitates the study of extreme events such as large increases in stock values during bull markets. We look at this use in the context of equity market behavior in the 1920s.

2. A Dating Algorithm

There is no widely accepted, formal definition of bull and bear markets in the finance literature. This is surprising given how often these terms are used to describe the state of the stock market. One possibility is to simply define bull and bear markets in exactly the same fashion as expansions and contractions in the business cycle literature. To this end we work with a definition of contractions and expansions in stock markets which emphasizes movements in stock prices between *local* peaks and troughs. This is in line with the business cycle literature and can be contrasted with the emphasis on *global* peaks and troughs in Hess and Iwata (1997). The definition essentially implies that the stock market has gone from a bull to a bear state if prices have declined since their previous (local) peak. This definition does not rule out sequences of negative price movements in stock prices during a bull market or positive ones in bear markets, but we will have to provide some extra rules to restrict the extent of these movements.

If one adopts the idea that bull and bear markets can be thought of as periods of expanding and contracting prices the definitions of these respective events might be done with the Bry-Boschan (BB) program. Watson (1994) essentially followed that route in looking at cycles in a wide variety of series. It is important to recognize that the BB program is basically a pattern recognition program and it seeks to isolate the patterns using a sequence of rules. Broadly these are of two types. First, a criterion is needed for deciding on the location of potential peaks and troughs. This is done by finding points which are higher or lower than a window of surrounding points. Secondly, durations between these points are measured and a set of censoring rules is then adopted which restricts the minimal lengths of any phase as well as those of complete cycles. Because one is simply seeking patterns in the data, the philosophy underlying the BB program is relevant to any series, but the nature of asset prices is sufficiently different to real quantities as to suggest that some modification may be needed in the precise way that pattern recognition is performed. In particular, the determination of a set of initial peaks and troughs of the business cycle is done by using data that is smoothed, and from which "outliers" have been removed, but this is not as attractive with asset price data as it is with data on monthly economic activity. In fact, the process of eliminating "outliers" may actually be suppressing some of the most important movements in the series. Considerations such as these lead us to make a number of modifications to the BB procedures.

Our first deviation from BB is not to smooth any of the series, while the second relates to the size of window used in making the initial location of turning points. In the BB program this is six. It is not entirely clear how to choose this parameter when dealing with asset prices. Taking a peak as an example, we would not want two local peaks to be included in the window, so it needs to be set so that this cannot happen. In the BB program the minimal length between peaks is taken to be 15 months so that this must represent an upper limit to the window size. In the BB program the window is actually set to six months whereas we eventually settled on eight months as the appropriate length for asset prices.

Our second deviation relates to some rule for deciding on the minimum time one can spend in any phase. In business cycle dating this is six months. To try to determine something that would be appropriate for stock prices we consider some of the earliest formal literature that emphasizes the terms “bull and bear markets”. This literature, Dow Theory, was developed by Charles Dow at the turn of the century. Dow theory saw the stock market as composed of three distinct movements and distinguished between

" The daily fluctuation... a briefer movement typified by the reaction in a bull market or the sharp recovery in a bear market which has been oversold... and the main movement .which decides the trend over a period of many months.”

W.P Hamilton, quoted in Rhea (1932) thought that the main or primary trend was

> “The broad upward and downward movements known as bull and bear markets” (Rhea, p. 12)

while the secondary reaction was

> “an important decline in a primary bull market or a rally in a primary bear market. These reactions usually last from three weeks to as many months”.

Since this paper shares with Dow theorists a fundamental interest in the primary movements, the quotations above point to a minimal length for a stock market phase of three months. We therefore set it at four months.[2]

Some minimum length to the complete cycle also needs to be prescribed. Dow Theory is somewhat vaguer about this. Dow defined a primary bull market as one with a broad upward movement, interrupted by secondary reactions, and averaging longer than two years. Furthermore, W.P. Hamilton says

> "There are the broad market movements; upwards or downwards, which may continue for years and are seldom shorter than a year at the least" (Hamilton, *Wall Street Journal*, Feb 26, 1909).

Twenty four months would therefore be one possibility but, with an eye on the hedging engaged in by Hamilton above, as well as the general recognition that bull and bear markets are unlikely to be equal in duration, the case for setting a shorter complete cycle than two years is strong. In business cycle dating the minimal cycle length is fifteen months, so we stay close to it at sixteen months. Moreover, this fits with the identification of original peaks and troughs as using a symmetric window of eight periods.

Finally, given the sharp movements that have been seen in stock prices it does seem as if some quantitative constraint needs to be appended to the rules above. Consider October 1987 for example. In terms of peaks and troughs the contraction only lasted 3 months, after which a recovery occurred, so that this would not be regarded as a bear market due to the duration of the price decline being too short. That seems unsatisfactory. Allowing bear markets to have less than a 3 month minimum duration would however almost certainly produce many spurious cycles. Hence an extra constraint that the minimal length of a phase (four months) can be disregarded if the stock price falls by 20% in a single month was appended to the rules. Appendix B sets out the rules used to establish the turning points of the series studied in this chapter.

3. Some Facts on Bull and Bear Markets

Fig 1 plots the natural log of the monthly stock price index lnP_t for the U.S. over the period 1835/1-1997/5. The series is equivalent to the S&P500 and the data sources are give

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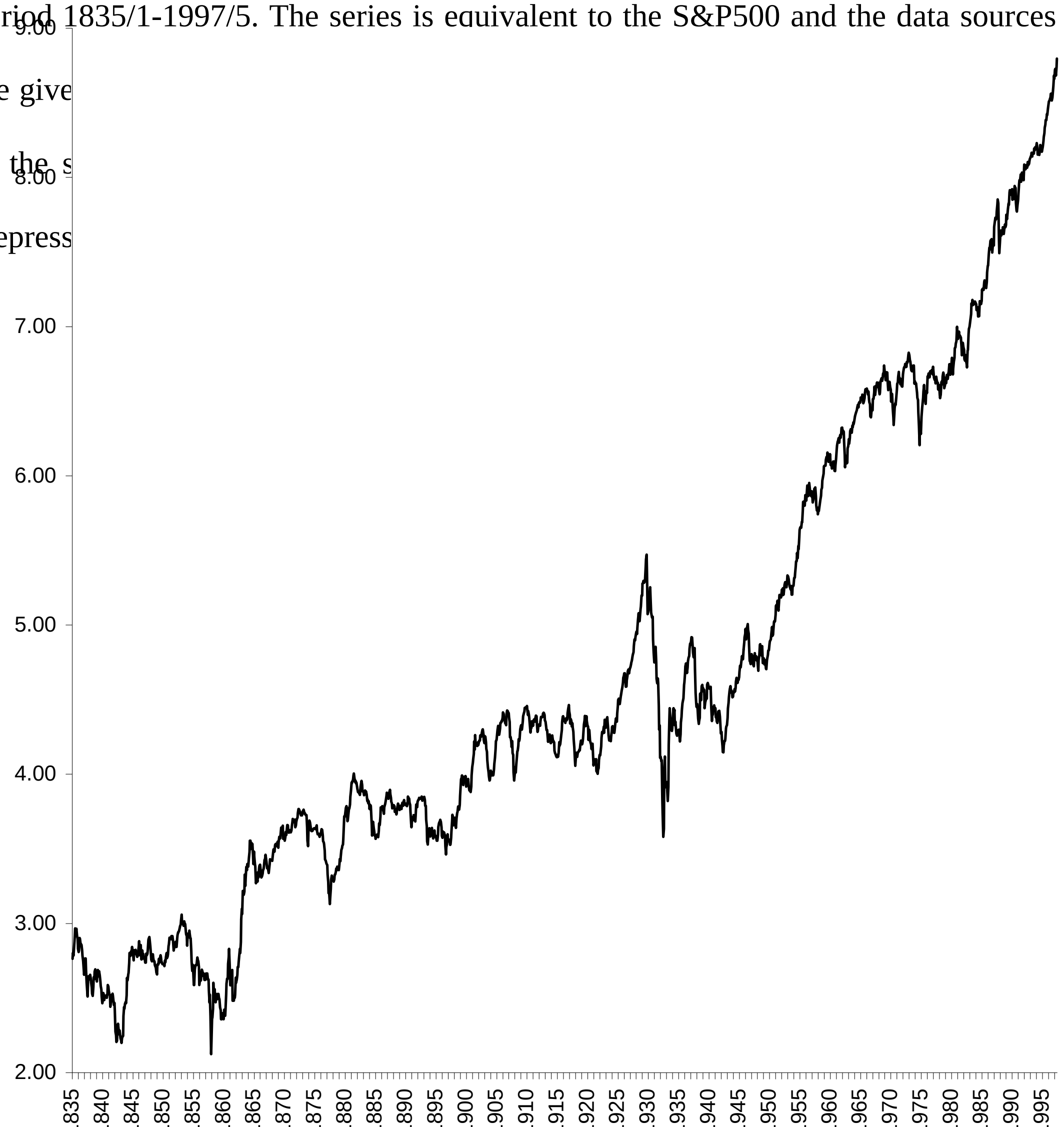
Figure 1. Log US Stock Prices Jan 1835 – May 1997



To summarize this history we apply the algorithm that incorporates the dating methods discussed above to the series. Once we establish where the turning points occur it is possible to summarize various characteristics of the movements between each of these points; such expansions and contractions are termed phases. We consider five such measures of the nature of these phases.

1. The average duration of each phase, *D*.
2. The average amplitude of each phase, *A*. For convenience, we define "amplitude" as the difference in the logs of the stock price from one turning point to another. This does not yield an exact measure of the actual percentage change in the equity price over a phase owing to the fact that these movements are sometimes large and therefore the approximation $ln(1+x)=x$ breaks down.
3. The average cumulated movements in $ln\ P_t$ over each phase, *C*.
4. The "shape" of the phases as measured by their departure from being a triangle. The index used for this purpose is the "excess" index in Harding and Pagan (2001), $EX=(C\text{-}.5A\text{-}.5(A\times D))/D$.
5. The fraction of expansions and contractions for which $A\geq .\ 0.18$ and $A\leq -0.22$. These numbers translate into increases in the equity price of more than *20%* and decreases of less than *20%* The motivation for considering such statistics is that some definitions of bull and bear markets require expansions and contractions that are of these magnitudes. We will refer to these as the B^+ and B^- proportions.

After defining S_t as a binary random variable taking the value unity if a bull market exists at time *t* and zero if it is a bear market, we can estimate the quantities above in the following way. First, the total time spent in an expansion is $\sum_{i=1}^{T} S_t$ and the number of peaks (hence expansions) is given by $NTP = \sum_{i=2}^{T} (1 - S_t) S_{t-1} + 1$. Therefore the average duration of an expansion will be[3]

$$\hat{D} = NTP^{-1} \sum_{t=1}^{T} S_t.$$

The average amplitude of expansions will be

$$\hat{A} = NTP^{-1} \sum_{t=1}^{T} S_t \Delta \ln P_t.$$

To get the cumulated change over any expansion we have to define $Z_t = S_t Z_{t-1} + S_t \Delta \ln P_t$, $Z_0 = 0$. Then Z_t contains the running sum of $\Delta \ln P_t$ provided $S_t=1$, with the sum being automatically re-set to zero whenever $S_t=0$. Hence the total of cumulated changes over all expansions is

$$TC = \sum_{t=1}^{T} Z_t = \sum_{t=1}^{T} \sum_{j=1}^{t} S_j \Delta \ln P_j$$

with the average being

$$\hat{C} = NTP^{-1} \sum_{t=1}^{T} Z_t.$$

All of these parameters can be found up to a factor of proportionality from regressions e.g. $\hat{C}$ comes from the regression of Z_t against unity. To get $\hat{C}$ one needs to adjust for an incorrect scaling factor e.g. the regression coefficient in the regression just described would need to be multiplied by $\frac{T}{NTP}$. The estimated excess is a function of all the quantities estimated above i.e.

$$E\hat{X} = (\hat{C} - 0.5\hat{A} - 0.5\hat{A}\hat{D}) / \hat{D}.$$

Finally, since the series $(1-S_t)S_{t-1}$ has unity at the peak of an expansion and zeros elsewhere, while Z_t contains the amplitude of each expansion at the point in time t, the amplitudes of expansions are the non-zero values of $(1-S_t)S_{t-1}Z_t$. Consequently,

$$B^{+} = NTP^{-1}\sum_{t=1}^{T} I[(1-S_t)S_{t-1}Z_t > 0.18],$$

where $I[a]=1$ if a is true and zero otherwise. Bear market statistics are found in the same way by replacing S_t with $1-S_t$. Since many of the statistics are non-linear functions of sample moments we can use the δ method to compute the asymptotic variances. Of course since A, C, D are all positive the distributions cannot be asymptotically normal unless one is testing a null hypothesis that their expectations have particular values.

Now the approach above is equivalent to the construction of Wald statistics for testing hypotheses. In many ways it is more convenient to use the LM approach since we are generally looking at whether a particular model produces statistics that are close to those of the data. In that case we can generally do a parametric bootstrap in order to compute p – values that reveal how close the model is to the data.

Table 1 provides the statistics just described for the U.S. for three sample sizes. The first uses the complete sample of observations available while the others work with particular sub-samples used in later sections.

The statistics of Table 1 are interesting. It is clear that bull markets tend to be longer than bear markets and the durations agree quite closely with those attributed to Hamilton in 1921. Over time it seems as if bear markets have become shorter and weaker while bull markets have become longer and stronger. The US stock market also exhibits expansions and contractions that deviate quite a lot from a triangle and this tendency has become more emphatic over time as well. The fact that there is a departure from a triangle in the evolution of the markets is also true for the U.S. business cycle, see Sichel (1994) and Harding and Pagan (2001). Finally, it is clear that most expansions become bull markets as defined by their ability to rise more than 20% while a much smaller fraction of contractions culminate in a fall in the market of more than 20%.

As a check on the dating algorithm, Table 2 compares our results for the US to some post-war stock market cycle dates quoted in Niemira and Klein(1994, Table 10.2, p431) that have been used by Chauvet and Potter (1997). Their results are in brackets. The correspondence is quite good, except for an extra contraction from April 71 to Nov 71 (the Niermira/Klein dating stops before the last contraction identified). There was a 10% contraction over this period and it may be that the Niermira/Klein results incorporate some censoring based on the magnitude of movements in share prices.

Finally, as mentioned earlier, most attention has been paid to the mean and volatility of $\Delta\ln P_t$. Slutsky (1937) and Fisher (1925) both emphasized that what seem to be regular ups and downs in a series can simply arise from stochastic variation. Fisher termed this phenomenon the "Monte Carlo cycle". Malkiel (1973) noted that a random walk in stock prices would produce cycles. Hence, as there is clearly going to be some connection between B&B market phenomena and these two moments, we present values for the mean μ and standard deviation σ of $\Delta\ln P_t$ for each of the three samples in Table 1. These quantities are used in simulations of the next section.

Table 3 shows that the mean capital gain has been increasing over time and the standard deviation has been falling. However, the decline in the latter is really quite small and statistically insignificant.

4. The Analytics of Bull and Bear Markets

To gain some appreciation of how the type of DGP determines B&B markets, return to how initial turning points in a series were selected. A peak was taken to have occurred at time t if the event

$$PK = [\ln P_{t-8}, ..., \ln P_{t-1} < \ln P_t > \ln P_{t+1}, ..., \ln P_{t+8}]$$

occurs, where P_t is the level of the stock price. One can then estimate the length of a complete cycle by $\frac{1}{\Pr(PK)}$, given that the number of complete cycles is one less than the number of peaks. What a turning point rule (for a peak at t) does is to describe the joint event ($S_{t+1}=0,\ S_t=1$). Now since Pr(PK) = Pr($S_{t+1}=0,\ S_t=1$) we have

$$\Pr(PK) = \Pr(S_{t+1} = 0 \mid S_t = 1)\Pr(S_t = 1).$$

Hence it is clear that, rather than describing how a turning point occurs, we might instead describe how one goes from the state S_t =1 to S_{t+1} =0. Such a rule was followed by Lunde and Timmermann (2000). . The difference between the two rules resides in the need to specify Pr(S_t=1) i.e. in terms of dating a given realization the need to know the initial state $S_{0.}$ Both methods have their attractions. The fact that one does not need to guess at S_0 means that the turning point method appeals. However, by describing what would cause a transition between states rather than a turning point may make it easier to take account of the magnitude of price changes as a determinant of changes in states. To do so via the dating method one needs to incorporate the magnitude restriction as a censoring operation as was done in one of the steps above.

To understand the implication of the asset price DGP for bull and bear market phenomena we concentrate upon how the DGP characteristics impact upon the probability of the event *PK* occurring. Since this event depends upon the long difference $\ln P_{t+j} – \ln P_t$, and these can be written as the sum of first differences, $\{\Delta \ln P_{t+k}\}^{j}_{k=1}$, it is clear that the probability of *PK* will depend upon the joint distribution of $\{\Delta \ln P_{t+k}\}^{8}_{k=-8}$. Hence, to determine that probability one requires a specification of the DGP for $\Delta \ln P_t$. For example, if $\Delta \ln P_t$ was $N(\mu\ ,\sigma^2)$ then the Pr(*PK*) would be solely a function of $\mu\ /\sigma$, since the turning points in $\ln P_t$ are identical to those in $\ln(P_t/\sigma)$. Consequently, it is likely that the probability will rise with μ (the mean capital gain) and decline with σ. Of course, there is more to the dating rules than that. After the initial turning points are found a set of censoring operations is applied that will change the probability of "final" turning points. Unfortunately, it becomes very hard to assess the precise impact of those operations analytically and so we will be forced to resort to numerical simulation. Nevertheless, the insight obtained from looking at what determines the initial turning points is extremely valuable in analyzing bull and bear markets. In particular, it is clear that, regardless of the model for $\Delta\ln P_t$, the ratio of μ to σ will be a key determinant of cycle characteristics. As an illustration of this point note that the movements in this ratio in Table 3 are very suggestive about the actual changes over time in bull and bear market characteristics noted in Table 1. It is clear that any theoretical model which claims to provide an explanation of historical bull and bear markets will also have to be capable of reproducing the historical values of μ and σ. Since μ is related to the equity premium one must be able to replicate that as well as the volatility of capital gains. Whether this is sufficient is something that we investigate further in the next section.

5. Some Statistical Models of Returns

As the previous section showed it is the DGP of $\Delta\ln P_t$ that is the key to understanding bull and bear markets. To this end we might categorize the potential DGP's into those for which the capital gain is a martingale and those for which it is not. The simplest martingale model would just be the basic random walk with drift.

$$\Delta \ln P_t = \mu + \sigma\varepsilon_{t,} \qquad (1)$$

where ε_t is n.i.d.(0,1)[4]. Columns two and three of Table 4 provide a summary of the bull and bear markets that would be seen in realizations of (1) when viewed through the dating filter described earlier[5]. In the simulations $\mu=0.0042$ and $\sigma=0.0458$ are taken from the U.S. data for 1899/1-1997/5 in Table 3. It is clear that the random walk with drift does quite well at replicating the bull and bear markets actually observed, but the pure random walk ($\mu=0$) fails quite badly. In fact, with the latter we would expect symmetry in the characteristics of the phases, since the probabilities of an initial peak and a trough occurring would be identical. The only exception to this comes with respect to the B^+ and B^- statistics. There the different censoring thresholds (-0.22 and 0.18) utilized to ensure that B&B market movements produce a 20% movement in P_t are the sources of the asymmetries in these proportions. If the censoring points were set to -0.2 and 0.2 then the fraction would be 0.73 in both cases. Given this result on the importance of μ it is clear that explaining it will be a key element in getting the nature of bull and bear markets right. A notable deficiency with the random walk model is its implication that phases should, on average, look like triangles, whereas this is clearly not so.

A possible extension to the basic random walk model is motivated by the fact that (1) implies that capital gains are normally distributed, while the sample excess kurtosis for the 1899-1997 period is 9.2. Consequently, one wants to adopt a DGP that produces realizations for $\Delta \ln P_t$ from a non-normal density. One response would be to change the density for ε_t to others with fatter tails e.g. Student's t, but it is more interesting to generate the excess kurtosis "endogenously". Some standard ways of doing that are to treat $\Delta \ln P_t$ as either a GARCH(1,1) or EGARCH(1,1) process i.e. σ in (1) is replaced by σ_t which varies with the past history of returns. Accordingly, these models were fitted to US capital gains over 1889/1-1997/5 yielding:

$$\sigma_t^2 = .000087 + .13 e_{t-1}^2 + .83 \sigma_{t-1}^2,$$

$$\ln \sigma_t^2 = -.314 - .06 \varepsilon_{t-1} + .26(|\varepsilon_{t-1}| - \frac{2}{\sqrt{2\pi}}) + .95 \ln \sigma_{t-1}^2.$$

In order to perform a valid comparison with the random walk model we also make the means of the GARCH and EGARCH capital gains identical to those in the data i.e. μ*=0.0042*. Columns four and five of Table 4 then process the simulated output from the GARCH and EGARCH processes. Given the symmetry of the GARCH process it is not surprising that it has little effect upon mean durations, but in general it seems that the GARCH model actually does worse than the random walk. Given that the GARCH specification produces fatter tails in the distribution of $\Delta \ln P_t$ it is a somewhat surprising outcome that bull and bear markets are slightly less extreme under it. Of course it has to be remembered that it is *cumulated* shocks that are important for bull and bear markets and the GARCH model is just as likely to produce a large positive shock as a negative one and these operate to offset one another. The EGARCH model tends to provide a better match to most of the phase characteristics than the GARCH model does. In particular, it is the only model that has the ability to produce shapes of phases that resemble the data. Apart from this last qualification the conditional volatility models don't seem to add very much to the explanation provided by the random walk with drift model.

The above models have $\Delta\ln P_t$ being a martingale difference. It has long been observed that there is little linear dependence in $\Delta\ln P_t$, motivating the search for some non-linear structure. One might fit some general non-linear models, such as neural networks or threshold autoregressions, but in some ways it would be nicer to be able to produce the non-linearity in a framework that preserves the flavor of the topic being examined. Because of the emphasis being laid upon the two types of markets, it is useful to try to obtain the requisite non-linearity by utilizing the literature on hidden layer Markov chains. Hamilton's work (1989) is the best known example of this in econometrics, although in other fields there have been many other versions.

In its simplest form Hamilton's model replaces (1) with

$$\Delta \ln P_t = (\mu_0 + \sigma_0 e_t)(1 - z_t) + (\mu_1 + \sigma_1 e_t) z_t, \qquad (2)$$

where z_t is a random variable taking the values of zero and unity whose evolution is governed by a Markov chain with transition probabilities $p_{00}=\Pr(z_t=0|z_{t-1}=0)$ and $p_{11}=\Pr(z_t=1|z_{t-1}=1)$. Which state corresponds to which type of market is essentially arbitrary. For the purpose of presentation and comparison of results we identify the bull state as that which has a higher mean capital gain and label it the state corresponding to $z_t=1$, although we stress again this is quite arbitrary. Many applications of Hamilton's model and its extensions have been made in the econometric literature. Pagan and Schwert (1990) applied the basic model to US stock returns from 1835 until 1925. Recently attempts have been made to generalize this model to allow for the transition probabilities to depend upon the length of time spent in a particular state i.e. to produce duration dependence. Maheu and McCurdy (2000) is a good example. They fitted a model in which the transition probabilities had the same format as in Durland and McCurdy (1994), namely

$$\Pr(z_t = j \mid z_{t-1} = j, d_t) = \frac{\exp(\psi_{1j} + \psi_{2j} d_t)}{1 + \exp(\psi_{1j} + \psi_{2j} d_t)}, \qquad (3)$$

where d_t, the duration of time spent in the *j*-th state in the current phase at time t, is constrained to not exceed 16. The model they preferred was called DDMS-DD and we took the parameters from their Table 5 to simulate it. One problem is that they fit the model to monthly US returns rather than to capital gains. Whilst the volatility of the returns series is much the same as capital gains since the dividend yield shows relatively small monthly variation, the mean of returns is higher than that of capital gains. Hence we adjusted the μ_0 *and* μ_1 parameter estimates in their model by a scaling factor of 1.55 so that the overall mean of the simulated data agreed with that in Table 4. By keeping the mean and variance of the simulated returns equal to that of the data we are therefore solely studying the effect of introducing duration dependence into the model. Data is simulated from this model in column 6 of Table 4. It provides no improvement on the results from the martingale models[6]. Moreover, it fails to capture the shapes of bull and bear markets any better than the other models. Indeed the outcomes are really rather disappointing as there is reason for thinking that E[*EX*] is sensitive to duration dependence; DDMS-DD certainly does better on this dimension than the random walk models but no better than an E-GARCH model. Comparing the EGARCH and DDMS-DD model one is struck by the fact that it behaves very much like a more extreme version of EGARCH

6. Some Economic Models

6.1. General Analysis

Most economic models of stock prices can be expressed as

$$P_t^r = E_t[\sum_{j=1}^{\infty} IMRS_{t+j,t} D_{t+j}^r],$$

where $P_t^r = P_t/P_{ct}$ is the real stock price at the end of period t, $D_t^r = D_t/P_{ct}$ are real dividends paid in time t, $IMRS_{t+j,t}$ is the inter-temporal marginal rate of substitution between time t and $t+j$ and P_{ct} is a consumption price deflator. It is useful to write $D^r_{t+j} = D^r_t(1+g_{t+j,t})$ so that the expression for P_t has the form

$$P_t = D_t E_t[\sum_{j=1}^{\infty} IMRS_{t+j,t}(1+g_{t+j,t})] = \quad (4)$$

$$= D_t K_t \quad (5)$$

reflecting the conditioning of the expectation upon information at time t. Then, in real terms,

$$\Delta \ln P_t^r = \Delta \ln D_t^r + \Delta(\ln K_t - E(\ln K_t)) \quad (6)$$

and, in nominal terms,

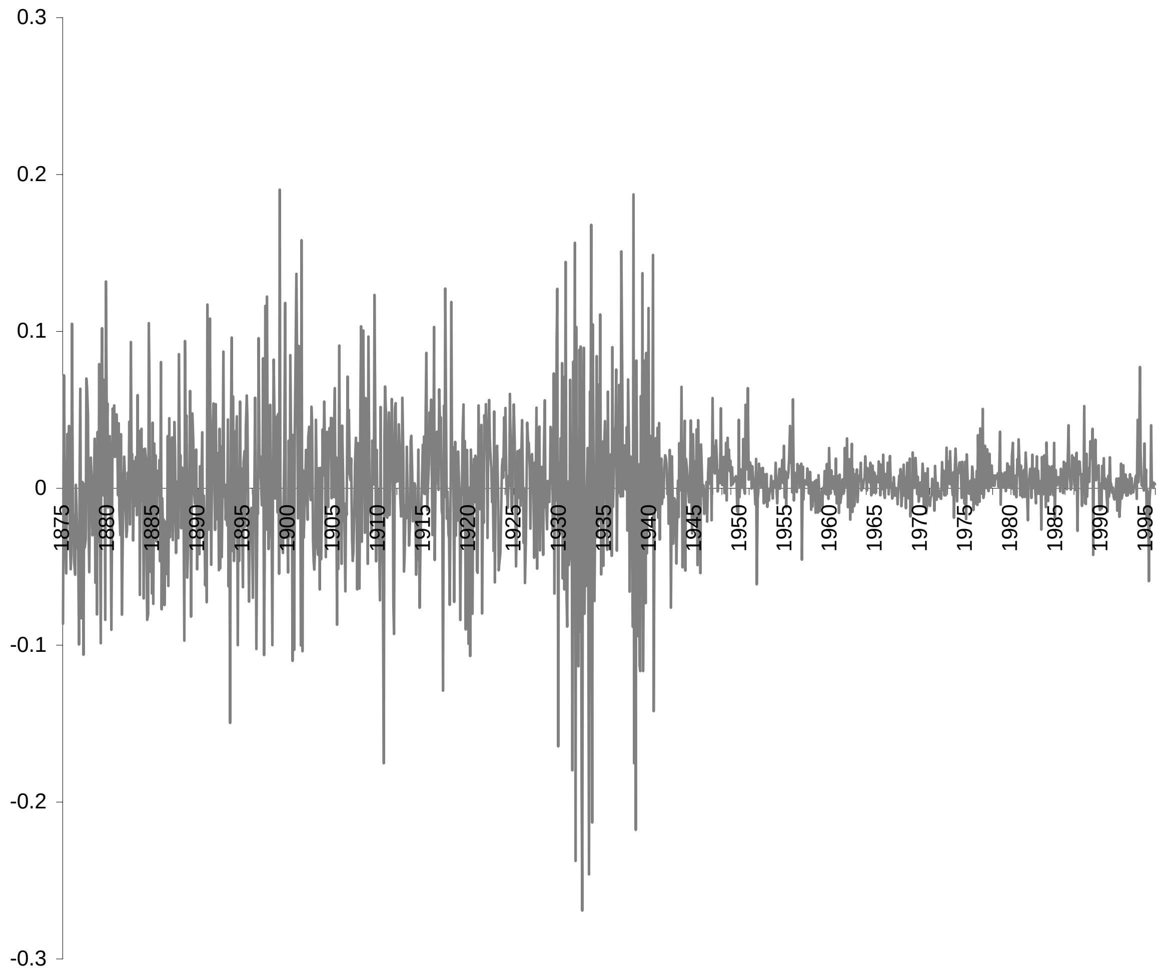
$$\Delta \ln P_t = \Delta \ln D_t + \Delta(\ln K_t - E(\ln K_t)) \quad (7)$$

$$= \Delta \ln D_t + \Delta \xi_t, \quad (8)$$

provided $\Delta \ln K_t$ is a second-order stationary process. Most of the economic models which have been advanced to explain B&B markets can be interpreted through (7), owing to our earlier observation that the characteristics of B&B markets stem from the nature of $\Delta\ln P_t$. Effectively, these models make assumptions which determine the processes generating $\Delta\ln D_t^r$, $\Delta\ln D_t$ and $IMRS_{t+j,t}$.

To give some feel for the way in which (7) may be used, consider initially the case that $\Delta\ln D_t \sim N(\mu_D, \sigma_D^2)$, $\Delta\ln D_t^r \sim N(\mu_D^r, \sigma_D^{2r})$ and there is a constant discount rate. Then $IMRS_{t+j,t} = (1/(1+r))^j$ and $K_t = \sum_{j=1}^{\infty}(\frac{1+\mu_D^r}{1+r})^j$, implying that $\Delta\ln K_t = 0$ and the log of capital gains will be a random walk with mean μ_D and volatility σ_D^2. Moving away from that case the mean equality remains but the volatility of $\Delta\ln P_t$ will now depend upon the relative strength of the volatilities of $\Delta\ln D_t$ and $\Delta\xi_t$. Thus economic models will be needed whenever there is a gap between the observed moments of the capital gains and nominal dividend growth processes. To gain some appreciation of the likelihood of any such difference it can be noted that, over the period 1875/1-1997/5, the ratios μ/μ_D and σ/σ_D were 1.3 and 1.03 respectively, while, over the post-WW2 era, they become 1.12 and 2.97 respectively. Hence, to explain post-WW2 markets one needs a substantial contribution from $\ln K_t$ to the volatility of capital gains. Plots of the series show this decline in volatility in a striking way - see fig 2. Another conclusion that can be drawn from (7) is that the nature of B&B markets will not depend upon $E(\ln K_t)$. Hence investigations that focus upon explaining this quantity e.g. Campbell and Cochrane (1999a) are not directly addressing B&B markets.

Figure 2. Dividend Growth from S&P Index, Jan 1875 – Dec 1995



As mentioned above economic models tend to concentrate upon the behavior of either the IMRS or dividends when explaining B&B phenomena, although there are some models, notably Campbell and Kyle (1993) and Lux and Marchesi (1998), which focus upon mechanisms involving decisions by heterogeneous agents. These mechanisms are such as to make the price/dividend process K_t very volatile. In the latter models equity prices are not the sum of discounted dividends, since only a fraction of agents adhere to fundamentals in making decisions. In the next two sections we consider some economic models that focus upon the IMRS, before moving on to dividend explanations.

6.2. The Gordon – St. Armour Model

As noted above we will be concerned here with models in which $IMRS_{t+j,t}$ is allowed to follow a stochastic process. Gordon and St Armour (GSA) (2000) make it a two state process, representing states of optimism and pessimism. In GSA the utility function has the form $\Theta \frac{(\Theta^{-1}C_t)^{1-\gamma}}{1-\gamma}$ and it is γ which can assume one of two states, with corresponding values γ_0 and γ_1. Thus the aggregate $IMRS_{t+j,t}$ has the form

$$IMRS_{t+j,t} = \psi_{00,j} IMRS^{0,0}_{t+j,t} + \psi_{01,j} IMRS^{0,1}_{t+j,t},$$

where $\psi_{kl,j}$ are probability weights determined by the transition probabilities from state k to l ($k,l=0,1$) and

$$IMRS^{k,l}_{t+j,t} = \beta^j (\Theta^{-1}C_{t+j})^{1-\gamma_l} / (\Theta^{-1}C_t)^{1-\gamma_k}$$

for $k,l=0,1$. There are some econometric issues raised by GSA's work. They assume that C_t is an $I(1)$ process since it comes from a VAR in differences. Since we can write

$$IMRS^{k,l}_{t+j,t} = \beta^j (\Theta^{-1}C_{t+j} / C_t)^{1-\gamma_l} (\Theta^{-1}C_t)^{\gamma_k - \gamma_l}$$

it is clear that this makes $IMRS^{k,l}_{t+j,t}$ the product of functions of an $I(0)$ variable C_{t+j}/C_t and an $I(1)$ variable C_t. Because C_t is treated as strictly exogenous it is unlikely that this would have any impact upon the distribution of the MLE of any parameters of the model that are being estimated, as one can always treat it as fixed by conditioning upon it. However, it may be that the K_t computed using $IMRS_{t+j,t}$ is not *I(0)* and so the log of the price/dividend ratio, which equals K_t, may not be *I(0)* i.e. lnP_t and lnD_t may not co-integrate.

Given a joint process for real dividend and consumption growth we can simulate Gordon and St Armour's model. This produces a value for K_t. Then, as nominal dividends follow a random walk, we can calibrate the process with values of μ_D and σ_D used in GSA (μ_D=0.0048, σ_D=0.012), and subsequently generate simulated values for P_t. The latter are then passed through our dating algorithm to find the average characteristics of bull and bear markets implied by the GSA model. These are given in Table 5 along with the actual characteristics for their estimation period. Clearly, the model statistics are not at all like actual bull and bear markets. Examining the simulated data it emerges that $\Delta \ln P_t$ has a mean and standard deviation of 0.0047 and 0.0364 respectively, while the actual values over their data period are 0.0051 and 0.0432, so it does not appear that the failure to match historical B&B markets is due to a failure to get these two moments right. To check this conclusion the last column of Table 5 examines simulated data from a process $\Delta \ln P_t \sim N(0.0047,0.0364)$. Comparing this column with the preceding one shows that the strong bull and bear markets coming from the GSA model have to be due to some difficulties with the higher order moments of $\Delta \ln P_t$. Indeed, inspection of simulated data shows that the problem seems to be that K_t jumps whenever γ changes value and this has a profound effect upon the probabilities of getting a turning point. It is interesting to note that, whilst μ=0.0047 and μ_D=0.0048 are virtually the same (as expected), the GSA model amplifies the standard deviation quite a lot - from σ_D=0.012 to σ=0.036 - a very desirable outcome.

6.3. The Campbell and Cochrane Model

Campbell and Cochrane (CC) (1999a) also produce a model that allows the $IMRS_{t+j,t}$ to change over time. However, unlike the GSA solution, it now changes continuously. Specifically they make

$$IMRS_{t+j,t} = \delta(\frac{S_{t+1}}{S_t}\frac{C_{t+1}}{C_t})^{-\gamma},$$

where C_t is consumption and the state $\ln S_t$ evolves as a heteroskedastic autoregressive process. To simulate data from their model they need to make some assumptions about the parameters $-\delta$ (the risk free rate) and γ–as well as the nature of the processes generating consumption and dividends. The logs of these latter variables are taken to be random walks with drift. Correlation between the innovations of the two series is allowed. The mean of the monthly real dividend growth rate was set to that of consumption and was quantified from statistics on consumption over 1899-1992. For this reason we will utilize data on bull and bear markets over 1899/1-1997/5 as the benchmark to assess this model. For later reference CC's calibration assumes that μ_D=0.00398 and σ_D=0.0388.[7] CC assess their model by its ability to replicate the equity premium, exp(E(ln(P_t/D_t)) and the volatility of ln(P_t/D_t). Thus our experiment provides an assessment from an alternative perspective. As we noted earlier the second of the quantities used in their assessment methods is irrelevant to the nature of B&B markets.

Table 6 gives the results. It is apparent that the CC model captures the data quite well and is superior to the random walk model in showing some ability to capture the phenomenon captured by the "excess" statistic. As the CC model is known to produce a leverage effect in volatility (something the EGARCH model was designed to capture) this result would be expected given the outcomes in Table 4 for the EGARCH model. In many ways its performance is reminiscent of what one gets with the EGARCH model. Thus it seems a promising candidate for further analysis of the nature of B&B markets.

6.4. Model Emphasizing Dividend Behavior

Another approach is to focus upon the dividend process as the cause of B&B markets. The issue then becomes what type of process one should specify for dividend growth. Assuming that real dividends follow a random walk of the form

$$\Delta \ln D_t^r = \ln(1+g_t) = \mu_D^r + \varepsilon_t, \qquad (9)$$

with $IMRS_{t+1,t}=(1+r)^{-1}$, then $K_t=[1-r\times\exp(\mu_D{}^r+½\sigma_\varepsilon{}^2)]^{-1}$, provided $r\times\exp(\mu_D{}^r+½\sigma_\varepsilon{}^2)<1$. Thus the price-dividend ratio becomes a constant which depends on the volatility of dividend growth as well as its mean. To make K_t stochastic therefore requires a more complicated process than (9) for real dividend growth. A simple way of introducing some stochastic variation into K_t is to allow ε_t to be a GARCH process. Then K_t will depend on a discounted sum of all the higher order conditional moments. This extension is problematic since the forecast of a conditional moment normally converges to the unconditional one as the forecast horizon lengthens, but very few unconditional moments exist for GARCH processes, so it may be very unlikely that the sum in (4) actually converges i.e. K_t may have no moments.

Another way to induce some stochastic variation into K_t is to allow for some serial correlation in ln($1+g_t$) i.e. the model for dividend growth becomes

$$\Delta \ln D_t^r = \ln(1+g_t) = \mu_D^r + \phi(\ln(1+g_{t-1}) - \mu_D^r) + \varepsilon_t .$$

Then one might attempt to evaluate (4) with this specification. This is quite difficult since $1+g_{t+j,j} = \prod_{k=1}^{j}(1+g_{t+k})$ and so

$$K_t = \sum_{k=1}^{\infty} \beta^k E_t \prod_{i=1}^{k}(1+g_{t+i}),$$

where β=$IMRS_{t+1,t}$. If ε_t is N(0,σ_ε^2) then $1+g_{t+k,k} = \prod_{j=1}^{k}(1+g_{t+j})$ is log-normal with ln ($1+g_{t+k,k}$) having moments

$$\mu_k = \mu_D^r k + \frac{\phi(1-\phi)^k}{1-\phi}(\ln(1+g_t) - \mu_D^r),$$

$$\sigma_k^2 = \frac{\sigma_\varepsilon^2}{(1-\phi)^2}(k - 2\phi\frac{1-\phi^k}{1-\phi} + \phi^2\frac{1-\phi^{2k}}{1-\phi^2}).$$

Therefore

$$E_t \prod_{i=1}^{k}(1+g_{t+i}) = \exp[\mu_D^r k + \frac{\phi(1-\phi^k)}{1-\phi}(\log(1+g_t) - \mu_D^r) + \frac{1}{2}\sigma_\upsilon^2]$$

and

$$K_t = (\frac{1+g_t}{\exp \mu_D^r})^{\frac{\phi}{1-\phi}} A \sum_{k=1}^{\infty}[(\beta \exp(\mu_D^r + \frac{\sigma_\varepsilon^2}{2(1-\phi)^2}))^k (\exp(\frac{-\phi^2}{1-\phi^2}\frac{\sigma_\varepsilon^2}{2(1-\phi)^2}))^{(\phi^{2k})} B^{(\phi^2)},$$

where

$$A = \exp(\frac{\sigma_\varepsilon^2}{2(1-\phi)^2})(\frac{\phi^2}{2(1-\phi)^2} - \frac{\phi}{1-\phi})),$$

$$B = \exp(\frac{\phi}{1-\phi}\frac{\sigma_\varepsilon^2}{(1-\phi)^2})(\frac{\exp \mu_D^r}{1+g_t})^{\frac{\phi}{1-\phi}}.$$

K_t is an infinite sum of different powers of a single lognormal variable. However, the term $\exp(\mu_D^r/(1+g_t))^{\phi/(1-\phi)}$ in B makes it hard to find the exact value of K_t. If, however, the sum is approximated by setting $B=1$ then

$$K_t = C(1+g_t)^{\frac{\phi}{1-\phi}},$$

and so the price-dividend ratio is stochastic and depends upon the growth rate of dividends at the point that the expectation is being taken.[8] Moreover, as ϕ rises i.e. the degree of persistence of shocks into the dividend growth rate increases, K_t will rise.

Given the formula above for K_t and using the relation between real equity prices and real dividends in (6) it follows that

$$\Delta \ln P_t^r = \mu_D^r + (1-\phi)^{-1}\varepsilon_t,$$

Now, we might expect that volatility in nominal and real equity prices are much the same due to stickiness in consumer prices, so that $\Delta \ln P_t$ will be well approximated by $N(\mu_D, (\sigma/(1-\phi))^2)$. Thus the introduction of serial correlation into the dividend process amplifies the volatility of equity prices and $\frac{\text{var}(\Delta \ln P_t)}{\sigma_D^2}$ will be $\frac{1+\phi}{1-\phi}$. This latter effect produces a dilemma for an investigator. By increasing ϕ it is possible to realize much larger values of $\ln K_t$ and thereby increase the magnitude of realizations of $\ln P_t$. But the rise in volatility in $\Delta \ln P_t^r$ also means that the average bull market becomes shorter and of smaller amplitude. Thus the explanation of a particular episode may compromise the ability to explain average market outcomes. Some amplification of volatility is desirable however since there is not enough volatility in dividend growth itself to explain the volatility in stock prices. As we observed earlier the ratio of σ to σ_D for post-war data is around 3, so that a value of ϕ=0.8 would be needed to produce the correct volatility in capital gains, given the observed dividend volatility. This is quite a high degree of persistence and, although it is smaller than that used by Barsky and de Long (1993), it is subject to the same criticism that has been made of their argument.

6.5. Studying Extreme Events

The history of asset price movements is replete with instances of extreme behavior e.g. the movement in equity prices in the US during the 1920s. Extreme events in asset markets are interesting for a number of reasons. One is that they provide a very demanding test of any postulated model. Another is that they are frequently used to shed light on the importance of bubbles within these markets. The latter motivation has produced several papers which have enquired into the nature of the bull and bear markets of the 1920s. Donaldson and Kamstra (1996) is probably the best known of these. They argued that the high price-dividend ratios of the 1920s were a consequence of the dividend processes at that time. To establish this fact they fit models to $y_t = \frac{1+g_t}{1+r_t}$, where r_t is an interest rate on low-risk commercial paper plus a constant equity premium and $K_t = E_t[\sum_{j=1}^{\infty}\prod_{k=1}^{j} y_{t+k}]$. The models have the form

$$(1-\rho_1 L-\rho_2 L^2)(1-\beta L)y_t = a+(1-\rho_1 L-\rho_2 L^2)\psi_{t-1}+u_t, \qquad (10)$$

where ψ_{t-1} is a non-linear function of past y_t and u_t has a GARCH structure. They recursively estimate the parameters of this process.

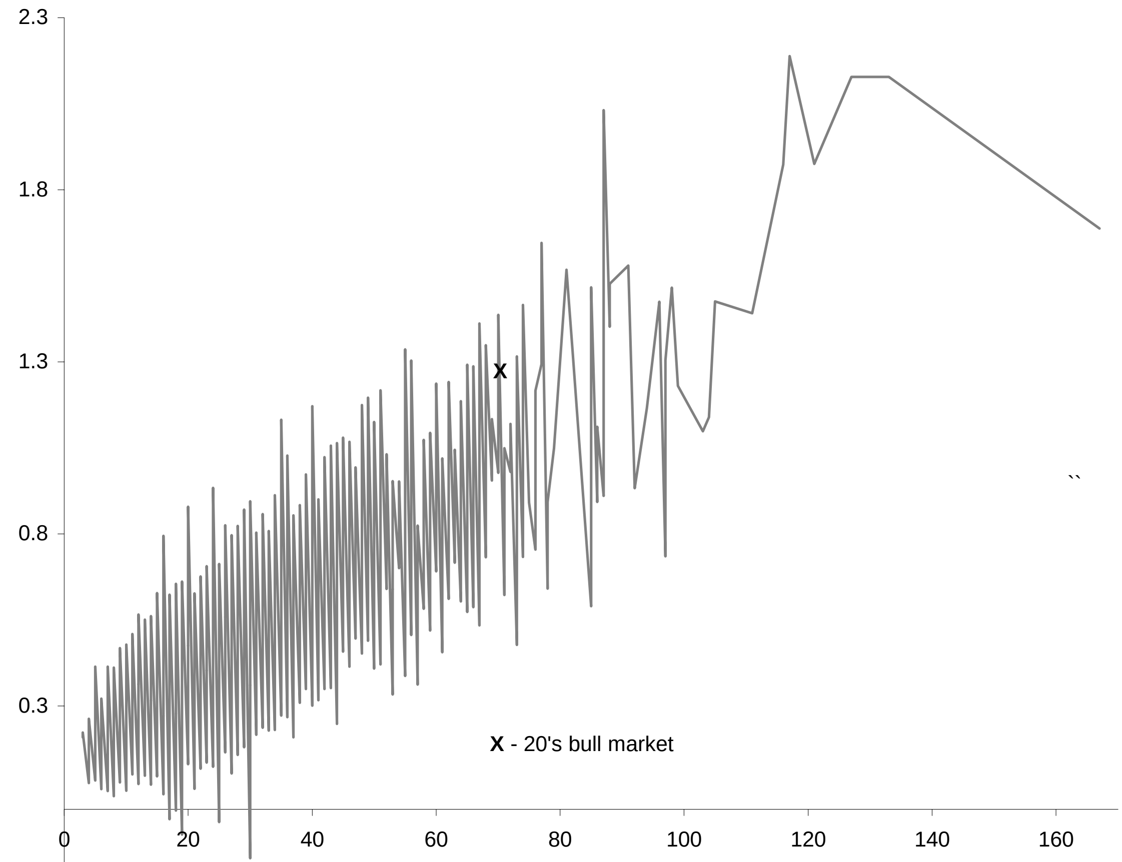
Since the objective is to find a large value of K_t during the 1920s bull market, we might begin by examining the degree of persistence in the y_t process. Ignoring the non-linearity in (10), persistence could be measured by the roots of the equation $(1-\rho_1 L-\rho_2 L^2)(1-\beta L)=0$ and, from their Table 3, these roots are very close to unity (the dominant root is 1.02 using estimates over samples from 1899/1 up to 1919/12 and 1925/12). Consequently, although the analysis of the previous section had ln(1+ g_t) rather than y_t evolving as an AR(1) structure, we might expect that the result found there would still hold i.e. the strong persistence has the potential for producing a large value of *K*. One of their simulations (reported in their figure 4) effectively sets $\phi=0.97$ and such an outcome can indeed be observed. Of course this was Barsky and de Long's point and the question which arose in comment on their paper was what the evidence was for such persistence in dividend growth. Yearly real dividend growth does not show any. Monthly dividends are harder to analyze owing to their seasonality and occasional very large spikes. Donaldson and Kamstra removed these effects by performing seasonal adjustment with the X-11 program followed by smoothing operations to eliminate spikes in the series. It is this adjusted series that become y_t. Thus there has to be a question mark over the origin of the persistence they find. It should also be noted that Donaldson and Kamstra maintain that the non-linearity and GARCH effects in (10) are very important to getting a large enough K_t. We find it hard to evaluate this argument since they compute K_t by simulation methods and so it is necessary to assume that all moments of this random variable exist, and we have already pointed out that this is problematic with GARCH structures.

We turn to another way of looking at the issue of whether the 1920s can be explained by fundamentals, asking whether a particular model is capable of generating outcomes such as occurred at that period in time i.e. rather than focussing upon the average B&B market characteristics as in earlier sections we look at the likelihood of extreme movements being generated by a particular model. Let us concentrate first on bull markets. Because we have a method for demarcating the periods of a bull market it is possible for us to simulate from a given process for equity prices and then determine whether any observed movement in a bull market is likely to have come from the assumed process. The computation is like that used in value at risk analysis.

A number of questions might then be asked. First, was the amplitude and the duration of the 1920s bull market unusual? Second, was the particular conjunction of those two characteristics observed at that time unusual? Essentially this analysis asks whether it was possible for there to be a sequence of dividend outcomes and a realization of K_t which supports a bull market defined with a specific amplitude and duration. It differs from Donaldson and Kamstra's work in that it does not directly address whether the actual dividends and likely value of K_t during the 1920s were such as to produce the necessary outcome for K_t.

To examine the first of these questions we select a model to simulate from and then summarize the history of amplitudes and durations of bull and bear markets from a long simulation. We have produced 3000 bull and bear markets in this way by simulating a random walk model with the μ and σ values from 1835/1-1997/5. Fig 3 cross plots the amplitudes against the durations of the bull markets; also marked on this graph is the 1920's configuration.

Figure 3. Amplitudes vs Durations of Bull Markets from Simulated Random Walk Model



It is clear that the 1920s bull market is not an improbable event when equity prices evolve as a random walk with the historical μ and σ. Another way to compare model output and data is to fit a linear relation between the amplitudes and the durations from the simulated data and to then place that relation on a similar graph to fig 3, along with historical bull market characteristics. Figure 4 does this and it shows that the random walk is quite a satisfactory model for bull markets. Its main deficiency is the inability to explain some long bull markets that failed to produce a large rise in stock prices. Fig 5 is the equivalent of fig 4 for bear markets and it is equally clear that the bear market at the end of the 1920s would never have been predicted by the random walk model. Thus, even though this model is capable of producing quite large declines in share prices, they only come with long duration bear markets and this one was quite short.[9]

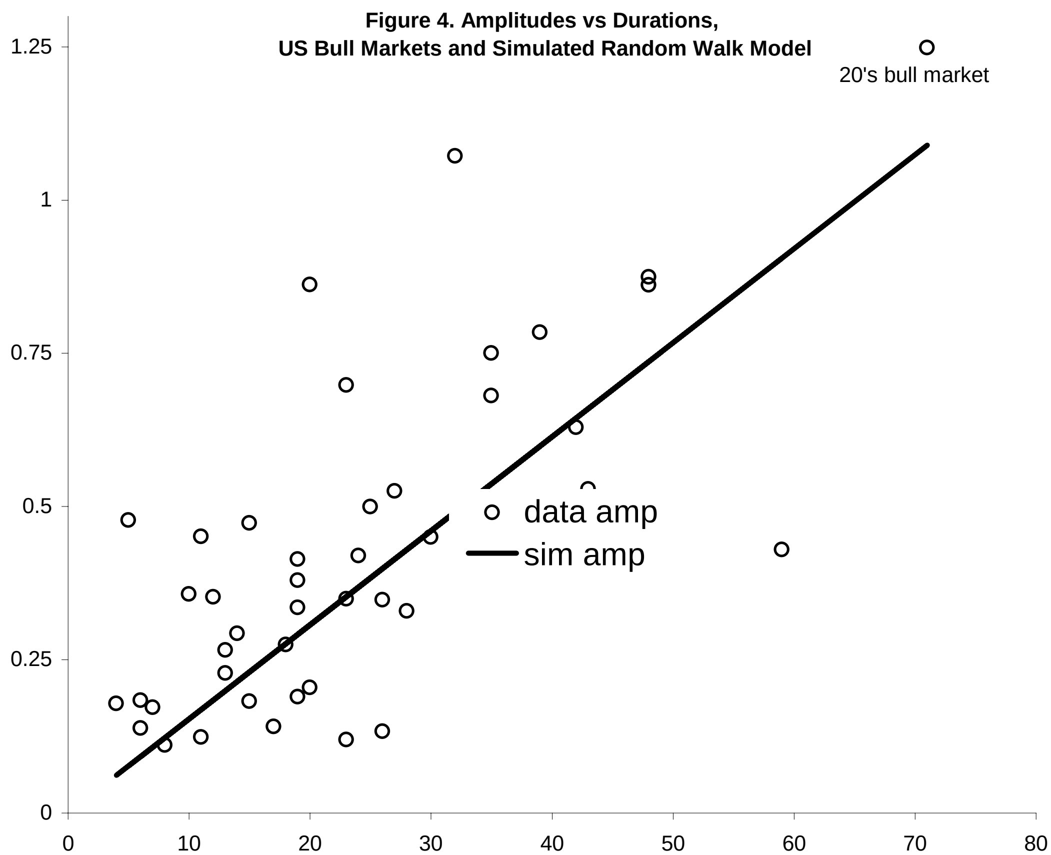
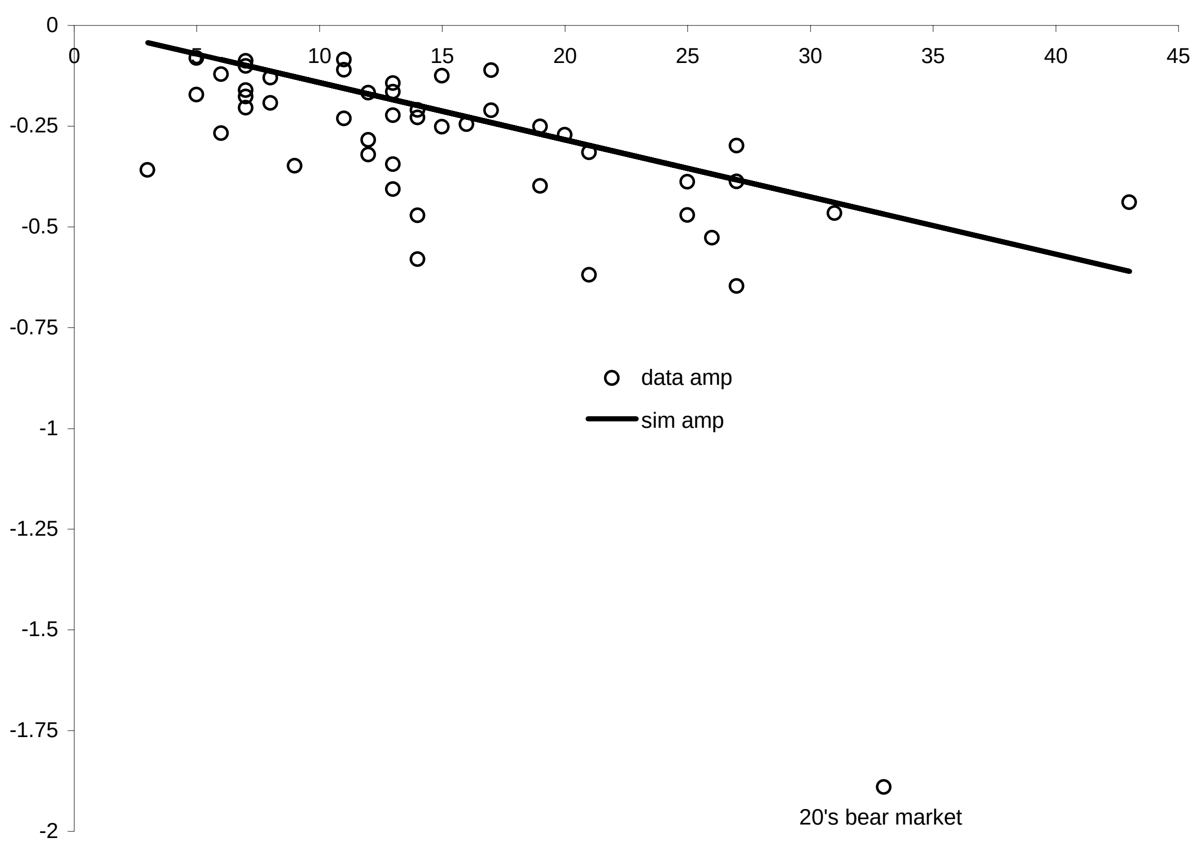


Figure 5. Amplitude vs Duration,
US Bear Markets and Simulated Random Walk Model



7. Conclusion

We have tried to present a framework that might be used for studying bull and bear markets in asset prices. To do this we first defined the idea of local peaks and troughs in asset prices and then observed that the proposed definitions meant that the characteristics of such markets come from the stochastic process driving capital gains. A number of statistical and economic models were then used to evaluate whether they were capable of producing bull and bear markets like those seen over a long period of time in the U.S. However, many other models might be used to generate a process for capital gains, realizations of which may be fed into our dating process, thereby enabling us to directly study the factors which give rise to bull and bear markets. For example, VAR models have sometimes been proposed that have asset prices as one of the variables within the system so that it would be possible to study which of the shocks that drive the VAR are responsible for bull and bear markets.

Appendix A. Stock Market Data

The US data is over 1835/1-1997/5 and consists of combining series from Schwert (1990) from 1835/1-1870/12 and the S&P index thereafter. From 1871/1-1956/12 this data was taken from series 11011 in the NBER macroeconomic database. Missing observations in 1914 due to the closure of the NYSE at the outbreak of WW1 were linearly interpolated. Dividends are those derived from a comparison of the S&P index with and without dividends and were obtained from Allan Timmerman.

Appendix B. Procedure for Programmed Determination of Turning Points.

1. Determination of initial turning points in raw data.
   A. Determination of initial turning points in raw data by choosing local peaks (troughs) as occurring when they are the highest (lowest) values in a window eight months on either side of the date.
   B. Enforcement of alternation of turns by selecting highest of multiple peaks (or lowest of multiple troughs).
2. Censoring operations (ensure alternation after each).
   A. Elimination of turns within 6 months of beginning and end of series.
   B. Elimination of peaks (or troughs) at both ends of series which are lower or higher).
   C. Elimination of cycles whose duration is less than 16 months.

D. Elimination of phases whose duration is less than 4 months (unless fall/rise exceeds 20%).

3. Statement of final turning points

Table 1. Statistics on Bull and Bear Markets in US Stock Price Data*

	1835/1-1997/5	1889/1-1997/5	1945/1-1997/5
Bear Duration	16	14	12
Bull Duration	25	25	26
Bear Amplitude	-0.32	-0.32	-0.24
Bull Amplitude	0.43	0.45	0.46
Bear Cumulated	-2.72	-2.66	-1.60
Bull Cumulated	7.28	7.71	7.70
Bear Excess	0.025	0.022	0.016
Bull Excess	0.019	0.026	0.030
B^-	0.61	0.53	0.40
B^+	0.83	0.88	0.93

*Amplitudes are % changes, durations are in months.

Table 2. Post-War US Stock Market Cycles: Two Dating Methods

(Niermira/Klein in brackets)

Peak	*Trough*
1946/5 (1946/4)	1948/2 (1948/2)
1948/6 (1948/6)	1949/6 (1949/6)
1952/12 (1953/1)	1953/8 (1953/9)
1956/7 (1956/7)	1957/12 (1957/12)
1959/7 (1959/7)	1960/10 (1960/10)
1961/12 (1961/12)	1962/6 (1962/6)
1966/1 (1966/1)	1966/9 (1966/10)
1968/11 (1968/12)	1970/6 (1970/6)
1971/4	1971/11
1972/12 (1973/1)	1974/9 (1974/12)
1976/12 (1976/9)	1978/2 (1978/3)
1980/11 (1980/11)	1982/7 (1982/7)
1983/6 (1983/10)	1984/5 (1984/7)
1987/8 (1987/9)	1987/11 (1987/12)
1990/5 (1990/6)	1990/10 (1990/10)
1994/1	1994/6

Table 3. Mean and Standard Deviation of Capital Gains

	1835/1 – 1997/5	1889/1 – 1997/5	1945/1 – 1997/5
μ	0.0031	0.0042	0.0066
σ	0.044	0.0458	0.0404
μ/σ	0.07	0.09	0.16

Table 4. US Bull and Bear Markets Generated by Various Statistical Models*

	Data	*RW* $\mu=0$	*RW* $\mu\neq0$	*GARCH* $\mu\neq0$	*EGARCH* $\mu\neq0$	*DDMS-DD*
Dur Bear	15	20	16	15	15	15
Dur Bull	26	20	25	26	26	27
Amp Bear	-0.31	-0.34	-0.27	-0.26	-0.30	-0.28
Amp Bull	0.45	0.34	0.44	0.43	0.47	0.48
Cum Bear	-2.71	-4.46	-2.70	-2.53	-2.87	-2.64
Cum Bull	7.84	4.46	7.65	7.71	8.92	9.67
Ex Bear	0.020	-0.000	-0.000	-0.000	0.005	0.005
Ex Bull	0.024	0.000	0.000	0.000	0.011	0.008
B^-	0.51	0.70	0.58	0.49	0.56	0.49
B^+	0.86	0.76	0.85	0.81	0.86	0.82

*Amplitudes are % changes, durations are in months.

Table 5. US Bull and Bear Markets Generated by Gordon-St. Aromour Model, Equivalent Random Walk and Data (1960/1 – 1992/6)*

	Data	*GSA*	*RW*
Bear Duration	19	11	15
Bull Duration	29	54	27
Bear Amplitude	-0.54	-0.12	-0.19
Bull Amplitude	0.64	0.43	0.39
Bear Cumulated	-6.43	-0.3	-1.73
Bull Cumulated	12.85	18.6	7.70
Bear Excess	0.019	0.000	0.000
Bull Excess	0.029	0.001	0.000
B^-	0.71	0.23	0.32
B^+	0.86	0.67	0.78

*Amplitudes are % changes, durations are in months.

Table 6. US Bull and Bear Markets Generated by Campbell-Cochrane Model, Equivalent Random Walk and Data (1899/1 – 1997/5)*

	Data	*GSA*	*RW*
Bear Duration	15	16	16
Bull Duration	26	23	25
Bear Amplitude	-0.31	-0.32	-0.27
Bull Amplitude	0.45	0.47	0.44
Bear Cumulated	-2.71	-3.11	-2.70
Bull Cumulated	7.84	7.67	7.65
Bear Excess	0.022	0.006	0.000
Bull Excess	0.026	0.009	0.000
B^-	0.51	0.67	0.58
B^+	0.86	0.89	0.85

*Amplitudes are % changes, durations are in months.

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[1] Pagan (1998) does this exercise for two other countries, the U.K. and Australia. There are many similarities to the US in the types of markets for these countries but, because most theoretical models have been calibrated to U.S. data, we focus on it in the current paper.

[2] It might be thought that the minimum phase length would be eight months since that would be an implication of the rule used to get the initial dates but later we allow a deviation from normal phase lengths based on quantitative movements in the stock price index.

[3] Some difficulties arise due to incomplete phases at the both ends of the sample. Since we actually measure the features of completed phases the summation should run from the beginning of the first completed phase until the end of the last one rather than over $1,...,T$. Since the asymptotics is not affected by that modification we use the complete sum in these formulae.

[4] Strictly speaking this is not a martingale unless one removes the drift but we will keep the terminology.

[5] Our frame of reference for discriminating between models is the "cycle information". Of course (1), with values of μ and σ from Table 3, can almost certainly be rejected as failing to replicate other features of the data. The simplest would just be the magnitude of $\Delta \ln P_t$ in any month. Since three standard deviations from the mean is an implausible outcome for $\Delta \ln P_t$, using US estimates of μ and σ over 1835/1-1997/5 would imply that a value of $\Delta \ln P_t$ less than -.127 (a 12.7% contraction) should rarely come up, whereas it actually occurs about 1% of the time in the US data.

[6] Although it should be noted that Maheu and McCurdy estimated the model with data from 1835 and not 1899.

[7] We used a program available on John Cochrane's web page to generate the data. The value CC assigned to μ_D^r comes from per capita real consumption data and to get μ_D this needs to be inflated with the rate of price inflation. Using the PPI yearly inflation rate over 1899-1997 as a deflator, the latter adds 0.0024 to their μ_D^r =0.00158.

[8] Because μ_D and g_t are monthly values it is likely that B will be very close to unity even for quite high values of ϕ. For example if μ_D^r=0.004, g_t=0.006, ϕ =0.8 then B=0.992.

[9] It is worth remembering that we use local turning points to demarcate the phases. Although there was a strong recovery from the trough in June 1932 to a peak in February 1934, the level had not returned to the peak of the 1920's. The market had essentially only recovered to its level of 1924.