Estimation of Multiproduct Cost Function for Multiple Technology Industry Using Gibbs Sampling

Maria Ana E. Odejar

Kansas State University

Abstract. In this study the assumption of simultaneous existence of multiple technologies of production is incorporated in modeling multiproduct cost function. This particular assumption is essential especially for an industry undergoing a transition period of adopting a new technology or regulatory changes. A Bayesian method using Gibbs sampling is developed to estimate the multiproduct cost function as a stochastic switching regression model. The estimated multiproduct cost function is utilized to evaluate the degree and direction of overall and product-specific economies of scale and scope of banks in Kansas offering agricultural loans. The standard regression model and the stochastic switching regression model provide coefficients differing in magnitude and sign which are also reflected in varying magnitude and direction of the overall and product-specific economies of scale and scope. These results imply that the traditional regression model may provide misleading estimates of the cost function coefficients under multiple technologies of production. Another interesting result is that the stochastic switching regression model provides coefficients with smaller standard errors.

Key Words: multiproduct cost function, economies of scale and scope, agricultural bank industry, Gibbs sampling

JEL Classification: C, D, E, G, H, M

1. Introduction

Multiproduct cost functions play an important role in applied economics. Cost functions are used to evaluate the degree and direction of economies of scale and scope of a particular industry. The traditional procedure is to take a sample of firms from a particular industry and observations on cost, output quantities, and input prices are used to estimate the coefficients of the standard regression model. The underlying assumption is that all firms in the industry have identical technology and consequently the same long run cost function. Previous studies indicate that this assumption is not realistic particularly for an industry undergoing a transition period after regulatory changes and innovations have been introduced. Adoption and diffusion process of new technology takes several years or decades. Moreover, it is very unlikely for firms to simultaneously respond to new innovations . To avoid the consequences of model specification error, it is essential to incorporate into the model the possibility of the existence of multiple technologies. If it is known which firms are using a particular technology, it is trivial to estimate several cost functions corresponding to the different technologies a particular industry is using. Without this information, estimation of multiproduct cost functions that account for the simultaneous existence of multiple technologies require more appropriate methods of estimation.

The main objective of this research is to use the stochastic switching regression model to estimate multiproduct cost functions under multiple technologies of production. Since parameter estimates have to be updated as new data becomes available, Bayesian analysis is used. This procedure is implemented using the Markov Chain Monte Carlo method, Gibbs sampling.

2. The Stochastic Switching Regression Model

The stochastic switching regression model is basically a finite mixture of regression models. It is assumed that an observation may be generated by the ith regression model depending on the probability value λ_i . That is,

(1) $y_j = \mathbf{x}_{ij}' \mathbf{b}_i + \varepsilon_{ij}$ $y_j \in$ Regime i with probability λ_i $0 < \lambda_i < 1$

$i = 1, 2, \ldots, s, \quad j = 1, 2, \ldots, n$

The errors ε_{ij} are normally and independently distributed as normal with mean 0 and variance σ_i^2 i.e. $\varepsilon_{ij} \sim$ iid N $(0, \sigma_i^2)$. The vector of parameters is $\mathbf{q} = \{\lambda_1, \ldots, \lambda_s, \mathbf{b}_1, \ldots, \mathbf{b}_s, \sigma_1^2, \ldots, \sigma_s^2\}$.

In the absence of information as to which observations follow which regression model, estimation of the regression parameters becomes very complicated and cumbersome. In the past, three methods have been introduced to estimate the stochastic switching regression model.

One method is by the method of moments discussed by Day (1969) and Cohen (1967). The sample moments are equated to the corresponding theoretical central moments about the mean. However, this method does not provide standard errors of the estimates.

Another method, introduced by Quandt and Ramsey (1978) is the moment generating function estimation method. It minimizes the sum of squared deviations of the sample from the theoretical moment generating function to derive parameter estimates. The moment generating function method produces estimates closer to the true values than estimates from the method of moments. However, there is a problem of choosing a satisfactory value of q in the moment generating function, $E[e^{qY}]$, which affects the asymptotic covariance matrix and the quality of numerical approximation including the ease with which convergence is achieved.

Another method is to implement maximum likelihood estimation using iterative algorithms. According to Hartley (1978), the Expectation Maximization (EM) of Dempster and Rubin (1977) can provide maximum likelihood estimates of the switching regression parameters. However, the EM algorithm has some known drawbacks. When the components are not well separated or when initial values are far from the true values, the EM algorithm converges intolerably slowly (Celeux and Diebolt 1985). To remedy this problem, Celeux and Diebolt modified the EM algorithm by adding a stochastic step to prevent the iteration from staying in an unstable stationary point of the likelihood function. Thus the modified algorithm, Stochastic Expectation Maximization (SEM), avoids the slow convergence observed in the EM algorithm. Another well known criticism pertains to the maximum likelihood of the switching regression itself. According to Maddala and Nelson (1975), Kiefer (1978), Swamy and Mehta (1975) and Quandt and Ramsey (1978), the maximum likelihood function for the switching regression model is unbounded at the edges of the parameter space. Phillips (1991) showed that mild constraints on the regime variances can be imposed to force the likelihood to be bounded. The global maximizer of the likelihood function on the constrained parameter space is consistent, efficient and asymptotically normally distributed. However, when it comes to testing parameter stability, there is still a problem regardless of how the null hypothesis is stated. The reason is that under the null hypothesis that the mixing parameter λ is zero or that the regression coefficients are identical for the two regimes , normality cannot be assured because the information matrix is singular [Phillips (1991)]. Another criticism on the maximum likelihood method is that it does not provide parameter estimates accurate enough to be useful for small samples and even for moderately large samples. Hosmer (1973) conducted a Monte Carlo experiment which showed this disadvantage of using the maximum likelihood method.

3. Markov Chain Monte Carlo Methods

Another method for estimating the parameters of the stochastic switching regression model is by Bayesian estimation. This method is particularly useful when there is some prior knowledge of the parameters as suggested by economic theory or previous research. If there is no prior knowledge, one of the methods discussed earlier can be used to obtain prior information. Markov Chain Monte Carlo (MCMC) methods can be used to obtain Bayesian estimates of the stochastic switching regression parameters. MCMC methods aim to summarize the features of a distribution by sampling indirectly from the distribution. These methods are most valuable for complicated distributions such as high dimensional joint distributions for which it is infeasible to sample from. One type of MCMC methods is Gibbs sampling. A Markov chain $\mathbf{q}^{(1)}, \mathbf{q}^{(2)},..., \mathbf{q}^{(m)}, ...$ is constructed by sampling from the full conditional distributions which are easy to sample from. The equilibrium distribution of the Markov chain is the desired joint posterior distribution which is

$$g(\theta \mid Y) \propto \prod_{j=1}^{n} \left[\sum_{i=1}^{s} \lambda_i \varphi_i \left[y_j \,\middle|\, \left(\beta_i, \sigma_i^2\right) \right] \right] g(\lambda_1, \ldots, \lambda_s) \quad \prod_{i=1}^{s} \left[g(\beta_i \mid \sigma_i^2)\, g(\sigma_i^2) \right]$$

where:

$$f_i \left[y_j \,\middle|\, \left(\beta_i, \sigma_i^2\right) \right] = \frac{1}{\left(2\pi\sigma_i^2\right)^{1/2}} \exp\left(-\frac{1}{2\sigma_i^2} \left(y_j - x_{ij}' \beta_i \right)^2 \right)$$

for the stochastic switching regression model. Ergodic averaging of the Markov Chain $\theta^{(m)}$ or the function $h(\theta)^{(m)}$ provides a consistent estimator of the parameters θ or a function $h(\theta)$. The algorithm is allowed to iterate until stationarity is achieved at the a th iteration, before Gibbs sampling results are included in the estimation method, i.e.

$$\hat{E}\left[h(\theta_k)\right] = \frac{\sum_{m=a+1}^{t} h(\theta_k)^{(m)}}{t - a} .$$

The Gibbs sampling algorithm for a two component stochastic switching regression model, proceeds as follows:

(1) Prior distributions:

(a) The prior distribution for the mixing parameter lambda, the probability that an observation is generated by regression model 1, is a Beta distribution with parameters α_1 and α_2 , $\lambda \sim Beta\,(\alpha_1, \alpha_2)$.

(b) The prior distribution of the regression coefficient β_i conditional on σ_i^2 is a multivariate normal distribution with mean $\mathbf{A}_i$ and variance-covariance matrix $\sigma_i^2 \mathbf{Q}_i$, $\beta_i \mid \sigma_i^2 \sim N(\mathbf{A}_i, \sigma_i^2 \mathbf{Q}_i)$, i = 1, 2 .

(c) The marginal prior distribution of the variance σ_i^2 is an inverse-

gamma distribution with parameters $\gamma_i/2$ and $v_i/2$, $\sigma_i^2 \sim IG(\gamma_i/2, v_i/2)$,

$i = 1, 2$.

Define the latent variables

$z_{ij} = 1$ if y_j is from model i, $i = 1, 2$

$= 0$ otherwise

(2) Start with initial values $\lambda^{(0)}$, $\mathbf{b}_1^{(0)}$, $\sigma_1^{2(0)}$, $\mathbf{b}_2^{(0)}$, $\sigma_2^{2(0)}$ and prior

zŠ

ÿÿ

D

19Ì D

\ \ – λ from the Beta distribution with parameters $\alpha_1 + n_1$ and $\alpha_2 + n_2$, that is, $\lambda \sim \text{Beta}(\alpha_1 + n_1,\ \alpha_2 + n_2)$

where:

$$a_1 = \frac{\mu_\lambda^{2}(1-\mu_\lambda)}{\sigma_\lambda^{2}} - \mu_\lambda$$

$$a_2 = \frac{(1-\mu_\lambda)\left[\mu_\lambda(1-\mu_\lambda)-\sigma_\lambda^{2}\right]}{\sigma_\lambda^{2}}$$

μ_λ = prior information about the mean of the mixing parameter

σ_λ^{2} = prior information about the variance of the mixing parameter

(b) Generate σ_i^{2} as an inverse-gamma distribution with parameters

$(\gamma_i + k_i + n_i)/2$ and $(\nu_i + \mathbf{E}_i)/2$, that is,

$\sigma_i^{2} \sim IG[(\gamma_i + k_i + n_i)/2,\ (\nu_i + \mathbf{E}_i)/2]$

where:

$$\boldsymbol{E}_i = (\boldsymbol{Y}_i - \boldsymbol{X}_i \boldsymbol{b}_i)'(\boldsymbol{Y}_i - \boldsymbol{X}_i \boldsymbol{b}_i) + (\boldsymbol{b}_i - \boldsymbol{A}_i)'\boldsymbol{Q}_i^{-1}(\boldsymbol{b}_i - \boldsymbol{A}_i)$$

$$\gamma_i = 2\left[2 + \frac{m_{s_i^2}^{\,2}}{V(s_i^{2})}\right] \qquad i=1, 2$$

and

$$\nu_i = 2\left[m_{s_i^2} + \frac{m_{s_i^2}^{\,3}}{\boldsymbol{V}(s_i^{2})}\right] \qquad i=1, 2$$

(c) Generate $\mathbf{b}_i$ as a multivariate normal distribution with mean $\mathbf{C}_i$ and covariance $\sigma_i^{2}(\boldsymbol{X}_i'\boldsymbol{X}_i + \boldsymbol{Q}_i^{-1})^{-1}$, that is, $\mathbf{b}_i \sim N[\mathbf{C}_i,\ \sigma_i^{2}(\boldsymbol{X}_i'\boldsymbol{X}_i + \boldsymbol{Q}_i^{-1})^{-1}]$

where:

$$\boldsymbol{C}_i = (\boldsymbol{X}_i'\boldsymbol{X}_i + \boldsymbol{Q}_i^{-1})^{-1}(\boldsymbol{X}_i'\boldsymbol{Y}_i + \boldsymbol{Q}_i^{-1}\boldsymbol{A}_i)$$

and $\boldsymbol{Q}_i$ is a matrix with diagonal elements Var $(\beta_{il}) / \sigma_i^{2}$, l = 1, 2, ... , k_i

i=1, 2

Steps (2) to (4) produce a sequence $\boldsymbol{\theta}^{(0)}, \boldsymbol{\theta}^{(1)}, \boldsymbol{\theta}^{(2)}, \ldots \boldsymbol{\theta}^{(m)} \ldots, \boldsymbol{\theta}^{(t)}$ where $\boldsymbol{\theta}^{(m)} = \{ \lambda^{(m)}, \boldsymbol{\beta}_1^{(m)}, \sigma_1^{2(m)}, \boldsymbol{\beta}_2^{(m)}, \sigma_2^{2(m)} \}$ which is a realization of a Markov Chain. The algorithm is allowed to iterate until stationarity is achieved. The first a values of the parameters are excluded to correct for correlated values of the iterations and also so that estimates will be invariant of the initial values. The last t - a values of the parameters are averaged to obtain Bayesian estimates of the parameters and their corresponding standard deviations. For this study the value of t , the total number of iterations is 6000 and the first a = 200 parameter values are excluded from estimation. Coefficients are considered significant (highly significant) if the interval between the 2.5% (0.5%) and 97.5% (99.5%) percentiles of the t-a Gibbs samples from the marginal posterior distribution of $\boldsymbol{\beta}_i$ exclude zero. The marginal posterior distribution is estimated as

$$f(\theta_k) \approx \frac{\sum_{m=a+1}^{t} f\left[\theta_k \middle| \left(\theta_1^{(m)}, \ldots \theta_{k-1}^{(m)}, \theta_{k+1}^{(m)} \ldots \theta_r^{(m)}\right)\right]}{t-a}$$

and the estimate of the predictive density for a new set of regressor values $\mathbf{X}^f$ is

$$\int f\left(Y^f \middle| \theta, X^f\right) g\left(\theta \middle| Y, X\right) d\theta \approx \frac{\sum_{m=a+1}^{t} f\left(Y^f \middle| \theta^{(m)}, X^{(f)}\right)}{t-a} .$$

4. Methodology

The multiproduct cost function assuming the existence of multiple technologies of production can be expressed as a stochastic switching regression model with the number of components equal to the number of distinct technologies. Each component of the cost function is of the flexible fixed cost quadratic cost function (FFC).

(2) $$C_j = \lambda\, C_1^*(Y_j, P_j)' + (1-\lambda)\, C_2^*(Y_j, P_j)$$

where:

$$C_i^*(Y_j, P_j^*, \theta_i) = \alpha_{i0} + \sum_{r=1}^{m-1} \tau_{ir} \ln P_r + \sum_{k=1}^{v} \alpha_{ik} Y_k + 1/2 \sum_{r=1}^{m-1}\sum_{r'=1}^{m-1} \eta_{irr'} \ln P_r \ln P_{r'} + 1/2 \sum_{k=1}^{v}\sum_{k'=1}^{v} \alpha_{ikk'} Y_k Y_{k'} + \sum_{k=1}^{v}\sum_{r=1}^{m-1} \gamma_{ikr} Y_k \ln P_r + \varepsilon_{ij}, \quad i = 1, 2.$$

Symmetry has been imposed on the FFC cost function. Because of the form of the FFC components of the cost function, linear homogeneity in input prices cannot be imposed [Friedlaender, Winston and Wang (1983) and Cohn, Rhine and Santos (1989)].

The outputs used are agricultural loans, non-agricultural real estate loans, non-agricultural loans, deposits on total transaction accounts and deposits on total non-transaction accounts. The inputs are interest rate, wage rate and occupancy rate. The data for this study are obtained from the Federal Reserve Call Report. The sample consists of 462 Kansas banks with agricultural loans in 1989. The choice of the sample period is dictated by technological and regulatory changes that occurred in the 1980s. Technology breakthroughs in computer, communication and electronic transfer systems have critical impact on costs incurred by banks.

Moreover, the Depository Institutions Deregulation and Monetary Control Act of 1980 and the Depository Institutions Act of 1982 have severely changed banks' processes. The explicit interest rate ceilings imposed by Regulation Q were phased out in 1986.

Economies derived from the size or scale of a firm's operations and also from simultaneous production of several different outputs of a single enterprise are evaluated.

Overall economies of scale is determined as

$$S_n(Y) = C(Y) \Big/ \sum_{i=1}^{n} Y_i C_i(Y)$$

where

$$C_i(Y) = \partial C(Y) \Big/ \partial Y_i \quad .$$

Returns to scale are increasing, constant or decreasing if S_N is greater, equal or less than one, respectively. S_N measures how the cost of the current bundle changes if the bundle remains in the same proportion as size changes. Product specific economies of scale is evaluated as the ratio of average incremental cost over its marginal cost

$$S_i(Y) = AIC_i \Big/ C_i(Y) = IC_i(Y) \Big/ Y_i C_i(Y)$$

where $IC_i(Y) = C(Y) - C(Y_{N-i})$ and $Y_{N-i} = (Y_1, \ldots Y_{i-1}, 0\, Y_{i+1}, \ldots, Y_N)$.

Economies of scope relative to a set of outputs *T* is defined as

$$SC_T(Y) = [C(Y_T) + C(Y_{N-T}) - C(Y)] / C(Y) \; .$$

This measures the relative increase in cost that would occur from splintering production of *Y* into separate groups. If $SC_T(Y)$ is greater (less) than zero, economies (diseconomies) of scope exist. Overall and product specific economies of scale and scope are evaluated at the mean bank size.

5. Results and Discussion

Summary statistics for the variables used in estimating the multiproduct cost function of Kansas banks with agricultural loans are shown in table 1. All the outputs have large standard deviations implying the banks are of different sizes. A stochastic switching regression cost function with two components reflecting two simultaneous technologies is estimated. Tables 2 and 3 show that coefficients for the standard regression model and the stochastic switching regression model do not only differ in magnitude but in their signs as well. It is interesting to note that that the coefficients of the stochastic switching regression model have smaller standard errors than those of the standard regression model. The difference in modeling the multiproduct cost function by the standard regression and the stochastic switching regression is reflected in the total costs in table 4, marginal costs in table 5, and economies of scale measures in table 6. With the assumption of simultaneous existence of multiple technologies of production incorporated through the stochastic switching regression model, the data apparently manifests different degrees and or direction of scale and scope at the mean bank size than those of the standard regression model.

6. Conclusion and Recommendation

This study, developes a method of estimating and updating the multiproduct cost function of a particular industry assuming simultaneous existence of multiple technologies of production. Bayesian estimation of the cost function is implemented using Gibbs sampling, a Markov Chain Monte Carlo method. The standard regression model and the stochastic switching regression models provide very different estimates of the cost function and consequently different magnitudes and or direction of overall and product specific economies of scale and scope. These results imply that the traditional regression method may produce misleading estimates of the cost function coefficients under multiple technologies of production. The stochastic switching regression model provide coefficients with smaller standard errors than those of the standard regression model.

The method presented here can also be used to estimate the mixing parameter for several years to monitor how the rate at which technology is adopted changes over time. Moreover, the cost function can also be estimated in the presence of more than two technologies. Instead of sampling from a Bernoulli distribution, samples of the latent variable should be drawn from the Multinomial distribution. Also, instead of taking samples of the mixing parameters from the Beta distribution, the Dirichlet distribution should be used for sampling. For future research, an efficient computational technique such as a hybrid of algorithms that incorporates a method determining the number of components of the stochastic switching regression model corresponding to the number of distinct simultaneous technologies of production and estimation of component parameters and the mixing parameters needs to be developed.

References

Beard, R., Caudill, S., and Gropper, D. (1991), "Finite Mixture Estimation of Multiproduct Cost Functions," *Review of Economics and Statistics*, 73, 654-664.

Celeux, G., and Diebolt, J. (1985), "The SEM Algorithm: A Probabilistic Teacher Algorithm Derived from the EM Algorithm for the Mixture Problem," *Computational Statistics Quarterly*, 73-82.

Cohen, A. Clifford (1967), "Estimation in Mixtures of Two Normal Distributions," *Technometrics*, 9, 15-28.

Cohn, E., Rhine, S., and Santos, M. (1989), "Institutions of Higher Education as Multiproduct Firms: Economies of Scale and Scope," *This Review,* 71, 284-290.

Day, N. E. (1969), "Estimating the Components of a Mixture of Normal Distributions," *Biometrika*, 56, 463-474.

Dempster, A. P., Laird, N. M. and Rubin, D. B. (1977), "Maximum Likelihood from Incomplete Data Via the EM Algorithm," *Journal of the Royal Statistical*

Society (B), 39, 1-38.

Diebolt, J., and Robert, C. (1994), "Estimation of Finite Mixture Distributions through Bayesian Sampling," *Journal of the Royal Statistical Society (B)*, 56, 363-375.

Fair, R., and Jaffee, D. (1972) "Methods of Estimation for Markets in Disequilibrium," *Econometrica*, 40, 497-514.

Friedlaender, A., Winston, C., and Wang, K. (1983), "Costs, Technology and Productivity in the U. S. Automobile Industry," *Bell Journal of Economics*, 14,1-20.

Geman, S. and Geman, D. J. (1984), "Stochastic Relaxation, Gibbs Distribution and the Bayesian Restoration of Images," *IEEE Transactions on Pattern Analysis and Machine Intelligence 6*, 721-741.

Hartley, M. (1978), "Comment" (on "Estimating Mixtures of Normal Distributions and Switching Regressions," by Quandt and Ramsey), *Journal of the American Statistical Association*, 73, 738-741.

Kiefer, N.M. (1978), "Discrete Parameter Variation : Efficient Estimation of a Switching Regression Model," *Econometrica* 46, 427-434.

Maddala, G. S., and Nelson, F. D. (1975), "Switching Regression Models with Exogenous and Endogenous Switching," *American Statistical Association Proceedings of Business and Economics Statistics Section*, 423-426.

Phillips, P. (1991), "A constrained Maximum-Likelihood Approach to Estimating

Switching Regressions," *Journal of Econometrics*, 48,240-261.

Phillips, P. (1991), "A Note on Testing for Switching Regressions," *Economics Letters,* 35, 31-33.

Quandt, R. and Ramsey, J. (1978), "Estimating Mixtures of Normal Distributions and Switching Regressions," *Journal of the American Statistical Association*, 73, 730-738.

Robert, C. (1994), *The Bayesian Choice,* New York: Springer-Verlag New York Inc.

Smith, A. F. M. , and Robert, G. O. (1993), "Bayesian Computation via the Gibbs Sampler and Related Markov Chain Monte Carlo Methods," *Journal of the Royal Statistical Society (B)*, 55, 3-23.

Swamy, P. and Mehta, J. (1975), "Bayesian and Non-Bayesian Analysis of Switching Regressions and of Random Coefficient Regression Models," *Journal of the American Statistical Association*, 70, 593-602.

Table 1

Summary Statistics for the Regression Variables

Variable Name	**Label**	**Mean**	**Standard Deviation**
AGLOANS (Y_1)	Agricultural Production Loans	493.3186	583.1380
RELOANS (Y_2)	Agricultural Real Estate Loans	815.8054	2274.0600
OTLOANS (Y_3)	Non-Agricultural Real Estate Loans	1015.6000	3273.1100
TRANDEP (Y_4)	Deposits -Total Transaction Accounts	1122.0500	2289.0200
NTRNDEP (Y_5)	Deposits-Total Non-Transaction Accounts	2959.2700	5741.4400
INTRATE (P_1)	Interest Rate	0.0530	0.0078
WAGETR (P_2)	Wage Rate	28.2204	6.3674
OCCURTE (P_3)	Occupancy Rate	0.3175	0.2749

Table 2

Estimates of the Regression Coefficients for the Standard Regression Model

VARIABLES	COEFFICIENTS	STANDARD ERRORS
INTERCEPT	-8709.878277 *	4379.594273
AGLOANS	0.266861 ns	0.809981
RELOANS	0.468745 ns	0.681584
OTLOANS	1.702714 *	0.818664
TRANDEP	-1.386187 *	0.648787
NTRNDEP	2.981970 **	0.523506
$AGLOANS^2$	-0.000052 ns	0.000032
AGLOANS*RELOANS	0.000019 ns	0.000058
AGLOANS*OTLOANS	-0.000416 **	0.000050
AGLOANS*TRANDEP	0.000015 ns	0.000053
AGLOANS*NTRNDEP	0.000156 **	0.000037
$RELOANS^2$	0.000038 *	0.000018
RELOANS*OTLOANS	-0.000236 **	0.000027
RELOANS*TRANDEP	0.000178 **	0.000024
RELOANS*NTRNDEP	0.000006 ns	0.000028
$OTLOANS^2$	-0.000137 **	0.000020
OTLOANS*TRANDEP	0.000033 ns	0.000033
OTLOANS*NTRNDEP	0.000192 **	0.000030
$TRANDEP^2$	0.000057 *	0.000027
TRANDEP*NTRNDEP	-0.000083 **	0.000024

VARIABLES	COEFFICIENTS	STANDARD ERRORS
$NTRNDEP^2$	-0.000033 *	0.000013
Ln INTRATE	-3633.254536 ns	1913.491380
Ln WAGETR	1711.969216 ns	1270.744463
Ln OCCURTE	-803.730854 *	392.646153
Ln $INTRATE^2$	-397.104990 ns	253.052716
Ln INTRATE*ln WAGETR	306.970747 ns	293.909316
Ln INTRATE*ln OCCURTE	-135.922928 ns	106.800339
Ln $WAGETR^2$	-100.537669 ns	121.678191
Ln WAGETR*ln OCCURTE	124.652781 *	59.428739
Ln $OCCURTE^2$	-5.894861 ns	11.833675
AGLOANS*ln INTRATE	-0.079192 ns	0.245396
AGLOANS*ln WAGETR	-0.172134 ns	0.132274
AGLOANS*ln OCCURTE	0.032333 ns	0.040280
RELOANS*ln INTRATE	0.440900 *	0.174544
RELOANS*ln WAGETR	0.247939 ns	0.134236
RELOANS*ln OCCURTE	0.009785 ns	0.037286
OTLOANS*ln INTRATE	0.084092 ns	0.196094
OTLOANS*ln WAGETR	-0.334813 *	0.167772
OTLOANS*ln OCCURTE	0.114085 **	0.034773
TRANDEP*ln INTRATE	-0.200267 ns	0.183774
TRANDEP*ln WAGETR	0.454533 **	0.125753
TRANDEP*ln OCCURTE	-0.083714 ns	0.048723
NTRNDEP*ln INTRATE	0.824956 **	0.139109
NTRNDEP*ln WAGETR	0.019467 ns	0.087387
NTRNDEP*ln OCCURTE	-0.013013 ns	0.022379

Table 3

Stochastic Switching Regression Coefficients [1]

VARIABLES	COMPONENT 1 [2]	COMPONENT 2 [2]
INTERCEPT	28141.950000 ** (926.112780)	1.583388 ns (1364.8954)
AGLOANS	-1.015875 ** (0.275626)	-2.158000 ** (0.291936)
RELOANS	-0.305199 ns (0.362888)	0.402309 ns (0.300963)
OTLOANS	-1.419685 ** (0.377745)	-0.368041 ns (0.332597)
TRANDEP	6.041401** (0.437314)	3.524290 ** (0.303747)
NTRNDEP	0.698526 ** (0.153564)	1.996372 ** (0.158940)
$AGLOANS^2$	-0.000068 ns (0.000053)	0.000139 ns (0.000083)
AGLOANS*RELOANS	-0.000092 ns (0.000158)	0.000244 ns (0.000206)
AGLOANS*OTLOANS	-0.000360 * (0.000145)	-0.000292 ns (0.000208)
AGLOANS*TRANDEP	-0.000020 ns (0.000096)	-0.000382 * (0.000149)
AGLOANS*NTRNDEP	0.000169 * (0.000077)	0.000130 ns (0.000082)
$RELOANS^2$	-0.000047 ns (0.000076)	0.000105 ns (0.000103)
RELOANS*OTLOANS	-0.000172 * (0.000086)	-0.000277 * (0.000110)
RELOANS*TRANDEP	0.000060 ns (0.000106)	0.000227 * (0.000104)
RELOANS*NTRNDEP	0.000082 ns (0.000072)	-0.000086 ns (0.000119)
$OTLOANS^2$	-0.000117 ns (0.000084)	-0.000204 * (0.000091)
OTLOANS*TRANDEP	0.000021 ns (0.000074)	-0.000122 ns (0.000086)

VARIABLES	COMPONENT 1 [2]	COMPONENT 2 [2]
OTLOANS*NTRNDEP	0.000163 ns (0.000089)	0.000337 ** (0.000109)
TRANDEP ^{2}	0.000110 ns (0.000088)	0.000274 ** (0.000092)
TRANDEP*NTRNDEP	-0.000121 ns (0.000074)	-0.000215 * (0.000090)
NTRNDEP ^{2}	-0.000024 ns (0.000022)	-0.000017 ns (0.000033)
ln INTRATE	12493.717000 **(441.980240)	2109.714500 ** (554.536000)
ln WAGETR	-4932.983000 ** (293.008400)	2136.203000 ** (530.319800)
ln OCCURTE	392.160040 ** (96.962159)	81.092331 ns (148.593400)
ln INTRATE ^{2}	1364.807800 ** (67.673919)	665.229180 ** (74.747168)
ln INTRATE*ln WAGETR	-1144.361000 ** (82.284666)	700.423370 ** (116.302654)
ln INTRATE*ln OCCURTE	97.384053 ** (28.940038)	141.646740 ** (39.009368)
ln WAGETR ^{2}	191.734170 ** (40.983441)	31.542682 ns (72.333408)
ln WAGETR*ln OCCURTE	-25.696420 ns (21.551094)	130.519790 ** (32.25075)
ln OCCURTE ^{2}	-6.987082 ns (4.274385)	17.058000 * (7.872254)
AGLOANS*ln INTRATE	-0.353596 ** (0.088947)	-0.874497 ** (0.086722)
AGLOANS*ln WAGETR	0.046096 ns (0.058766)	-0.243859 ** (0.055475)
AGLOANS*ln OCCURTE	0.152808 ** (0.035801)	-0.221220 ** (0.026029)
RELOANS*ln INTRATE	0.080366 ns (0.084504)	0.529220 ** (0.070463)
RELOANS*ln WAGETR	0.364204 ** (0.117952)	0.278998 ** (0.072850)
RELOANS*ln OCCURTE	0.337980 ** (0.021762)	-0.264152 ** (0.079556)
OTLOANS* ln INTRATE	-0.482899 ** (0.124964)	-0.670476 ** (0.084148)
OTLOANS* ln WAGETR	0.038944 ns (0.086557)	-0.492666 ** (0.032828)
OTLOANS* ln OCCURTE	0.026596 ns (0.024174)	-0.004039 ns (0.074675)
TRANDEP* ln INTRATE	1.914006 ** (0.110045)	1.364913 ** (0.067243)
TRANDEP* ln WAGETR	0.171156 ns (0.100573)	0.541129 ** (0.045510)

VARIABLES	COMPONENT 1 [2]	COMPONENT 2 [2]
TRANDEP* ln OCCURTE	-0.016247 ns (0.029883)	0.184468 ** (0.036449)
NTRNDEP*ln INTRATE	0.106319 * (0.046976)	0.397671 ** (0.031344)
NTRNDEP* ln WAGETR	0.016037 ns (0.035667)	-0.061768 ** (0.013143)
NTRNDEP*ln OCCURTE	-0.097698 ** (0.010722)	0.008604 ** (0.000000)
MIXING PARAMETER	0.501091 ** (0.052334)	0.498909 ** (0.052334)

[1] Standard errors are enclosed in parentheses .

[2] * , ** and ns indicate significant at the 5%, 1% level and not statistically significant respectively

Table 4

Total Costs

COSTS	**STANDARD REGRESSION MODEL**	**STOCHASTIC SWITCHING REGRESSION MODEL**
$C(Y)$	3062.090	3475.476
$C(Y_1)$	31.286	43.212
$C(Y_2)$	117.017	241.497
$C(Y_3)$	167.895	-64.205
$C(Y_4)$	1087.147	1501.943
$C(Y_5)$	1700.386	1613.550
$C(Y_{N-1})$	3095.000	3529.087
$C(Y_{N-2})$	3055.000	3333.808
$C(Y_{N-3})$	2782.964	3266.702
$C(Y_{N-4})$	2141.1569	2639.495
$C(Y_{N-5})$	917.496	1525.869

* $Y_{N-i} = (Y_1, \ldots, Y_{i-1}, 0, Y_{i+1}, \ldots, Y_N)$

Table 5

Marginal Costs

MARGINAL COSTS	STANDARD REGRESSION MODEL	STOCHASTIC SWITCHING REGRESSION MODEL
C_{Y1}	-0.0211	-0.1431
C_{Y2}	0.0269	0.1620
C_{Y3}	0.3433	0.3817
C_{Y4}	0.8253	0.4002
C_{Y5}	0.8108	1.2173

Table 6

Scale Economies

VARIABLES	SCALE ECONOMIES	
	STANDARD REGRESSION	**STOCHASTIC SWITCHING REGRESSION MODEL**
OVERALL	0.8308	0.7722
AGLOANS	1.3417	0.7594
RELOANS	-0.0454	1.0719
OTLOANS	0.8755	0.5386
TRANDEP	0.9972	1.8617
NTRNDEP	0.9406	0.5412

Table 7

Scope Economies

VARIABLES	SCOPE ECONOMIES	
	STANDARD REGRESSION	**STOCHASTIC SWITCHING REGRESSION MODEL**
OVERALL	0.0325	0.0205
AGLOANS	0.0210	0.0279
RELOANS	0.0359	0.0287
OTLOANS	-0.0363	-0.0785
TRANDEP	0.0543	0.1916
NTRNDEP	-0.1451	-0.0967