Medium-Term Estimation of the Almost Ideal Demand System in Japan.

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Abstract

This paper defines the medium term as the residual component of time series after extracting secular trend and seasonal variation. To select an optimal detrending method, the paper applies a distance metric (integrated mean squared error), which measures the distortionary effect of linear filters on the spectrum of detrended time series. The metric identified substantial distortions of conventional detrending methods (including first-differencing and deterministic linear detrending), and singled out the Hodrick-Prescott and Baxter-King filters as the least-distorting ones. The paper illustrates the consequences of alternative detrending approaches by estimating the Almost Ideal Demand System in Japan.

Keywords: detrending; spectral analysis; Hodrick-Prescott filter; Baxter-King filter; Gibbs effect; Almost Ideal Demand System.

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Introduction.

The Almost Ideal Demand System of Deaton and Muellbauer (1980) has become a widespread tool for analyzing consumer behavior. However, many of its applications to time series data failed to account properly for the trending nature of price and income variables, producing results that are often typical for spurious regressions. This outcome was recently verified by Ng (1997), who demonstrated that the commonly occurring persistence in the estimated residuals of the Almost Ideal Demand System may be an indicator of unit root (non-stationarity) in prices and income.

To avoid the problem of non-stationary data, the most common solution is to first-difference the original time series. This transformation removes the unit root, but it also substantially amplifies the high-frequency component of time series (including noise), and possibly – obscuring significant relationships at the medium frequency band[1]. Besides, the first-differencing is appropriate for removing only stochastic trend, and induces the over-differencing effect when the deterministic trend is present.

In this paper I point out the advantages of an alternative approach to detrending time series. It relies on optimal symmetric linear filters as a flexible and robust tool to extract *both* deterministic and stochastic trends, thus making redundant the problematical distinction between difference-stationary and trend-stationary processes in finite samples. Such filters were suggested by Hodrick, Prescott (1997) and Baxter, King (1999). As shown by King, Rebelo (1993) and Baxter, King (1999), the Hodrick-Prescott (HP) and Baxter-King (BK) filters not only extract the deterministic linear trend, but also can render stationary integrated processes up to order *I(4)* and *I(2)*, respectively.

[1] This outcome was illustrated by Baxter (1994).

In the past these filters were often criticized by Harvey, Jaeger (1993), Cogley, Nason (1995) and Guay, St-Amant (1997) and others for generating spurious cycles (i.e., "Slutsky effect") and for amplifying the spectrum of detrended data, especially at business cycle frequencies. Addressing the criticism, I discuss in this paper various modifications to the HP and BK filters, which are designed to improve their approximation to filters with ideal shape.

To evaluate the distortionary effect of alternative detrending filters, I use a distance metric, which is based on the integrated mean square error (MSE) of approximating the ideal high-pass filter. The metric is often used in the signal-processing literature to design filters with optimal properties in the least-squares sense. In this paper I applied the metric for the identification of the least distorting filter among prevalent detrending methods. Finally, the paper illustrates possible consequences of applying distortionary detrending methods by estimating the Almost Ideal Demand System for major consumption categories in Japan. Estimation results with good high-pass filters turned out, in Tolstoy's words, all alike, while bad filters were flawed in their own way. Canova (1998) made a similar comparison of detrending methods with respect to the "stylized facts" of real busyness cycle theory. The sensitivity study of detrending methods in the context of regression analysis appears to be a novel contribution.

The paper is organized as follows. Section 1 briefly discusses properties of conventional linear filters, focusing on their ability to remove unit root and to approximate the ideal high-pass filter. Section 2 introduces the modified version of integrated MSE metric to measure distortionary effects of linear filters, and applies it to common detrending methods. Section 3 briefly outlines the specification of the Almost Ideal Demand System. Section 4 reports results of the sensitivity study with different detrending methods. Section 5 concludes.

Section 1. Design of optimal high-pass filters.

Let x_t be zero mean stationary process with autocorrelation functions $\gamma_x(s) = \mathrm{cov}(x_t, x_{t-s})$.

Define the autocorrelation generating function by $g_x(z) = \sum_{s=-\infty}^{\infty} \gamma_x(s) z^s$. Then population spectrum (i.e., power spectral density function) is given by

$$S_x(w) = \frac{1}{2\pi} g_x(e^{-iw}) = \frac{1}{2\pi} \sum_{s=-\infty}^{\infty} \gamma_x(s) e^{-iws} \qquad (1)$$

where $w = [-\pi, \pi]$ denotes frequency in radians[2], and i is an imaginary number $\sqrt{-1}$.

Population spectrum is a convenient decomposition of the population variance by discrete frequencies w, with the integral from $-\pi$ to π summing up to the variance of x_t:

$$\int_{-\pi}^{\pi} S_x(w) dw = 2\int_{0}^{\pi} S_x(w) dw = \gamma_x(0) = \sigma^2 \qquad (2)$$

Thus, the area under the population spectrum $S_x(w)$ equals to the variance of x_t.

Define linear filter as a weighted moving average of x_t with weights h_s and output $y_t = \sum_{s=-\infty}^{\infty} h_s x_{t-s}$, with the constraint $\sum_{s\sim-\infty}^{\infty} |h_s| < \infty$ to assure that the variance of the transformed variable y_t is finite. Symmetric filters are identified by weights $h_s = h_{-s}$. The majority of linear filters considered in the paper are symmetric, with the exception of asymmetric first differencing filter (since its weights are $h_o = 1$ and $h_1 = -1$, and zero otherwise).

A well-known result in the frequency-domain analysis relates the population spectrum of filtered output y_t to the population spectrum of input x_t and the frequency response function $H(w)$. The latter, in turn, is calculated by the Fourier transform of filter weights h_s:

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[2] So that if *p* is cycle's period, then *w=2π/p*.

$$H(w) = h(e^{-iw}) = \sum_{s=-\infty}^{\infty} h_s e^{-iws} \tag{3}$$

Then the spectrum of output $S_y(w)$ is related to the spectrum of input $S_x(w)$ by

$$S_y(w) = |H(w)|^2 S_x(w) \tag{4}$$

where $H(w)$ denotes the frequency transfer function of linear filter with weights h_s, while $|H(w)|^2$ denotes the power transfer function of the linear filter. These $H(w)$ and $|H(w)|^2$ are convenient for evaluating consequences of filtering. $H(w)$ quantifies how a linear filter affects the standard deviation of output y_t at frequency w (compared with the corresponding variance of input x_t). Similarly, $|H(w)|^2$ measures the effect on the variance of y_t.

Since secular trend is essentially the low-frequency component of time series, the power transfer function is useful in the design of optimal filters that remove the low frequency band without affecting the variance at other frequencies. Such filters are called high-pass filters. In particular, I define the trend component of time series as one with period of 32 quarters and longer (with the corresponding frequency band $0 \le w_0 \le \pi/16$). This cutoff frequency was also selected by Baxter and King (1999), who justified the choice by reference to the average duration of US business cycles that rarely exceeded 32 quarters (as defined by the NBER chronology). The cutoff of 32 quarters has become widely shared among other business-cycle researchers.[3].

The removal of secular trend can be achieved by the high-pass filters that eliminate low-frequency component of time series (with frequencies up to w_0) and 'pass through' the

[3] Hassler, Lundvik, Persson, and Söderlind (1994) and Stock, Watson (1999) also selected 32 quarters as a cutoff between the secular trend and business cycle components. On the other hand, Canova (1998) postulated the cutoff of 30 quarters.

high-frequency component without affecting its spectrum for frequencies in excess of w_0. The ideal high-pass filter then has power transfer function

$$\left|H^*_{HP}(w)\right|^2 = \begin{cases} 1 & if \quad w \geq w_0 \\ 0 & if \quad w < w_0 \end{cases} \tag{5}$$

The construction of the ideal filter requires the infinite sequence of filter weights h_s, which in practice is not possible due to the finite length of available observations (Koopmans, 1995, p. 177). However, the ideal $\left|H^*(w)\right|^2$ can be used as a benchmark to evaluate the distorting effect of high-pass filters with finite (truncated) filter weights.

Consider the power transfer functions of major linear filters and their relationship with ideal power transfer function $\left|H^*(w)\right|^2$.

1) *Symmetric moving average filter*.

Define MA(m) as a moving average filter with weights truncated to *m*. Then $MA^{LP}(m)$ is a low-pass filter that determines the trend component x_t^g as follows:

$$x_t^g = \sum_{s=-m}^{m} h_s x_{t-s} = \frac{1}{2m+1}\sum_{s=-m}^{m} x_{t-s} \ ,$$

from which the cyclical component of x_t is given by $x_t^c = x_t - x_t^g$.

Frequency response function of $MA^{LP}(m)$ filter equals

$$\begin{aligned} H(w) = \sum_{s=-m}^{m} h_s e^{-iws} &= \frac{1}{2m+1}\left(e^{-iwm} + e^{-iw(m-1)} + \ldots + e^{-iw} + 1 + e^{iw} + \ldots + e^{iwm}\right) = \\ &= \frac{1}{2m+1}\left(1 + 2\sum_{s=1}^{m}\cos(ws)\right), \end{aligned} \tag{6}$$

where the last expression is based on the identity $e^{-iw} + e^{iw} = 2\cos(w)$.

It follows that the power transfer function of the high-pass moving average filter $MA^{HP}(m)$ is given by $\left(1 - \frac{1}{2m+1}\left(1 + 2\sum_{s=1}^{m}\cos(ws)\right)\right)^2$. In particular, consider the effect of

$MA^{HP}(m)$ on the unit root component of time series (which corresponds to zero frequency). Since $\cos(0)=1$, $|H(w)|^2$ of the $MA^{HP}(m)$ filter is also zero. Thus, the high-pass filter $MA^{HP}(m)$ removes the unit-root component in the spectrum of output time series x_t^c, and this property is preserved for any choice of *m*.

Panel 1 of fig. 1 plots $|H(w)|^2$ of the $MA^{HP}(12)$ and $MA^{HP}(20)$ filters, as well as $|H^*(w)|^2$ of the ideal filter with $w_0=\pi/16$. Though for both filters $|H(0)|^2=0$, they also induce substantial oscillatory movements in the spectrum of x_t^c. Such oscillations demonstrate the so - called Gibbs effect, which is typical for Fourier series approximations of a discontinuous function, such as the one, given by (5). Various approaches to alleviate the Gibbs effect will be discussed shortly.

2) *First difference filter.*

As previously noted, this filter can be considered as an asymmetric MA filter with weights $h_o=1$ and $h_1=-1$, and zero otherwise. From (6) it follows that the filter's frequency response function is $H(w)=1-e^{-iw}$, and its power transfer function is $|H(w)|^2=|(1-e^{-iw})|^2=2-2\cos(w)$. Panel 2 of figure 1 plots $|H(w)|^2$ of the first difference filter together with $|H(w)|^2$ of deterministic linear detrending.

Similarly to $MA^{HP}(m)$, the first differencing removes unit root, but it also substantially amplifies the variance of output time series at high frequencies, introducing extra noise in the filtered data. Also note that $|H(w)|^2$ of first difference filter is much less than unity for the frequency band $\pi/16\le w\le\pi/8$ (which corresponds to cycles between 4 and 8 years), demonstrating the so-called "compression effect", as defined by Baxter, King (1999), on the variance of detrended data at business cycle frequencies.

On the other hand, $|H(w)|^2$ of deterministic linear detrending does not involve any reweighing of frequencies. In particular, since its $|H(0)|^2 \neq 0$, the simple detrending procedure does not remove unit root component of time series.

3) *Approximate ideal high-pass filter.*

Weights h_s^* for the filter are calculated by the inverse Fourier transform of $H^*(w)$ for the ideal low-pass filter:

$$H_{LP}^*(w) = \begin{matrix} 1 & if & |w| \leq w_0 \\ 0 & if & |w| > w_0 \end{matrix} \tag{7}$$

with h_s^* calculated by

$$h_s^* = \frac{1}{2\pi} \sum_{j=-\pi}^{\pi} H^*(w) dw, \quad j = 0, \pm 1, \pm 2, \pm 3, ... \tag{8}$$

It can be shown (Granger, Hatanaka, 1964, p. 137; Koopmans, 1995, p. 177) that h_s^* in (8) can be expressed by

$$h_s^* = \begin{cases} w_0 / \pi & for \quad s = 0 \\ \sin(sw_0)/(s\pi) & for \quad s = \pm 1, \pm 2, ..., \infty \end{cases} \tag{9}$$

Then the high-pass version of the ideal filter $I^{HP}(\infty)$ can be obtained by subtracting the output of the ideal low-pass filter $I^{LP}(\infty)$ from the original time series (i.e., similar to the case of $MA^{HP}(m)$ filter).

The optimality of $I^{HP}(\infty)$ depends on the assumption of the infinite sequence of filter weights h_s^*. After filter weights are truncated for some m (so that $h_s^* = 0$ for $s > m$), the finite approximation to the ideal filter $AI^{HP}(m)$ will differ substantially from its infinite version $I^{HP}(\infty)$ (Koopmans, 1995, p. 179). Nevertheless, the comparison of power transfer functions of $MA^{HP}(12)$ and $AI^{HP}(12)$ (panel 1 of fig. 2) demonstrates that $AI^{HP}(12)$ has much smaller oscillations in $|H(w)|^2$. On the other hand, the approximate high-pass filter fails to remove the unit root component in time series. For instance, $|H(0)|^2 = 0.016$ for *AI^HP(12)*, and $|H(0)|^2 = 0.014$ for *AI^HP(20)*.

4) Baxter-King high-pass filter.

The filter is derived from $AI^{HP}(m)$ filter, with the additional constraint on filter weights, designed to eliminate the unit-root component of time series. To preserve the unit-root eliminating property $|H(0)|^2 = 0$, the necessary and sufficient condition for high-pass filters is $\sum_{s=-m}^{m} h_s^* = 0$ (King, Rebelo, 1993, p. 216; Baxter, King 1999, p. 592). Conversely, the condition for low-pass filters is $\sum_{s=-m}^{m} h_s^* = 1$.

The latter restriction is imposed by adjusting the weights h_s^* of (9):

$$\widetilde{h}_s^* = h_s^* + \frac{1 - \sum_{s=-m}^{m} h_s^*}{2m+1} \tag{10}$$

for each h_s^*. With these weights we obtain the Baxter-King low-pass filter $BK^{LP}(m)$. The output of the corresponding high-pass filter $BK^{HP}(m)$ can be obtained by subtracting the output of $BK^{LP}(m)$ from the original time series[4].

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[4] An alternative approach is to change the filter weights of $BK^{LP}(m)$ as follows: $\widetilde{z}_s^* = 1 - \widetilde{h}_s^*$ for $s = 0$, and otherwise $\widetilde{z}_s^* = -\widetilde{h}_s^*$. Then $\sum_{s=-m}^{m} \widetilde{z}_s^* = 0$ for high-pass filters is satisfied automatically, and the output of $BK^{HP}(m)$ is derived directly by the application of filter weights $\widetilde{z}_s^*$ to the original time series.

Panel 2 of figure 2 compares power transfer functions of $AI^{HP}(12)$ and $BK^{HP}(12)$ filters. The difference between $|H(w)|^2$ of these filters is rather small, except that now $|H(0)|^2 = 0$ for $BK^{HP}(12)$ filter.

5) Baxter-King high-pass filter with "sigma correction".

As shown in panel 2 of figure 2, $BK^{HP}(12)$ $BK^{HP}(12)$ still exhibits significant side lobs (the Gibbs effect) due to the discontinuity in $H^*_{LP}(w)$ in (7) at frequency w_0. It is possible to alleviate the distortionary effect by replacing the discontinuous $H^*_{LP}(w)$ by a smoother function that changes less abruptly from one to zero in the neighborhood of w_0. Hamming (1998) discusses various approaches to alleviate the Gibbs effect, including:

1) Lanczos' σ-factors (p. 110)[5];
2) Hann window (p. 117);
3) Hamming window (p. 119);
4) Kaiser window (p. 189) .

Using the integrated MSE metric of approximating the ideal high-pass filter (to be discussed in the next section), I found that distortionary effect of the Lanczos' σ-factors was always smaller compared with other alternatives[6]. To save space, I will report results only for the best modification of $BK^{HP}(m)$ filter.

The calculation of filter weights in the Baxter-King filter with σ-adjustment (which I refer as $BKS^{HP}(m)$) proceeds as follows:

1. For a given truncation parameter *m*, calculate filter weights h_s^* by (9).
2. Compute $h_s^\sigma = \sigma_{s,m} h_s^*$, where $\sigma_{s,m} = \dfrac{\sin(2\pi s)/(2m+1)}{2\pi s/(am+1)}$ is the Lanczos' σ-factor.

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[5] This approach to suppress side lobs of BK filter was also discussed by Woitek (1998).

[6] Results of these calculations are available upon request.

3. Apply Baxter-King adjustment (10) to h_s^σ to satisfy $\left|H(0)\right|^2 = 0$.

4. Change filter weights as follows: $\tilde{z}_s^\sigma = 1 - \tilde{h}_s^\sigma$ for $s = 0$, and otherwise $\tilde{z}_s^* = -\tilde{h}_s^*$.

5. Apply $\tilde{z}_s^\sigma$ in the symmetric MA(m) filter with m leads and lags[7].

Figure 3 plot power transfer functions of $BKS^{HP}(12)$ *and* $BKS^{HP}(20)$ filters. In both filters the σ-adjustment makes spurious oscillations much less pronounced. Besides, for frequencies below $w_0 = \pi/16$ the power transfer function of $BKS^{HP}(12)$ is nearer to zero than the one of $BK^{HP}(12)$. In other words, $BKS^{HP}(12)$ has a smaller "leakage" of the frequencies which it is designed to suppress. Conversely, the $BKS^{HP}(20)$ filter does not reduce leakage at frequencies below w_0, (panel 2 of figure 3), reducing the benefit of σ-adjustment for $m = 20$.

6) <u>Hodrick-Prescott (HP) filter</u>. Properties of the filter have been extensively discussed in the literature. Harvey, Jaeger (1993) showed the filter is optimal for the following unobserved components (UC) model:

$$x_t = x_t^g + x_t^c$$
$$x_t^g = x_{t-1}^g + \beta_{t-1}$$
$$\beta_t = \beta_{t-1} + \varsigma_t$$

with x_t^c (the cycle component) $\mathrm{NID}(0,\sigma_c^2)$, $\varsigma \sim \mathrm{NID}(0,\sigma_g^2)$, and ς_t assumed independent of x_t^c. Under these conditions the smoothing parameter of HP filter becomes $\lambda = \sigma_c^2 / \sigma_g^2$. In the case of quarterly data, Hodrick, Prescott (1997) postulated $\lambda = 1600$, making reference to their "prior view" that 5 per cent standard deviation of x_t^c is as large as 1/8 per cent standard deviation of ς_t. This essentially arbitrary choice of λ was often criticized. For example, Nelson, Plosser (1982, p. 257) estimated that $\sqrt{\lambda}$ was likely to be constrained between five

[7] Filter weights for $BK^{HP}(m)$ and $BKS^{HP}(m)$ filters (m=12,16, 20) are given in table 1.

and six. Similarly, Pedersen (1998) calculated the optimal λ for a wide range of AR(1) and AR(2) models, and found it most often in the range of 1000-1050.

King, Rebelo (1993) interpreted the HP filter as a symmetric MA filter with a frequency transfer function for the cyclical component

$$H(w) = \frac{4\lambda[1-\cos(w)]^2}{1+4\lambda[1-\cos(w)]^2} \quad (11)$$

The HP filter removes unit root (regardless any value of parameter λ, $H(0)=0$). Besides, the power transfer function of the filter quickly approaches unity for frequencies above $w_0 = \pi/16$, as shown in panel 1 of figure 4 (for the case of $\lambda = 1600$). However, compared with $BKS^{HP}(12)$ filter, the leakage of HP(1600) is larger for frequencies $w < \pi/16$. On the other hand, when λ is set to 1000[8], the power HP(1000) filter is closer to the ideal high-pass filter at low frequencies (panel 2 of figure 4). On the other hand, the less optimal shape of $|H(w)|^2$ of HP(1000) for $w > \pi/16$ is less of a concern. Not a great deal of power is concentrated at these high frequencies in "typical spectral shape" of economic series (Granger, 1964), thus alleviating resulting distortions to the spectrum of output series.

It is often neglected that (11) for Hodrick-Prescott filter was derived under the assumption of infinite span of available observations. In finite samples the filter's weights are limited by the sample size, so the $|H(w)|^2$ of HP filter may differed substantially from the pattern, depicted in figure 4. The disparity is especially pronounced for observations at both ends of sample, as illustrated by Baxter, King (1999). Due to the lack of finite-sample counterpart for formula (11), this formula will be used in the paper. Yet it is important to remember that the asymptotic formula provides essentially only the lower bound on the distortionary effect of HP filter, which can be somehow larger in finite samples. Besides HP

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[8] Which is in the range of optimal λ, recommended by Pedersen (1998).

filter, no other linear filters which were discussed in this section, requires an asymptotic justification to derive its power transfer function.

Section 2. Integrated MSE metric to measure distortions of linear filters.

In this section I will compare the distortionary effects of major linear filters, using a modified version of a distance metric, recently discussed by Pedersen (1998). In the signal-processing literature the metric is known as the integrated MSE of approximating optimal filters (Koopmans, 1995, p. 188; Porat, 1997, p. 290). The metric compares the output spectrum $S_y(w)$ of a linear filter to the corresponding output spectrum $S_y^*(w)$ of the ideal high-pass filter H_{HP}^* , as given by (5).

Let $S_y^*(w) = \left|H_{HP}^*(w)\right|^2 S_x(w)$ be the spectrum of output at frequency w after applying the ideal high-pass filter. Using the ideal filter as a benchmark, the distortion in the spectrum of output time series $S_y(w)$ at frequency w is given by

$$\Delta S_y(w) = \left|S_y^*(w) - S_y(w)\right| = \left|\left|H_{HP}^*(w)\right|^2 - \left|H_{HP}(w)\right|^2\right| S_x(w) \tag{12}$$

Integrating (12) over $w = [-\pi, \pi]$ yields the metric of the distortionary effects of linear filters on $S_y(w)$ for the whole frequency interval:

$$Q = \int_{-\pi}^{\pi} \Delta S_y(w) dw = 2\int_{0}^{\pi} \Delta S_y(w) dw = 2\int_{0}^{\pi} \left|\left|H_{HP}^*(w)\right|^2 - \left|H_{HP}(w)\right|^2\right| S_x(w) dw \tag{13}$$

where $dw = w_i - w_{i-1}$. Following Pedersen (1998), I will refer to the metric as Q-statistic. The statistic depends on the difference between power spectral functions of an evaluated and the ideal high pass filter, which is weighted by the spectrum $S_x(w)$ of input time series. Using (2), $S_y^*(w)$ can be interpreted as the true variance of the cyclical component of x_t. Thus, the Q-statistic essentially measures how these true and estimated variances are close to each other, with less distorting filters producing smaller values of the Q-statistics.

Pedersen (1998) applied the Q-statistic for the comparison of major linear filters. Input time series were represented by several AR(1) and AR(2) processes, for which analytical expressions for $S_x(w)$ are available. In contrast, I suggest to calculate the statistic with the *estimated* spectrum $\hat{S}_x(w)$ of actual time series, using both non-parametric and parametric approaches. Then one can check the sensitivity of relative ranking by Q-statistic to the alternative approaches to estimate $\hat{S}_x(w)$.

The original version of Q-statistic is not normalized, so it is particularly informative on the extend of relative distortions among investigated linear filters. One approach, similar to the R^2-statistic, is to normalize (13) by $S_y^*(w)$. However, since $S_y^*(w)$ would be the same for a given $S_x(w)$, it is also not particularly useful. Given that the Q-statistics would be normally calculated for several alternative filters, resulting in a set of $Q_1, Q_2, ..., Q_p$ statistics, I suggest to normalize them by the *minimum* Q-statistics among these p linear filters.

To evaluate linear filters, which were discussed in section 1, I utilize as input time series x_t the actual data that will be further used for the estimation of Almost Ideal Demand System in Japan. These time series included consumption shares Sh_i and price deflators P_i for eight major consumption commodities[9], as well as real income *Inc*.

Q-statistics for these time series were calculated with two alternative spectral estimates of $\hat{S}_x(w)$: the non-parametric Thompson's multitaper method, and the parametric

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[9] The consumption categories included: 1)food, beverages, tobacco; 2) clothing, footwear; 3) rent, fuel, power; 4) furniture, household operation; 5) medical care; 6) transport and communication; 7) recreation and entertainment; 8) other consumption.

Burg's method, using Signal Processing Toolbox 5.0 of Matlab (commands 'pmtm' and 'pburg', respectively). To save space, I will report Q-statistics, based on the Burg's estimates of $\hat{S}_x(w)$ [10].

The Burg's method requires selection the order of $AR(p)$ process. Although it is often recommended to choose *p* on the basis of minimum information criteria (such as AIC), Monte Carlo evidence with actual time series often indicate that a different approach – by setting *p* to a fixed proportion of sample size *n* (such as *n*/3) – may be more sensible (Percival, Walden, 1993, p. 437). In preliminary trials *p* was set to 10, 20, and 30. With $p = 30$ spectral estimates started to become too volatile, so I opted for $AR(20)$ process as a compromise.

Tables 1a and 1b report Q-statistics for linear filters, based on the Burg's spectral estimates of $\hat{S}_x(w)$ [11]. In addition to filters, discussed in section 1, the tables report results for the HP filter with optimal choice of λ [12].

Consider the relative performance of filters in the case of Sh_1. The smallest Q-statistic is produced by $BK^{HP}(20)$ filter, while the HP filter with optimal parameter $\lambda = 854$ (as shown in the last row of table 1a) turned out the second with $Q = 1.04$. *HP(opt)* was followed by *HP(1000)*[13] and *HP(1600)*. Note that for all variations of BK and BKS filters the Q-statistic always fell short of 1.5, while for the first-difference filter it was as high as 3.02. The

[10] Results for non-parametric spectral estimates turned out very similar to ones, reported in the paper. They are available upon request from the author.

[11] Before calculating the Q-statistics, I used seasonal adjustment by X-12 filter to remove spectral peaks at seasonal frequencies.

[12] The optimal λ is one that yields the smallest Q-statistic during the grid among feasible values of λ, (Pedersen, 1998).

[13] Value of λ, recommended by Pedersen (1998) on the basis of his study of AR(1) and AR(2) processes.

application of the linear trend and $AI^{HP}(m)$ filters was especially distorting, primarily as a result of their failure to eliminate the unit root component with zero frequency.

For detrended Sh_2 time series the best filter was *HP(opt)*, followed by two other Hodrick-Prescott filters. $BK^{HP}(20)$ this time was fourth, while distortions in the first-difference filter became even more pronounced, with $Q = 7.67$.

Now I will summarize results for all 17 time series. The clear frontrunner turned out to be the *HP(opt)* filter, since it had the smallest Q-statistic in 13 cases. $BK^{HP}(20)$ was the best three times, while *HP(1600)* – once. If filters are compared by their median rank, HP(opt) was again the best, achieving the median rank of 1 among examined filters. The HP(1000) filter had median rank 2, while for HP(1600) the median rank was 4. $BK^{HP}(20)$ and $BKS^{HP}(12)$ had median ranks 3 and 5, respectively.

There was a noteworthy link between the benefits of the Lanczos' σ-adjustment in $BKS^{HP}(12)$ and the estimated optimal value of λ parameter, used in the *HP(opt)* filter. When optimal λ turned out much smaller than the 'default' 1600 (such as in the case of price deflators), the use of $BKS^{HP}(12)$ filter with the σ-adjustment yielded *smaller* distortions compared with $BK^{HP}(12)$ filter (for example, compare their Q-statistics for P_1, table 1b)[14]. Recall that $\lambda = \sigma_c^2 / \sigma_g^2$, so that a smaller λ indicates a relatively larger variance of the growth component x_t^g. With more power concentrated in the frequency band $0 \le w \le w_0$, the Q-statistic gives more weight to relatively smaller leakage of $BKS^{HP}(12)$ compared with $BK^{HP}(12)$ in this frequency band (as shown in panel 1, Figure 3).

On the other hand, the $BKS^{HP}(20)$ filter is not nearer to zero compared with $BK^{HP}(20)$ at the low-frequency band (panel 2 of figure 3). This resulted in more distorting application of

[14] This result contradicts Woitek's assertion that the σ-adjusted BK filter is no longer optimal in the least-squares sense (1998, p. 5).

the $BKS^{HP}(20)$ filter compared with its unadjusted counterpart in cases when the optimal λ in the *HP(opt)* filter approached zero.

Not surprisingly, the most distorting filter turned out to be the deterministic linear trend. $MA^{HP}(m)$ filters induced smaller distortions than IA^{HP}, but both filters still were ranked at the bottom. As for the first-difference filter, its medium rank among 17 detrending methods in tables 1a and 1b was only 11.

Using these results for Q-statistics, I selected 5 filters to evaluate the sensitivity of estimated Almost Ideal Demand System to various detrending methods. The filters included relatively less distorting HP(*opt*), HP(*1600*) and $BKS^{HP}(12)$ filters, as well as more traditional deterministic linear trend and first-differencing, plus the original data without detrending.

Section 3. Specification and Estimation of Almost Ideal Demand System.

I considered major consumption categories from the National Accounts statistics of Japan. All data were seasonally-adjusted by X-12 filter. Original time series were quarterly, with sample covering 1970:2-1999:1. The actual estimation sample was shortened to 1973:2-1996:1 due to the use of $BKS^{HP}(12)$ filter. Dropping 3 years of data also alleviated the problem of Hodrick-Prescott filter at the ends of sample period.

I estimated the following standard specification of the Almost Ideal Demand System:

$$w_{it} = \alpha_i + \beta_i \log(E_t / P_t) + \sum_{j=1}^{k} \gamma_{ij} \log(P_{jt}) \tag{14}$$

where w_{it} is consumption share of commodity i, E_t is total expenditures, P_t and P_{jt} are deflators for total consumption and i^{th} commodity, respectively. To avoid non-linearity in the demand system, I used Stone's index

$$\log(P_t) = \sum_{j=1}^{k} w_{jt} \log(P_{jt}) \tag{16}$$

The demand system allows one to test the following restrictions that imply rational consumption behavior:

1. Homogeneity (no money illusion): $\sum_{j=1}^{k} \gamma_{ij} = 0$ for each i^{th} commodity;
2. Symmetry of the substitution matrix: $\gamma_{ij} = \gamma_{ji}$.

Due to the substantial evidence that tests of homogeneity and symmetry restrictions may have seriously distorted nominal size, I also applied Monte Carlo tests, using moving block bootstrap (MBB) with overlapping blocks. The bootstrap procedure is effective for taking into account the serial dependence between observations of unknown form. In this respect it is similar to the HAC estimator of Newey and West (1987)[15]. MBB was implemented, using 'resample' command in Eviews 4.0.

Section 4. Estimation results with alternative detrending methods.

I ran several specification checks to verify that major assumptions of the linear regression model were not violated. The lack of simultaneity bias was verified by the Hausman test in its "artificial regression" version, as discussed by Davidson, MacKinnon (1993, p. 239). I ran the Hausman test twice, with different sets of instruments. First, I used independent variables at lag 4, and second – just the ranks of independent variables. Tables 2a and 2b reports p-values for these Hausman tests. Significance level was set at 5% significance level.

In the first test the exogeneity assumption was rejected in each category with original (i.e., not detrended) and linearly detrended time series. When Hodrick-Prescott filters were applied, there were 3 rejections for food, rent & power, and furniture & household operation.

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[15] Fitzenberger (1998) compared the moving-block bootstrap with the Newey-West HAC estimator of variance, and found that the latter produced a better (thought still incomplete) adjustment to eliminate the downward bias in variance estimation.

With adjustment by the $BKS^{HP}(12)$ filter the null hypothesis was rejected twice. Results in table 2b are very similar, but with the fewer cases of significant p-values across detrending methods. Since the simultaneity bias does not appear to be a serious problem with data, I continued using the OLS estimator.

Other specification tests were the Jarque-Bera test for normality of residuals and White's test for heteroskedasticity. The Jarque-Bera's test (table 3) identified the failure of the normality assumption in just one category of consumption (rent & fuel) in almost all data transformations (except linear detrending, where the null hypothesis was never rejected). However, at the 10 per cent significance level, there were 3 rejections of the null for the first-difference filter.

As for White's test (table 4), most of its significant p-values occurred for the original and linearly detrended data, with other detrending methods producing just one significant p-value (as in the previous test, it was rent & fuel).

Results of testing the homogeneity restriction are summarized in table 5. Its p-values are based on the F-distribution, with no correction for the likely serial correlation. For the original data the result is very similar to the original estimation of the demand system by Deaton and Muellbauer (1980), with as many as five rejections of the homogeneity restriction. When linear detrending and first-differencing is applied, the number of rejections of the null drops to four in both cases. Interestingly, with HP(*opt*), HP(*1600*) and $BKS^{HP}(12)$ there is just one rejection of the null (rent & fuel), which in fact was the consumption category with the most serious problems in the preceding specification testing, so that the result may be attributed to the severe misspecification of this equation.

Tables 6a and 6b report results of testing the homogeneity restriction by MBB with block sizes 16 and 4, respectively. These p-values were calculated after resampling blocks of actual data as described by Fitzenberger (1998, p. 245-246). Due to high computational cost, I used only 99 replications, which is, however, sufficient to calculate the exact critical values (or p-values) at 1 per cent significance level.

Specifically, in each replication I calculated the usual F-statistic for the homogeneity restriction. After obtaining the simulated null distribution of the test statistic, its p-value was estimated by the quantile of the actual F-statistic for homogeneity restriction. As evident in tables 6a and 6b, the application of MBB results in the clear-cut confirmation of the theoretical restriction, with essentially all p-values becoming highly insignificant. There is a similar contrast in the case of testing of the symmetry restriction in table 7[16].

Apparently, the most striking differences between alternative detrending methods turned out in the parameter estimates of the Almost Ideal Demand System, as shown in tables from 8a to 8f.

Results of Table 8a, where the original time series were used, illustrate the ubiquitous spurious regressions, with both R^2 and DW statistics very close to unity. There is also a large number of parameter estimates that appear to be "significant". Results in Table 8b (with deterministic linear detrending) are very similar, since this detrending still preserves stochastic trend in regression residuals.

On the other hand, detrending with the first difference filter produced exactly the opposite estimation results (Table 8c). Most R^2 statistics hardly exceeded 0.100, while DW statistic indicated the prevalence of negative autocorrelation. Only 10 price parameters have absolute t-ratios larger than 2.0, and there are fewer significant estimates of uncompensated income elasticity.

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[16] Selvanathan (1995) also reported very different results of the conventional and Monte Carlo tests in the demand analysis of OECD countries. Instead of MBB Salvanathan applied a parametric bootstrap with OLS residuals generated independently according to the estimated covariance matrix of residuals.

On the other hand, parameter estimates with *HP(1600)*, *HP(opt)* and $BKS^{HP}(12)$ filters turned out very close to each other, placing the group of filters in the middle between the extremes of linear detrending and first-differencing. The removal of time trend by these high-pass filters usually resulted in R^2 statistics of about 0.350, with DW statistics slightly less than 2.0. On the other hand, the number of significant estimates of price parameters was, respectively, 18, 15, 14, thus exceeding the corresponding number with first-differenced data, but still fewer than in extreme cases of spurious regressions, when the unit root component with frequency zero was left in the original time series.

Section 5. Conclusion.

In this paper I found that the consequences of using various detrending methods may substantially affect the results of demand analysis. In particular, the conventional duo of deterministic linear detrending and first - differencing are not satisfactory for the removal of secular trend component, since their power transfer functions provides a poor approximation to the ideal high-pass filter with a cutoff at typical business cycle frequency of 32 quarters. Applying the integrated MSE metric, I measured the distortionary effects of major detrending procedures with respect to "the typical spectral shape" of actual economic time series, and found that HP(*opt*), $BK^{HP}(20)$, and $BKS^{HP}(12)$ induced the smallest distortions in the spectrum of filtered time series. These high-pass filters proved to be versatile tools to deal with the non-stationary time series without masking a number of significant relationships in the estimated Japanese demand system.

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References.

Baxter, M., 1994. Real exchange rates and real interest differentials: Have we missed the business-cycle relationship? Journal of Monetary Economics 33, 5-37.

Baxter, M., King R. G., 1999. Measuring Business Cycles Approximate Band-Pass Filters for Economic Time Series. Review of Economics and Statistics 81(4), 575-593.

Canova, F., 1998. Detrending and business cycle facts. Journal of Monetary Economics 41(3), 475-512.

Cogley, T., Nason J. M., 1995. Effects of the Hodrick-Prescott filter on trend and difference stationary time series: Implications for business cycle research. Journal of Economic Dynamics and Control 19, 253-278.

Davidson R., MacKinnon J., 1993. Estimation and Inference in Econometrics. Oxford University Press, Oxford.

Deaton, A., Muellbauer J., 1980. An Almost Ideal Demand System. American Economic Review 70, 312-326.

Fitzenberger, B., 1998. The Moving Blocks Bootstrap and Robust Inference for Linear Least Squares and Quantile Regressions. Journal of Econometrics 82(2), 235-287.

Granger C.W.J., Hatanaka M., 1964. Spectral Analysis of Economic Time Series, Princeton University Press, Princeton.

Granger, C., 1966. The Typical Spectral Shape of an Economic Variable. Econometrica 34, 150-161.

Guay, A., St-Amant, P., 1997. Do the Hodrick-Prescott and Baxter-King filters provide a good approximation of business cycles? Working Paper No. 53, Center for Research on Economic Fluctuations and Employment (CREFE).

Hamming R.W., 1998. Digital Filters. Dover Publications, Mineola (New York).

Harvey, A. C., Jaeger A., 1993. Detrending, stylized facts and the business cycle. Journal of Applied Econometrics 8, 231-247.

Hassler J., Lundvik T., Persson T., Söderlind P., 1994. The Swedish Business Cycle: Stylized Facts over 130 Years. In: Bergstrom V., Vredin A. (Eds.), Measuring and Interpreting Business Cycles. Oxford University Press, Oxford, pp. 9-108.

Hodrick, R. J., Prescott, E. C., 1997. Postwar U.S. Business Cycles: An Empirical Investigation. Journal of Money, Credit, and Banking 29, 1–16.

Jarque, C., Bera A., 1980. Efficient Tests for Normality, Homoskedasticity, and Serial Independence of Regression Residuals. Economics Letters 6, 255–259.

King, R.G., Rebelo, S.T., 1993. Low Frequency Filtering and Real Business Cycles. Journal of Economic Dynamics and Control 17, 207-231.

Koopmans, L. H., 1974. The Spectral Analysis of Time Series. Academic Press, San Diego.

Nelson, C. R., Plosser C. I., 1982. Trends and Random Walks in Macro-Economic Time Series: Some Evidence and Implications. Journal of Monetary Economics 10, 139-162.

Newey, W. West K., 1987. A Simple Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix. Econometrica 55, 703–708.

Ng, S., 1997. Accounting for Trends in the Almost Ideal Demand System. Department of Economics, Boston College and C.R.D.E., University of Montreal.

Pedersen T. M., 1998. The Hodrick-Prescott Filter, the Slutzky Effect, and the Distortionary Effect of Filters. University of Copenhagen, Institute of Economics, working paper 98-09.

Percival D.B., Walden A.T., 1993. Spectral Analysis for Physical Applications. Multitaper and Conventional Univariate Techniques. Cambridge University Press, Cambridge.

Porat, B., 1997. A Course in Digital Signal Processing. John Wiley, New York.

Selvanathan, S., 1995. Hypothesis Testing in Demand Analysis. In: Selvanathan, E. A., Clements, K. W., (Eds.) Recent developments in applied demand analysis: Alcohol, advertising and global consumption. Springer, Heidelberg, pp. 155-189.

Stock, J.H., Watson, M.W., 1999. Business Cycle Fluctuations in U.S. Macroeconomic Time Series. In: Taylor J.B., Woodford M. (Eds.) Handbook of Macroeconomics, North Holland, Amsterdam.

Woitek, U., 1998. A Note on the Baxter-King Filter. University of Glasgow, Discussion Papers in Economics, No 9813.

Table 1a. Q-statistic for linear filters

	SH_1	SH_2	SH_3	SH_4	SH_5	SH_6	SH_7	SH_8
Linear trend	10942	10360	16796	3538	12581	6316	15299	8225
First difference	3.02	7.67	3.66	5.46	7.76	3.77	8.86	8.71
HP(1600)	1.17	1.32	1.14	1.03	1.34	1.17	1.04	1.00
HP(1000)	1.06	1.08	1.03	1.00	1.09	1.05	1.00	1.01
HP(opt)	1.04	1.00	1.02	1.00	1.00	1.02	1.00	1.07
$MA^{HP}(12)$	1.91	2.77	2.02	1.93	2.54	1.71	2.18	1.76
$MA^{HP}(16)$	3.92	5.47	4.03	3.01	4.85	3.31	3.80	2.59
$MA^{HP}(20)$	6.83	9.75	7.07	4.58	8.36	5.53	6.22	3.59
$AI^{HP}(12)$	171.70	162.46	263.39	56.37	198.15	99.91	239.71	130.46
$AI^{HP}(16)$	346.19	327.30	531.59	112.65	399.26	200.83	483.60	262.19
$AI^{HP}(20)$	148.01	140.07	226.99	48.57	170.63	86.05	206.67	112.27
$BK^{HP}(12)$	1.36	1.61	1.36	1.26	1.53	1.28	1.36	1.26
$BK^{HP}(16)$	1.34	1.56	1.34	1.21	1.49	1.25	1.31	1.19
$BK^{HP}(20)$	1.00	1.18	1.00	1.09	1.13	1.00	1.10	1.12
$BKS^{HP}(12)$	1.42	1.40	1.40	1.38	1.38	1.31	1.50	1.42
$BKS^{HP}(16)$	1.45	1.48	1.43	1.28	1.44	1.32	1.42	1.28
$BKS^{HP}(20)$	1.36	1.40	1.34	1.17	1.36	1.26	1.30	1.18
Optimal lambda for HP filter	854	647	868	1172	601	767	1100	767

Table 1b. Q-statistic for linear filters

	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	*Inc*
Linear trend	576	709	5384	240	1317	1592	921	777	277548
First difference	2.75	2.99	6.69	2.79	5.39	4.90	3.28	2.68	6.61
HP(1600)	1.51	1.78	2.03	1.56	1.36	2.15	1.59	1.63	1.08
HP(1000)	1.16	1.29	1.41	1.18	1.09	1.47	1.20	1.23	1.00
HP(opt)	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$MA^{HP}(12)$	3.27	4.06	8.24	3.15	3.60	6.74	3.80	3.55	2.40
$MA^{HP}(16)$	7.63	9.84	22.19	7.12	8.18	17.12	9.15	8.68	4.44
$MA^{HP}(20)$	14.81	19.32	47.65	13.42	16.09	35.37	18.14	17.19	7.94
$AI^{HP}(12)$	7.48	9.16	64.93	3.36	16.70	17.79	11.62	9.47	43664
$AI^{HP}(16)$	14.46	17.83	134.70	5.79	33.59	36.26	23.09	18.76	88247
$AI^{HP}(20)$	6.76	8.23	59.27	3.00	15.17	16.42	10.52	8.69	37576
$BK^{HP}(12)$	1.91	2.29	4.24	1.85	2.02	3.49	2.15	2.10	1.43
$BK^{HP}(16)$	1.91	2.28	4.27	1.83	2.00	3.52	2.16	2.10	1.40
$BK^{HP}(20)$	1.09	1.21	1.55	1.10	1.17	1.49	1.15	1.14	1.06
$BKS^{HP}(12)$	1.62	1.80	3.04	1.54	1.69	2.55	1.76	1.72	1.47
$BKS^{HP}(16)$	1.89	2.19	4.09	1.78	1.94	3.32	2.11	2.07	1.47
$BKS^{HP}(20)$	1.80	2.11	3.91	1.70	1.83	3.19	2.01	1.99	1.34
Optimal lambda for HP filter	529	446	381	519	624	363	500	443	1004

Table 2a. P-values for the Hausman test with 4 lags of independent variables

	Original data	Linear detrend.	First difference	HP(1600)	HP(opt)	$BKS^{HP}(12)$
Food, beverages, tobacco	0.000	0.001	0.000	0.000	0.000	0.000
Clothing, footwear	0.000	0.000	0.432	0.196	0.407	0.357
Rent, fuel, power	0.000	0.000	0.700	0.011	0.014	0.361
Furniture, household operation	0.026	0.001	0.436	0.038	0.034	0.045
Medical care	0.000	0.000	0.987	0.193	0.268	0.224
Transport and communication	0.000	0.000	0.630	0.117	0.253	0.270
Recreation, entertainment	0.030	0.005	0.195	0.142	0.103	0.285

Table 2b. P-values for the Hausman test with ranks of independent variables

	Original data	Linear detrend.	First difference	HP(1600)	HP(opt)	$BKS^{HP}(12)$
Food, beverages, tobacco	0.218	0.282	0.099	0.117	0.053	0.409
Clothing, footwear	0.002	0.768	0.729	0.801	0.949	0.357
Rent, fuel, power	0.000	0.525	0.180	0.024	0.008	0.059
Furniture, household operation	0.878	0.371	0.669	0.327	0.344	0.159
Medical care	0.000	0.004	0.430	0.038	0.190	0.027
Transport and communication	0.000	0.007	0.863	0.308	0.020	0.305
Recreation, entertainment	0.007	0.042	0.858	0.586	0.773	0.489

Table 3. P-values for the Jarque-Bera normality test

	Original data	Linear detrend.	First difference	HP(1600)	HP(opt)	$BKS^{HP}(12)$
Food, beverages, tobacco	0.421	0.235	0.068	0.275	0.203	0.261
Clothing, footwear	0.958	0.829	0.473	0.904	0.807	0.926
Rent, fuel, power	0.001	0.273	0.000	0.013	0.004	0.033
Furniture, household operation	0.535	0.648	0.800	0.980	0.993	0.977
Medical care	0.594	0.337	0.070	0.513	0.611	0.775
Transport and communication	0.440	0.329	0.466	0.438	0.361	0.257
Recreation, entertainment	0.460	0.644	0.958	0.488	0.530	0.421

Table 4. P-values for the White test for heteroskedasticity

	Original data	Linear detrend.	First difference	HP(1600)	HP(opt)	$BKS^{HP}(12)$
Food, beverages, tobacco	0.577	0.623	0.581	0.378	0.196	0.482
Clothing, footwear	0.017	0.504	0.863	0.275	0.087	0.186
Rent, fuel, power	0.000	0.007	0.000	0.002	0.000	0.018
Furniture, household operation	0.685	0.567	0.133	0.865	0.627	0.434
Medical care	0.016	0.588	0.296	0.671	0.810	0.801
Transport and communication	0.097	0.000	0.337	0.277	0.439	0.510
Recreation, entertainment	0.148	0.907	0.943	0.847	0.937	0.815

Table 5. P-values for the test of the homogeneity restriction

	Original data	Linear detrend.	First difference	HP(1600)	HP(opt)	$BKS^{HP}(12)$
Food, beverages, tobacco	0.455	0.520	0.383	0.307	0.245	0.584
Clothing, footwear	0.001	0.117	0.025	0.807	0.312	0.822
Rent, fuel, power	0.000	0.000	0.000	0.000	0.008	0.000
Furniture, household operation	0.105	0.325	0.017	0.146	0.261	0.225
Medical care	0.005	0.010	0.041	0.209	0.338	0.635
Transport and communication	0.038	0.042	0.477	0.559	0.833	0.987
Recreation, entertainment	0.000	0.000	0.477	0.349	0.434	0.836
Total demand system	0.000	0.000	0.000	0.006	0.090	0.008

Table 6a. P-values for the test of the homogeneity restriction by MBB (with block length 16)

	Original data	Linear detrend.	First difference	HP(1600)	HP(opt)	$BKS^{HP}(12)$
Food, beverages, tobacco	0.670	0.620	0.840	0.740	0.650	0.640
Clothing, footwear	0.540	0.590	0.610	0.880	0.600	0.920
Rent, fuel, power	0.680	0.720	0.280	0.620	0.510	0.690
Furniture, household operation	0.730	0.680	0.670	0.310	0.340	0.360
Medical care	0.340	0.760	0.290	0.650	0.610	0.820
Transport and communication	0.400	0.480	0.480	0.640	0.990	0.770
Recreation, entertainment	0.180	0.570	0.840	0.410	0.480	0.600
Total demand system	0.730	0.920	0.490	0.820	0.860	0.840

Table 6b. P-values for the test of the homogeneity restriction by MBB (with block length 4)

	Original data	Linear detrend.	First difference	HP(1600)	HP(opt)	$BKS^{HP}(12)$
Food, beverages, tobacco	0.580	0.700	0.720	0.690	0.620	0.660
Clothing, footwear	0.320	0.430	0.600	0.890	0.520	0.920
Rent, fuel, power	0.430	0.450	0.370	0.490	0.360	0.490
Furniture, household operation	0.630	0.660	0.600	0.470	0.500	0.420
Medical care	0.430	0.530	0.520	0.510	0.540	0.620
Transport and communication	0.310	0.480	0.580	0.560	0.990	0.670
Recreation, entertainment	0.200	0.520	0.900	0.380	0.460	0.660
Total demand system	0.500	0.750	0.490	0.730	0.800	0.690

Table 7. P-values for the test of the symmetry restriction

	Original data	Linear detrend.	First difference	HP(1600)	HP(opt)	$BKS^{HP}(12)$
Asymmetric test statistic	0.000	0.000	0.047	0.000	0.000	0.000
MBB with block length 16	0.610	0.590	0.630	0.630	0.500	0.600
MBB with block length 4	0.580	0.640	0.830	0.810	0.720	0.820

Table 8a. parameter estimates with the original time series

	Const	P1	P2	P3	P4	P5	P6	P7	P8	Inc	R-sq	DW
Food	1.94	0.10	0.04	-0.11	-0.08	-0.04	0.04	0.03	0.00	-0.16	0.997	1.982
	(16.66)	(5.10)	(2.85)	(-7.27)	(-8.27)	(-5.81)	(3.38)	(2.19)	(0.11)	(-15.00)		
Clothing	-0.25	0.01	0.01	-0.07	0.02	-0.01	0.05	0.06	-0.10	0.03	0.925	1.091
	(-1.77)	(0.30)	(0.43)	(-3.68)	(1.61)	(-1.26)	(3.32)	(3.32)	(-4.87)	(2.24)		
Rent, fuel	1.26	-0.02	0.05	0.25	-0.08	-0.01	-0.11	-0.15	0.15	-0.10	0.976	0.733
	(6.85)	(-0.56)	(2.06)	(10.44)	(-5.75)	(-0.60)	(-5.97)	(-6.39)	(5.86)	(-5.77)		
Furniture, household	-0.06	0.08	-0.02	0.07	0.03	-0.02	0.00	-0.03	-0.13	0.01	0.910	1.370
	(-0.53)	(4.30)	(-1.15)	(4.55)	(3.67)	(-2.43)	(-0.22)	(-2.25)	(-8.30)	(1.03)		
Medical care	0.43	0.01	-0.07	0.05	0.00	0.03	-0.02	-0.04	0.07	-0.03	0.887	0.711
	(2.81)	(0.25)	(-3.59)	(2.60)	(-0.32)	(3.20)	(-0.98)	(-1.89)	(3.33)	(-2.15)		
Transport	-0.77	-0.10	0.05	-0.15	0.00	0.00	0.07	0.01	0.10	0.08	0.881	0.844
	(-4.17)	(-3.14)	(2.06)	(-6.05)	(-0.07)	(-0.41)	(3.45)	(0.57)	(3.71)	(4.78)		
Recreation	-1.39	0.05	0.00	-0.07	0.01	-0.01	-0.03	0.05	-0.05	0.14	0.977	1.342
	(-8.93)	(1.81)	(0.04)	(-3.27)	(0.48)	(-1.54)	(-1.59)	(2.38)	(-2.15)	(9.72)		

Table 8b. Parameter estimates with data, detrended by the deterministic linear trend

	Const	P1	P2	P3	P4	P5	P6	P7	P8	Inc	R-sq	DW
Food	0.00	0.12	0.03	-0.10	-0.08	-0.04	0.03	0.03	0.00	-0.14	0.891	1.934
	(2.53)	(5.67)	(2.14)	(-6.14)	(-8.49)	(-5.61)	(2.05)	(2.41)	(0.01)	(-9.26)		
Clothing	0.00	0.04	0.01	-0.03	0.01	-0.01	0.01	0.00	-0.04	0.10	0.604	1.040
	(-0.46)	(1.77)	(0.33)	(-1.75)	(1.35)	(-1.17)	(0.55)	(0.19)	(-2.74)	(6.07)		
Rent, fuel	0.00	-0.07	0.06	0.20	-0.08	-0.01	-0.05	-0.07	0.08	-0.20	0.845	0.443
	(3.20)	(-2.48)	(3.03)	(9.30)	(-6.47)	(-1.11)	(-3.00)	(-3.92)	(3.97)	(-10.23)		
Furniture, household	0.00	0.07	0.00	0.06	0.03	-0.02	0.00	-0.05	-0.11	0.00	0.779	1.250
	(2.63)	(3.26)	(0.09)	(3.67)	(3.56)	(-2.58)	(0.33)	(-3.61)	(-7.20)	(0.07)		
Medical care	0.00	-0.04	-0.04	0.01	0.00	0.02	0.03	-0.03	0.07	-0.10	0.856	1.042
	(2.13)	(-1.70)	(-2.13)	(0.74)	(0.12)	(2.94)	(1.71)	(-1.76)	(3.73)	(-5.48)		
Transport	0.00	-0.04	0.00	-0.11	-0.01	0.00	0.02	0.02	0.08	0.15	0.535	1.084
	(-2.68)	(-1.24)	(-0.07)	(-4.12)	(-0.47)	(0.23)	(0.83)	(1.10)	(3.47)	(6.56)		
Recreation	0.00	0.04	-0.01	-0.09	0.01	-0.01	0.00	0.09	-0.09	0.10	0.870	1.247
	(-2.16)	(1.31)	(-0.36)	(-3.93)	(0.67)	(-1.55)	(-0.26)	(4.94)	(-4.32)	(4.94)		

Table 8c. Parameter estimates with data, detrended by the first difference filter

	P1	P2	P3	P4	P5	P6	P7	P8	Inc	R-sq	DW
Food	0.10	-0.02	-0.04	0.01	-0.03	0.01	0.00	-0.05	-0.09	0.268	2.553
	(3.15)	(-0.60)	(-1.09)	(0.29)	(-2.61)	(0.53)	(0.11)	(-1.20)	(-3.14)		
Clothing	0.01	0.02	-0.04	-0.03	0.00	-0.01	-0.01	0.02	0.04	0.057	2.541
	(0.47)	(0.85)	(-1.16)	(-1.49)	(-0.36)	(-0.43)	(-0.72)	(0.59)	(1.55)		
Rent, fuel	0.01	0.01	0.22	-0.05	0.00	-0.04	-0.03	-0.01	-0.12	0.540	1.758
	(0.75)	(0.49)	(9.15)	(-3.29)	(-0.55)	(-2.98)	(-2.88)	(-0.43)	(-7.70)		
Furniture, household	0.02	-0.05	0.00	0.07	-0.01	0.00	-0.04	-0.04	0.07	0.181	2.362
	(0.92)	(-2.28)	(0.14)	(3.36)	(-0.98)	(-0.02)	(-2.16)	(-1.26)	(3.12)		
Medical care	0.01	-0.01	0.01	-0.03	0.03	0.00	-0.01	0.03	-0.03	0.119	2.562
	(0.33)	(-0.33)	(0.44)	(-1.34)	(3.11)	(-0.13)	(-0.80)	(0.97)	(-1.26)		
Transport	0.00	0.00	-0.03	0.00	0.00	0.02	0.00	0.03	0.00	-0.047	2.464
	(-0.11)	(-0.11)	(-0.68)	(-0.02)	(-0.36)	(0.94)	(0.14)	(0.81)	(-0.18)		
Recreation	-0.01	0.00	-0.04	0.00	0.02	-0.01	0.04	-0.02	0.05	0.053	2.715
	(-0.25)	(0.04)	(-0.90)	(-0.16)	(1.19)	(-0.36)	(1.96)	(-0.45)	(1.60)		

Table 8d. Parameter estimates with data, detrended by the HP(1600) filter

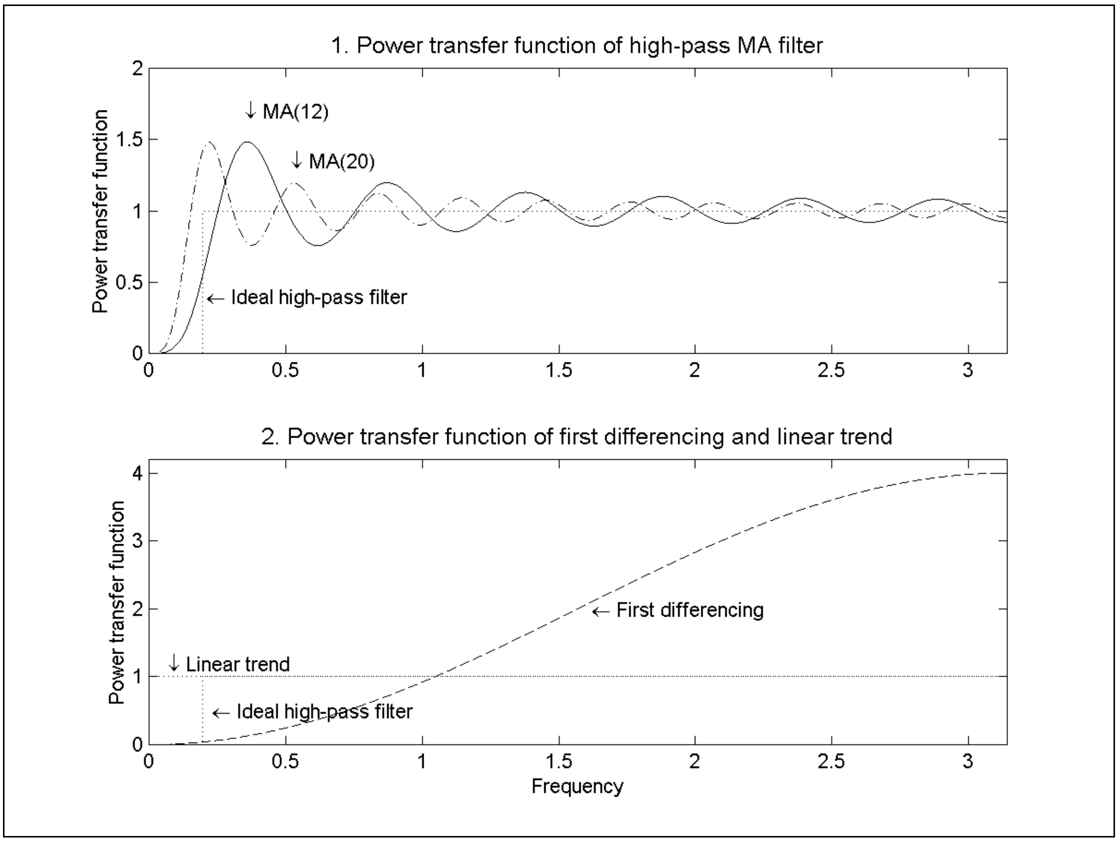
	Const	P1	P2	P3	P4	P5	P6	P7	P8	Inc	R-sq	DW
Food	0.00	0.11	0.02	-0.05	-0.06	-0.03	0.03	0.03	0.00	-0.12	0.763	1.876
	(0.48)	(4.75)	(0.86)	(-1.75)	(-3.91)	(-3.56)	(2.11)	(1.46)	(-0.11)	(-4.96)		
Clothing	0.00	0.01	0.04	0.01	-0.02	-0.01	-0.01	-0.01	0.00	0.05	0.336	1.582
	(0.18)	(0.50)	(2.54)	(0.34)	(-1.67)	(-1.19)	(-0.61)	(-0.98)	(0.03)	(2.40)		
Rent, fuel	0.00	-0.02	-0.01	0.21	0.00	-0.02	-0.02	-0.02	-0.05	-0.12	0.797	1.249
	(0.47)	(-1.18)	(-1.06)	(11.21)	(0.05)	(-2.90)	(-2.68)	(-2.44)	(-2.83)	(-8.52)		
Furniture, household	0.00	0.06	0.01	0.03	0.02	-0.02	-0.01	-0.04	-0.10	0.02	0.545	1.586
	(0.48)	(2.80)	(0.88)	(0.88)	(1.92)	(-2.20)	(-0.49)	(-2.33)	(-3.89)	(1.09)		
Medical care	0.00	-0.02	-0.05	-0.06	0.02	0.02	0.02	0.03	0.00	-0.02	0.387	1.668
	(0.36)	(-0.79)	(-2.99)	(-2.28)	(1.64)	(2.31)	(1.82)	(1.96)	(-0.03)	(-0.98)		
Transport	0.00	-0.04	-0.02	-0.04	0.00	0.01	0.03	-0.02	0.09	0.08	0.137	1.227
	(-0.43)	(-1.35)	(-0.72)	(-1.04)	(0.19)	(0.56)	(1.95)	(-0.80)	(2.54)	(2.85)		
Recreation	0.00	0.00	0.01	0.02	-0.01	0.00	-0.01	0.03	-0.01	0.04	-0.007	1.749
	(-0.39)	(0.12)	(0.36)	(0.53)	(-0.59)	(-0.09)	(-0.38)	(1.49)	(-0.27)	(1.36)		

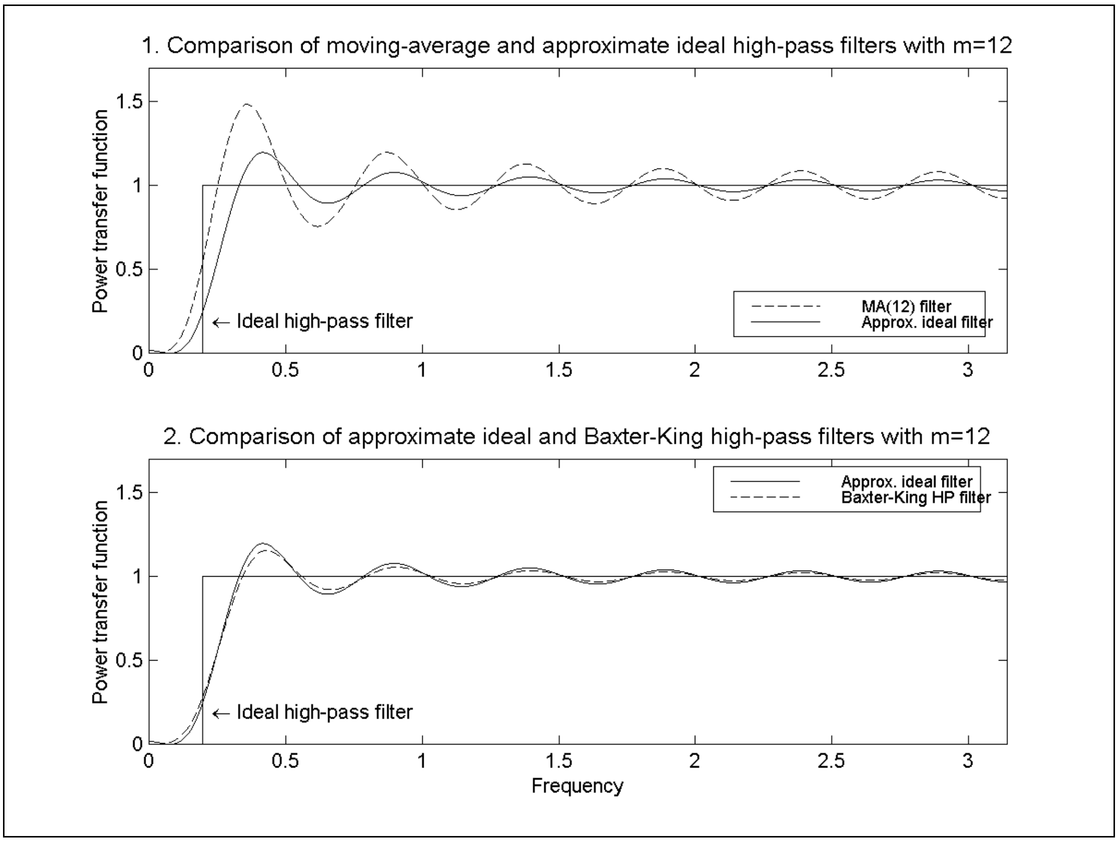
Table 8e. Parameter estimates with data, detrended by the HP(opt) filter

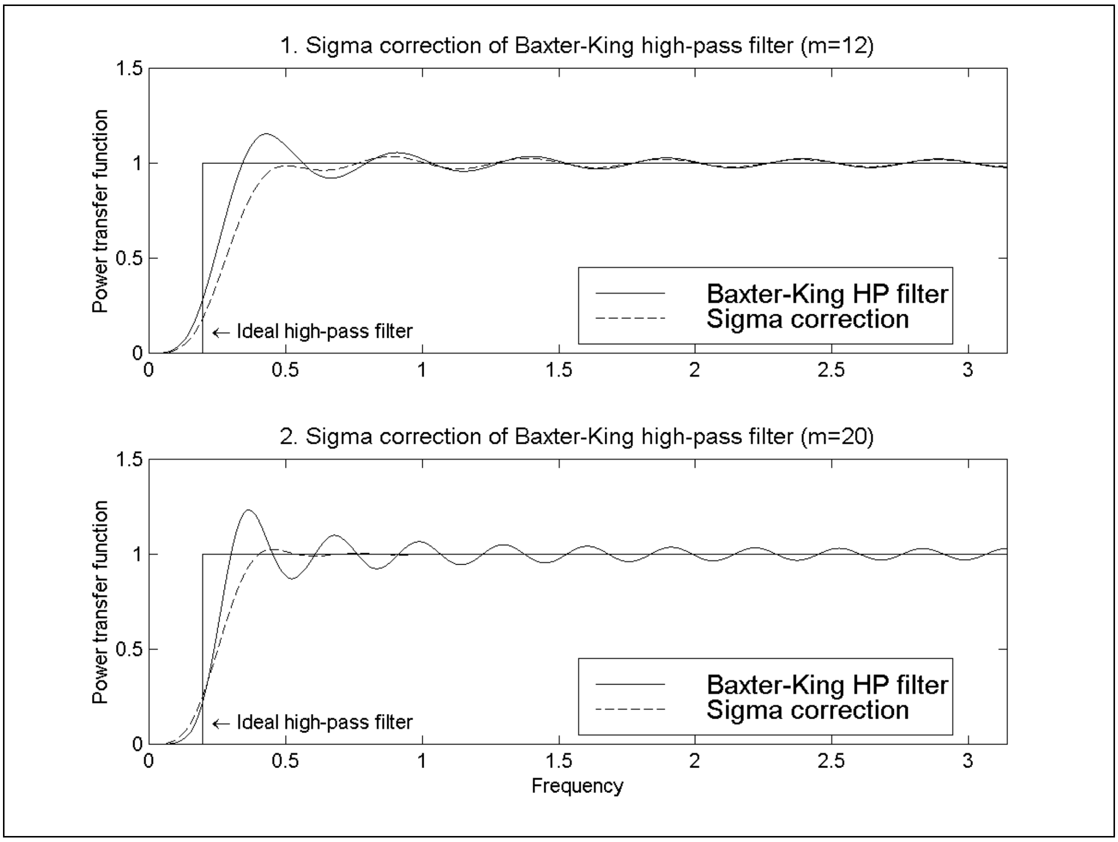
	Const	P1	P2	P3	P4	P5	P6	P7	P8	Inc	R-sq	DW
Food	0.00	0.12	0.01	-0.05	-0.06	-0.03	0.03	0.03	0.00	-0.12	0.673	1.798
	(0.57)	(4.82)	(0.45)	(-1.29)	(-3.47)	(-2.43)	(1.75)	(1.35)	(0.05)	(-4.29)		
Clothing	0.00	0.02	0.04	0.04	-0.02	-0.01	-0.01	-0.01	-0.02	0.05	0.310	1.766
	(0.21)	(0.90)	(2.39)	(1.31)	(-1.46)	(-0.89)	(-0.86)	(-0.81)	(-0.63)	(2.40)		
Rent, fuel	0.00	-0.02	-0.01	0.20	-0.01	-0.01	-0.02	-0.03	-0.05	-0.14	0.767	1.194
	(0.17)	(-1.28)	(-0.68)	(9.68)	(-1.20)	(-2.38)	(-2.50)	(-2.53)	(-2.55)	(-9.27)		
Furniture, household	0.00	0.06	0.02	0.04	0.03	-0.02	-0.01	-0.05	-0.10	0.05	0.547	1.665
	(0.33)	(2.69)	(0.83)	(1.10)	(2.09)	(-1.67)	(-0.74)	(-3.06)	(-3.54)	(1.89)		
Medical care	0.00	-0.02	-0.05	-0.06	0.02	0.02	0.02	0.02	0.01	-0.01	0.336	1.878
	(0.40)	(-0.96)	(-2.80)	(-1.84)	(1.69)	(2.64)	(1.54)	(1.49)	(0.22)	(-0.32)		
Transport	0.00	-0.04	-0.02	-0.06	0.01	0.00	0.03	0.00	0.06	0.05	0.061	1.272
	(-0.32)	(-1.49)	(-0.80)	(-1.39)	(0.61)	(-0.25)	(2.16)	(0.07)	(1.70)	(1.57)		
Recreation	0.00	0.01	0.01	0.02	-0.01	0.00	-0.01	0.03	-0.02	0.03	-0.013	1.795
	(-0.33)	(0.29)	(0.41)	(0.42)	(-0.66)	(-0.12)	(-0.39)	(1.63)	(-0.43)	(0.92)		

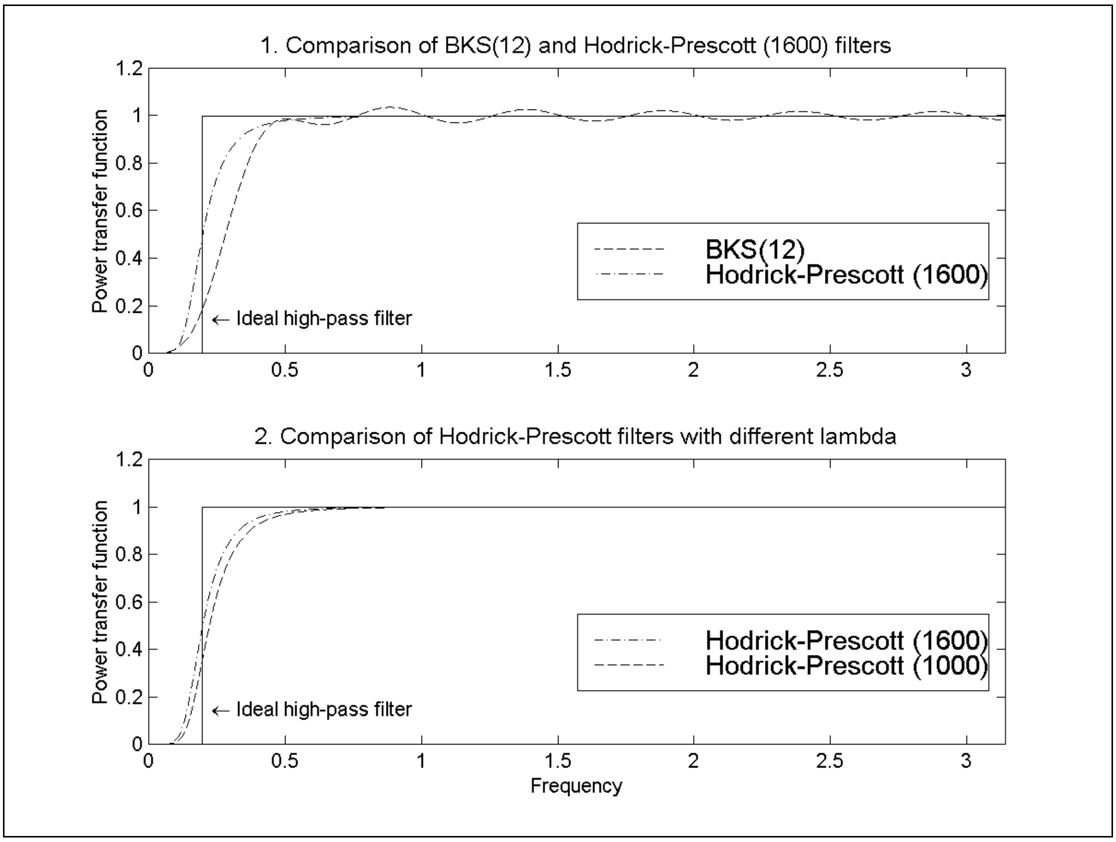
Table 8f. Parameter estimates with data, detrended by the $BKS^{HP}(12)$ filter.

	Const	P1	P2	P3	P4	P5	P6	P7	P8	Inc	R-sq	DW
Food	0.00	0.11	0.00	-0.06	-0.05	-0.03	0.03	0.02	0.00	-0.11	0.637	1.872
	(0.35)	(4.47)	(0.14)	(-1.80)	(-2.95)	(-2.86)	(1.84)	(1.00)	(-0.08)	(-4.36)		
Clothing	0.00	0.02	0.04	0.01	-0.03	-0.01	-0.01	-0.01	-0.01	0.05	0.324	1.713
	(0.58)	(1.09)	(2.65)	(0.51)	(-1.96)	(-0.85)	(-0.84)	(-1.04)	(-0.55)	(2.20)		
Rent, fuel	0.00	-0.02	0.00	0.20	-0.01	-0.01	-0.02	-0.03	-0.03	-0.13	0.776	1.273
	(-0.66)	(-1.23)	(0.10)	(10.69)	(-1.37)	(-2.33)	(-2.62)	(-2.64)	(-1.94)	(-8.54)		
Furniture, household	0.00	0.06	0.01	0.03	0.03	-0.01	-0.01	-0.05	-0.09	0.04	0.503	1.678
	(0.80)	(2.60)	(0.69)	(1.20)	(1.77)	(-1.64)	(-0.56)	(-2.99)	(-3.55)	(1.65)		
Medical care	0.00	-0.02	-0.05	-0.04	0.02	0.02	0.02	0.02	0.01	-0.01	0.323	1.727
	(0.35)	(-1.10)	(-2.76)	(-1.46)	(1.59)	(2.81)	(1.51)	(1.23)	(0.48)	(-0.41)		
Transport	0.00	-0.04	-0.01	-0.05	0.01	0.00	0.03	0.00	0.06	0.05	0.071	1.399
	(-0.43)	(-1.65)	(-0.46)	(-1.54)	(0.53)	(0.31)	(1.94)	(0.13)	(1.95)	(1.67)		
Recreation	0.00	0.02	0.00	0.00	-0.01	0.00	-0.01	0.04	-0.03	0.03	0.000	1.777
	(-0.75)	(0.71)	(0.08)	(-0.13)	(-0.42)	(-0.31)	(-0.39)	(2.01)	(-0.97)	(0.95)		


1. Power transfer function of high-pass MA filter
Power transfer function
↓ MA(12)
↓ MA(20)
← Ideal high-pass filter
2. Power transfer function of first differencing and linear trend
Power transfer function
← First differencing
↓ Linear trend
← Ideal high-pass filter
Frequency


1. Comparison of moving-average and approximate ideal high-pass filters with m=12
Power transfer function
← Ideal high-pass filter
MA(12) filter
Approx. ideal filter
2. Comparison of approximate ideal and Baxter-King high-pass filters with m=12
Power transfer function
Approx. ideal filter
Baxter-King HP filter
← Ideal high-pass filter
Frequency


1. Sigma correction of Baxter-King high-pass filter (m=12)
Power transfer function
Baxter-King HP filter
Sigma correction
← Ideal high-pass filter
2. Sigma correction of Baxter-King high-pass filter (m=20)
Power transfer function
Baxter-King HP filter
Sigma correction
← Ideal high-pass filter
Frequency


1. Comparison of BKS(12) and Hodrick-Prescott (1600) filters
Power transfer function
BKS(12)
Hodrick-Prescott (1600)
← Ideal high-pass filter
2. Comparison of Hodrick-Prescott filters with different lambda
Power transfer function
Hodrick-Prescott (1600)
Hodrick-Prescott (1000)
← Ideal high-pass filter
Frequency