On the Theory of the Price- and Quality-Setting Firm with Uncertain Demand

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ABSTRACT

This paper examines the effects of demand uncertainty on price and product quality for the price- and quality-setting firm in the context of a general profit relation. Comparative-statics predictions, when faced with a simple increase in risk, depend on the specific functional forms of demand uncertainty and of the cost function, and on whether the expected utility function is supermodular or submodular.

Keywords: The Price- and Quality-Setting Firm, Demand Uncertainty, a simple increase in risk

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1. Introduction

One normally expects that if a firm decreases the price of product, it also decreases the quality. However, when facing with demand uncertainty, under some conditions a firm that reduces the price raises product quality where a firm sets price and quality simultaneously. This prediction may be counter-intuitive. We confirm this situation for activities that are submodular with respect to price and product quality in the expected utility function.

The effects of changes in demand uncertainty on price, factor demands, or the supply decision of monopoly or monopolistic competition have been investigated in an extensive literature initiated by Leland (1972, 1975) and developed by Ishii (1979, 1981, 1991). While investigating a variety of behavioral modes in the monopoly under stochastic demand and stating restrictions on risk preferences in terms of the degrees of risk aversion for determinate comparative-statics predictions of changes in risk, most of these previous studies consider only single choice variable; price or quantity.

However, Ishii (1979, 1991) examined the optimal input choices of the price/quantity adjusting monopoly or the effects of changes in demand uncertainty on the output of individual firms, on the number of firms in the industry, and on industry output in the context of a monopolistic competition model with two decision variables. His model is different from that of the price/quantity setting introduced by Leland. By introducing two choice variables, De Vany (1976) extended the work of Leland (1972), who examined a price-setting monopoly firm facing uncertain demand, and analyzed how a monopoly firm controls both price and quality to maximize the expected profit. Ibarra-Salazar (1995) also extended the results of De Vany by assuming that the firm's objective is to maximize the expected utility of profit rather than the expected profit. However, the profit function in their model is very specific, and their results depend on the specific functional form. Specifically, the cost function used by Ibarra-Salazar, which is additively separable in output and quality, is unrealistic. Moreover, the cost function, which is multiplicatively separable in output and quality, is equally implausible. It is more appropriate to assume that the quality of products depends not only on variable factors (i.e., number of workers) but also on fixed factors (i.e., workers' skill, and nice equipments and structures). Also, the comparative-statics results of previous studies have been examined in the model with less general changes in the random variable. In this paper, we introduce an unrestricted cost function and analyze the effects of more general type of changes in the random parameter.

The motivation of this paper suggests that a firm, while facing with an increase in demand-related risk, tends to spread its risk by controlling both the price and product quality. When a firm reduces the price and improves the quality in response to an increase in risk, the output of a firm is increased. Actually a monopoly firm controls its own product's quality in addition to the price or quantity. While faced with a difficulty in raising the price because of a potential entrant or competition with rivals in realistic circumstances, a monopolist or an oligopoly firm in fact adjust its product's quality or consider product differentiation. This paper extends the model of the price- or quantity-setting firm with one choice variable to the generalized model of the price- and quality-setting firm with two choice variables under demand uncertainty, and analyzes the effects of a simple increase in risk on price and quality in the context of general types of demand uncertainty and the cost function. We also illustrate four kinds of the specific functional form of demand uncertainty and the cost function.

In the next section, we set out the model of price- and quality-setting firm facing random demand and introduce the definition of a simple increase in risk. Comparative-statics results for changes in risk on price and quality of product are derived in section 3. Conclusions are presented in the final section.

2. The Model

Consider uncertain demand denoted by $Q(p,q;\tilde{x})$, where p is the price of the product with a positive output, q is product quality, and x

$\tilde{}$ is a random variable. It is assumed that $Q_p < 0$, $Q_{pp} \leq 0$, $Q_q > 0$, and $Q_{qq} \leq 0$. The cases of an additive and of a multiplicative uncertain demand are represented by $Q(p,q;\tilde{x}) = Q(p,q) + \tilde{x}$ and $Q(p,q;\tilde{x}) = Q(p,q)\tilde{x}$, respectively.

We investigate the profit relation for the price- and quality-setting firm, in which π takes the form

(1) $$\pi(p,q;\tilde{x}) = pQ(p,q;\tilde{x}) - C(Q,q),$$

where C(Q,q) is the cost function. We assume that the cost function is monotone increasing and convex; C_Q, $C_q > 0$, C_{QQ}, $C_{qq} > 0$, and $C_{QQ}C_{qq} - C_{Qq}^2 > 0$. This unrestricted cost function includes both an additively separable case and a multiplicatively separable case. For example, an additively separable cost function in output and the quality is given by $C(Q,q) = cQ(p,q;\tilde{x}) + f(q)$, where c is the marginal cost and f(q) is fixed cost with $f_q > 0$, used in Ibarra-Salazar (1995). A multiplicatively separable cost function in output and the quality such that $C(Q,q) = c(q)Q(p,q;\tilde{x})$, where c(q) is the marginal cost with $c_q(q) > 0$ and $c_{qq}(q) \geq 0$, implies that the marginal cost is invariant with respect to output level (Q) but increases positively with product quality. Note that the two choice variables are profit substitutes (complements) in the sense that the cross-partial derivatives π_{pq} is negative (positive).

The objective of the firm is assumed to choose price, p and quality, q to maximize expected utility

(2) $$H = Eu(\pi(p,q;\tilde{x})) = \int u(\pi(p,q;\tilde{x}))dF(x),$$

where F(x) denotes the cumulative distribution function of the random variable $\tilde{x}$.[1] The utility function $u(\pi)$ is assumed to be thrice differentiable with $u' > 0$ and $u'' \leq 0$. The profit function $\pi(p,q;\tilde{x})$ is assumed three times differentiable with $\pi_{pp} < 0$, $\pi_{qq} < 0$, and $\pi_{pp}\pi_{qq} - \pi_{pq}^2 > 0$. This condition on π, combined with $u'' \leq 0$, ensures that the second-order condition for the maximization problem is satisfied. To simplify the discussion, we will focus on the case where $\pi_x > 0$. This assumption, combined with $u' > 0$, indicates that higher values of the random variable are preferred to lower values. To focus on interior solutions to the maximization problem, it is assumed that both $\pi_p = 0$ and $\pi_q = 0$ are satisfied for some finite p and q for all x.

The first- and second-order conditions for optimum of (2) can be written as:

(3) $$H_p(p,q) = \partial Eu(\pi)/\partial p = Eu'(\pi)\pi_p = \int u'(\pi)\pi_p dF(x) = 0$$

(4) $$H_q(p,q) = \partial Eu(\pi)/\partial q = Eu'(\pi)\pi_q = \int u'(\pi)\pi_q dF(x) = 0,$$ and

(5) $$H_{pp} = E[u'(\pi)\pi_{pp} + u''(\pi)\pi_p^2] < 0$$

$$H_{qq} = E[u'(\pi)\pi_{qq} + u''(\pi)\pi_q^2] < 0$$

$$H_{pq} = E[u'(\pi)\pi_{pq} + u''(\pi)\pi_p\pi_q]$$

$$D = H_{pp}H_{qq} - H_{pq}^2 > 0.$$

Note that the expected utility function is supermodular (submodular) with respect to price and product quality, which is given and interpreted in Milgrom and Roberts (1990), in the sense that the cross-partial derivatives H_{pq} is nonnegative (nonpositive).

In this paper we focus on a simple increase in risk introduced by Meyer and Ormiston (1989) which is a subclass of a Rothschild and Stiglitz increase in risk established by Rothschild and Stiglitz (1970). This type of changes in risk described below can be defined by transforming $\tilde{x}$ deterministically.

<u>Definition</u>: The deterministic transformation t(x) represents a simple increase in risk for a random variable given by F(x) if the function $k(x) \equiv t(x) - x$ satisfies

(a) $\int k(x)dF(x) = 0$

(b) $\int_0^s k(x)dF(x) \leq 0$ for all s

(c) $k'(x) \geq 0$.

The simple increases in risk described in Definition are carried out using a nonlinear deterministic transformation of the random variable and generalize the mean-preserving linear transformation given by k(x) = (-

1)(x

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x

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) and k'(x) = (- 1) ≥ 0, where

x

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is the mean of

x

~ and a nonrandom parameter is greater than or equal to one, introduced by Sandmo (1971). The simple increase in risk includes the Sandmo linear transformation represented as an increase in as a special case. The transformation t(x) is assumed to be nondecreasing, continuous, and piecewise differentiable. Meyer and Ormiston (1989) show that if the function k(x) satisfies the first two conditions, then it reduces expected utility for all risk averse decision makers. Thus, the transformation can be interpreted as an increase in risk in the Rothschild and Stiglitz sense. Note that previous studies with one random-two choice-one outcome variable model only consider quite restrictive changes in

x

~; that is, an increase in risk represented by an increase in and a global increase in risk.

3. The Comparative Statics Analysis

In this section, consider the effect of a simple increase in risk on price and quality of product. Let $H_p(p,q;\ )$ denote the derivative with respect to the choice variable p of the expected utility when the random variable is transformed according to t(x) = x + k(x), where   is a nonrandom parameter with $0 \le\ \ \le 1$; that is, $H_p(p,q;\ ) = \int u'(\pi(x+ k(x),p,q))\pi_p(x+ k(x),p,q)dF(x)$. Note that $H_p(p,q;0) = 0$ for the initial optimal values of p and q. From (3) and (4), the following comparative-statics predictions are given concerning the effect on p and q of this increase in risk:

$$(6)\qquad \partial p/\partial \ |_{\theta=0} = (1/D)[-\ (\partial H_p/\partial \ |_{\theta=0})H_{qq} + (\partial H_q/\partial \ |_{\theta=0})H_{pq}]$$

$$(7)\qquad \partial q/\partial \ |_{\theta=0} = (1/D)[-\ (\partial H_q/\partial \ |_{\theta=0})H_{pp} + (\partial H_p/\partial \ |_{\theta=0})H_{pq}],$$

where $\partial H_p/\partial \ |_{\theta=0}$ and $\partial p/\partial \ |_{\theta=0}$ represent the effect of simple increases in risk on H_p and p, respectively. Note that if the signs of $\partial H_p/\partial \ |_{\theta=0}$ and $\partial H_q/\partial \ |_{\theta=0}$ are known, the determinate comparative-statics predictions depend on whether the expected utility function H(p,q) is supermodular ($H_{pq} \ge 0$) or submodular ($H_{pq} \le 0$).

The following Lemmas state the conditions for the signs of

$\partial H_p/\partial \;|_{\theta=0}$ and $\partial H_q/\partial \;|_{\theta=0}$.

Lemma 1: $\partial H_p/\partial \;\leq (\geq)\; 0$ when the random variable is transformed according to $t(x) = x + \; k(x)$, where $t(x)$ represents a simple increase in risk if

(a) $u(\pi)$ displays decreasing absolute risk aversion (DARA)

(b) $\pi_x > 0$ and $\pi_{xx} \leq 0$

(c) $\pi_{px} \geq (\leq)\; 0$ and $\pi_{pxx} \leq (\geq)\; 0$.

Proof: See the Appendix.

Lemma 2: $\partial H_q/\partial \;|_{\theta=0} \;\leq\; (\geq)\; 0$ when the random variable is transformed according to $t(x) = x + \; k(x)$, where $t(x)$ represents a simple increase in risk if

(a) $u(\pi)$ displays decreasing absolute risk aversion (DARA)

(b) $\pi_x > 0$ and $\pi_{xx} \leq 0$

(c) $\pi_{qx} \geq (\leq)\; 0$ and $\pi_{qxx} \leq (\geq)\; 0$.

Proof: Using the same procedures as for the case of Lemma 1 and $\partial H_q/\partial \;|_{\theta=0}$ = $\int u'(\pi)\pi_{qx}k(x)dF(x) - \int A(\pi)u'(\pi)\pi_q\pi_x k(x)dF(x)$, complete the proof.

Our first result concerns case in which the profit relation is represented by a general form of demand uncertainty and the cost function. In this case, the Proposition shows that the cross-partial derivative H_{pq} in equation (5)

$$(8) \qquad H_{pq} = \int u'(\pi)\pi_{pq}dF(x) - \int A(\pi)u'(\pi)\pi_p\pi_q dF(x)$$

plays a decisive role in determining the sign of the effect on p and q.

<u>Proposition</u>: When the random variable is transformed according to $t(x) = x + k(x)$, where $t(x)$ represents a simple increase in risk, a price- and quality-setting firm displaying DARA increases (decreases) the price and increases (reduces) the quality if

(a) $H_{pq} \geq 0$

(b) $\pi_{px} \leq (\geq)\ 0$, $\pi_{pxx} \geq (\leq)\ 0$

(c) $\pi_{qx} \leq (\geq)\ 0$, $\pi_{qxx} \geq (\leq)\ 0$,

but increases (decreases) the price and decreases (increases) the quality if

(a)' $H_{pq} \leq 0$

(b)' $\pi_{px} \leq (\geq)\ 0$, $\pi_{pxx} \geq (\leq)\ 0$

(c)' $\pi_{qx} \geq (\leq)\ 0$, $\pi_{qxx} \leq (\geq)\ 0$.

<u>Proof</u>: From (6) and (7), by Lemma 1 and Lemma 2, we can show that the signs of $\partial p/\partial \,|_{\theta=0}$ and $\partial q/\partial \,|_{\theta=0}$ are obtained according to the sign of H_{pq}.

Proposition gives conditions sufficient to yield determinate comparative-statics results concerning the effect on p and q of a simple increase in risk. The firm exhibits DARA, which is generally thought to be a reasonable assumption concerning risk preferences. The condition (a) and (a)' on the sign of H_{pq} are also added to allow determinate statements to be made concerning the effect on price and quality of a change in the random parameter.

The intuition behind Proposition implies that there exists the trade-off between price and product quality when the expected utility function with DARA is submodular with respect to price and product quality in equation (8). For this to be true, the sign of $\partial H_p/\partial \,|_{\theta=0}$ and $\partial H_q/\partial \,|_{\theta=0}$ must be different. Specifically, if the sign of $\partial H_p/\partial \,|_{\theta=0}$ is negative and that of $\partial H_q/\partial \,|_{\theta=0}$ is positive, then a firm decreases price and increases product quality. When this effects of simple increases in risk on H_p and H_q have different sign, the second term on the right-hand side of equation
(8),
$\int A(\pi)u'(\pi)\pi_p\pi_q dF(x)$ is negative.[2] Hence, if price and product quality are substitutes in profits ($\pi_{pq} < 0$) and this substitutability in the first term of equation (8) must be strong enough to offset the effect of risk aversion in the second term, which tends to make the choice variables substitutes in generating expected utility, then price and product quality are substitutes in the expected utility function.

Our next results refer to four cases in which the profit function displays the specific functional forms of demand uncertainty and the cost function to analyze the possible impacts on the firm's price and quality. First, in case of a multiplicatively uncertain demand function and a multiplicatively separable cost function in output and a function of product quality, From (1), the model should be slightly changed, following $C(Q,q) = c(q)Q(p,q;\tilde{x}) = c(q)Q(p,q)\tilde{x}$. Then, the firm's profit which is assumed to be nonnegative is rewritten as

$$(9) \qquad \pi(p,q;\tilde{x}) = (p-c(q))Q(p,q)\tilde{x}.$$

In equation (9), we get $\pi_p = [(p-c(q))Q_p + Q]x$, $\pi_q = (p-c(q))Q_q - c_qQ]x$, and $\pi_x = (p-c(q))Q > 0$. From (3) and (4), it follows that $\pi_p = 0$ and $\pi_q = 0$ for all $\tilde{x}$. Hence, $\pi_{px} = 0$ and $\pi_{qx} = 0$ for all $\tilde{x}$. A simple increase in risk does not affect the price and quality of product, since $\partial H_p/\partial \,|_{\theta=0} = 0$ and $\partial H_q/\partial \,|_{\theta=0} = 0$, by Lemma

1 and Lemma 2. Thus, the price- and quality-setting firm, with a simple increase in risk, does not influence price and quality regardless of the sign of H_{pq}.

Second, assuming that uncertain demand function is additive and the cost function is multiplicatively separable in output and the quality, and the marginal cost varies linearly positive with product quality, $C(Q,q) = qQ(p,q;\tilde{x}) = q[Q(p,q)+ \tilde{x}]$, then the profit relation takes the form

(10) $$\pi(p,q;\tilde{x}) = (p-q)[Q(p,q)+ \tilde{x}].$$

In this case, since $\pi_x = (p-q) > 0$, $\pi_{xx} = 0$, $\pi_{px} = 1 > 0$, $\pi_{qx} = -1 < 0$, $\pi_{pxx} = 0$, and $\pi_{qxx} = 0$, by Lemma 1 and 2, we have $\partial H_p/\partial \,|_{\theta=0} < 0$ and $\partial H_q/\partial \,|_{\theta=0} > 0$. It is clear to see that $\partial H_p/\partial \,|_{\theta=0} = - \partial H_q/\partial \,|_{\theta=0}$ since $\pi_p = -\pi_q$ and $\pi_{px} = -\pi_{qx}$ for all $\tilde{x}$. Hence, by Proposition, the firm exhibiting DARA, when faced with a simple increase in risk, will decrease price and increase quality if price and product quality prove to be substitutes in expected utility function (i.e., the expected utility function is submodular, $H_{pq} < 0$).[3]

Third, consider that the type of demand uncertainty is additive and the cost function is additively separable in output and the quality, $C(Q,q)= cQ(p,q;\tilde{x}$

$) + f(q) = c[Q(p,q)+ \tilde{x}] + f(q)$, used in Ibarra-Salazar (1995), then the profit relation takes the form

(11) $\pi(p,q;\tilde{x}) = (p-c)[Q(p,q)+ \tilde{x}] - f(q)$.

Since $\pi_x = (p-c) > 0$, $\pi_{xx} = 0$, $\pi_{px} = 1 > 0$, and $\pi_{pxx} = 0$, by Lemma 1, we have $\partial H_p/\partial \,|_{\theta=0} < 0$. It is clear to see that $\partial H_q/\partial \,|_{\theta=0} = 0$ since $\pi_{qx} = 0$ and $\pi_q = 0$ for all $\tilde{x}$. Hence, the firm exhibiting DARA, when faced with a simple increase in risk, will decrease price, but increase, leave constant, or reduce quality of product according as the sign of H_{pq} is negative, zero, or positive.

Finally, consider that the form of demand uncertainty is additive and the shape of the cost function is additive in output and a function of product quality, $C(Q,q) = c(q)Q(p,q;\tilde{x}) + f(q) = c(q)[Q(p,q)+ \tilde{x}] + f(q)$, then the profit relation takes the form

(12) $\pi(p,q;\tilde{x}) = (p-c(q))[Q(p,q)+ \tilde{x}] - f(q)$.

Since $\pi_x = (p-c(q)) > 0$, $\pi_{xx} = 0$, $\pi_{px} = 1 > 0$, $\pi_{qx} = -c_q < 0$, $\pi_{pxx} = 0$, and $\pi_{qxx} = 0$, by Lemma 1 and Lemma 2, we have $\partial H_p/\partial \,|_{\theta=0} < 0$ and

$\partial H_q/\partial \,|_{\theta=\Theta}$ > 0. The firm displaying DARA, in responses to a simple increase in risk, will increase, leave constant, or reduce both price and quality of product when the sign of H_{pq} is negative, zero, or positive. Hence, the effects of a simple increase in risk on price and quality of product depend on whether the expected utility function H(p,q) is supermodular or submodular.

In specific forms of function we have considered, it is not quite difficult to determine the effects of changes in risk on the firm's product price and quality. On the other hand, while the conditions stated in Proposition are rather restrictive, they include the specific models with two choice variables presented by the type of demand uncertainty and by the shape of the cost function as special cases.

4. Conclusions

This paper analyzes the effects of demand uncertainty on price and quality of firm's product for the price- and quality-setting firm in the framework of a general profit function and derives the conditions sufficient to yield comparative-statics results of a simple increase in risk. This paper also illustrates four cases of the specific profit function represented by several forms of demand uncertainty and the cost function.

In the cases we have considered, three facts have been exploited to establish comparative-statics effects on price and quality. First, definite comparative-statics predictions are possible for the general profit relation of a firm with two choice variables by specifying whether the expected utility function is supermodular or submodular. Second, the results obtained in this paper depend on the specific functional forms of demand uncertainty and the cost function. Finally, a firm that decreases the price raises the quality in response to an increase in risk when the expected utility function is submodular with respect to price and product quality. As a consequence, the output level of a firm is increased. Although these comparative-statics predictions are counter-intuitive, their results are potentially useful in adjusting firm's strategy about price and product quality.

Footnotes

1. Note that the limits of integration encompassing the support set for the random variable $\tilde{x}$ are suppressed in Equation (2).

2. To prove that the second term on the right-hand side of equation
(8),
$\int A(\pi)u'(\pi)\pi_p\pi_q dF(x)$ is negative, consider the case that the sign of $\partial H_p/\partial \,|_{\theta=0}$ is negative and that of $\partial H_q/\partial \,|_{\theta=0}$ is positive. Note that π_p (π_q) is increasing (decreasing) functions of the random variable x. If $\pi_i(s(i))=0$ (i=p, q), then $\pi_p(x) \geq 0$ for all x > s(p) and $\pi_q(x) \leq 0$ for all x > s(q). Let m = max{s(p), s(q)}. Suppose that m = s(q) > x(p), then $\pi_p(x) \geq 0$ and $\pi_q(x) \leq 0$ for all x > m. The second term of H_{pq} in equation (8) an be rewritten as:
$\int A(\pi)u'(\pi)\pi_p\pi_q dF(x) = \int^m A(\pi)u'(\pi)\pi_p\pi_q dF(x) + \int_m A(\pi)u'(\pi)\pi_p\pi_q dF(x)$. Since $\int_m A(\pi)u'(\pi)\pi_p\pi_x dF(x) \leq 0$, $\pi_p(x) \geq 0$, and $\pi_q(x) \leq 0$ for all x > m, Then, $\int A(\pi)u'(\pi)\pi_p\pi_q\, dF(x) \leq \int^m A(\pi)u'(\pi)\pi_p\,\pi_q dF(x)$.

Since from the first-order condition (3), if $\int u'(\pi)\pi_p dF = \int^m u'(\pi)\pi_p dF + \int_m u'(\pi)\pi_p dF = 0$ and $\int_m u'(\pi)\pi_p dF > 0$, then $\int^m u'(\pi)\pi_p\, dF < 0$, and let $(x)= A(\pi(x))\pi_q(x) > 0$, for all x < m, $d\,/dx = A_\pi\pi_x\pi_q + A\pi_{qx} < 0$, that is, (x) is a decreasing function of x, for all x < m. Then $0 > \int^m u'(\pi)\pi_p dF = \int^m A(\pi)u'(\pi)\pi_p\pi_q dF \geq \int A(\pi)\, u'(\pi)\pi_p\pi_q dF$.

3. To prove the condition that the two decision variables are

profit substitutes instead the expected utility is submodular, rearrange the expressions at (6) and (7), since $\partial H_p/\partial \,|_{\theta=0} = -\partial H_q/\partial \,|_{\theta=0} < 0$, to arrive at

$$\partial p/\partial \,|_{\theta=0} = (1/D)[-(H_{pq} + H_{qq})(\partial H_p/\partial \,|_{\theta=0})]$$

$$\partial q/\partial \,|_{\theta=0} = (1/D)[(H_{pq} + H_{pp}](\partial H_p/\partial \,|_{\theta=0})].$$

Note that $\pi_p + \pi_q = 0$, for all $\tilde{x}$. From (5), we have $H_{pq} + H_{qq} = Eu'(\pi)(\pi_{pq} + \pi_{qq})$ and $H_{pq} + H_{pp} = Eu'(\pi)(\pi_{pq} + \pi_{pp})$. Hence, $\partial p/\partial \,|_{\theta=0} < 0$ and $\partial q/\partial \,|_{\theta=0} > 0$ if price and product quality are substitutes in profits ($\pi_{pq} < 0$).

Appendix: Proof of Lemma 1

To establish the sign of $\partial H_p/\partial \,|_{\theta=0}$, differentiating this gives $\partial H_p/\partial \,|_{\theta=0} = \int [u'(\pi)\pi_{px} + u''(\pi)\pi_p\pi_x]k(x)dF(x) = \int u'(\pi)\pi_{px}k(x)dF(x) - \int A(\pi)u'(\pi)\pi_p\pi_x k(x)\, dF(x)$, where $A(\pi) = -u''(\pi)/u'(\pi)$ is absolute risk aversion. First of all, we prove the case of $\partial H_p/\partial \,|_{\theta=0} \le 0$, which gives the local result. We first establish the sign on the first integral of $\partial H_p/\partial \,|_{\theta=0}$. Integrating the first integral by parts, and let $(x)= u'(\pi(x))\pi_{px}(x) > 0$. Since $d /dx = u''(\pi)\pi_{px}\pi_x + u'(\pi)\pi_{pxx} \le 0$, and the conditions (a) and (c) on the definition of a simple increase in risk, the first term of $\partial H_p/\partial \,|_{\theta=0}$ is nonpositive for all .

Next, consider the sign of the second integral. Note that π_p and k are increasing functions of the random variable x. If $\pi_p(s(p))=0$, then $\pi_p(x) \ge 0$ for all $x > s(p)$. If $k(t(k))=0$, then $k(x) \ge 0$, for all $x > t(k)$. Let $M = \max\{s(p), t(k)\}$. The second term of $\partial H_p/\partial \,|_{\theta=0}$ can be rewritten as: $\int A(\pi)u'(\pi)\pi_p\pi_x k(x)dF = \int^M A(\pi)u'(\pi)\pi_p\pi_x k(x)dF + \int_M A(\pi)u'(\pi)\pi_p\pi_x k(x)dF$. Since $\int$

$_M$ $A(\pi)\,u'(\pi)\pi_p\pi_x k(x)dF \geq 0$, $\pi_p(x) \geq 0$, and $k(x) \geq 0$ for all $x > M$,

Then, $\int A(\pi)u'(\pi)\pi_p\pi_x k(x)dF \geq \int^M A(\pi)u'(\pi)\pi_p\pi_x k(x)dF$.

Now if $M = s(p) \geq t(k)$, then $\int A(\pi)u'(\pi)\pi_p\pi_x k(x)dF \geq \int^M A(\pi)$ $u'(\pi)\pi_p\pi_x k(x)dF$ =

$\int$

^{M} $k(x)dF \geq 0$, since from the condition (a) of Definition, if $\int$

$k(x)dF$ =

$\int$

M

$k(x)dF + \int_M k(x)dF = 0$ and $\int_M k(x)dF > 0$, then $\int^M k(x)dF < 0$ and

let $(x)= A(\pi(x))u'(\pi(x))\pi_p\pi_x < 0$, for all $x \leq M = s(p)$, d /dx =

$A_\pi\pi_x u'(\pi)\pi_p\pi_x + Au''(\pi)\pi_x\pi_p\pi_x + Au'(\pi)\pi_{px}\pi_x + Au'(\pi)\pi_p\pi_{xx} \geq 0$, that is, (x) is a increasing function of x, for all $x \leq M = s(p)$. This is true for all .

On the other hand, if $M = t(k) \geq s(p)$, then $\int A(\pi)u'(\pi)\pi_p\pi_x\, kdF \geq$

$\int$

M

$A(\pi)u'(\pi)\pi_p\pi_x k(x)dF$ =

$\int^M u'(\pi)\pi_p dF \geq 0$, since from the first- order condition (3), if $\int u'(\pi)\pi_p dF = \int^M u'(\pi)\pi_p dF + \int_M u'(\pi)\pi_p dF = 0$, and $\int_M u'(\pi)\pi_p dF > 0$, then

$\int^M u'(\pi)\pi_p dF < 0$ and let (x)= $A(\pi(x))\pi_x k(x) < 0$ for all $x \leq M = t(k)$, d /dx = $A_\pi \pi_x \pi_x k(x) + A\pi_{xx} k(x) + A\pi_x k'(x) \geq 0$. Thus, we have $\partial H_p/\partial \,|_{\theta=0} \leq 0$.

Next, in case of $\partial H_p/\partial \,|_{\theta=0} \geq 0$. This proof is similar to that of $\partial H_p/\partial \,|_{\theta=0} \leq 0$. Meyer and Ormiston (1989) show that under the conditions of Lemma 1, $\partial H_p/\partial \,|_{\theta=0} \leq (\geq)\ 0$.

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