Small firms: How many is too many*

Arghya Ghosh† and Souresh Saha‡

June 29, 2001

Preliminary and Incomplete

*We thank Tom Holmes and Arijit Mukherji for their help and encouragement during the initial stages of the work. We also acknowledge helpful comments from seminar participants at the University of Sydney. The usual disclaimer applies.

†Address: School of Economics, University of New South Wales, Sydney 2052, Australia; Tel: 61-2-9385-1347; e-mail: a.ghosh@unsw.edu.au

‡Address: Department of Economics, The National University of Singapore, 10 Kent Ridge Crescent, Singapore 119260; Tel: 65-874-8911; Fax: 65-775-2646; e-mail: ecsss@nus.edu.sg

Abstract

We consider entry regulation with asymmetric costs. It is known that with economies of scale, absent entry regulation more firms enter than is optimal. We show under general demand conditions with Cournot competition that asymmetry in costs also leads to excess entry. Our proof of this result is novel. Further, disallowing entry by firms more efficient than already-existing firms might be welfare-improving! We also consider Stackelberg competition. With leadership by low-cost(high-cost) firm, we find free entry to be optimal(inoptimal). However, with endogenous choice of form of competition, a low-cost, leader firm might forego leadership making regulation optimal.

JEL classification: L51, L52
Keywords: Business-stealing effect, socially optimal entry, free-entry equilibrium, excess entry, entry regulation

1 Introduction

What happens when the number of firms in an industry increases? Typically, this causes total industry output to rise though each "already-existing firm" - firm(s) in the industry prior to increase in the total number of firms - lowers output. Each entering firm considers only its post-entry profit. Since entrants take away some business from existing firms - referred to in the literature as *business-stealing effect*[1] - which they ignore while deciding on entry, social value of entry falls short of its private value. Nonetheless, absent scale effects entry improves welfare(measured as sum of consumer and producer surplus).

However, it has been shown that not imposing any restrictions on entry by firms - *free-entry equilibrium* - is socially suboptimal with increasing returns to scale - for example, if entrants incur a fixed cost to enter the industry. More precisely, as noted by Kim(1997), in addition to economies of scale, if firms produce a homogeneous good, post-entry game is characterized by quasi-Cournot conjectures and each individual firm's output is decreasing in the number of firms, more firms enter - i.e. there is *excess entry* - in a free-entry equilibrium than the welfare-maximizing number of firms - *socially optimal entry*. This suggests that restricting number of firms entering an industry - *entry regulation* - might improve welfare.

A fair amount of work[2] exists on excess entry and entry regulation. To the best of our knowledge, the literature has only considered the case where all firms have the same cost function. In reality however, an industry often consists of firms that differ in costs. This could be due to a host of reasons, for example idiosyncratic differences between firms, differential treatment by government or as a result of outcome of R&D with uncertainty. Empirical research[3] strongly suggests existence of inter-industry cost differences across firms. In this paper, we consider entry regulation with asymmetric costs.

We consider a world with constant marginal cost of production and no fixed costs[4]. We assume that one firm is low-cost while all other firms have a higher cost of production. Also, as in the excess entry literature, we assume that all firms are profit-maximizing and the government can only control the

[1] See for example, Mankiw and Whinston(1986).

[2] See for example Von Weizsacker(1980a, 1980b), Perry(1984), Mankiw and Whinston(1986), Suzumura and Kiyono(1987), Kim(1997), Berry and Waldfogel(1999).

[3] See for example Foster, Haltiwanger and Krizan(1998).

[4] We abstract away from fixed costs to clearly distinguish the factors driving our results from the case with increasing returns to scale.

number of entering firms.

We show under fairly general demand conditions and with Cournot competition that asymmetry in production costs across firms in an industry also leads to excess entry[5]. This is driven by the fact that in our framework, entry by an inefficient firm leads to a shift in output from the efficient firm to the inefficient entrant and a corresponding welfare loss which is ignored by the entrant.

The above-mentioned welfare loss is given by the product of production-efficiency loss per unit of output shift(cost difference between efficient and inefficient firms) and the amount of output shifted. For small cost differences, this welfare loss is low due to the small per unit production-efficiency loss while for large cost differences, it is low because amount of business stolen by the entrant is small. Hence, we find that welfare gain from entry regulation is non-monotone with respect to cost difference between efficient and inefficient firms. Also, for large enough cost differences, we find that it is optimal not to allow any inefficient firm to enter.

Our proof of the excess entry theorem under general demand conditions and Cournot competition is novel. We use the well-known result in the public enterprise literature that as a Stackelberg leader, a welfare-maximizing inefficient public sector firm competing with efficient, profit-maximizing private firms produces less than in the corresponding Cournot equilibrium. We show that as the number of inefficient firms tends towards infinity, the outcome of a Cournot game between one efficient firm and a number of inefficient firms where all firms are profit-maximizing tends to the outcome of a Cournot game between a profit-maximizing, efficient firm and a welfare-maximizing, inefficient, public sector enterprise. Then we show that these two results imply that free entry is suboptimal with Cournot competition and asymmetric production costs.

We then consider entry regulation when firms enter and choose quantities in two different stages(one firm enters in the first stage and the rest in the second stage) - i.e. compete in Stackelberg fashion - and the government can only regulate the number of entrants in the second stage. Till date, the excess entry literature has focused mainly on "one-stage simultaneous move games"(for example Cournot competition) - an exception being Kim(1997).

[5]This is true provided all entering firms do not have access to the most efficient technology. Of course, if all potential entrants have the most efficient technology, presence of some inefficient firms does not matter. In such a case, absent scale effects, free entry is socially optimal.

This does not allow for discussion of issues like possibility of strategic manipulation of government entry regulation decision by firms when government lacks commitment.

Unlike Cournot, we find that with Stackelberg competition and efficient firm as leader, free entry is optimal. This is true even if the government lacks commitment - modeled as government taking entry regulation decision after the leader firm chooses output, i.e. the leader firm has the option of trying to influence the government decision through its output choice. The intuition behind this result is as follows. Since in this case, output of the efficient firm is fixed prior to choice by inefficient firms and does not change following entry by inefficient firms, the welfare loss driving the excess entry result with Cournot competition(this has been discussed earlier) vanishes and hence not regulating entry is always optimal. However, if an inefficient firm is the leader, we find that this is equivalent to our Cournot set-up with a reduced demand and hence free entry is suboptimal.

Further, we analyze the endogenous choice between a Cournot and a Stackelberg game in the presence of entry regulation. We consider the case where one firm can but is not forced to be a leader and has the option of foregoing leadership - if it undertakes(foregoes) leadership, we have a Stackelberg(Cournot) game.

We find that compared to the case where the form of competition is exogenous, the outcome of the game changes dramatically when the efficient firm is the leader and the government lacks commitment. In this case, the efficient firm faces a trade-off between exercising its leadership option and being the leader in a Stackelberg game but facing all potential entrants (by argument for the corresponding case above where form of competition is exogenous, government allows free entry) or of foregoing leadership and ending up in a Cournot game but facing a smaller number of firms (by argument for the Cournot game discussed earlier, regulating number of inefficient entrants is optimal in such a case).

In our framework, all firms are profit-maximizing and output choices are strategic substitutes. Still, for a very general demand structure, we find that there always exists cost-differences for which the efficient firm foregoes leadership! This can be understood from the fact(discussed earlier) that for large enough cost-differences in a Cournot game, it is optimal not to allow any inefficient firms to enter which clearly leads to higher profits for the efficient firm than if it exercises its leadership option.

We also consider the special case of linear demand. This provides us with sharper results. With linear demand and Cournot competition, we characterize the optimal industry structure. We find that with more than two different cost levels, not allowing entry by some firms more efficient than already-existing firms might be welfare-improving! The intuition behind this result is as follows. Consider an industry with three kinds of firms - low-cost, medium-cost and high-cost. Also assume that the industry starts off with a certain number of low-cost and a certain number of high-cost firms. Further, all entering firms are medium-cost and only their number can be regulated. In such a case, an entering firm steals output both from high-cost firms(which is good) and from low-cost firms(which is bad). If the cost difference between medium-cost and high-cost firms is sufficiently smaller compared to that between the medium-cost and the low-cost firms, the effect of output stolen from low-cost firms might dominate and it might be optimal not to allow some medium-cost firms to enter although the high-cost firms continue in business under such regulation.

We also find that the optimal number of inefficient firms is decreasing in the cost difference between efficient and inefficient firms while welfare gains from regulation are small(largest) for small or large(moderate) values of cost difference. For Stackelberg competition with endogenous leadership choice, we are able to characterize the precise condition under which an efficient leader firm foregoes leadership. We also find that if an inefficient firm is the leader, it always undertakes leadership and free entry is suboptimal.

Apart from its obvious contribution to the excess entry literature, this paper is also connected with several other strands of the IO literature. We have already highlighted our contribution to the growing literature on endogenous choice of moves in an oligopoly framework[6].

Our work is also connected to the public enterprise literature[7]. As discussed earlier, our proof of the excess entry theorem with general demand conditions and Cournot competition uses a well-known result from this literature. Further, Pal(1998) has considered a multi-stage oligopoly framework with efficient, profit-maximizing firms and an inefficient, welfare-maximizing public sector firm and endogenous choice of moves and finds conditions under which the public sector firm will choose to be a follower. Note that in a Stackelberg set-up, infinitely many inefficient profit-maximizing firms moving after an efficient leader firm is equivalent to a welfare-maximizing public

[6]For an example of some work in this area, see Anderson and Engers(1992) or Pal(1998).

[7]See for example, Beato and Mascollel(1984), DeFraja and DelBono(1989), Pal(1998).

sector firm moving after efficient, profit-maximizing firms. Thus our result that free entry is optimal in a Stackelberg game with efficient firm as leader is related to the findings of Pal(1998). Further, in our framework with Stackelberg competition and infinitely many inefficient firms, the inefficient firms end up producing nothing just as the public sector enterprise when it chooses to move second in Pal(1998) - in fact the resulting outcome in our case is the same as in Pal(1998).

While a lot of the work on industrial policy and regulation(in various forms) has considered identical firms, there is a fairly old and ever-increasing literature that has focused on cost asymmetry amongst firms and its policy implications. For example, Lahiri and Ono(1988) and more recently Zhao(2001) have shown that with asymmetric costs, a small reduction in production cost of a high-cost firm might reduce welfare. Similarly, it is known that with asymmetric costs, a merger might be welfare-improving even absent any synergies or savings in fixed costs[8]. Our paper obviously adds to this literature as well.

Both the results mentioned in the paragraph above are due to a shift in output from efficient to inefficient firms. As discussed earlier, this production efficiency loss drives our excess entry results. It is interesting to note that while the possibility of welfare improvement following cost reduction or a merger holds only for a certain range of possible values of cost difference, for entry regulation it holds even for arbitrarily small values of cost difference.

The paper is organized as follows. Section 2 lays down our basic framework. Sections 3 and 4 contain our excess entry results under general demand conditions with Cournot and Stackelberg competition respectively. Section 5 considers linear demand. Section 6 concludes.

2 Model

Consider an industry consisting of 1 efficient firm - henceforth firm e - and an infinite number of inefficient firms[9] producing a homogeneous product with constant marginal cost of production and no fixed costs. Marginal cost of production of firm e(each inefficient firm) is $c_e(c > c_e)$. We will typically consider the case with firm e and N inefficient firms and also consider(when

[8]See for example, Williamson(1968), Farell and Shapiro(1990).

[9]Later on, we consider the case with a finite number of inefficient firms. Our results with an infinite and with a finite number of inefficient firms are qualitatively the same.

needed) limiting values of various variables as N→ ∞. With firm e and N inefficient firms, let q_i, $i \in \{1, 2, ..N\}(q_e)$ denote output of i-th inefficient firm(firm e) and $P(Q)$ where $Q = \sum_{i=1}^{N} q_i + q_e$ denote inverse demand for the product. We assume that the inverse demand curve satisfies the following assumptions.

Assumption 1:(A1) $P(Q)$ is twice continuously differentiable.
Assumption 2:(A2) There exists $\bar{Q} \in (0, \infty)$ such that $P(Q) = 0$ for $Q \geq \bar{Q}$ and $-\infty < P'(Q) < 0$ for $Q < \bar{Q}$.
Assumption 3:(A3) $P'(Q) + QP''(Q) \leq 0 \; \forall Q \in [0, \bar{Q})$.
Assumption 4:(A4) $P(0) > c_e$.

(A1) - (A3) ensure that with Cournot competition, a unique equilibrium exists[see Gaudet and Salant(1991)[10]]. (A4) implies that firm e(efficient firm) produces in equilibrium. Let $P_m(c_e) \equiv$ monopoly price when firm e is the sole producer[11]. Assumption 5 below ensures that with Cournot competition and absent entry regulation, inefficient firms enter.

Assumption 5:(A5) $P_m(c_e) > c$.

3 Cournot

Suppose firm e and N inefficient firms are allowed to enter[12]. We begin by solving for the corresponding Cournot equilibrium.

3.1 Preliminary calculations

Firm i solves

$$\max_{q_i \geq 0} [P(q_i + Q_{-i}) - c_i] q_i \tag{1}$$

[10]Gaudet and Salant's conditions are weaker than those mentioned here. We make some additional assumptions - e.g. $P'(Q) > -\infty$ to illustrate our results as simply as possible.
[11]It is shown in the appendix that given (A2) and (A3), $P_m(c_e)$ is unique.
[12]Note that it can never be optimal to disallow firm e to enter.

In equation(1), $c_i = c_e$ if $i = e$; else $c_i = c$ while $Q_{-i} \equiv$ sum of output of all firms except i.

Let q_i^E, Q_{-e}^E and $Q^E \equiv$ denote output of firm i, aggregate output of the inefficient firms and the aggregate output respectively in equilibrium. Given assumptions (A1) - (A5) the unique equilibrium is strictly interior. To characterize the equilibrium, let us consider optimization problems that firms solve [given by (1)]. First-order conditions for this problem are(using the fact that the equilibrium is interior)

$$\frac{\partial \pi_e}{\partial q_e} = P(Q) - c_e + q_e P'(Q) = 0; \frac{\partial \pi_i}{\partial q_i} = P(Q) - c + q_i P'(Q) = 0, i \neq e \quad (2)$$

Adding first-order conditions(one for each firm) yields

$$(N+1)P(Q) - Nc - c_e - QP'(Q) = 0 \quad (3)$$

Using (A2) and (A3), we find that[13] the right hand side of (3) is strictly decreasing in Q. Hence Q that satisfies (3) $\equiv Q^E$ is unique - this Q^E is the industry output in the unique Cournot equilibrium. Substituting $Q = Q^E$ in the first-order conditions yield q_i's in the Cournot equilibrium.

Lemma 1: *(i)* $q_e^E = -\frac{P(Q^E)-c_e}{P'(Q^E)}$ *and for* $i \neq e$, $q_i^E = -\frac{P(Q^E)-c}{P'(Q^E)}$.
(ii) $\forall i$, q_i^E *is continuous in* N *and hence* $Q_{-e}^E = Nq_i^E$ *is continuous in* N *as well.*

As N increases, it is easy to check from (3) that $P(Q) \to c$. Let q_i^∞, Q_{-e}^∞ and Q^∞ denote limiting values of q_i^E, Q_{-e}^E and Q^E as $N \to \infty$.

Lemma 2: *(i)* $\lim_{N\to\infty} P(Q^E) = P(Q^\infty) = c$
(ii) $q_e^\infty = -\frac{c-c_0}{P'(Q^\infty)}$
(iii) $i \neq e$, $q_i^\infty = 0$.
(iv) $Q_{-e}^\infty = \lim_{N\to\infty} Nq_i^E > 0$[14]

[13]See Appendix.

[14]Though each inefficient firm's output tends to 0[see (iii)], aggregate output produced

3.2 Welfare considerations

Welfare $\equiv$ W is given by the sum of consumer and producer surplus. Thus

$$W = \int_0^Q P(\hat{Q})d\hat{Q} - P(Q)Q + (P(Q) - c_e)q_e + (P(Q) - c)(Q_{-e}) \quad (4)$$

where Q, q_e and Q_{-e} denote total output, output of firm e and total output produced by all inefficient firms respectively.

Suppose, one more inefficient firm is allowed to enter. Let us denote this firm as $N+1$ - th inefficient firm and its output as q_{N+1}. Its profit $\equiv \pi_{N+1} = [P(Q) - c]q_{N+1}$. Consider this firm contemplating entry. Evaluated at $q_{N+1} = 0$

$$\frac{d\pi_{N+1}}{dq_{N+1}} = P(Q) - c \quad (5)$$

However, social value of entry at $q_{N+1} = 0$ is

$$\frac{dW}{dq_{N+1}} = [P(Q) - c_e]\frac{dq_e}{dq_{N+1}} + [P(Q) - c]\frac{d[Q^E_{-e}]}{dq_{N+1}} \quad (6)$$

Comparing (5) and (6) shows that social value of entry is always less than private value(to the firm)[15], provided output choices are strategic substitutes ($\frac{dq_j}{dq_k} < 0,\ j \neq k$). This does not depend on values of cost parameters. In particular, it holds if $c = c_e$ - i.e. all firms are equally efficient. However the two cases - $c = c_e$ and $c \neq c_e$ - have different implications for desirability of free entry.

From(5), it follows that the firm will enter if P(Q) $-$ c $>$ 0. If $c = c_e$, social value of entry is $[P(Q) - c]\frac{dQ}{dq_{N+1}}$. Assuming "net business-creation"[16] -

by inefficient firms is positive in the limit[see (iv)]. This is a general feature of the Cournot model and was noted by Mankiw and Whinston(1986).

[15]This is due to the well-known "business-stealing" effect mentioned earlier.

[16]This holds in our framework.

i.e. increase in q_{N+1} outweighs decrease in aggregate output of the rivals$(Q - q_{N+1})$ - as discussed earlier, we see that in this case whenever a firm enters, entry is welfare-improving.

If $c \neq c_e$, social value of entry is $[P(Q) - c]\frac{d[Q_{-e}]}{dq_{N+1}} + [P(Q) - c_e]\frac{dq_e}{dq_{N+1}}$. In our framework, "net business-creation effect" still remains - aggregate output increases following entry. However output reshuffled between inefficient firms is valued at $P(Q) - c$ while business stolen from firm e is valued at $P(Q) - c_e$. Note that $[P(Q) - c] > 0 \forall N$ - thus with free entry, all inefficient firms enter. However, for large N, $[P(Q) - c] \to 0$ but since $c > c_e$, $P(Q) - c_e$ is bounded away from 0. Since $\frac{dq_e}{dq_{N+1}} < 0$, this suggests that free entry might be inoptimal.

However, note that unless $\frac{dq_e}{dq_{N+1}}$ is bounded away from 0 as $N \to \infty$ which need not be the case[17], both the terms in the expression above for social value of entry tend towards zero as $N \to \infty$ and hence we cannot conclude whether free entry would be optimal or not in our general framework[18]. To tackle this problem, we consider a different scenario.

3.3 Mixed oligopoly

Now we consider a game between a welfare-maximizing but inefficient public sector firm and a profit-maximizing, efficient firm. We denote output produced by the public sector firm as Q_g. We further assume that the two firms compete in a Stackelberg fashion with the public sector firm as the leader - i.e the public sector firm chooses Q_g first following which the efficient firm chooses q_e. Given Q_g, q_e solves

$$\max_{q_i \geq 0} [P(q_i + Q_g) - c_i] q_i \tag{7}$$

First-order condition for this problem is

$$\frac{\partial \pi_e}{\partial q_e} = P(Q) - c_e + q_e P'(Q) = 0; \tag{8}$$

[17] In fact, this is not true with linear demand.

[18] Of course this might be possible if we make further assumptions regarding the inverse demand curve. For example, if we assume demand to be linear, then we can explicitly compute the social value of entry - given by the expression above - and show that free entry is indeed inoptimal.

Since firm e's first-order condition is same as before, the best-response function q_e is unique and the public sector firm can choose $Q_g = Q^E_{-e}$ corresponding to any value of N, it follows that

Lemma 3: *The public sector firm can replicate the Cournot outcome with N inefficient firms corresponding to any value of N.*

Since the public sector firm is welfare-maximizing, it chooses Q_g to maximize

$$W = \int_0^{q_e+Q_g} P(\hat{Q})d\hat{Q} - P(Q)Q + (P(Q) - c_e)q_e + (P(Q) - c)Q_g \quad (9)$$

Differentiating W with respect to Qg yields

$$\frac{\partial W}{\partial Q_g} = (P(Q) - c) + (P(Q) - c_e)\frac{\partial q_e}{\partial Q_g} \quad (10)$$

For any $Q_g > Q^{\infty}_{-e}$, $Q > Q^{\infty}_{-e}$ and hence $P(Q) < P(Q^{\infty}) = c$. Differentiating (8) yields

$$\frac{\partial q_e}{\partial Q_g} = -1 + \frac{P'(Q)}{P''(Q)q_e + 2P'(Q)}; \quad (11)$$

As long as $Q_g > 0$, $P''(Q)q_e + 2P'(Q) < P''(Q)Q + 2P'(Q) \leq 0$. This together with the fact that $-\infty < P'(Q) < 0$ implies that $-1 < \frac{\partial q_e}{\partial Q_g} < 0$. So, for any such value of Q_g, output reduction is welfare improving. For $Q_g = Q^{\infty}_{-e}$, the first term - $[P(Q) - c]$ - vanishes but since $\frac{\partial q_e}{\partial Q_g} < 0$, $\frac{\partial W}{\partial q_e} < 0$. Hence, the optimal value of $Q_g < Q^{\infty}_{-e}$. The existence of an optimal Q_g follows from the continuity of the maximand(W) function and the reaction function of the efficient firm which in turn follow from assumptions $(1) - (3)$. Let $Q^*_g \equiv$ optimal Q_g[19]. We have shown that

[19]This could possibly have several different values. Note that optimal value of Q_g need

Lemma 4:*In the Stackelberg quantity game between welfare-maximizing public sector firm and firm e with the public sector firm as the leader,* ${Q_g}^* < Q^{\infty}_{-e}$.

3.4 Excess entry results

Let us consider the set of possible outcomes in the Cournot game of subsection 3.1 assuming that N is any non-negative real number(we are ignoring the integer constraint) and the set of outcomes for the mixed duopoly game of the previous subsection. Note that any outcome of the mixed duopoly game cannot be achieved in the Cournot game through a suitable choice of N. For example, consider $Q_g > Q^{\infty}_{-e}$. This leads to $P(Q) < c$. Since profit-maximizing, inefficient firms will not produce in such a scenario and it is assumed that $P_m(c_e) > c$(A5), this cannot arise in the Cournot game. From this and Lemma 3, it follows that

Lemma 5 *The set of outcomes in the Cournot game of subsection 3.1 where N is any non-negative real number(we are ignoring the integer constraint) is a proper subset of the set of outcomes for the mixed duopoly game of subsection 3.3.*

Since, $Q^E_{-e} = 0$ for $N = 0$ and $Q^E_{-e} \equiv Nq^E_i$ is continuous in N(Lemma 1), it follows that any outcome of the mixed oligopoly game such that $0 \leq Q_g < Q^{\infty}_{-e}$(note that $\lim_{N\to\infty} Q^E_{-e} \equiv Q^{\infty}_{-e}$) can be achieved in the Cournot game through a suitable choice of N. Hence from Lemmas 4 and 5, it follows that

Theorem I: *If demand function and cost parameters satisfy assumptions (A1)- (A5) and post-entry game is Cournot, there exists $N^* < \infty$ such that $W(N^*) \geq W(N) \forall N < \infty$ and $W(N^*) > W^{\infty}$*[20].

Further, consider entry by an inefficient firm(say Firm 1, whose output $\equiv q_1$) into a market with firm e the sole producer. With Cournot competition following entry, at $c = P_m(c_e)$

---

not be unique with general demand. With linear demand and ignoring integer constraint, it is unique.

[20] N^* may not be unique.

$$\frac{dW}{dq_1} = [P_m(c_e) - c] + [P_m(c_e) - c_e]\frac{dq_e}{dq_1} < 0 \tag{12}$$

From (12), it follows that

Corollary to Theorem I: *There exists* $c^* \in (c_e, P_m(c_e))$ *such that* $N^* = 0$ *for* $c \geq c^{*}$[21].

4 Stackelberg environment

So far we have considered Cournot competition. Now consider a 3-stage framework where quantity choices by firms are made sequentially in two stages. Everything else is the same as before. In the first quantity-choosing stage - **Stage 1** - a pre-designated firm - the leader - can choose any quantity it wants to; this choice becomes common knowledge. Only this firm is allowed to choose its quantity during Stage 1. Following Stage 1, all firms allowed to enter in the second quantity-choosing stage -**Stage 2** - choose quantities simultaneously. There is one more stage - **Stage G** - in which the government decides on entry regulation; it can only restrict the number of firms entering in Stage 2.

We consider variants of this game reflecting different levels of commitment power of the government and restrict ourselves to subgame perfect Nash equilibria(SPNE). Also, we consider both the case where the form of competition is exogenous and the case where the form of competition is endogenous. With endogenous choice of form of competition, it is not that the leader must choose quantity in Stage 1 - **exercise leadership** - if it chooses to do so, we have a Stackelberg game. It can also decide not to exercise leadership - **forego leadership** following which(if it is allowed to enter in Stage 2), it chooses quantity simultaneously with all other firms allowed to enter in Stage 2 - i.e. we have a Cournot game. In other words, the leader can but is not forced to be the leader and its choice determines whether we have a Stackelberg or a Cournot game. We analyze both where firm e and where an inefficient firm is the leader.

[21]Note that no inefficient firms will enter if $c \geq P_m(c_e)$.

4.1 Firm e as the leader:

We will start by assuming that the form of competition is exogenous. As before, let $q_e \equiv$ output of firm e. Given q_e from Stage 1, each q_i, $\{i = 1, 2, ...N\}$ where N inefficient firms are allowed to enter solves

$$\max_{q_i \geq 0} [P(q_e + Q_{-e}) - c_i] q_i \tag{13}$$

The first-order condition for firm i, $i \neq e$ is:

$$\frac{\partial \pi_i}{\partial q_i} = P(q_e + Q_{-i}) - c_i + P'(q_e + Q_{-i}) q_i = 0 \tag{14}$$

Assume $P(q_e) < c$ so that the inefficient firms will enter. Adding the first-order conditions and taking limit yields $lim_{N \to \infty} P(Q) = c$. Hence with free entry, for any $q_e < P^{-1}(c)$, following Stage 2 price equals c. Since, $P_m(c_e) > c(\mathbf{A5})$ and π_e is strictly concave[follows from assumptions (A2) and (A3)], it follows that

Lemma 6: *With Stackelberg competition with firm e as leader and free entry in Stage 2* $q_e = P^{-1}(c)$, $q_i = 0$ *for* $i \neq e$ *and price equals c..*

Government moves before Stage 1: This is equivalent to the government taking a decision regarding entry regulation and committing to stick to it regardless of the choice made by the leader firm. Suppose the government restricts the number of firms to $N^r < \infty$. Note that price will not be less than c. Further, we might have some of the output being produced by inefficient firms. Thus, compared with free entry, entry regulation cannot increase welfare in this case.

So far we have assumed the form of competition to be exogenously given. Suppose firm e if it wants to can forego its leadership option - in such a case, it chooses q_e simultaneously with inefficient firms that are allowed to enter in Stage 2. If the government commits to free entry, then from the outcome with Cournot competition(see Section 3.1) and Lemma 6 above, it follows that it is inoptimal for firm e to forego leadership. Given this and the discussion in the paragraph above, it is clearly optimal for the government to commit to free entry. Thus we have shown that

Proposition 1: *With government commitment - modeled as government making entry decisions prior to the quantity-choosing stages -*

(i) firm e always undertakes leadership.

(ii) free entry is optimal.

(iii) outcome is as described in Lemma 6.

Government moves between Stage 1 and Stage 2: Though this might seem slightly artificial, it is a way to model lack of commitment on the part of the government. If firm e commits to some quantity choice q_e in Stage 1, the government moving after firm e has no reason to restrict entry of inefficient firms since there is no business-stealing from firm e. Hence, if form of competition is exogenously given, free entry is optimal and outcome will be as described by Lemma 6.

If form of competition is endogenous, if firm e undertakes leadership, then by the argument above, free entry is optimal. However, if firm e foregoes leadership, the game reduces to a Cournot game. From Theorem 1 it follows that entry regulation is optimal in such a case. Thus

Lemma 7: *Absent government commitment - modeled as government making entry decisions between the two quantity-choosing stages - free entry is optimal if firm e exercises leadership while entry regulation is optimal if firm e foregoes leadership.*

Now consider the case where $c - c_e$ is large. From corollary to Theorem 1, it follows that in such cases $N^* = 0$. Hence by foregoing leadership, firm e can become a monopolist. While exercising leadership, firm e can deter entry but given that $c < P_m(c_e)$ it cannot enjoy monopoly profits. Thus

Proposition 2: *Absent government commitment, for large cost differences*

(i) firm e foregoes leadership.

(ii) entry regulation is optimal.

4.2 Inefficient firm as the leader

Suppose the inefficient leader firm undertakes leadership. Let q_l denote its quantity choice. Following its choice, suppose firm e and M other inefficient firms are allowed to enter and let q_e and q_i, $i \in \{1, 2, ..M\}$ denote their output choices. The resultant price is $P(Q)$ where $Q = \sum_{i=1}^{M} q_i + q_e + q_l$. Clearly, the leader firm will not choose $q_l > P^{-1}(c)$. From our calculations in Section 3.1, it follows that if $q_l = P^{-1}(c)$, firm e enters and price is less than c. Hence, the leader firm chooses $q_l < P^{-1}(c)$. The game that follows following Stage 1 in this case satisfies all the assumptions of the game considered in Section 3 and hence by Theorem 1, we get

Proposition 3: *If the inefficient leader firm undertakes leadership, entry regulation is optimal.*

With endogenous choice of form of competition, whether the inefficient leader firm undertakes or foregoes leadership depends among other things on a comparison between the total number of inefficient firms allowed to enter in the two cases[22]. We look at this with linear demand in the next section.

5 Linear demand

The cost structure is same as in previous sections. Suppose N inefficient firms are allowed to enter. We assume that the inverse demand function[23] is given by

$$P = a - Q \tag{15}$$

where Q denotes aggregate output sold - $Q = q_e + \sum_{i=1}^{N} q_i$. We assume[24] that $a - c_e > 0$ and $c < \frac{a+c_e}{2}$.

5.1 Preliminary calculations:

Given q_j, $q_i (i \neq j)$ solves

[22]The chances of the inefficient leader firm being allowed to enter if it foregoes leadership would also matter. With leadership by firm e, we do not need to consider this since it is never optimal not to let firm e enter.

[23]To be more precise, the inverse demand function is $P = max\{a - Q, 0\}$. This specification satisfies assumptions (A1) - (A3) in section 3. Hence a unique Cournot equilibrium exists.

[24]These correspond to (A4) and (A5) respectively.

$$\max_{q_i \geq 0} \pi_i = [a - Q - c_i]q_i \tag{16}$$

First order conditions for this problem are(using Lemma 1)

$$\frac{\partial \pi_e}{\partial q_e} = a - 2q_e + \sum_{i \neq e} q_i - c_e = 0; \frac{\partial \pi_i}{\partial q_i} = a - 2q_i + \sum_{j \neq i} q_i - c = 0, i \neq e \tag{17}$$

In the Cournot equilibrium with linear demand,

$$q_e^E = \frac{a - c_e + N[c - c_e]}{N + 2}; q_i^E = \frac{a - c_e - 2[c - c_e]}{N + 2}, i \neq e \tag{18}$$

$$Q^E = \frac{N(a - c) + [a - c_e]}{N + 2}; P^E = \frac{a + c_e + Nc}{N + 2} \tag{19}$$

Since all inefficient firms produce the same output in equilibrium, we will denote $q_i^E = q^E \forall i \neq e$. Welfare($W$) in this case is given by

$$W = \frac{Q^{E2}}{2} + N[q^E]^2 + [q_e^E]^2 \tag{20}$$

In (20), $\frac{Q^{E2}}{2}$ is consumer surplus(CS), $N[q^E]^2$ is total profits of inefficient firms and $[q_e]^2$ is profit of firm e. Treating N as a continuous variable, we get

$$\frac{dq^E}{dN} = -\frac{[a + c_e - 2c]}{[N + 2]^2} < 0; \frac{dq_e^E}{dN} = -\frac{[a + c_e - 2c]}{[N + 2]^2} < 0; \frac{dQ^E}{dN} = \frac{[a + c_e - 2c]}{[N + 2]^2} > 0 \tag{21}$$

Thus, an increase in N lowers the existing individual firms' output. However, increase in output due to entry outweighs output reduced by existing firms and hence aggregate output increases.

Consumer surplus increases due to entry while industry profit Π where $\Pi = N[q^E]^2 + [q_e^E]^2$ declines. For large N, we find that the loss in industry profits dominates the gain in consumer surplus and hence entry reduces welfare (W).

5.2 Optimal industry structure

Let $N^* \equiv$ socially optimal number of inefficient firms with infinitely many potential inefficient entrants.

Treating N as a continuous variable, using (20) and differentiating W with respect to N and setting this equal to 0 using equilibrium values for various quantities from (18) and (19) gives

Proposition 4: $\mathrm{N}^* = max\{\frac{a-c_e}{c-c_e} - 4, 0.$

Note that $\frac{c-c_e}{a-c_e}$ is a measure of the cost-differential between the inefficient firms and firm e. Also, given our assumptions, $\frac{c-c_e}{a-c_e} \in (0, \frac{1}{2})$. From Proposition 4, it follows that

Corollary to Proposition 4:

i) N^* is finite and non-increasing in cost-differential between inefficient firms and firm e- $\frac{c-c_e}{a-c_e}$[25].

ii) $\lim_{c \to c_e} N^* = \infty$.

It is interesting to note that although policies to implement a certain market structure have not been undertaken in the United States, it has been done elsewhere. For example, the Spanish government and MITI in Japan have undertaken policies to achieve a desired market structure.

[25] N^* is non-increasing and not strictly decreasing in $\frac{c-c_e}{a-c_e}$ since $N^* = 0$ for a range of suitably high values of $\frac{c-c_e}{a-c_e}$.

5.3 Welfare gains from entry regulation:

With identical costs across firms there is no need for entry regulation since there is no loss in production efficiency due to entry - i.e. free entry is socially optimal. On the other hand if the costs are too different firms will not enter on their own and hence entry regulated equilibrium and free entry equilibrium yield identical outcomes. In general the welfare gains from entry regulation matters only for moderate cost differences. However it is difficult to say anything more for a general demand function. Hence we restrict our discussion to linear demand. Welfare with entry regulation (W^*) is denoted as

$$W^* = \frac{[N^*(a-c)+(a-c_e)]^2}{2(N^*+2)^2} + \frac{[(a-c_e)+N^*(c-c_e)]^2}{(N^*+2)^2} + \frac{N^*[(a-c_e)-2(c-c_e)]^2}{(N^*+2)^2} \tag{22}$$

Welfare with free entry $\equiv W^\infty = \lim_{N\to\infty} W$ is given by $\frac{(a-c)^2}{2} + (c-c_e)^2$. If cost-differential is too high, with entry regulation, only firm e is allowed to enter and W^* is welfare with firm e as the sole producer. Without entry regulation, any inefficient firm can(and will) enter but due to high cost-differential, firm e is almost a monopoly. Hence, in this case, welfare gain from entry regulation is small. Similarly, when $c \to c_e$, $N^* \to \infty$ and from standard continuity argument, it follows that $lim_{N^*\to\infty} W(N^*) \to W^\infty$. Thus welfare gains from entry regulation are insignificant for too high or too low cost-differential. For moderate values of $c - c_e$, welfare gains from entry regulation are higher than at extreme values. We find that

Proposition 5: *Given a and c_e, the welfare gains from entry regulation($W^* - W^\infty$) is non-monotone in c.*

This suggests that government should be most concerned about entry regulation for moderate cost-differential. Gain from regulation can be significant in such cases depending on market size - in fact, ratio of welfare under optimal entry to that under free entry can be as high as 1.125[26].

[26] With socially optimal number of firms, we find that profit of an inefficient firm that gains entry is increasing in cost differential.

5.4 More than one efficient firm:

So far, we have assumed only one efficient firm. Adding more efficient firms does not alter the reasoning for our excess entry result - namely, shift in output from efficient to inefficient firms. With E efficient firms ($E > 1$), "business-stealing effect" assumes more importance because usually[27] as E increases, business stolen from efficient firms increase. Hence, we would expect entry regulation in general to be more stringent with a greater number of efficient firms. For linear demand we find

$$N^*(E) = max\{\frac{\frac{a-c_e}{c-c_e} - (E+1)^2}{E}, 0\} \tag{23}$$

where $N^*(E) \equiv$ optimal number of inefficient firms with E efficient firms. From (23), we get

Proposition 6: *Given a, c_e and c, $N^*(E)$ is weakly decreasing in E.*

5.5 More than one type of inefficient firm:

Consider the scenario with 1 efficient firm - firm e once again. However,in addition to firm e, suppose there are K - $K \geq 1$ - types of inefficient firms with unit costs $c_1, c_2, ...c_K$ respectively where $c_e < c_1 < c_2.... < c_K < P_m(c_e)$. While calculations regarding the excess entry result become more complicated, free entry remains inoptimal. There are three possible cases.

Case 1: Only most inefficient firms can be regulated

Let $N_k \equiv$ number of firms with cost c_k. Following analysis as in section 3, we find that given $N_k, k \in \{1, 2, ..., K-1\}$, $N_K^* < \infty$. The reasoning is as before - beyond a certain number of firms of type K, gains due to entry of a type K firm are outweighed by the loss due to business stolen from types 1, 2, ... $K-1$ firms.

Case 2: All inefficient firms can be regulated

[27] We say "usually" and not "always" since with an increase in E, each efficient firm's output and hence business stolen from each efficient firm is lower. However, there are more efficient firms to steal from. The latter effect often dominates the former leading to greater "production-efficiency loss", greater the value of E.

Consider $K = 2$ and suppose government can restrict entry of both inefficient types. Suppose in an entry-regulated equilibrium, highest-cost type firms - firms with cost c_2 - produce a positive amount. Let $\hat{P} \equiv$ equilibrium price. Alternatively, government can set $N_2^* = 0$. Since $\hat{P} > c_2 > c_1$, government can choose sufficiently large N_1 such that equilibrium price and hence equilibrium quantity is same as before. Since $q_e = -\frac{P(Q)-c_e}{P'(Q)}$ - firm e produces as before. Welfare is higher due to the shift in output from type 2 to type 1 firms. Hence, type 2 firms produce nothing in any equilibrium under entry regulation in such a case. The result extends for any $K \geq 3$.

Case 3: Only "medium-cost" inefficient firms can be regulated

Alternatively, consider K = 2 once again but suppose that the government can regulate entry of only type 1 firms. Number of firms with cost c_e and c_2 that the industry starts off with are given. However, infinitely many potential entrants with cost c_1 exist. Will the government allow all potential entrants to enter? The answer is once again NO. Assume $c_1 = c_2$. Then, effectively there are two types and entry should be restricted due to loss in production-efficiency. This will also be true if c_1 is close to c_2. What is more interesting is that (see Appendix) in an equilibrium with entry regulation, type 2 firms might produce a positive amount while more efficient type 1 firms are not allowed to enter. Thus we have shown that

Proposition 7: *With more than one type of inefficient firms*

i) if only the most inefficient firms can be regulated, free entry is still inoptimal.

ii) if entry by all inefficient firms can be regulated, entry of all except for the most efficient amongst the inefficient types should be banned.

iii) if only the "medium-cost" firms can be regulated, it might be optimal not to let some of these firms enter even though more inefficient firms produce a positive amount.

5.6 Finite number of entrants

With linear demand the implication of finite number of entrants in Cournot environment is obvious. The entry regulation might not be necessary. Since $W(N)$ is strictly concave, if the number of potential entrants is less than N^*, then government will allow all the firms to enter. With non-concavity of $W(N)$ this might not hold.

In the Stackelberg environment with efficient firm as the leader, unlike the other case(infinite number of potential entrants) entry may not be deterred. If $c - c_e$ is very low efficient firm will be unwilling to deter entry. In such cases

$$q_e = \frac{[a - c_e] + N[c - c_e]}{2} \tag{24}$$

In (24) above N denote the number of potential entrants. Though entry is not deterred note that the leader increases output corresponding to an increase in N. In anticipation of business-stealing the leader firm creates more business. Thus the flavor of infinite potential entrants still remains.

6 Conclusion

In the excess entry literature, it has been shown that with economies of scale, free entry can lead to more firms entering than is socially optimal. We show under very general demand conditions that another factor - asymmetry in production costs across firms - can lead to excess entry. The proof of the excess entry result is novel and related to the literature on public enterprise. Though gains from entry regulation exists for arbitrarily small cost differences these gains are of some importance only for moderate cost-differences. With linear demand we characterize the optimal industry structure. Further we show that optimal number of inefficient firms is decreasing in cost-differences. With Stackelberg competition whether free entry is optimal depends on the leader's cost level. If an inefficient firm is the leader free entry is inoptimal while the reverse is true an efficient firm is the leader free entry is optimal. A surprising feature of the Stackelberg environment is that with endogenous choice of leadership and lack of government commitment an efficient firm might forego leadership. Foregoing the leadership changes the game to Cournot environment rendering free entry inoptimal once again.

The discussion in the paper assumes that the cost- differences are given. But an efficient firm might share the technology (if it is not costly) with some inefficient firms. In particular assume it can license technology to the inefficient firms. Will the results change once we allow for endogenous cost differences? The answer is not necessarily. Absent government entry regulation efficient firm would license the technology to some(but not all) of the inefficient firms. If the licensing equilibrium is such that firms still producing with the inefficient technology drops out of the market then there is no need for entry regulation. However if the output produced by the inefficient firms is positive in the free-entry equilibrium then the case for entry regulation exists. This is due to the fact no matter how many efficient firms are there as long as that number is finite there is always a case for entry regulation.

7 Appendix

To be completed.

8 References

Incomplete

Beato, P. and Mas-Collell, A. “The marginal cost pricing as a regulation mechanism in mixed markets.” in Marchand, M., Pestieau, P., and Tulkens, H. eds., *The Performance of Public Enterprises. 1984* Amsterdam: North-Holland.

Defraja, G. and Delbono, F. “Alternative strategies of a public enterprise in oligopoly.” *Oxford Economic Papers, 1989, 41*

Farrell, Joseph and Shapiro, Carl. “Horizontal Mergers: An Equilibrium Analysis” *American Economic Review, March 19990, Vol. 80, Issue 1*, pp. 107-126.

Gaudet, Gerard and Salant, Stephen. “Uniqueness of Cournot Equilibrium: New Results from Old Methods.” *Center for Research on Economic and Social Theory Working Paper, August, 1988, 89-10.*

Ghosh , Arghya and Saha, Souresh. “Horizontal Mergers: A Welfare Analysis”, mimeo, December 1999.

Kim, J. “Inefficiency of Subgame Optimal Entry Regulation.” *RAND Journal of Economics, Vol.28, NO.1, Spring 1997*, pp. 25-36.

Lahiri, S. and Ono, Y. “Helping Minor Firms Reduces Welfare.” *The Economic Journal, December 1988, Vol.98*, pp. 1199-1202.

Mankiw, N.G. and Whinston, M.D. “Free Entry and Social Inefficiency.” *RAND Journal of Economics, Vol. 17, NO.1, Spring 1986*, pp. 48-58.

Perry, M.K. “Scale Economies, Imperfect Competition and Public Policy.” *Journal of Industrial Economics, Vol. 32, 1984*, pp. 313-333.

Suzumura, K. and Kiyono, K. "Entry Barriers and Economic Welfare." *Review of Economic Studies, Vol. 54 1987*, pp. 157-167.

von Weizacker, C.C. "A Welfare Analysis of Barriers to Entry." *Bell Journal of Economics, Vol. 11 1980a*, pp. 399-420.

————*Barriers to Entry: A Theoretical Treatment. 1980b* Berlin: Springer-Verlag.

Zhao, Jingang. "A Characterization for the Negative Welfare Effects of Cost Reduction in Cournot Oligopoly" Forthcoming in *International Journal of Industrial Organization. 1999a.*