Conditional Beta Estimation and Forecasting with Panel Data Methods*

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Abstract

Standard approaches to the estimation and testing of conditional CAPM models with time-varying or random beta have ignored the potential panel nature of …nancial data. We test for whether or not homogeneity restrictions on the time-variation component of multifactor betas and on the slope parameters for the conditioning variables can be rejected. We …nd that such homogeneity restrictions are not rejected, and show that there are resultant bene…ts for testing conditional CAPM and forecasting expected returns and beta. Further, this panel approach yields more precise parameter estimates, and a greater understanding of the signi…cance of both conditional variables and multi-factors.

JEL Classi…cations: G12; C23; C52.

Key words: conditional CAPM; panel data; heterogeneity; beta estimation; time varying and random beta; multifactor CAPM.

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1 Introduction

The traditional methods of estimating beta and testing conditional CAPM and multi-beta conditional CAPM models, with or without time-varying or random beta, typically do not enable researchers to exploit the potential panel nature of financial data sets. The most enduring method of estimating and testing these models is the two-pass Fama and MacBeth (1973) approach.[1] Other standard methods include the Generalized Method of Moments (GMM) approach of Harvey (1989) and MacKinlay and Richardson (1991)[2] and the Seemingly Unrelated Regression (SUR) approach of Gibbons (1982) and Gibbons, Ross and Shanken (1989).[3] Although these last two methods enable the researcher to use full information to simultaneously estimate beta and test the CAPM or conditional CAPM, they, like the first method, invariably specify heterogeneous coefficients on the conditional variables, without testing whether or not this is statistically optimal.[4] In this paper a different approach to the testing and estimation of conditional CAPM and multi-beta CAPM with time-varying or random beta[5] is proposed: a panel approach in which almost all aspects of the model are allowed to be homogeneous across all portfolios in the sample.

To date, theory says little about which conditional variables to use in the estimation and testing of conditional (and time-varying or random beta) CAPM models, or which additional factors to use in a multi-beta CAPM model; it also does not guide researchers as to whether the coefficients on these conditional variables should be heterogeneous or homogeneous.[6] Thus, in terms of

[1] This approach performs time-series estimation of beta in the first step, and then uses these beta estimates in the second stage cross-sectional regression of returns on the time-series estimates of beta along with any other variables thought to be correlated with the cross-section of returns. Examples of this approach include, inter alia, Banz (1981), Ferson and Harvey (1991), Fama and French (1992), Brennan, Chordia and Subrahmanyam (1998), and Ferson and Harvey (1999). The two-pass method suffers the classic "errors in variables" (EIV) problem. Shanken (1992) discusses the properties of this two-pass method using Ordinary Least Squares (OLS) and Generalized Least Squares (GLS), and compares the GLS version with the Gauss-Newton Maximum-Likelihood approach of Gibbons (1982). Related studies are Jagannathan and Wang (1998) and Ahn and Gadarowski (1999).

[2] For applications of the GMM approach see, inter alia, Jagannathan and Wang (1996) who offer an empirical comparison of the stacked GMM estimation approach with the two-pass method. Harvey (1989), Ferson and Harvey (1991, 1993) and Ferson and Korajczyk (1995) use the GMM approach to study time-variation in beta, and Ghysels (1998) uses the method to test for structural breaks and to evaluate pricing errors when such time-variation is mis-specified.

[3] Papers in a similar vein to Gibbons (1982) include Shanken (1990), Gibbons and Ferson (1985), and Elton, Gruber and Blake (1995). Wheatley (1989) provides a critique of Gibbons and Ferson (1985).

[4] A notable exception is Harvey (1989), who tests for the homogeneity of the conditional market price of risk across assets. Although the analytic framework he outlines is a panel in this sense, he only performs the estimation and test for individual assets, and not for a panel of assets.

[5] In the multi-factor CAPM context, beta is a vector and not a scalar.

[6] Berk (1995) shows that size-related regularities should be apparent in an economy and why size can explain that part of the cross-section of returns which remains unexplained due to model mis-specification. He maintains that size-related measures should be included in cross-sectional tests to detect model mis-specifications.

specifying conditional CAPM models, it is fair game to impose homogeneity restrictions on the conditional variables, and test for whether or not this is legitimate from a statistical point of view. Beta will remain heterogeneous even if (some or all of) its time-varying components are homogeneous, so the spirit of CAPM is not violated. If it is legitimate to impose homogeneity restrictions on the conditional variables, then there is potentially much to be gained, such as: 1) beta can be estimated with more precision than that obtained using other estimation and testing approaches; 2) the acquisition of di¤erent, and possibly more appropriate, quantitative values for beta; 3) greater power in testing the conditional CAPM since fewer nuisance parameters are included in the model; 4) improved power in testing for the presence of time-varying and random beta; 5) better out-of-sample forecasts of expected return; 6) better out-of-sample forecasts of beta;[7] and 7) improved understanding of the signi…cance of both conditional variables and multi-factors.

In this paper, a conditional form of Fama and French's (FF) (1993, 1995, 1996) three-factor model à la Ferson and Harvey (1999) is analyzed. Using this model has the advantage of facilitating the comparison of the new results with previous results, and it also avoids the speci…cation of a functional form for the expected market premiums.[8] An allowance is made for possible time-variation in both alpha and beta, and tests for whether or not the coe¢cients on the time-varying (conditional) components of alpha and beta are homogeneous are performed. Essentially, alpha and beta are modelled as having …xed-e¤ects with possible homogeneous time-variation in the parlance of the panel econometrics literature. The CAPM literature is also extended by testing for whether or not beta has a random, as well as a time-varying, component in the conditional FF model. Here, a modeling approach akin to Hildreth and Houck's (1968) random-coe¢cients methodology is employed but in a panel data setting.[9]

The Ferson and Harvey (1999) model embeds many classic models as special cases when the coe¢cients are heterogeneous or zero: unconditional (static) CAPM; the unconditional FF model; and the conditional single-factor CAPM. Thus, we also investigate whether the imposition of homogeneity constraints provides better forecasts of expected returns and beta than some of the previous models and econometric approaches.[10] As a natural by-product of this investigation, new evidence is provided on the testing of conditional CAPM, the conditional FF model of Ferson and Harvey (1999) and our conditional random-beta version of their model as well. Although it is beyond the scope of the present paper, this new approach to estimation of conditional (and multi-

[7] To the knowledge of the authors, very few papers have considered the properties of different model speci…cations in out-of-sample forecasting of beta.

[8] See Edwin J. Elton's 1999 American Finance Association Presidential Address for a call to develop better measures of expected returns.

[9] Although Ohlson and Rosenberg (1982), who model time-variation in beta, could, in principle, have estimated their model in a panel fashion, their study e¤ectively performed a time-series analysis. Further, they do not test for whether or not this stochastic parameter approach is better in a statistical sense than, say, models of time-varying but non-stochastic beta.

[10] In so doing, pricing errors are evaluated as in Ghysels (1998).

factor) CAPM will bear upon those areas which employ CAPM-based analysis: including, but not exclusive to, weighted-average cost of capital calculations, portfolio manager or fund performance measurement, the determination of the under or over-valuation of securities and investment projects, the analysis of international market integration and more generally risk analysis.

The results indicate that null hypotheses of zero time-variation in alpha and beta tend to be rejected in favor of homogeneous time-variation in those coefficients. Further the null of homogeneous time-variation in alpha and beta generally cannot be rejected against more heterogeneous alternatives. This last result does not hold if there is very little heterogeneity specified in other parts of the model; in this case, homogeneity in either alpha or beta can be rejected slightly more than 50% of the time in favor of heterogeneity. In order to avoid statistical problems associated with using multiple hypothesis tests[11], we also consider using model selection criteria to select the most appropriate model from the available candidates.[12] These methods suggest that the best models are the models with homogeneous intercept and time-varying alpha and beta, and the same plus randomness in beta. The FF model is also selected often (zero time-variation in alpha or beta), but the Ferson and Harvey (1999) version of the FF model with heterogeneous intercept and time-variation in alpha and beta is the least often selected.

Employing these findings on the appropriateness of homogeneous coefficients, whether or not such coefficient restrictions improve the model's ability to forecast returns and beta vis-à-vis the Ferson and Harvey (1999) and FF models is investigated. Within the context of this panel framework, the relevance of the additional two FF factors is also examined.

In terms of forecasting returns out of sample, the best performing models are those with zero time-variation in the coefficients. Overall, restricting the intercept to be homogeneous gives the best forecast, followed by heterogeneous and zero intercept. For the low portfolios (those that lie in the bottom quintile for size and book-to-market values), homogeneous intercept and time-variation in alpha performs well. The fully-heterogeneous model of Ferson and Harvey (1999) performs consistently worse than all other models in forecasting returns across all time periods and portfolios.

To forecast beta, modelling homogeneous time-variation in beta tends to yield better results than the more extreme assumptions of zero or heterogeneous time-variation in beta. Additional homogeneity in alpha is helpful for forecasting beta on the SMB and HML factors.

[11] Ferson and Harvey (1999) have a nice discussion of the data-mining criticisms often levied against this literature. They stress that there is no reason why a spurious time-series correlation with returns would yield a spurious ability to predict stock returns over time.

[12] Model selection criteria have, to the knowledge of the authors, not yet been employed is selecting between the various specifications of the CAPM. Such an analysis differs from traditional hypothesis testing schemes in that multiple hypotheses can be "tested" simultaneously - the null hypothesis model is not favored by the use of a low significance level (like 5%). Since a large number of potential specifications are available, the use of model selection criteria should provide considerable insight. See Granger, King and White (1995) for a full discussion.

Further, zero restrictions placed on the coefficients of the FF factors are always very strongly rejected in a panel context. Examination of the estimation results for the different models confirms this result: fully heterogenous parameter specifications lead to insignificant t-statistics on the FF factors for all portfolios, even in cases where all other models (including those with zero and homogenous time-variation in alpha and beta) evidence significant t-statistics. Generally, there is wide dispersion in both the estimates of beta and their associated t-statistics across all models compared.

The paper is organized as follows. Section 2 introduces the empirical methods to be employed in this paper. Here, we outline the model, the estimation procedure, the various hypotheses relating to testing of the model(s) and the homogeneity of coefficients, and the criteria for the best model and the best forecasts of expected return and beta. Section 3 discusses the data. Empirical results are discussed and presented in Section 4, along with a comparison of estimation results for the panel model, the FF model and the Ferson and Harvey (1999) model. The final section concludes.

2 Methodology

2.1 General Model

As in Ferson and Harvey's (1999) model (FH), we allow the returns generating process to be linear in the FF risk factors:

$$\begin{aligned} r_{i,t+1} = E_t(r_{i,t+1}) + \beta'_{i,t}\{r_{p:t+1} - E_t(r_{p,t+1})\} + \varepsilon_{i,t+1}; \\ E_t(\varepsilon_{i,t+1}) = 0; \\ E_t(\varepsilon_{i,t+1} r_{p,t+1}) = 0; \end{aligned} \quad (1)$$

where $r_{i,t+i}$ is the excess return on asset i, $E_t(.)$ indicates a conditional expectation given an information set available at time t, $r_{p,t+i}$ is a 3×1 vector of excess returns on the three FF risk factors: the market excess return, the excess return on a portfolio that is long in high book-to-market stocks and short in low book-to-market stocks (HML), and the excess return of a portfolio that is long in small, or low market capitalization, firms, and short in large, or high market capitalization, firms (SMB).[13] Equation (1) represents the unexpected return as a linear regression on the unexpected excess factor returns, hence the β_{it} are conditional betas of the return r_i on the factors.

Departing from FH, we assume the following model for the conditional expected returns and betas:

[13] Ferson and Harvey (1999) provide an excellent survey of the controversy involving the inclusion of these last two factors.

$$
\begin{aligned}
E_t(r_{i,t+1}) &= \alpha_{it} + \beta'_{it} E_t(r_{p,t+1}); \\
\beta_{it} &= b_{0i} + b'_{1i} Z_t + u_{it}; \\
E_t(u_{i,t}) &= 0; \\
\alpha_{it} &= a_{0i} + a'_{1i} Z_t;
\end{aligned}
\tag{2}
$$

where Z_t is an $L \times 1$ vector of mean-zero information variables (also known as conditioning or instrumental variables) available to the analyst at time t, α_{0i} is scalar, α_{1i} is $1 \times L$, β_{0i} is 3×1, β_{1i} is $3 \times L$ and u_{it} is an unobserved iid error term. Combining equations (1) with (2) gives the econometric model used in the present paper:

$$
r_{i,t+1} = (a_{0i} + a'_{1i} Z_t) + (b_{0i} + b'_{1i} Z_t + u_{it}) r_{p,t+1} + \varepsilon_{i,t+1}. \tag{3}
$$

All of the models considered here are nested within this general framework. This formulation, along with the usual assumption of Gaussianity, allows us to test parameter restrictions using least squares and likelihood based methods.

2.2 Homogeneity Testing

The homogeneity tests employed are standard. Ordinary least squares (OLS) is used to estimate (3) for all i portfolios simultaneously, with and without imposing homogeneity of the time-varying coefficients. Using the regression errors from the restricted and unrestricted regressions, F-statistics are calculated to evaluate whether or not the null of the homogeneity restriction can be rejected. For the case where we restrict a_{1i} and b_{1i} to be jointly homogenous across i, and u_{it} to be zero, for example, the restricted regression equation takes the form:

$$
r_{i,t+1} = (a_{0i} + a'_1 Z_t) + (b_{0i} + b'_1 Z_t) r_{p,t+1} + \varepsilon_{i,t+1}, \tag{4}
$$

which is a panel model with fixed effects on both alpha and beta, while the unrestricted regression has the general form:

$$
r_{i,t+1} = (a_{0i} + a'_{1i} Z_t) + (b_{0i} + b'_{1i} Z_t) r_{p,t+1} + \varepsilon_{i,t+1}. \tag{5}
$$

Tests such as this one, as well as a number of tests that exploit the general nature of (3) are performed in this paper. Note that the random component on beta is dropped in this exposition; the initial focus is on the time-varying version of the model, and not on the random beta version. The F-test for the null of homogeneous a'_1 and/or b'_1 across i against the general alternative is:

$$
F_{((NT-k_r)(NT-k_{ur}),(NT-k_{ur}))} = \frac{(RSS_r - RSS_{ur})/(NT-k_r)(NT-k_{ur})}{(RSS_{ur})/(NT-k_{ur})}, \tag{6}
$$

and we reject the null for large values of F. In (6), RSS_{ur} and RSS_r denote the residual sum of squares for the unrestricted (5) and restricted (4) regressions respectively; k_{ur} and k_r are the number of estimated slope parameters in the respective models, N is the number of portfolios contained in the data set and T is the number of time periods used to estimate each model. The F-test is calculated for 242 different starting times, where the restricted and unrestricted regressions are run on a rolling sample of data of length 100 periods.[14] The behavior of these tests across the rolling sample is also studied.

In addition to homogeneity tests on the time-varying coefficients, Likelihood Ratio (LR) statistics are employed to test for randomness in beta, and whether or not such randomness, if present, is homogeneous or heterogeneous. The likelihood function for the general model specification in (3), assuming Gaussian innovations, is of the form

$$L(\theta) = (2\pi)^{-NT/2} |\Omega|^{-1/2} \exp\left(\frac{-\epsilon' \Omega^{-1} \epsilon}{2} \right) \quad (7)$$

where $\epsilon = (\epsilon_{1;2}, \cdots, \epsilon_{1;T+1}, \cdots, \epsilon_{N;2}, \cdots, \epsilon_{N;T+1})'$ is a stacked vector containing the errors from (3) and

$$\Omega = \mathrm{diag}(1 + \theta_1 r^2_{p;2}, \cdots, 1 + \theta_1 r^2_{p;T+1}, \cdots, 1 + \theta_N r^2_{p;2}, \cdots, 1 + \theta_N r^2_{p;T+1}). \quad (8)$$

If θ_i is restricted to be zero for all i, the classical linear regression model results. A further restriction is possible since θ_i can be restricted to equal θ_j for all $i \neq j$. In this case, we have the situation where the variance in the randomness of the betas is assumed to be constant for all portfolios contained in the data set. Naturally, θ_i can be expressed as a ratio of variances and therefore must be non-negative. A more powerful one-sided test is therefore more appropriate for testing restrictions in this framework. In order to test these restrictions, we use the standard likelihood ratio test statistic

$$LR = 2(\log \widehat{L}_C - \log \widehat{L}_r) \quad (9)$$

which is asymptotically a probability weighted mixture of χ^2 distributions under the null hypothesis (see Gouriéroux, Holly and Monfort (1982) for details). Here, $\widehat{L}_C$ is the inequality-constrained, maximized likelihood and $\widehat{L}_r$ is the equality-constrained, maximized likelihood; we reject the null for large values of LR.

2.3 CAPM and Conditional CAPM Testing

Traditional tests of the CAPM consist of testing whether or not the intercept term is statistically different from zero. These tests are performed in the panel

[14] Shanken (1992) recognizes that while the literature uses such rolling betas to eliminate potential spurious cross-sectional relationships that are due to statistical dependence between returns and estimated betas, in fact contemporaneously estimated betas and mean returns are uncorrelated at the least, and independent under stricter assumptions (i.e. joint normality of the asset returns and factors). He also notes that this approach allows for changes in the true beta over time, and hence may be more robust to mis-specification error.

estimation context as they are with most methods, by using F-tests and t-tests. Regardless of the specification for beta, for the associated form of CAPM to hold statistically, it must be the case that the intercept is not significantly different from zero. Weaker forms of the test of a particular CAPM in a conditional context include testing whether or not the slopes on lagged instruments are equal to zero. If the model in question represents a minimum variance portfolio in a conditional context, both the intercepts and the slopes on the lagged instruments should be equal to zero. So, a strict form of the test is that a_{it} equals zero; a weaker version states that just the slopes on the conditional variables, a_{1i}^{0}, equal zero. In the panel context, the set of alternative hypotheses under which the null of CAPM or conditional CAPM is tested is richer. This set includes homogeneous, heterogenous and hybrid (heterogeneous time-variation in market beta and homogeneous time-variation in the FF factor betas) alternatives. One of the potential benefits of this technique is improved power in testing for whether or not conditional CAPM holds. This is due to the fact that if the general model is restricted, fewer degrees of freedom are devoted to the estimation of nuisance parameters. In this context it is also possible to test a time-varying and random beta version of the FF model as in Equation 3.

2.4 Model Selection

The discussion above has introduced a large number of possible CAPM specifications. Since finance theory does not guide us as to which model has more or less validity within the conditional or multi-factor CAPM framework, a statistical solution to the problem of model selection is desirable. In this paper we employ two different model selection criteria, each with very different properties, to select the most appropriate model from a range of candidates. The two criteria considered are Akaike's (1973) Information Criterion (AIC):

$$AIC_i = \log \hat{L}_i - k_i$$

and Schwarz's (1978) Bayesian Information Criterion (BIC):

$$BIC_i = \log \hat{L}_i - \frac{k_i \log NT}{2}.$$

These criteria are computed for each candidate model and that with the largest value is chosen. Here, k_i is the number of estimable parameters contained in model i and $\log \hat{L}_i$ is the maximized log-likelihood derived from model i. The relevant properties of these criteria are as follows. If the true model is contained in the set of candidates and is fixed and of finite dimension as $NT \to \infty$, BIC will select the true model with certainty asymptotically. BIC is therefore defined as a consistent model selection criterion. AIC is inconsistent in that it has a positive probability of selecting an overfitted[15] model asymptotically but will not fit an underfitted model in the limit. With fixed-effects-type models, the

[15] A model that nests the true model but contains extraneous parameters.

true process is not fixed and finite as $NT \to \infty$ since the number of fixed effects contained in the true model will be unbounded as $N \to \infty$: Shibata (1976) and Knight (1989) explore the asymptotics of AIC under these circumstances and find them to be optimal. Further, many authors have found that BIC is too severe on large models when applied in small (finite) samples (see Meese and Geweke (1984) for an example). In recent years, AIC has become more popular largely for these reasons. A further consideration is that AIC is relatively more favorable for large models than is BIC. This suggests that FH would be favored by AIC - if the panel models are chosen more readily by AIC, this would suggest that they are probably more appropriate specifications.

2.5 Forecasts of Returns and Beta

To determine how well the various models perform in terms of forecasting expected returns out of sample, the root mean squared one-step-ahead forecast error (RMSFE) is calculated using the forecast errors from the various models:

$$RMSFE = \sqrt{\frac{1}{n^0} \sum_{it} (r_{i;t+1} - \widehat{r_{i;t+1}})^2}; \quad (10)$$

where n^0 denotes the number of periods being forecasted or conversely the number of different rolling regressions employed in the study (i.e. 242) and $\widehat{r_{i;t+1}}$ indicates the predicted value of $r_{i;t+1}$, using information available at time t.

Out-of-sample forecasting of beta is somewhat unusual since the true value of beta, at any point in time, is never known with certainty; rather, it must be estimated using linear regression or some other statistical estimation procedure. Since the true model of expected returns is unknown, we do not know which of the available beta estimates is most appropriate. For this reason, several target models are arbitrarily chosen and are used to determine how well a model's forecast of beta predicts a target model's beta. Rolling regressions facilitate the analysis of how well the particular model would forecast at different points in time over the entire sample. Basically, the first rolling time series sample (i.e. observations $t = 1; \cdots; 100$) is used to estimate the beta for the forecasting model; the target beta is then estimated using the next 100 observations (i.e. $t = 101; \cdots; 200$). This process is then repeated by rolling the sample forward one observation at a time. The target beta ($\widehat{\beta'_{it+1}}$) takes the place of the actual beta and the beta from the forecasting model ($\widehat{\beta_{it}}$) is used as the predictor. Then the RMSFE is calculated as usual:

$$RMSFE = \sqrt{\frac{1}{n^0} \sum_{it} (\widehat{\beta'_{it+1}} - \widehat{\beta_{it}})^2}: \quad (11)$$

but in this case $n^0 = 142$.

Where betas are assumed to be time-varying, the average value of beta across 100 time series observations is used both as the target and the predictor.[16] This assumption allows us to directly compare the forecasting properties of time-varying betas with constant betas; we feel, however, that the use of time-varying parameter models for short term forecasting of beta may be a fruitful avenue for future research.

3 Data

The data consist of monthly returns on U.S. common stock portfolios from December 1964 through July 1993. These portfolios are constructed à la Fama and French (1993). There are 25 such portfolios. Individual equities are placed into five portfolios according to two independent criteria: the value of their prior equity market capitalization and their book to market value. The first portfolio consists of shares in the bottom 20% of the distribution of size and book to market value, while the 25th portfolio consists of shares in the top 20% of the distribution of these characteristics. A more thorough discussion of the construction of these portfolios and their summary statistics can be found in Ferson and Harvey (1999). Their data set is used for comparability purposes.

The conditioning variables (lagged), Z_t, used in this study are: 1) the difference between the lagged values of a one-month and a three-month treasury bill (as in Campbell (1987), Harvey (1989), Ferson and Harvey (1991, 1999)); 2) the dividend yield of the Standard and Poor's 500 (S&P 500) index (see Fama and French (1988) and Ferson and Harvey (1999)); 3) the spread between Moody's Baa and Aaa rated corporate bond yields (per Keim and Stambaugh (1986), Fama (1990) and Ferson and Harvey (1999)); 4) the spread between the 10-year and one-year treasury bond yields (see Fama and French (1989) and Ferson and Harvey (1999)); and 5) the one-month Treasury bill yield lagged one period (see Fama and Schwert (1977), Ferson (1989), Breen, Glosten and Jaganathan (1989) and Ferson and Harvey (1999)). Again, these variables are used in order to compare our results with those of previous studies.

4 Empirical Results

4.1 Is the Panel Framework Appropriate?

The first question to consider is whether or not the imposition of homogeneity restrictions is statistically valid. If it is, for which model specifications is it appropriate? There are three factors, and five conditional variables. There are many permutations of the question, from holding just one of the myriad of coefficients homogeneous to holding all except the fixed effect for the first excess

[16] We also considered using the first sample estimate of beta, the midpoint value of beta and the endpoint value of beta but the average was found to provide vastly superior forecast errors for all model specifications.

return factor (the market) homogeneous. Rather than mechanically exhausting the set of possibilities, we focus on the following types of questions. What is the evidence in favor of homogeneous time-varying beta over heterogeneous time-varying beta? Homogeneous time-varying alpha over heterogeneous time-varying alpha? Does this evidence change over time? The question of whether or not beta is random is also addressed in this framework, but will be discussed in Section 4.6. Once it has been established that homogeneity restrictions are appropriate, we can revisit such questions as: Does conditional CAPM hold, or do panel methods force a stronger rejection of CAPM? Are the additional FF factors super‡uous? Which version of the model does the data choose? Which model performs best at forecasting out-of-sample returns and beta? How do the coe¢cient estimates with homogeneity constraints compare to coe¢cients estimated without such constraints?

4.1.1 Time-varying Beta

In Table 1, evidence is presented on the question of whether or not the coe¢cients on time-varying beta are homogeneous. The null hypothesis of no time-varying beta (b_1 ´ zero) is evaluated against three di¤erent alternatives: homogeneous time-varying beta (b_1 ´ hom), an hybrid of heterogenous time-varying market factor with homogeneous time-varying beta for the SMB and HML factors (b_1 ´ hyb), and heterogeneous time-varying beta (b_1 ´ het). This is performed for the following speci…cations of a_0 and a_1: both alphas homogeneous ((a_0; a_1) ´ (hom; hom)), both alphas heterogeneous ((a_0; a_1) ´ (het; het), as in the time-varying alpha case of FH), homogeneous intercept with heterogeneous coe¢cients on the conditional variables ((a_0; a_1) ´ (hom; het)), heterogeneous intercept with homogeneous time-variation in alpha ((a_0; a_1) ´ (het; hom)), homogeneous intercept with zero time-variation in alpha ((a_0; a_1) ´ (hom; zero)), and heterogeneous intercept with zero time-variation in alpha((a_0; a_1) ´ (het; zero)). For all tests, b_0 remains heterogenous (b_0 ´ het) in line with CAPM theory. Table 1 provides the mean F-statistic over the 242 rolling regressions for these 36 di¤erent tests, along with the proportion of times the null hypothesis is rejected at the 5% level of signi…cance. The appropriate critical values are included.

The results in Table 1 suggest that the most consistent rejection of the null of zero time-variation in beta over time and across di¤erent speci…cations of alpha comes from the alternative that beta is time-varying but homogeneous across i. For this alternative, the proportion of rejections of the null is 100% over all subperiods and speci…cations of alpha. In comparison, the null is rejected less in favor of the alternative of heterogeneous time-varying beta as more heterogeneity in alpha is allowed (88%, 79% and 74% for (hom; hom), (hom; het), and (het; het) speci…cations of alpha respectively). The null tends to be rejected less over time as the proportion of rejections declines from 100% for the …rst subperiod to 70%, 56% and 50% for the last subperiod with progressively more heterogeneity assumed for alpha. These results lend credence to the notion that homogeneity restrictions on the model improve the power of the hypothesis tests employed.

Further, homogeneity restrictions generally cannot be rejected against heterogeneous alternatives, a phenomenon that becomes stronger over time. Regardless of alpha speci…cation, the null of homogeneous time-variation in beta against an hybrid alternative is rejected between 51 and 55% of the time over the entire sample. For the …rst third of the sample, the null is rejected 100% of the time and for the last third, rejection rates range from 0 to 4%. Similar results occur for the test of the null of homogeneous time-variation in beta against the alternative of heterogeneous b_1. Overall, the proportions of rejection range from 46 to 48%. The rejection proportions for this test are all 100% for the …rst third of the sample and 0% for the last third of the sample. The null of hybrid time-variation in beta against an heterogeneous alternative is rejected only around 37% of the time overall. It, too, faces rejection rates of around 100% for the …rst third of the data sample and 0% for the last third.

4.1.2 Time-varying Alpha

From Table 2 it is evident that the alternative that most often forces a rejection of the null of zero time-variation in alpha is homogeneous a_1. The F-statistic for this test lies around the value of 8 for all of the model speci…cations, with an associated critical value around 2:22. The average F-statistic for the test with the alternative of heterogenous time-variation in alpha is around 1:4, with a critical value of 1:23. Again, this supports the argument that homogeneity restrictions improve power.

Homogeneous time-variation in alpha cannot be rejected against the alternative of heterogeneous time-variation in alpha as long as the other model parameters (the intercept or the time-varying beta parameter) are speci…ed as having some heterogeneity. In such cases, the percentage of rejection of the null lies between 4 and 17%. When the intercept is restricted to be homogeneous and the time-varying beta is either zero, hybrid or homogeneous, the proportion of rejection of the null lies between 52 and 57%, indicating a slight preference for the heterogeneous time-varying alpha alternative to the homogeneous null. Over all model speci…cations, it is clear that the null of homogeneous time-varying alpha is rejected more over the last third than over the …rst third of the sample.

From this collective body of evidence we conclude that the data prefer homogeneous time-varying alpha and beta to either zero or heterogeneous time-variation. Since it is the case that homogeneity restrictions cannot be rejected in general, it is valuable to investigate whether or not such restrictions yield di¤erent results for the relevance of the HML and SMB portfolios, whether or not forecasts of expected return and beta can be improved by exploiting the panel nature of the data, and whether or not homogeneity constraints improve the power of conditional CAPM testing. Some of these questions are asked in the context of randomness in market beta as well.

4.2 Random Beta, Time-varying Beta and Alpha: Model Selection

In Tables 3 and 4, the question of which model is best over the entire sample and over time is addressed. Choosing models based on model selection criteria should help mitigate data-mining problems which may arise due to the sequential nature of hypothesis testing using F-tests. Although information criteria have not been widely used in the literature, we advocate this approach to choose which combination of factors and conditional variables are most appropriate. Table 3 contains summary results based on the AIC criterion for twenty-five model specifications. In this table, the proportion of times the model in question is selected by AIC across 242 starting times is reported. This is calculated over the entire sample, as well as for the three subperiods constituting the sample. "Best" indicates the proportion of times the particular model was ranked highest, "Top 3" indicates the proportion of times the model was ranked as one of the top 3 models, "Bottom 3" gives the proportion of times the model was listed in the bottom three places according to the model selection criterion, and "Worst" provides the proportion of times the model was ranked lowest.

The results are presented for homogeneous intercept (a_0) with either zero, homogeneous or heterogeneous time-varying alpha (a_1) and zero, homogeneous, hybrid or heterogeneous time-variation in beta (b_1); heterogeneous a_0 with either zero, homogeneous or heterogeneous a_1 and either zero, homogeneous, hybrid or heterogeneous time-variation in beta (b_1); and zero alpha and time-varying beta. Thus, the models considered include the FF model ($(a_0; a_1; b_0; b_1) \equiv (het; 0; het; 0)$; $(a_0; a_1; b_0; b_1) \equiv (0; 0; het; 0)$) which has no time-variation in alpha or beta, and the conditional FF model of Ferson and Harvey (1999) with heterogeneous time-varying alpha and/or beta ($(a_0; a_1; b_0; b_1) \equiv (het; het; het; 0)$; $(a_0; a_1; b_0; b_1) \equiv (het; het; het; het)$; and $(a_0; a_1; b_0; b_1) \equiv (het; 0; het; het)$). Table 4 contains the same information as Table 3, except that the model selection criterion employed is BIC.[17]

According to both criteria, the first best model overall is the "full panel": homogeneous intercept, homogeneous coefficients on the conditional factors (time-variation in alpha), heterogeneous beta - fixed effects in b_0 - and homogeneous time-variation in beta ($(a_0; a_1; b_0; b_1) \equiv (hom; hom; het; hom)$). It is chosen 45% of the time by AIC and 29% of the time by BIC. For AIC, the "full panel" falls in the "Top 3" ranking 90% of the time, and (het; hom; het; hom) falls in the "Top 3" 95% of the time. According to BIC, the "full panel" falls in the "Top 3" 84% of the time; more often than any other model. The results for the three subperiods suggest that, over time, models with more restrictions are more likely to be selected.

Conversely, the worst model according to AIC is in fact that with all heterogeneous coefficients (het; het; het; het), the model found in Ferson and Harvey (1999) to reject the conditional form of the FF model in the presence of time-variation in beta. This model is found to be the worst 44% of the time when

[17] The random beta versions are not included in this section; they are extensions of the (hom; hom; het; hom) model and are invariably chosen as the best model.

selected by AIC and 100% of the time when selected by BIC. This least parsimonious model lies in the "Bottom 3" most frequently according to both AIC (74%) and BIC (100%, tied with (hom; het; het; het)).

In summary, from Tables 3 and 4 it is evident that both the full panel and (het; hom; het; hom) models are selected most frequently and the worst model is generally (het; het; het; het). Over time, models that impose homogeneity or zero coefficient restrictions are preferred to those models that allow more coefficient heterogeneity. The BIC tends to be too parsimonious, as anticipated above, which is why models with zero restrictions on the time-varying elements are chosen quite frequently by BIC. This also explains why the fully heterogeneous model always performs the worst according to that criterion. In our opinion, AIC is the superior criterion, so the results on BIC should be discounted somewhat.

4.3 Conditional CAPM Testing Using Panel Methods

One of the important contributions of this paper is that more powerful tests of conditional CAPM are possible - in principle with or without the FF factors and random beta - because homogeneity restrictions on the parameters can be imposed. Sections 4.1 and 4.2 show that the "full panel" model cannot be rejected against more heterogeneous alternatives and is the most frequently selected model, along with random beta "full panel". Thus, it is valuable to include such models in the set of alternatives when it comes to testing conditional CAPM. In particular, it may be the case that the true null of zero time-varying alpha components may be rejected against the alternative of homogeneous time-varying alpha, while this null is accepted when the alternative is heterogeneous time-varying alpha.[18]

In Table 2, the answers to these types of questions can be found. F-tests of nulls of zero time-varying alpha (the weaker version of the conditional CAPM tests) against alternatives of homogeneous and heterogeneous time-varying alpha are presented, along with the proportion of times the null is rejected over the whole sample, the first third, second third and last third of the sample. All tests are performed with and without time-varying beta (all heterogeneous, homogeneous and hybrid versions).

The proportion of rejections of the null of zero time-variation in alpha against the alternative of homogeneous time-variation in alpha ranges from 88% to 93%, depending on the parameter specifications for the model. The proportion of rejections of the same null against an heterogeneous alternative ranges from 46% to 90%. The F-statistics for the former tests tend to be around the value of 8 (with critical values around 2:22), while the same statistics for the heterogeneous alternative lie around the 1:5 value (with critical values around 1:23). Essentially, the null of zero time-varying alpha is rejected against both of these alternatives, but the homogeneous alternative forces a stronger rejection

[18] Testing Conditional CAPM in the presence of randomness in beta may also yield different outcomes, but this is not explored here.

of the null because allowing for homogeneous alternatives lends greater power to the CAPM tests. The weak form of the conditional CAPM in the presence of the FF factors is rejected.[19]

From Table 2 it is also apparent that for the second third of the sample, the proportion of rejections of the zero time-varying alpha null against the alternative of heterogeneous time-variation in alpha is only around 27% for the case of heterogeneous intercept and time-varying beta and 17% for the case of heterogeneous intercept and homogeneous time-varying beta. Hence, for this subsample the weak-form of the conditional CAPM test implies that conditional CAPM holds. However, for the same null specifications over the same time-frame, the alternative of homogeneous time-variation in alpha is preferred to the null 99% and 100% of the time. Again, this illustrates the improved power of the test under the panel framework, and points to a potential weakness in standard CAPM tests that do not allow for homogeneous alternatives.

4.4 Useless Factors?

One of the problems with evaluating the usefulness of the FF factors using a two-pass approach is that the t-values of such factors may converge to infinity in a cross-sectional regression even when the true premium is zero (Kan and Zhang (1999), Jagannathan and Wang (1998), Ferson and Harvey (1999)). Since alternative estimation methods do not involve a pure cross-sectional regression stage, they do not face this criticism. Here, we investigate the question of whether or not the FF factors can be jointly or individually restricted to zero in the context of the "full panel" model since this model is selected most often by the information criteria, and such homogeneity restrictions improve the power of the tests vis-à-vis more heterogeneous versions of the same.[20] Essentially, zero restrictions placed on the coefficients of these factors are always resoundingly rejected. Thus, the FF factors are very important in a panel CAPM model.

4.5 Forecasting Expected Returns

The RMSFE and their associated rank for forecasting returns out of sample are listed in Table 5. The models that tend to forecast returns best according to RMSFE are the stripped-down FF versions of the model, with zero time-variation in alpha and beta, and zero or homogeneous intercepts; the worst, across all time periods, is always the heterogeneous time-varying alpha and beta model of Ferson and Harvey (1999). Over the entire sample period, across all portfolios, (hom; 0; het; 0) provides the best forecast of expected returns, followed by (het; 0; het; 0). For the first half of the sample, the best forecasting models are (0; 0; het; 0) and (hom; 0; het; 0). For the second half, the best models are (het; 0; het; 0) and (hom; 0; het; 0).

[19] Since the weak-form of the test is generally rejected, the strong-form of the test is not examined here.

[20] Although it is possible to do so, we do not examine the usefulness of the factors within a random beta model.

Table 5 also contains information on how well the various models forecast returns over time for the lowest portfolio (lowest quintile for both size and book-to-market ratio), the middle portfolio (third quintile of both sorting characteristics) and the highest portfolio (the largest quintile for both size and book-to-market ratio). For the low portfolio, the best forecasting model overall and for the first half of the sample is (hom; 0; het; 0), followed by the full panel, (hom; hom; het; hom). For the last half of the sample, the best model is (hom; hom; het; 0) followed by (hom; 0; het; 0).

Overall, (0; 0; het; 0) performs best for the middle portfolio, followed by (hom; 0; het; 0), which also forecasts best over the first sample half. The best forecasting model for the second half of the sample for this portfolio is (0; 0; het; 0). For the high portfolio, the best forecasting model overall and over the first half of the sample is also (0; 0; het; 0), followed by (hom; 0; het; 0). For the last half of the sample, the best forecasting models are (hom; het; het; 0) and (0; 0; het; 0).

Although Ferson and Harvey (1999) were not concerned with how well their model forecasted expected returns,[21] the above analysis suggests that their heterogeneous time-varying alpha and beta model performs consistently worse than other models, while the best models typically have zero time-variation at least in beta.

4.6 Random Market Beta

The average LR-statistic obtained for the test of the null of zero randomness in market beta against the alternative of homogeneous random market beta across the 242 starting times is 8:65. This statistic is asymptotically distributed as a probability weighted mixture of χ^2 distributions under the null hypothesis of no randomness in beta, such that χ^2_1 has 0.5 weight and a point mass at the origin (akin to χ^2_0) also has 0.5 weight. The critical value for the one-sided LR test at the 5% level of significance is therefore 2:706. Graph 1 illustrates the behavior of the statistic over time. It is apparent that in the second half of the sample the null is not rejected as much as it is in the first half or towards the sample's end.

The average LR-statistic for the null of no randomness in market beta against the alternative of heterogeneous randomness in market beta is 155. In this instance, one-sided critical values for the test are difficult to compute. Using a more conservative (in the sense the size is deliberately under-stated) two-sided test with 25 degrees of freedom at the 5% level of significance, the critical value is 37:65. The value of this statistic over time is in Graph 2. The LR-statistic is always above the critical value; the null is always overwhelmingly rejected.

The average LR-statistic for the test of homogeneous randomness in beta against the alternative of heterogeneous randomness in beta is 145. The critical value for the two-sided test with 24 degrees of freedom at the 5% level of

[21] Perhaps they were not interested in this issue since random walks (RW) often perform better than CAPM models at forecasting returns. It is likely that a RW would also outperform the more homogeneous model this paper introduces.

significance is 36.41. Thus, we conclude that the market beta displays heterogeneous randomness. For all of these tests, the values for the nuisance parameters are $(a_0; a_1; b_0; b_1) \equiv$ (hom; hom; het; hom).[22]

Graphs 3 and 4 display the average estimates of μ_i for the 25 portfolios present in the data. Note that for Portfolio 1, which includes firms that are both small and have low book-to-market values, the estimated value of μ_i is large relative to portfolios that are larger or have higher book-to-market values. Estimating μ_i seems to be important for firms that are particularly small and/or have a low book-to-market ratio.

4.7 Beta Forecasting

To evaluate how well the different models perform at forecasting beta out of sample, five target models are chosen and estimated using a one-step ahead sample of data. These estimated target betas are used as the actual betas in a RMSFE calculation. Twenty-seven models are used to forecast beta, including homogeneous and heterogeneous random beta versions of the full panel model. The estimated beta from these models are used as the predicted beta in the RMSFE calculations. In Tables 6 through 8 evidence is presented on how well these models predict the three factor betas for the five target models across all portfolios, the lowest portfolio, the middle portfolio and the highest portfolio.

The five target models are the FF models (zero time variation in alpha and beta, Target 1, (0; 0; het; 0) and Target 2, (het; 0; het; 0)), the Ferson and Harvey (1999) full heterogeneous time-variation version of their model, Target 3, (het; het; het; het), and the two models shown in sections 4.1 and 4.2 above to be the most valid in a statistical sense: Target 4, (hom; hom; het; hom), and Target 5, (hom; hom; het; hom; het). The first four elements of these coefficient vectors are defined as before; the last element indicates the assumed nature of the randomness modelled in beta. In the non-random beta models, this element is implicitly zero.

4.7.1 Market Beta

Table 6 presents the RMSFE and the associated rank of the forecasting model that best forecasts the beta on the market portfolio. According to the RMSFE, the best forecasting model across all portfolios for Targets 2 and 3 is the fully-heterogenous forecasting model without randomness in beta (het; het; het; het). The best forecasting model for the other three target models is (het; 0; het; hyb). Other models that perform well are (hom; 0; het; hyb) and (hom; het; het; het). Apparently, to forecast market beta for all portfolios, a fair degree of heterogeneity in the coefficients is desirable. Across all target models, the worst predictor in terms of RMSFE is (0; 0; het; 0), or the FF model.

[22] These values are chosen for the following reasons: 1) with more restrictions, there are less parameters to estimate, so it takes less time to estimate the restricted and unrestricted models; and 2) according to the F-tests above, homogeneity restrictions are generally better than no restrictions.

Interesting patterns arise when the RMSFE are analyzed on a portfolio-by-portfolio basis. The best predictor of market beta for the lowest portfolio is (hom; het; het; hyb) and the worst predictor is the model with heterogeneous randomness in beta. For the middle portfolio, the best predicting model for most target models is (0; 0; het; 0), and (hom; 0; het; hom) for Target 3 (the fully heterogeneous model), followed by the "full panel" model and (hom; 0; het; 0). The worst predictors for both the middle and the highest portfolios are (hom; het; het; het) and (het; het; het; het). The best predictors for the highest portfolio are (hom; 0; het; hom) followed by the "full panel," (hom; hom; het; hom), and (hom; 0; het; hyb). It appears that more successful prediction of market beta for middle size and book-to-market ratio stocks occurs when there is a fair bit of heterogeneity in the forecasting model's parameters, while the prediction of market beta for small and especially large size and book-to-market ratio stocks is best when the homogeneity restrictions are imposed.

4.7.2 SMB Beta

For purposes of forecasting out of sample beta on the SMB factor, the best overall model for all portfolios is the (het; hom; het; hom) for all but the random beta target model. The best forecasting model for that target is (het; hom; het; 0). The next best predicting models are the "full panel", (hom; hom; het; 0) and (het; het; het; hom). The worst predictor is the heterogenous random beta model. These results are in Table 7. Relative to the forecasting performance of the different models for market beta, successful forecasting of SMB beta is achieved when the forecasting models have more coefficient homogeneity.

The best forecasting model for the lowest portfolio for all target models except the one with random beta is (het; het; het; hom). The best predicting model for the random beta target is (het; het; het; 0). The worst forecasting models for all target betas is the "full panel" with heterogeneous random beta. For the middle portfolio, the best forecasting models are (hom; hom; het; 0) and (het; hom; het; 0), while the worst are (het; het; het; het), (het; het; het; hyb) and (hom; het; het; hom). Finally, to forecast beta on the SMB factor for the highest portfolio, it appears that models with a lot of homogeneity restrictions and random beta perform best: "full panel" with heterogeneous randomness in beta is the most preferred, followed by the same model with homogeneous randomness in beta. The predicting model with heterogeneous fixed effects and time-variation in alpha with no time-variation in beta (het; het; het; 0) is generally the worst. The worst model for forecasting Target 3 for the high portfolio is (hom; het; het; het).

4.7.3 HML Beta

In Table 8, the RMSFE for the forecasts of the HML beta in the target models are tabulated. Across all portfolios the best predicting model for all targets is (hom; 0; het; hom). The next best models are "full panel," and (het; 0; het; hom). For Targets 1 and 5 ((0; 0; het; 0) and (hom; hom; het; hom; het), respectively) the worst predicting model is (hom; het; het; het), while for the other targets it is (0; 0; het; 0). Again, more homogeneity is useful for forecasting this factor's beta compared with the market's beta.

For the low portfolio, the best predicting model is (het; hom; het; 0). The most suitable predicting model for the random beta full panel target, Target 5 is (het; het; het; 0). The worst model for predicting all targets for the low portfolio is the "full panel" with random beta. For the middle portfolio, (hom; 0; het; hom) is the best predicting model for all targets, followed by the "full panel and the "full panel" with homogeneous randomness in beta. Ranked lowest in forecasting ability is (het; het; het; het). The HML beta for the highest portfolio is best predicted by the "full panel" model with heterogeneous randomness in beta, and then by the "full panel" model with homogeneous randomness in beta. The worst predictors are the (het; 0; het; 0) and (het; hom; het; het) models.

Regarding forecast performance, the more heterogeneity in the forecasting model, the more over-parameterized it is; with zero time-variation in both alpha and beta, the models are under-parameterized. Homogeneity restrictions on the time-varying parameters alpha and/or beta seem to achieve better balance, especially for the HML and SMB factors, and for the high portfolio. The over-parameterized fully-heterogeneous FH model performs poorly for the middle and high portfolios for all factor betas.

4.8 A Comparison of Estimation Results

Using the results on model selection, a comparison of b_0 estimation results for the "full panel" model, the (het; hom; het; hom) model, the (hom; 0; het; hom) model, the "full panel" model with heterogeneous randomness in beta, (hom; hom; het; hom; het), the (hom; hom; het; 0) model, the FF model, (0; 0; het; 0), the (het; 0; het; 0) model, and the FH model, (het; het; het; het), is presented in Table 9.

From this table, it is evident that the precision with which the estimates of the beta coe¢ cients are estimated are improved greatly using panel methods where the coe¢ cients are restricted to be zero or homogeneous. In fact, the FH model often has insigni…cant t-statistics on the beta estimates and, on average, these t-statistics are the lowest of all the models considered. This is a direct consequence of the fact that this model has relatively few degrees of freedom because of the numerous parameters that need to be estimated. This is the case either when the equations are estimated singularly or when pooled estimation techniques are used. To further illustrate the point, the market beta for the low portfolio for the fully heterogeneous model is insigni…cant, while this is not the case for any of the other estimated models. The same result obtains for the SMB factor on the middle and low portfolios, and for the HML factor on the high portfolio. The models that generally have the highest t-statistics are (0; 0; het; 0), (hom; hom; het; 0), and (het; 0; het; 0). Exceptions occur for the factor estimates for the two portfolios which had insignificant t-statistics across all models: the SMB factor for the high portfolio and the HML factor for the low portfolio.

A glance over the coefficient estimates shows that there can be wide disparities in the coefficient estimates, too. The average disparity between the lowest and highest coefficients across all factors and portfolio categories is 0.21. The highest disparities occur for the HML factor and the lowest portfolios, both with average disparities of 0.46. The largest single disparity occurs for the low portfolio with the HML factor: 0.81. For the disaggregated portfolios, the "full panel" with heterogeneous randomness in beta, the model that would be most favored by information criteria, has the highest coefficient in 4 of 9 cases, while the fully heterogeneous model has the lowest coefficient with the same frequency.

Interestingly, across all portfolios and factors, the beta estimates are the same for the full panel and full heterogeneity models, but the full panel model betas are much more significant. Disaggregation points out further differences in these two models: different coefficient values obtain across all portfolios and factors, and the full panel model generally yields much more statistical significance than the fully-heterogeneous model.

Table 9 reveals some other interesting patterns as well. For the market and HML factors, the coefficients increase with the size of the portfolio, while for the SMB factor, the reverse pattern obtains. Over all portfolios, the coefficients on the market factor are larger than the coefficients for the SMB factor, which are in turn larger than the coefficients on the HML factor. The size of the t-statistics follow this pattern too.

5 Conclusion

Standard procedures for the estimation and testing of conditional CAPM models with time-varying or random beta typically do not exploit the panel nature of financial data sets. Using a model based on Ferson and Harvey's (1999) extension of the FF model to allow for time-varying alpha and beta, we show that the null hypothesis of homogeneity restrictions on the time-varying alpha and beta are generally not rejected, and that model selection criteria select models with homogeneity restrictions imposed upon the model parameters, and random beta versions of the same. We also demonstrate that zero randomness in beta can be rejected in favor of homogeneous or heterogeneous randomness, and that homogeneous randomness in beta is rejected in favor of heterogeneous randomness.

Since it is legitimate in a statistical sense to impose such homogeneity restrictions on the conditional variables, we illustrate that estimating and testing the conditional CAPM version of the FF model employing panel methods yields many benefits. First of all, the power of conditional CAPM tests are improved with the imposition of homogeneity constraints. We substantiate this with an example where in the last subsample of data, the weak form of the conditional

CAPM test could not be rejected on average against heterogenous time-variation in alpha, while the homogenous time-varying alpha alternative forced a strong rejection of CAPM for this subperiod.

Further, to forecast out of sample returns, models that impose homogeneity restrictions on the intercept and/or time-varying alpha perform well, though the FF model with zero intercept and time variation in alpha and beta perform best overall. The fully-heterogeneous model of Ferson and Harvey (1999) performs consistently worse than all other models in forecasting returns.

For forecasting beta, models with homogeneous time-variation in the intercept or alpha or beta perform well relative to the FF or FH models, especially for forecasting the beta on the HML and SMB factors. Modelling homogeneous time-variation in beta tends to yield better results than the more extreme assumptions of zero or heterogeneous time-variation in beta. Additional homogeneity in alpha is helpful for forecasting beta on the SMB and HML factors. For middle and high portfolios, the fully-heterogeneous model performs poorly for all three factors.

Finally, the panel framework provides a better understanding of the role of conditional variables and multi-factors, more precise estimates of beta relative to the FH-type fully-heterogenous models, and coe¢cient values that are qualitatively di¤erent to other standard pooled approaches to estimating beta.

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Table 1: *F-statistics and Proportions of Rejection for Tests on b_1, for Various Specifications of the Nuisance Parameters. Average Values over 242 Starting Periods are Presented, Along with a Breakdown of these Average Values Over the First, Second and Third Subsample of the Data. Critical Values for the Specific Test are Also Provided.*

Null. Hyp.	$H_o:b_1=zero$		$H_o:b_1=zero$		$H_o:b_1=zero$		$H_o:b_1=hom$		$H_o:b_1=hom$		$H_o:b_1=hyb$	
Alt. Hyp.	$H_a:b_1=hom$		$H_a:b_1=hyb$		$H_a:b_1=het$		$H_a:b_1=hyb$		$H_a:b_1=het$		$H_a:b_1=het$	
	F-stat.	*% Rej.*	*F-stat.*	*% Rej.*	*F-stat.*	*% Rej.*	*F-stat.*	*% Rej.*	*F-stat.*	*% Rej.*	*F-stat.*	*% Rej.*
Nuis. Param.	a_0 = het; a_1 = zero; b_0=het											
Overall	11.679	1.000	2.453	1.000	1.580	0.826	1.274	0.512	1.146	0.455	1.053	0.372
1st Third	10.372	1.000	2.599	1.000	1.919	1.000	1.587	1.000	1.532	1.000	1.394	0.988
2nd Third	16.274	1.000	2.941	1.000	1.662	0.852	1.238	0.531	1.041	0.358	0.941	0.123
3rd Third	8.351	1.000	1.811	1.000	1.153	0.625	0.994	0.000	0.861	0.000	0.821	0.000
Crit. Val.	1.671		1.216		1.136		1.229		1.138		1.166	
Nuis. Param.	a_0 = het; a_1 = hom; b_0=het											
Overall	11.426	1.000	2.443	1.000	1.590	0.789	1.294	0.512	1.166	0.463	1.069	0.372
1st Third	9.856	1.000	2.563	1.000	1.925	1.000	1.611	1.000	1.560	1.000	1.415	0.988
2nd Third	16.810	1.000	3.017	1.000	1.699	0.827	1.254	0.531	1.055	0.383	0.953	0.123
3rd Third	7.565	1.000	1.742	1.000	1.141	0.538	1.014	0.000	0.880	0.000	0.837	0.000
Crit. Val.	1.671		1.216		1.136		1.229		1.138		1.166	
Nuis. Param.	a_0 = hom; a_1 = het; b_0=het											
Overall	11.398	1.000	2.425	0.979	1.566	0.785	1.278	0.508	1.143	0.483	1.048	0.360
1st Third	9.834	1.000	2.491	1.000	1.860	1.000	1.538	1.000	1.496	1.000	1.368	0.951
2nd Third	16.722	1.000	3.016	1.000	1.686	0.790	1.263	0.506	1.045	0.444	0.936	0.123
3rd Third	7.590	1.000	1.759	0.938	1.146	0.563	1.029	0.013	0.884	0.000	0.837	0.000
Crit. Val.	1.671		1.217		1.137		1.229		1.139		1.166	
Nuis. Param.	a_0 = het; a_1 = het; b_0=het											
Overall	11.403	1.000	2.423	0.996	1.568	0.740	1.275	0.504	1.144	0.483	1.049	0.376
1st Third	9.850	1.000	2.526	1.000	1.878	1.000	1.572	1.000	1.512	1.000	1.373	0.988
2nd Third	16.722	1.000	2.992	1.000	1.681	0.716	1.238	0.506	1.039	0.444	0.938	0.136
3rd Third	7.588	1.000	1.743	0.988	1.139	0.500	1.013	0.000	0.877	0.000	0.834	0.000
Crit. Val.	1.671		1.217		1.137		1.229		1.139		1.166	
Nuis. Param.	a_0 = hom; a_1 = zero; b_0=het											
Overall	11.534	1.000	2.493	1.000	1.621	0.909	1.333	0.550	1.192	0.471	1.089	0.372
1st Third	10.271	1.000	2.586	1.000	1.921	1.000	1.585	1.000	1.538	1.000	1.403	0.988
2nd Third	16.048	1.000	3.034	1.000	1.728	0.951	1.361	0.605	1.114	0.407	0.986	0.123
3rd Third	8.243	1.000	1.851	1.000	1.209	0.775	1.049	0.038	0.921	0.000	0.876	0.000
Crit. Val.	1.671		1.216		1.136		1.229		1.138		1.166	
Nuis. Param.	a_0 = hom; a_1 = hom; b_0=het											
Overall	11.281	1.000	2.484	1.000	1.632	0.884	1.353	0.554	1.212	0.483	1.105	0.376
1st Third	9.757	1.000	2.550	1.000	1.926	1.000	1.609	1.000	1.565	1.000	1.424	1.000
2nd Third	16.575	1.000	3.110	1.000	1.766	0.951	1.378	0.617	1.129	0.444	0.997	0.123
3rd Third	7.463	1.000	1.782	1.000	1.198	0.700	1.068	0.038	0.940	0.000	0.892	0.000
Crit. Val.	1.671		1.216		1.136		1.229		1.138		1.166	

Table 2: *F-statistics and Proportions of Rejection for Tests on a_1, for Various Specifications of the Nuisance Parameters. Average Values over 242 Starting Periods are Presented, Along with a Breakdown of these Average Values Over the First, Second and Third Subsample of the Data. Critical Values for the Specific Test are Also Provided.*

Null. Hyp. *Alt. Hyp.*	$H_o: a_1$ =zero $H_a: a_1$ =hom		$H_o: a_1$ =zero $H_a: a_1$ =het		$H_o: a_1$ =hom $H_a: a_1$ =het		$H_o: a_1$ =zero $H_a: a_1$ =hom		$H_o: a_1$ =zero $H_a: a_1$ =het		$H_o: a_1$ =hom $H_a: a_1$ =het	
Nuis. Param.	a_0 = het; b_0 =het; b_1 = zero						a_0 = het; b_0 =het; b_1 = het					
	F-stat.	*% Rej.*	*F-stat.*	*% Rej.*	*F-stat.*	*% Rej.*	*F-stat.*	*% Rej.*	*F-stat.*	*% Rej.*	*F-stat.*	*% Rej.*
Overall	8.562	0.888	1.250	0.558	0.946	0.070	8.019	0.930	1.250	0.467	0.968	0.041
1st Third	9.166	0.864	1.286	0.667	0.959	0.049	8.364	0.864	1.258	0.506	0.963	0.000
2nd Third	4.638	0.802	0.999	0.210	0.848	0.000	6.251	0.988	1.116	0.272	0.903	0.000
3rd Third	11.923	1.000	1.468	0.800	1.032	0.163	9.460	0.938	1.378	0.625	1.039	0.125
Crit. Val.	2.218		1.224		1.229		2.219		1.226		1.230	
Nuis. Param.	a_0 = het; b_0 =het; b_1 = hom						a_0 = het; b_0 =het; b_1 = hyb					
	F-stat.	*% Rej.*	*F-stat.*	*% Rej.*	*F-stat.*	*% Rej.*	*F-stat.*	*% Rej.*	*F-stat.*	*% Rej.*	*F-stat.*	*% Rej.*
Overall	7.803	0.934	1.278	0.579	1.006	0.103	7.926	0.934	1.279	0.570	1.002	0.116
1st Third	7.703	0.864	1.280	0.778	1.013	0.086	7.943	0.864	1.281	0.691	1.004	0.062
2nd Third	6.137	1.000	1.138	0.173	0.932	0.000	6.247	1.000	1.143	0.247	0.932	0.012
3rd Third	9.591	0.938	1.417	0.788	1.076	0.225	9.610	0.938	1.414	0.775	1.072	0.275
Crit. Val.	2.218		1.225		1.229		2.218		1.225		1.229	
Nuis. Param.	a_0 = hom; b_0 =het; b_1 = zero						a_0 = hom; b_0 =het; b_1 = het					
	F-stat.	*% Rej.*	*F-stat.*	*% Rej.*	*F-stat.*	*% Rej.*	*F-stat.*	*% Rej.*	*F-stat.*	*% Rej.*	*F-stat.*	*% Rej.*
Overall	8.483	0.884	1.493	0.843	1.199	0.517	7.979	0.926	1.350	0.669	1.072	0.165
1st Third	9.079	0.864	1.470	0.951	1.151	0.457	8.283	0.864	1.367	0.704	1.077	0.198
2nd Third	4.572	0.790	1.331	0.580	1.194	0.457	6.213	0.975	1.248	0.543	1.041	0.025
3rd Third	11.839	1.000	1.681	1.000	1.254	0.638	9.461	0.938	1.436	0.763	1.100	0.275
Crit. Val.	2.218		1.224		1.229		2.218		1.226		1.230	
Nuis. Param.	a_0 = hom; b_0 =het; b_1 = hom						a_0 = hom; b_0 =het; b_1 = hyb					
	F-stat.	*% Rej.*	*F-stat.*	*% Rej.*	*F-stat.*	*% Rej.*	*F-stat.*	*% Rej.*	*F-stat.*	*% Rej.*	*F-stat.*	*% Rej.*
Overall	7.723	0.930	1.536	0.893	1.275	0.570	7.861	0.930	1.472	0.835	1.204	0.521
1st Third	7.620	0.864	1.472	0.951	1.213	0.531	7.860	0.864	1.433	0.840	1.163	0.519
2nd Third	6.049	0.988	1.495	0.728	1.303	0.494	6.190	0.988	1.392	0.667	1.191	0.457
3rd Third	9.523	0.938	1.641	1.000	1.309	0.688	9.555	0.938	1.592	1.000	1.257	0.588
Crit. Val.	2.218		1.224		1.229		2.218		1.225		1.229	

Table 3: *"Best" Gives the Proportion of Times a Model was Selected as the Best Model; "Top 3" Gives the Proportion of Times a Model was Ranked Best, Second or Third; "Bottom 3" Similarly Provides the Proportion of Times a Model Fell to One of the Bottom Three Places; and "Worst" Gives the Proportion of Times a Model was Selected as the Worst Model. "Overall" Indicates That the Rankings Were Performed for the 242 Rolling Regressions. 1st Third, 2nd Third and 3rd Third Breaks this Down into the First, Second and Third Subsamples. The AIC Was Used To Determine the Rankings.*

Param. Val.	**Models**																								
a_0	zero	hom	hom	hom	hom	hom	hom	hom	hom	hom	hom	hom	hom	het	het	het	het	het	het	het	het	het	het	het	het
a_1	zero	zero	zero	zero	zero	hom	hom	hom	hom	het	het	het	het	zero	zero	zero	zero	hom	hom	hom	hom	het	het	het	het
b_0	het	het	het	het	het	het	het	het	het	het	het	het	het	het	het	het	het	het	het	het	het	het	het	het	het
b_1	zero	zero	hom	hyb	het	zero	hom	hyb	het	zero	hom	hyb	het	zero	hom	hyb	het	zero	hom	hyb	het	zero	hom	hyb	het
	Best																								
Overall	0.00	0.00	0.00	0.00	0.00	0.00	0.45	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00	0.00	0.00	0.42	0.03	0.00	0.00	0.00	0.00	0.00
1st Third	0.00	0.00	0.00	0.00	0.00	0.00	0.27	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.10	0.00	0.00	0.00	0.47	0.10	0.01	0.00	0.00	0.00	0.00
2nd Third	0.00	0.00	0.00	0.00	0.00	0.00	0.38	0.07	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.54	0.00	0.00	0.00	0.00	0.00	0.00
3rd Third	0.00	0.00	0.00	0.00	0.00	0.00	0.69	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.00	0.00	0.00	0.25	0.00	0.00	0.00	0.00	0.00	0.00
	Top 3																								
Overall	0.00	0.00	0.38	0.01	0.00	0.05	0.90	0.13	0.00	0.00	0.00	0.00	0.00	0.00	0.40	0.00	0.00	0.00	0.95	0.14	0.02	0.00	0.00	0.00	0.00
1st Third	0.00	0.00	0.22	0.00	0.00	0.00	0.80	0.26	0.00	0.00	0.00	0.00	0.00	0.00	0.36	0.01	0.00	0.00	0.93	0.36	0.06	0.00	0.00	0.00	0.00
2nd Third	0.00	0.00	0.32	0.04	0.00	0.00	0.96	0.14	0.00	0.00	0.00	0.00	0.00	0.00	0.54	0.00	0.00	0.00	0.93	0.07	0.00	0.00	0.00	0.00	0.00
3rd Third	0.00	0.00	0.61	0.00	0.00	0.14	0.94	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.30	0.00	0.00	0.01	1.00	0.00	0.00	0.00	0.00	0.00	0.00
	Bottom 3																								
Overall	0.08	0.05	0.00	0.00	0.28	0.00	0.00	0.00	0.03	0.42	0.00	0.00	0.54	0.07	0.00	0.00	0.31	0.00	0.00	0.00	0.01	0.47	0.00	0.00	0.74
1st Third	0.12	0.14	0.00	0.00	0.10	0.00	0.00	0.00	0.06	0.68	0.00	0.00	0.41	0.11	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.84	0.00	0.00	0.52
2nd Third	0.11	0.00	0.00	0.00	0.30	0.00	0.00	0.00	0.00	0.57	0.00	0.00	0.43	0.11	0.00	0.00	0.14	0.01	0.00	0.00	0.00	0.57	0.00	0.00	0.77
3rd Third	0.00	0.00	0.00	0.00	0.44	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.78	0.00	0.00	0.00	0.79	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.95
	Worst																								
Overall	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.00	0.00	0.00	0.08	0.00	0.00	0.00	0.00	0.41	0.00	0.00	0.44
1st Third	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.14	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.67	0.00	0.00	0.19
2nd Third	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.57	0.00	0.00	0.43
3rd Third	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00	0.00	0.00	0.24	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.71

Table 4: *"Best" Gives the Proportion of Times a Model was Selected as the Best Model; "Top 3" Gives the Proportion of Times a Model was Ranked Best, Second or Third; "Bottom 3" Similarly Provides the Proportion of Times a Model Fell to One of the Bottom Three Places; and "Worst" Gives the Proportion of Times a Model was Selected as the Worst Model. "Overall" Indicates That the Rankings Were Performed for the 242 Rolling Regressions. 1st Third, 2nd Third and 3rd Third Breaks this Down into the First, Second and Third Subsamples. The BIC Was Used To Determine the Rankings.*

Param. Val.	***Models***																								
a_0	*zero*	*hom*	*hom*	*hom*	*hom*	*hom*	*hom*	*hom*	*hom*	*hom*	*hom*	*hom*	*hom*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*
a_1	*zero*	*zero*	*zero*	*zero*	*zero*	*hom*	*hom*	*hom*	*hom*	*het*	*het*	*het*	*het*	*zero*	*zero*	*zero*	*zero*	*hom*	*hom*	*hom*	*hom*	*het*	*het*	*het*	*het*
b_0	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*
b_1	*zero*	*zero*	*hom*	*hyb*	*het*	*zero*	*hom*	*hyb*	*het*	*zero*	*hom*	*hyb*	*het*	*zero*	*hom*	*hyb*	*het*	*zero*	*hom*	*hyb*	*het*	*zero*	*hom*	*hyb*	*het*
	Best																								
Overall	0.11	0.00	0.37	0.00	0.00	0.23	0.29	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1st Third	0.33	0.00	0.10	0.00	0.00	0.12	0.44	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2nd Third	0.00	0.00	0.68	0.00	0.00	0.00	0.32	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3rd Third	0.00	0.00	0.33	0.00	0.00	0.58	0.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Top 3																								
Overall	0.14	0.54	0.78	0.00	0.00	0.49	0.83	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.10	0.00	0.00	0.00	0.12	0.00	0.00	0.00	0.00	0.00	0.00
1st Third	0.38	0.36	0.79	0.00	0.00	0.72	0.64	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.00	0.00	0.00	0.05	0.00	0.00	0.00	0.00	0.00	0.00
2nd Third	0.01	0.42	1.00	0.00	0.00	0.04	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.22	0.00	0.00	0.00	0.31	0.00	0.00	0.00	0.00	0.00	0.00
3rd Third	0.04	0.85	0.54	0.00	0.00	0.71	0.86	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Bottom 3																								
Overall	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.58	0.00	0.00	0.00	0.42	0.00	0.00	0.00	1.00
1st Third	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.64	0.00	0.00	0.00	0.36	0.00	0.00	0.00	1.00
2nd Third	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.47	0.00	0.00	0.00	0.53	0.00	0.00	0.00	1.00
3rd Third	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.64	0.00	0.00	0.00	0.36	0.00	0.00	0.00	1.00
	Worst																								
Overall	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00
1st Third	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00
2nd Third	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00
3rd Third	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00

Table 5: *RMSFE and Associated Rank for One-Step Ahead Forecasts of Returns. "All Portfolios" Indicates which Forecsting Models Perform Best Across All Portfolios; "Low Portfolio" Refers to the Portfolio with the Lowest Size and Book-to-Market Values; "Middle Portfolio" Indicates the Portfolio with the Middle Size and Book-to-Market Values; and "High Portfolio" is the Portfolio with the Highest Size and Book-to-Market Values. "O" Reports Average RMSFE Over all the 242 Rolling Regressions. "F" Reports Average RMSFE Across the First 121 Rolling Regressions; and A27"L" Reports Average RMSFE Across the Last 121 Rolling Regressions.*

	Forecasting Models																								
a_0	*zero*	*hom*	*hom*	*hom*	*hom*	*hom*	*hom*	*hom*	*hom*	*hom*	*hom*	*hom*	*hom*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*
a_1	*zero*	*zero*	*zero*	*zero*	*zero*	*hom*	*hom*	*hom*	*hom*	*het*	*het*	*het*	*het*	*zero*	*zero*	*zero*	*zero*	*hom*	*hom*	*hom*	*hom*	*het*	*het*	*het*	*het*
b_0	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*
b_1	*zero*	*zero*	*hom*	*hyb*	*het*	*zero*	*hom*	*hyb*	*het*	*zero*	*hom*	*hyb*	*het*	*zero*	*hom*	*hyb*	*het*	*zero*	*hom*	*hyb*	*het*	*zero*	*hom*	*hyb*	*het*
Time	*All Portfolios*																								
O	2.7	2.7	2.7	2.8	3.1	2.7	2.8	2.9	3.1	2.7	2.8	2.9	3.2	2.7	2.8	2.8	3.1	2.7	2.8	2.9	3.1	2.7	2.9	3	3.3
	3	*1*	8	13	20	4	10	16	22	6	12	18	**24**	*2*	9	14	21	5	11	17	**23**	7	15	19	**25**
F	2.7	2.7	2.8	2.9	3.3	2.7	2.8	3	3.3	2.8	2.9	3.1	3.4	2.7	2.8	2.9	3.3	2.7	2.8	3	3.3	2.8	2.9	3.1	3.5
	1	*2*	6	14	20	4	9	16	22	8	12	18	**24**	*3*	7	15	21	5	10	17	**23**	11	13	19	**25**
L	2.6	2.6	2.7	2.7	2.9	2.6	2.8	2.8	2.9	2.6	2.8	2.8	3	2.6	2.7	2.7	2.9	2.6	2.8	2.8	3	2.7	2.8	2.9	3
	6	*2*	9	11	20	5	14	17	22	*3*	12	18	**24**	*1*	8	10	21	4	13	16	**23**	7	15	19	**25**
	Low Portfolio																								
O	5.6	5.6	5.6	5.7	6.4	5.6	5.7	5.8	6.4	5.8	5.9	6	6.7	5.6	5.7	5.7	6.5	5.6	5.8	5.8	6.5	5.8	6	6.1	6.9
	3	*1*	4	8	20	*2*	9	13	21	11	16	17	**24**	5	7	10	22	6	12	14	**23**	15	18	19	**25**
F	5.7	5.7	5.8	5.9	6.9	5.7	5.9	6	6.9	6.1	6.2	6.3	7.3	5.8	5.8	5.9	7	5.8	5.9	6	7	6.1	6.3	6.4	7.6
	3	*1*	5	10	20	*2*	8	12	21	14	16	17	**24**	4	7	11	22	6	9	13	**23**	15	18	19	**25**
L	5.5	5.4	1.8	5.5	5.8	5.4	5.6	5.6	5.9	5.4	5.6	5.7	6	5.5	5.5	5.5	5.9	5.5	2.8	5.6	6	5.5	5.7	5.8	6.1
	5	*2*	8	6	20	*1*	13	12	21	*3*	14	17	**24**	8	9	11	22	7	13	16	**23**	10	18	19	**25**

Table 5 (cont.): *RMSFE and Associated Rank for One-Step Ahead Forecasts of Returns. "All Portfolios" Indicates which Forecsting Models Perform Best Across All Portfolios; "Low Portfolio" Refers to the Portfolio with the Lowest Size and Book-to-Market Values; "Middle Portfolio" Indicates the Portfolio with the Middle Size and Book-to-Market Values; and "High Portfolio" is the Portfolio with the Highest Size and Book-to-Market Values. "O" Reports Average RMSFE Over all the 242 Rolling Regressions. "F" Reports Average RMSFE Across the First 121 Rolling Regressions; and "L" Reports Average RMSFE Across the Last 121 Rolling Regressions.*

	Forecasting Models																								
a_0	*zero*	*hom*	*hom*	*hom*	*hom*	*hom*	*hom*	*hom*	*hom*	*hom*	*hom*	*hom*	*hom*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*
a_1	*zero*	*zero*	*zero*	*zero*	*zero*	*hom*	*hom*	*hom*	*hom*	*het*	*het*	*het*	*het*	*zero*	*zero*	*zero*	*zero*	*hom*	*hom*	*hom*	*hom*	*het*	*het*	*het*	*het*
b_0	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*
b_1	*zero*	*zero*	*hom*	*hyb*	*het*	*zero*	*hom*	*hyb*	*het*	*zero*	*hom*	*hyb*	*het*	*zero*	*hom*	*hyb*	*het*	*zero*	*hom*	*hyb*	*het*	*zero*	*hom*	*hyb*	*het*
Time	*Middle Portfolio*																								
O	1.7	1.7	1.9	1.9	2	1.7	1.9	2	2.1	1.7	1.9	2	2.2	1.7	1.9	2	2	1.7	1.9	2	2.1	1.7	1.9	2	2.2
	1	*2*	8	14	19	5	10	16	**23**	6	12	20	**24**	*3*	9	15	21	4	13	17	22	7	11	18	**25**
F	1.7	1.7	1.8	2	2.1	1.7	1.8	2	2.3	1.7	1.8	2	2.4	1.7	1.8	2	2.2	1.7	1.8	2	2.3	1.8	1.8	2	2.5
	2	*1*	8	16	20	5	9	15	**23**	6	13	18	**24**	*3*	10	17	21	4	12	14	22	7	11	19	**25**
L	1.7	1.7	1.9	1.9	1.8	1.7	2	1.9	1.9	1.7	2	1.9	1.9	1.7	1.9	1.9	1.8	1.7	2	1.9	1.9	1.7	1.9	1.9	1.9
	1	*3*	19	13	8	4	**23**	17	11	7	**24**	21	15	*2*	20	14	9	5	**25**	18	10	6	22	16	12
	High Portfolio																								
O	2.3	2.4	2.5	2.5	3	2.4	2.5	2.6	3	2.4	2.5	2.6	3.1	2.4	2.5	2.5	3	2.4	2.5	2.6	3	2.5	2.6	2.7	3.2
	1	*2*	8	14	20	5	11	16	22	6	13	18	**24**	*3*	7	12	21	4	9	15	**23**	10	17	19	**25**
F	2.7	2.8	2.9	3.1	3.6	2.8	2.9	3	3.6	2.9	3	3.1	3.8	2.8	2.9	3.1	3.6	2.8	2.9	3	3.6	2.9	3	3.2	3.9
	1	*2*	10	17	22	5	7	15	20	8	12	18	**24**	*3*	9	16	**23**	4	6	14	21	11	13	19	**25**
L	1.9	1.9	1.9	1.9	2.2	1.9	2	2	2.4	1.9	2	2	2.3	1.9	1.9	1.9	2.2	1.9	2	2	2.4	1.9	2	2	2.4
	2	8	6	11	21	5	15	14	**24**	*1*	13	17	22	7	4	*3*	20	9	16	12	**23**	10	18	19	**25**

Table 6: *RMSFE and Associated Rank for One-Step Ahead Forecasts of Target Market b. There are Five Target Models. Target 1 is (a0, a1, b0, b1)=(0, 0, het, 0); Target 2 is (a_0, a_1, b_0, b_1)=(het, 0, het, 0); Target 3 is (a_0,a_1,b_0,b_1)=(het, het, het, het); Target 4 is (a_0,a_1,b_0,b_1)= (hom, hom, het, hom); and Target 5 is (a_0,a_1,b_0,b_1, q_i)=(hom, hom, het, hom, het). "All Portfolios" Indicates which Forecsting Models Perform Best Across All Portfolios; "Low Portfolio" Refers to the Portfolio with the Lowest Size and Book-to-Market Values; "Middle Portfolio" Indicates the Portfolio with the Third Quintile Size and Book-to-Market Values; and "High Portfolio" is the Portfolio with the Highest Size and Book-to-Market Values.*

	Forecasting Models																										
a_0	*zero*	*hom*	*hom*	*hom*	*hom*	*hom*	*hom*	*hom*	*hom*	*hom*	*hom*	*hom*	*hom*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*hom*	*hom*
a_1	*zero*	*zero*	*zero*	*zero*	*zero*	*hom*	*hom*	*hom*	*hom*	*het*	*het*	*het*	*het*	*zero*	*zero*	*zero*	*zero*	*hom*	*hom*	*hom*	*hom*	*het*	*het*	*het*	*het*	*hom*	*hom*
b_0	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*
b_1	*zero*	*zero*	*hom*	*hyb*	*het*	*zero*	*hom*	*hyb*	*het*	*zero*	*hom*	*hyb*	*het*	*zero*	*hom*	*hyb*	*het*	*zero*	*hom*	*hyb*	*het*	*zero*	*hom*	*hyb*	*het*	*hom*	*hom*
q_i	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*hom*	*het*
Tgt.	*All Portfolios*																										
1	0.18	0.18	0.17	0.15	0.16	0.18	0.17	0.16	0.16	0.17	0.16	0.16	0.15	0.18	0.17	0.15	0.15	0.18	0.17	0.16	0.16	0.17	0.16	0.16	0.15	0.16	0.17
	27	24	17	2	6	26	20	10	12	21	13	7	4	23	16	1	5	25	19	9	11	22	14	8	3	15	18
2	0.22	0.21	0.20	0.19	0.19	0.22	0.21	0.19	0.19	0.21	0.20	0.19	0.18	0.21	0.20	0.19	0.19	0.21	0.20	0.19	0.19	0.21	0.20	0.19	0.18	0.20	0.20
	27	25	18	8	4	26	22	12	10	21	14	5	2	23	16	6	3	24	19	11	9	20	13	7	1	15	17
3	0.23	0.23	0.22	0.20	0.20	0.23	0.22	0.21	0.21	0.22	0.21	0.21	0.20	0.23	0.22	0.20	0.20	0.23	0.22	0.21	0.21	0.22	0.21	0.21	0.20	0.21	0.22
	27	24	17	6	5	26	20	12	11	22	14	7	2	23	16	4	3	25	18	10	9	21	13	8	1	15	19
4	0.22	0.21	0.20	0.19	0.19	0.21	0.20	0.19	0.19	0.20	0.19	0.19	0.19	0.21	0.20	0.19	0.19	0.21	0.20	0.19	0.19	0.21	0.19	0.19	0.19	0.19	0.20
	27	24	17	2	4	26	20	10	12	21	13	7	6	23	16	1	3	25	18	9	11	22	14	8	5	15	19
5	0.20	0.19	0.18	0.17	0.17	0.19	0.18	0.17	0.18	0.19	0.18	0.17	0.17	0.19	0.18	0.17	0.17	0.19	0.18	0.17	0.18	0.19	0.18	0.17	0.17	0.18	0.19
	27	24	17	2	4	26	19	7	12	21	13	5	9	23	16	1	3	25	18	6	11	22	14	8	10	15	20
	Low Portfolio																										
1	0.38	0.38	0.36	0.30	0.33	0.38	0.37	0.30	0.33	0.35	0.33	0.29	0.31	0.37	0.35	0.30	0.33	0.37	0.35	0.30	0.33	0.34	0.33	0.29	0.31	0.34	0.41
	26	24	20	4	9	25	21	6	13	17	12	1	8	22	18	3	11	23	19	5	14	15	10	2	7	16	27
2	0.46	0.45	0.44	0.37	0.40	0.45	0.44	0.38	0.41	0.42	0.41	0.36	0.38	0.44	0.43	0.37	0.40	0.44	0.43	0.37	0.41	0.42	0.40	0.37	0.38	0.42	0.48
	26	24	20	4	9	25	21	6	13	17	12	1	8	22	18	3	11	23	19	5	14	15	10	2	7	16	27
3	0.46	0.45	0.44	0.38	0.40	0.45	0.44	0.38	0.41	0.42	0.40	0.36	0.38	0.44	0.43	0.37	0.40	0.44	0.43	0.38	0.41	0.42	0.40	0.37	0.38	0.42	0.49
	26	24	20	4	10	25	21	6	13	17	12	1	8	22	18	3	11	23	19	5	14	15	9	2	7	16	27
4	0.43	0.42	0.41	0.34	0.37	0.42	0.41	0.35	0.37	0.39	0.37	0.33	0.35	0.41	0.39	0.34	0.37	0.41	0.40	0.34	0.37	0.39	0.37	0.34	0.35	0.39	0.45
	26	24	20	4	9	25	22	6	13	17	12	1	8	21	18	3	10	23	19	5	14	15	11	2	7	16	27
5	0.33	0.33	0.32	0.26	0.30	0.33	0.32	0.27	0.30	0.30	0.29	0.26	0.29	0.32	0.31	0.26	0.30	0.32	0.31	0.27	0.31	0.31	0.29	0.26	0.29	0.30	0.37
	26	24	20	3	11	25	22	6	15	14	8	1	7	21	17	2	13	23	19	5	18	16	10	4	9	12	27

Table 6 (cont.): *RMSFE and Associated Rank for One-Step Ahead Forecasts of Target Market b. There are Five Target Models. Target 1 is (a0, a1, b0, b1)=(0, 0, het, 0); Target 2 is (a_0, a_1, b_0, b_1)=(het, 0, het, 0); Target 3 is (a_0,a_1,b_0,b_1)=(het, het, het, het); Target 4 is (a_0,a_1,b_0,b_1)=(hom, hom, het, hom); and Target 5 is (a_0,a_1,b_0,b_1, q_i)=(hom, hom, het, hom, het). "All Portfolios" Indicates which Forecsting Models Perform Best Across All Portfolios; "Low Portfolio" Refers to the Portfolio with the Lowest Size and Book-to-Market Values; "Middle Portfolio" Indicates the Portfolio with the Third Quintile Size and Book-to-Market Values; and "High Portfolio" is the Portfolio with the Highest Size and Book-to-Market Values.*

	Forecasting Models																										
a_0	zero	hom	hom	hom	hom	hom	hom	hom	hom	hom	hom	hom	hom	het	het	het	het	het	het	het	het	het	het	het	het	hom	hom
a_1	zero	zero	zero	zero	zero	hom	hom	hom	hom	het	het	het	het	zero	zero	zero	zero	hom	hom	hom	hom	het	het	het	het	hom	hom
b_0	het	het	het	het	het	het	het	het	het	het	het	het	het	het	het	het	het	het	het	het	het	het	het	het	het	het	het
b_1	zero	zero	hom	hyb	het	zero	hom	hyb	het	zero	hom	hyb	het	zero	hom	hyb	het	zero	hom	hyb	het	zero	hom	hyb	het	hom	hom
q_i	zero	zero	zero	zero	zero	zero	zero	zero	zero	zero	zero	zero	zero	zero	zero	zero	zero	zero	zero	zero	zero	zero	zero	zero	zero	hom	het
Tgt.	*Middle Portfolio*																										
1	0.08	0.09	0.10	0.09	0.09	0.09	0.10	0.09	0.09	0.10	0.10	0.10	0.11	0.09	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.11	0.09	0.09
	1	3	15	8	4	9	13	7	2	25	24	12	27	6	19	18	23	16	17	14	20	22	21	11	26	10	5
2	0.09	0.09	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.11	0.11	0.10	0.11	0.10	0.10	0.10	0.11	0.10	0.10	0.10	0.11	0.11	0.10	0.10	0.11	0.10	0.10
	1	2	12	11	5	9	10	7	3	25	22	15	27	6	16	19	24	17	14	18	23	21	20	13	26	8	4
3	0.11	0.11	0.10	0.11	0.12	0.12	0.11	0.12	0.12	0.13	0.12	0.12	0.14	0.12	0.11	0.12	0.13	0.12	0.11	0.12	0.14	0.13	0.12	0.12	0.14	0.11	0.11
	5	3	1	9	14	10	2	11	19	23	15	18	27	12	4	17	24	20	8	21	25	22	13	16	26	7	6
4	0.10	0.10	0.10	0.10	0.11	0.11	0.10	0.11	0.11	0.12	0.11	0.11	0.13	0.11	0.11	0.11	0.12	0.11	0.11	0.11	0.12	0.12	0.11	0.11	0.12	0.11	0.10
	1	2	3	6	13	10	4	8	14	23	21	17	27	11	9	18	24	19	12	20	25	22	16	15	26	7	5
5	0.10	0.10	0.10	0.10	0.11	0.10	0.10	0.10	0.11	0.12	0.11	0.11	0.12	0.10	0.10	0.11	0.12	0.11	0.10	0.11	0.12	0.11	0.11	0.11	0.12	0.10	0.10
	1	2	3	8	13	9	4	10	14	23	21	17	27	11	7	19	24	18	12	20	25	22	16	15	26	6	5
	High Portfolio																										
1	0.11	0.10	0.08	0.08	0.11	0.09	0.08	0.09	0.11	0.11	0.10	0.10	0.12	0.11	0.09	0.09	0.11	0.10	0.09	0.09	0.11	0.11	0.10	0.10	0.13	0.08	0.08
	18	12	1	2	19	11	5	6	22	23	14	13	26	20	7	8	21	17	9	10	25	24	16	15	27	4	3
2	0.11	0.11	0.08	0.09	0.12	0.10	0.09	0.09	0.12	0.12	0.11	0.11	0.13	0.12	0.09	0.10	0.12	0.11	0.10	0.10	0.12	0.12	0.11	0.11	0.14	0.09	0.09
	18	12	1	2	19	11	5	7	22	23	14	13	26	20	6	8	21	17	9	10	25	24	16	15	27	4	3
3	0.11	0.10	0.08	0.09	0.11	0.10	0.09	0.10	0.12	0.13	0.12	0.11	0.13	0.11	0.09	0.10	0.12	0.11	0.10	0.10	0.12	0.13	0.12	0.11	0.14	0.09	0.09
	13	12	1	5	15	11	2	7	20	24	19	16	26	17	6	8	21	14	9	10	23	25	22	18	27	3	4
4	0.14	0.13	0.11	0.11	0.14	0.13	0.11	0.12	0.15	0.15	0.14	0.13	0.16	0.14	0.12	0.12	0.14	0.14	0.12	0.13	0.15	0.15	0.14	0.14	0.16	0.11	0.11
	16	12	1	3	20	11	2	7	22	24	14	13	26	19	6	8	21	18	9	10	23	25	17	15	27	4	5
5	0.13	0.12	0.10	0.11	0.13	0.12	0.10	0.11	0.14	0.14	0.13	0.13	0.15	0.13	0.11	0.11	0.14	0.13	0.11	0.12	0.14	0.15	0.13	0.13	0.16	0.10	0.11
	14	12	1	4	20	11	2	7	22	24	15	13	26	19	6	8	21	17	9	10	23	25	18	16	27	3	5

Table 7: *RMSFE and Associated Rank for One-Step Ahead Forecasts of Target SMB* b*. There are Five Target Models. Target 1 is (a0, a1, b0, b1)=(0, 0, het, 0); Target 2 is* (a_0, a_1, b_0, b_1)*=(het, 0, het, 0); Target 3 is* (a_0, a_1, b_0, b_1)*=(het, het, het, het); Target 4 is* (a_0, a_1, b_0, b_1)*= (hom, hom, het, hom); and Target 5 is* $(a_0, a_1, b_0, b_1, q_i)$*=(hom, hom, het, hom, het). "All Portfolios" Indicates which Forecsting Models Perform Best Across All Portfolios; "Low Portfolio" Refers to the Portfolio with the Lowest Size and Book-to-Market Values; "Middle Portfolio" Indicates the Portfolio with the Third Quintile Size and Book-to-Market Values; and "High Portfolio" is the Portfolio with the Highest Size and Book-to-Market Values.*

	Forecasting Models																										
a_0	*zero*	*hom*	*hom*	*hom*	*hom*	*hom*	*hom*	*hom*	*hom*	*hom*	*hom*	*hom*	*hom*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*hom*	*hom*
a_1	*zero*	*zero*	*zero*	*zero*	*zero*	*hom*	*hom*	*hom*	*hom*	*het*	*het*	*het*	*het*	*zero*	*zero*	*zero*	*zero*	*hom*	*hom*	*hom*	*hom*	*het*	*het*	*het*	*het*	*hom*	*hom*
b_0	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*
b_1	*zero*	*zero*	*hom*	*hyb*	*het*	*zero*	*hom*	*hyb*	*het*	*zero*	*hom*	*hyb*	*het*	*zero*	*hom*	*hyb*	*het*	*zero*	*hom*	*hyb*	*het*	*zero*	*hom*	*hyb*	*het*	*hom*	*hom*
q_i	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*hom*	*het*
Tgt.	*All Portfolios*																										
1	0.30	0.30	0.28	0.30	0.29	0.29	0.28	0.29	0.29	0.30	0.28	0.30	0.29	0.30	0.28	0.29	0.29	0.29	0.28	0.29	0.29	0.29	0.28	0.30	0.29	0.29	0.33
	26	24	5	20	18	17	2	15	13	23	6	25	14	21	3	16	11	12	1	8	7	19	4	22	9	10	27
2	0.32	0.31	0.30	0.31	0.31	0.31	0.30	0.31	0.31	0.31	0.30	0.31	0.31	0.31	0.30	0.31	0.31	0.31	0.29	0.30	0.30	0.31	0.30	0.31	0.31	0.31	0.35
	26	25	5	19	18	17	2	12	11	24	6	22	14	23	3	15	13	16	1	8	7	21	4	20	9	10	27
3	0.34	0.33	0.33	0.34	0.34	0.33	0.32	0.34	0.34	0.33	0.32	0.34	0.33	0.33	0.32	0.33	0.33	0.33	0.32	0.33	0.33	0.33	0.32	0.34	0.33	0.34	0.39
	20	15	6	25	26	9	4	22	24	10	5	23	14	12	3	17	18	7	1	13	16	8	2	19	11	21	27
4	0.31	0.31	0.30	0.31	0.31	0.30	0.30	0.31	0.31	0.30	0.30	0.31	0.31	0.30	0.30	0.31	0.30	0.30	0.29	0.30	0.30	0.30	0.30	0.31	0.30	0.31	0.35
	24	18	4	23	22	8	2	21	19	15	6	26	20	12	3	17	14	7	1	11	9	10	5	25	13	16	27
5	0.26	0.25	0.25	0.26	0.26	0.25	0.25	0.26	0.26	0.25	0.26	0.27	0.26	0.25	0.25	0.26	0.26	0.25	0.25	0.26	0.26	0.25	0.26	0.27	0.26	0.26	0.31
	14	8	9	23	20	2	10	24	22	6	12	26	19	3	4	16	13	1	7	17	15	5	11	25	18	21	27
	Low Portfolio																										
1	0.71	0.70	0.68	0.73	0.74	0.70	0.67	0.72	0.73	0.69	0.66	0.70	0.69	0.69	0.67	0.72	0.72	0.69	0.66	0.71	0.71	0.67	0.64	0.69	0.67	0.72	0.93
	18	16	8	25	26	14	6	23	24	9	2	15	10	13	5	20	21	11	3	17	19	7	1	12	4	22	27
2	0.76	0.76	0.73	0.78	0.79	0.75	0.72	0.77	0.78	0.74	0.71	0.75	0.74	0.75	0.72	0.77	0.77	0.74	0.71	0.76	0.76	0.73	0.70	0.74	0.72	0.77	0.96
	18	16	8	25	26	14	6	23	24	10	2	15	9	13	5	20	21	12	3	17	19	7	1	11	4	22	27
3	0.83	0.83	0.81	0.86	0.87	0.82	0.80	0.86	0.86	0.80	0.78	0.82	0.81	0.82	0.79	0.84	0.85	0.81	0.79	0.84	0.84	0.79	0.76	0.81	0.79	0.86	1.08
	17	16	9	25	26	14	7	23	24	8	2	15	11	13	6	20	21	10	3	18	19	5	1	12	4	22	27
4	0.71	0.71	0.69	0.74	0.75	0.70	0.68	0.74	0.74	0.69	0.66	0.71	0.69	0.70	0.67	0.72	0.72	0.69	0.67	0.71	0.72	0.67	0.65	0.70	0.67	0.73	0.95
	17	16	9	25	26	14	7	23	24	8	2	15	10	13	5	20	21	11	3	18	19	6	1	12	4	22	27
5	0.40	0.39	0.41	0.46	0.47	0.39	0.42	0.47	0.47	0.34	0.36	0.41	0.40	0.37	0.38	0.43	0.44	0.36	0.39	0.43	0.45	0.34	0.36	0.40	0.39	0.46	0.76
	13	10	16	23	25	9	17	24	26	2	4	15	12	6	7	18	20	5	8	19	21	1	3	14	11	22	27

Table 7 (cont.): *RMSFE and Associated Rank for One-Step Ahead Forecasts of Target SMB b. There are Five Target Models. Target 1 is (a0, a1, b0, b1)=(0, 0, het, 0); Target 2 is (a_0, a_1, b_0, b_1)=(het, 0, het, 0); Target 3 is (a_0, a_1, b_0, b_1)=(het, het, het, het); Target 4 is (a_0, a_1, b_0, b_1)= (hom, hom, het, hom); and Target 5 is $(a_0, a_1, b_0, b_1, q_i)$=(hom, hom, het, hom, het). "All Portfolios" Indicates which Forecsting Models Perform Best Across All Portfolios; "Low Portfolio" Refers to the Portfolio with the Lowest Size and Book-to-Market Values; "Middle Portfolio" Indicates the Portfolio with the Third Quintile Size and Book-to-Market Values; and "High Portfolio" is the Portfolio with the Highest Size and Book-to-Market Values.*

	Forecasting Models																										
a_0	zero	hom	hom	hom	hom	hom	hom	hom	hom	hom	hom	hom	hom	het	het	het	het	het	het	het	het	het	het	het	het	hom	hom
a_1	zero	zero	zero	zero	zero	hom	hom	hom	hom	het	het	het	het	zero	zero	zero	zero	hom	hom	hom	hom	het	het	het	het	hom	hom
b_0	het	het	het	het	het	het	het	het	het	het	het	het	het	het	het	het	het	het	het	het	het	het	het	het	het	het	het
b_1	zero	zero	hom	hyb	het	zero	hom	hyb	het	zero	hom	hyb	het	zero	hom	hyb	het	zero	hom	hyb	het	zero	hom	hyb	het	hom	hom
q_i	zero	zero	zero	zero	zero	zero	zero	zero	zero	zero	zero	zero	zero	zero	zero	zero	zero	zero	zero	zero	zero	zero	zero	zero	zero	hom	het
Tgt.	*Middle Portfolio*																										
1	0.08	0.08	0.09	0.10	0.10	0.07	0.10	0.10	0.10	0.08	0.11	0.11	0.10	0.08	0.09	0.09	0.09	0.07	0.10	0.10	0.10	0.07	0.11	0.11	0.11	0.10	0.10
	7	6	10	13	12	2	18	21	19	4	25	26	22	5	8	11	9	1	17	20	16	3	23	27	24	15	14
2	0.09	0.09	0.09	0.10	0.10	0.08	0.10	0.10	0.10	0.08	0.11	0.11	0.10	0.08	0.09	0.09	0.09	0.08	0.10	0.10	0.10	0.08	0.11	0.11	0.11	0.10	0.10
	7	6	11	13	15	3	18	21	20	4	25	26	22	5	8	10	9	1	17	19	16	2	23	27	24	14	12
3	0.10	0.09	0.13	0.13	0.12	0.09	0.14	0.14	0.13	0.10	0.15	0.15	0.14	0.09	0.13	0.13	0.12	0.09	0.14	0.14	0.14	0.10	0.15	0.15	0.15	0.14	0.14
	5	2	10	11	8	1	18	20	14	7	27	24	17	4	12	13	9	3	21	22	15	6	25	26	23	19	16
4	0.10	0.09	0.12	0.12	0.12	0.09	0.13	0.12	0.12	0.09	0.13	0.13	0.13	0.09	0.12	0.12	0.12	0.09	0.13	0.13	0.13	0.09	0.13	0.13	0.14	0.12	0.13
	7	4	9	8	11	1	17	15	16	6	26	22	24	5	13	10	12	2	21	19	20	3	25	23	27	14	18
5	0.10	0.09	0.12	0.12	0.12	0.09	0.13	0.13	0.13	0.10	0.14	0.13	0.13	0.10	0.12	0.12	0.12	0.09	0.13	0.13	0.13	0.09	0.14	0.13	0.14	0.13	0.13
	7	4	9	8	10	1	17	16	15	6	26	22	24	5	13	11	12	2	21	18	19	3	25	23	27	14	20
	High Portfolio																										
1	0.28	0.28	0.26	0.24	0.25	0.27	0.25	0.23	0.25	0.29	0.27	0.25	0.27	0.28	0.27	0.25	0.26	0.28	0.26	0.24	0.26	0.29	0.27	0.26	0.27	0.22	0.20
	24	23	15	4	9	21	10	3	7	26	17	8	18	25	16	6	14	22	13	5	11	27	20	12	19	2	1
2	0.29	0.28	0.26	0.24	0.25	0.28	0.26	0.23	0.25	0.29	0.27	0.25	0.27	0.29	0.27	0.25	0.26	0.28	0.26	0.24	0.26	0.29	0.27	0.26	0.27	0.22	0.20
	24	23	15	4	9	21	10	3	7	26	17	8	18	25	16	6	13	22	14	5	11	27	20	12	19	2	1
3	0.21	0.21	0.20	0.18	0.21	0.20	0.20	0.18	0.21	0.22	0.21	0.20	0.23	0.22	0.21	0.20	0.22	0.21	0.20	0.20	0.22	0.22	0.21	0.21	0.23	0.16	0.15
	20	18	8	4	17	11	7	3	16	24	15	10	27	22	13	6	23	14	9	5	21	25	19	12	26	2	1
4	0.29	0.29	0.27	0.25	0.27	0.28	0.27	0.25	0.27	0.30	0.28	0.27	0.29	0.29	0.28	0.26	0.28	0.29	0.27	0.26	0.27	0.30	0.29	0.28	0.29	0.23	0.21
	24	20	11	4	9	17	8	3	7	26	18	10	22	25	16	6	14	19	13	5	12	27	23	15	21	2	1
5	0.24	0.23	0.22	0.20	0.22	0.23	0.21	0.20	0.21	0.24	0.23	0.22	0.23	0.24	0.22	0.21	0.22	0.23	0.22	0.21	0.22	0.25	0.23	0.22	0.23	0.18	0.16
	24	22	10	4	9	17	7	3	8	26	18	12	23	25	16	6	15	19	11	5	13	27	21	14	20	2	1

Table 8: *RMSFE and Associated Rank for One-Step Ahead Forecasts of Target HML* b *. There are Five Target Models. Target 1 is (a0, a1, b0, b1)=(0, 0, het, 0); Target 2 is* (a_0, a_1, b_0, b_1)*=(het, 0, het, 0); Target 3 is* (a_0,a_1,b_0,b_1)*=(het, het, het, het); Target 4 is* (a_0,a_1,b_0,b_1)*= (hom, hom, het, hom); and Target 5 is* (a_0,a_1,b_0,b_1, q_i)*=(hom, hom, het, hom, het). "All Portfolios" Indicates which Forecsting Models Perform Best Across All Portfolios; "Low Portfolio" Refers to the Portfolio with the Lowest Size and Book-to-Market Values; "Middle Portfolio" Indicates the Portfolio with the Third Quintile Size and Book-to-Market Values; and "High Portfolio" is the Portfolio with the Highest Size and Book-to-Market Values.*

	Forecasting Models																										
a_0	*zero*	*hom*	*hom*	*hom*	*hom*	*hom*	*hom*	*hom*	*hom*	*hom*	*hom*	*hom*	*hom*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*hom*	*hom*
a_1	*zero*	*zero*	*zero*	*zero*	*zero*	*hom*	*hom*	*hom*	*hom*	*het*	*het*	*het*	*het*	*zero*	*zero*	*zero*	*zero*	*hom*	*hom*	*hom*	*hom*	*het*	*het*	*het*	*het*	*hom*	*hom*
b_0	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*	*het*
b_1	*zero*	*zero*	*hom*	*hyb*	*het*	*zero*	*hom*	*hyb*	*het*	*zero*	*hom*	*hyb*	*het*	*zero*	*hom*	*hyb*	*het*	*zero*	*hom*	*hyb*	*het*	*zero*	*hom*	*hyb*	*het*	*hom*	*hom*
q_i	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*zero*	*hom*	*het*
Tgt.	*All Portfolio*																										
1	0.20	0.19	0.18	0.18	0.20	0.19	0.18	0.18	0.20	0.19	0.19	0.19	0.21	0.20	0.18	0.19	0.20	0.19	0.18	0.19	0.20	0.19	0.19	0.19	0.21	0.18	0.20
	24	16	1	3	21	12	2	6	22	18	10	14	27	19	5	9	23	15	7	11	25	17	8	13	26	4	20
2	0.24	0.23	0.22	0.22	0.23	0.23	0.22	0.22	0.23	0.23	0.22	0.22	0.24	0.23	0.22	0.22	0.23	0.23	0.22	0.22	0.23	0.23	0.22	0.22	0.24	0.22	0.24
	27	19	1	3	18	14	4	7	22	16	9	12	25	20	2	8	21	15	5	11	23	17	10	13	24	6	26
3	0.24	0.23	0.22	0.22	0.23	0.23	0.22	0.22	0.24	0.23	0.23	0.23	0.24	0.23	0.22	0.22	0.24	0.23	0.22	0.22	0.24	0.23	0.23	0.23	0.24	0.22	0.24
	27	18	1	3	20	12	4	8	23	16	10	13	25	19	2	5	21	14	6	9	24	17	11	15	26	7	22
4	0.23	0.22	0.20	0.20	0.22	0.21	0.20	0.21	0.22	0.21	0.21	0.21	0.23	0.22	0.20	0.21	0.22	0.21	0.21	0.21	0.22	0.21	0.21	0.21	0.23	0.20	0.22
	27	18	1	3	20	12	2	6	22	17	10	14	26	19	4	8	23	15	7	11	24	16	9	13	25	5	21
5	0.21	0.20	0.19	0.19	0.21	0.20	0.19	0.20	0.21	0.21	0.20	0.20	0.22	0.21	0.19	0.20	0.21	0.20	0.20	0.20	0.21	0.20	0.20	0.20	0.22	0.19	0.21
	25	15	1	4	21	11	2	6	22	18	9	14	27	20	5	10	23	16	7	12	24	17	8	13	26	3	19
	Low Portfolio																										
1	0.29	0.28	0.27	0.29	0.34	0.27	0.27	0.29	0.34	0.26	0.26	0.30	0.34	0.26	0.26	0.29	0.34	0.25	0.26	0.29	0.34	0.25	0.26	0.30	0.33	0.30	0.41
	13	12	10	16	24	9	11	15	23	3	8	18	22	4	7	17	26	1	5	14	25	2	6	19	21	20	27
2	0.40	0.39	0.39	0.41	0.44	0.38	0.39	0.41	0.44	0.37	0.37	0.41	0.44	0.37	0.37	0.40	0.44	0.36	0.37	0.40	0.44	0.37	0.37	0.41	0.43	0.41	0.52
	13	12	11	17	26	9	10	16	25	3	7	18	22	8	6	15	24	1	4	14	23	2	5	19	21	20	27
3	0.33	0.32	0.32	0.33	0.37	0.31	0.32	0.33	0.37	0.31	0.31	0.35	0.37	0.30	0.30	0.33	0.37	0.30	0.30	0.33	0.37	0.31	0.31	0.35	0.37	0.34	0.43
	13	12	10	16	23	8	11	17	25	6	9	19	26	2	3	15	21	1	4	14	22	5	7	20	24	18	27
4	0.33	0.32	0.31	0.33	0.37	0.31	0.31	0.33	0.37	0.30	0.30	0.34	0.37	0.30	0.30	0.33	0.37	0.29	0.30	0.33	0.37	0.29	0.30	0.34	0.36	0.34	0.44
	14	12	10	16	25	9	11	17	26	3	8	18	22	5	6	15	24	1	4	13	23	2	7	19	21	20	27
5	0.26	0.26	0.26	0.28	0.31	0.25	0.26	0.28	0.31	0.25	0.26	0.29	0.32	0.26	0.26	0.29	0.32	0.25	0.26	0.29	0.32	0.25	0.25	0.29	0.32	0.28	0.37
	13	11	9	16	22	5	6	15	21	3	7	19	24	8	12	20	26	2	10	17	25	1	4	18	23	14	27

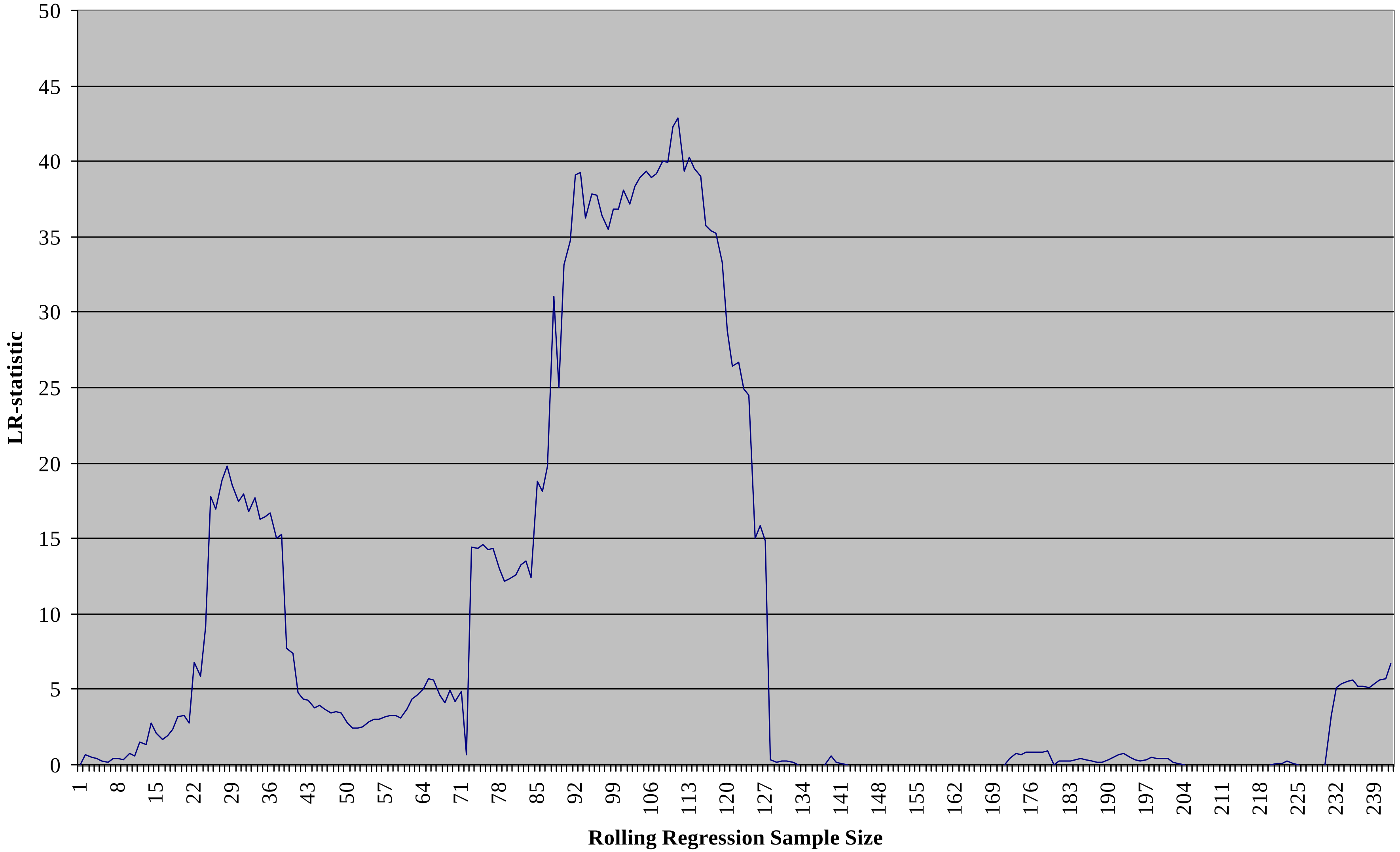
Table 8 (cont.): *RMSFE and Associated Rank for One-Step Ahead Forecasts of Target HML b. There are Five Target Models. Target 1 is (a0, a1, b0, b1)=(0, 0, het, 0); Target 2 is (a_0, a_1, b_0, b_1)=(het, 0, het, 0); Target 3 is (a_0,a_1,b_0,b_1)=(het, het, het, het); Target 4 is (a_0,a_1,b_0,b_1)=(hom, hom, het, hom); and Target 5 is (a_0,a_1,b_0,b_1, q_i)=(hom, hom, het, hom, het). "All Portfolios" Indicates which Forecsting Models Perform Best Across All Portfolios; "Low Portfolio" Refers to the Portfolio with the Lowest Size and Book-to-Market Values; "Middle Portfolio" Indicates the Portfolio with the Third Quintile Size and Book-to-Market Values; and "High Portfolio" is the Portfolio with the Highest Size and Book-to-Market Values.*

	Forecasting Models																										
a_0	zero	hom	hom	hom	hom	hom	hom	hom	hom	hom	hom	hom	hom	het	het	het	het	het	het	het	het	het	het	het	het	hom	hom
a_1	zero	zero	zero	zero	zero	hom	hom	hom	hom	het	het	het	het	zero	zero	zero	zero	hom	hom	hom	hom	het	het	het	het	hom	hom
b_0	het	het	het	het	het	het	het	het	het	het	het	het	het	het	het	het	het	het	het	het	het	het	het	het	het	het	het
b_1	zero	zero	hom	hyb	het	zero	hom	hyb	het	zero	hom	hyb	het	zero	hom	hyb	het	zero	hom	hyb	het	zero	hom	hyb	het	hom	hom
q_i	zero	zero	zero	zero	zero	zero	zero	zero	zero	zero	zero	zero	zero	zero	zero	zero	zero	zero	zero	zero	zero	zero	zero	zero	zero	hom	het
Tgt.	*Middle Portfolio*																										
1	0.21	0.20	0.17	0.18	0.21	0.19	0.17	0.19	0.22	0.21	0.19	0.20	0.23	0.21	0.18	0.19	0.22	0.20	0.18	0.19	0.22	0.21	0.19	0.20	0.23	0.18	0.19
	21	14	1	5	22	12	2	7	23	19	9	15	26	18	4	10	24	16	6	13	25	20	11	17	27	3	8
2	0.22	0.21	0.18	0.19	0.22	0.20	0.18	0.20	0.22	0.22	0.20	0.21	0.23	0.22	0.19	0.20	0.23	0.21	0.20	0.21	0.23	0.22	0.20	0.21	0.24	0.19	0.20
	22	14	1	5	21	9	2	8	23	19	10	16	26	18	4	11	24	15	7	13	25	20	12	17	27	3	6
3	0.28	0.26	0.24	0.25	0.27	0.26	0.24	0.25	0.28	0.27	0.26	0.27	0.29	0.27	0.25	0.26	0.28	0.27	0.25	0.26	0.28	0.27	0.26	0.27	0.29	0.24	0.25
	22	14	1	5	20	9	2	8	23	19	10	15	26	18	4	11	24	16	7	13	25	21	12	17	27	3	6
4	0.24	0.22	0.20	0.21	0.24	0.22	0.20	0.22	0.24	0.24	0.22	0.23	0.25	0.24	0.21	0.22	0.24	0.23	0.22	0.22	0.25	0.24	0.22	0.23	0.25	0.21	0.21
	22	13	1	4	18	9	2	7	23	20	10	16	26	19	5	11	24	15	8	14	25	21	12	17	27	3	6
5	0.24	0.22	0.20	0.21	0.24	0.22	0.20	0.21	0.24	0.23	0.22	0.23	0.25	0.23	0.21	0.22	0.24	0.23	0.21	0.22	0.24	0.24	0.22	0.23	0.25	0.20	0.21
	21	13	1	5	20	9	2	8	23	19	10	16	26	18	4	11	24	15	7	14	25	22	12	17	27	3	6
	High Portfolio																										
1	0.17	0.16	0.15	0.15	0.17	0.16	0.15	0.15	0.17	0.16	0.14	0.15	0.17	0.18	0.16	0.16	0.17	0.17	0.16	0.16	0.17	0.16	0.14	0.15	0.16	0.13	0.13
	25	18	7	10	20	17	8	9	21	16	4	6	22	27	14	11	23	26	15	12	24	13	3	5	19	2	1
2	0.19	0.18	0.16	0.16	0.18	0.18	0.16	0.16	0.18	0.17	0.16	0.16	0.19	0.19	0.17	0.17	0.19	0.19	0.17	0.17	0.19	0.17	0.16	0.16	0.18	0.14	0.14
	24	18	6	9	20	17	7	10	21	16	4	8	22	27	13	11	23	26	15	12	25	14	3	5	19	2	1
3	0.18	0.17	0.15	0.16	0.17	0.17	0.15	0.16	0.17	0.16	0.15	0.15	0.17	0.18	0.16	0.16	0.17	0.18	0.16	0.16	0.17	0.16	0.15	0.15	0.16	0.14	0.13
	26	22	8	10	20	18	7	9	19	14	5	6	21	27	16	12	24	25	15	11	23	13	4	3	17	2	1
4	0.19	0.18	0.16	0.16	0.18	0.17	0.16	0.16	0.19	0.17	0.16	0.16	0.19	0.19	0.17	0.17	0.19	0.19	0.17	0.17	0.19	0.17	0.16	0.16	0.18	0.14	0.13
	22	18	3	8	19	14	6	10	21	16	7	9	24	27	15	11	25	23	17	12	26	13	4	5	20	2	1
5	0.18	0.17	0.15	0.15	0.18	0.16	0.15	0.15	0.18	0.16	0.15	0.15	0.18	0.18	0.16	0.16	0.18	0.18	0.16	0.16	0.18	0.16	0.15	0.15	0.18	0.13	0.12
	22	18	5	9	19	14	4	8	20	15	7	10	25	24	16	11	26	23	17	13	27	12	3	6	21	2	1

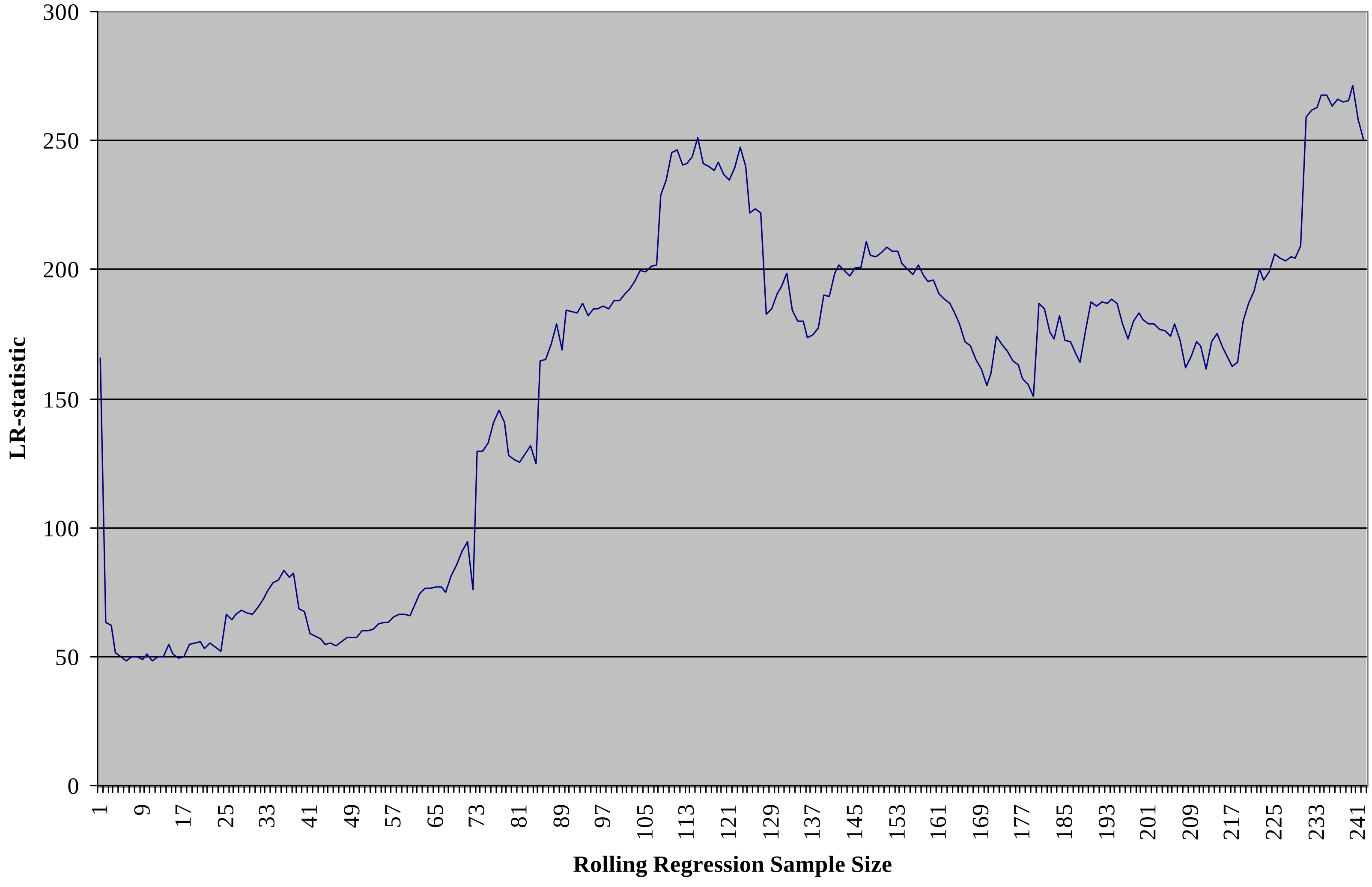
Table 9: *Comparison of Market, SMB and HML b_0 Estimates and t-statistics for Nine Models. Estimates Provided For All Portfolios; Low Portfolio; Middle Portfolio; and the High Portfolio. The Low Portfolio has the Lowest Values of Size and Book-to-Market Values. Middle Has the Third Quintile Values; and the High Portfolio Has the Highest Size and Book-to-Market Values.*

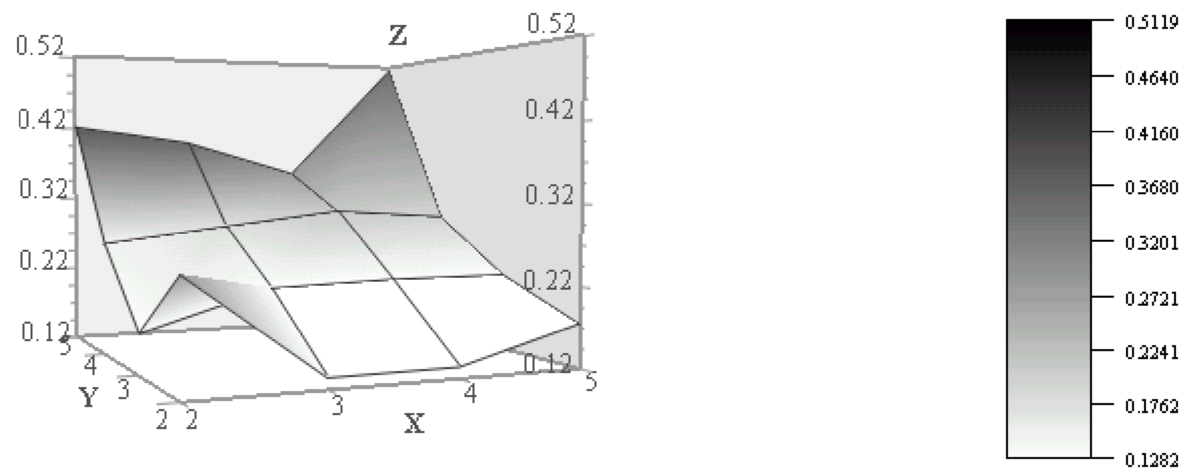
	b_0 Est.	*t-stat.*	*b_0 Est.*	*t-stat.*	*b_0 Est.*	*t-stat.*
Model	*Market*		*SMB*		*HML*	
(a_0,a_1,b_0,b_1,q_i)	*All Portfolios*		*All Portfolios*		*All Portfolios*	
(0,0,het,0,0)	0.91	18.14	0.56	6.33	0.21	2.60
(hom,0,het,hom,0)	0.75	7.57	0.66	3.45	0.18	1.19
(hom,hom,het,0,0)	0.89	17.81	0.54	6.21	0.18	2.18
(hom,hom,het,hom,0)	0.76	7.21	0.60	2.88	0.18	1.29
(het,0,het,0,0)	0.90	17.20	0.56	6.41	0.19	2.31
(het,hom,het,hom,0)	0.76	7.21	0.60	2.94	0.18	1.28
(het,het,het,het,0)	0.76	2.53	0.60	0.93	0.18	0.40
(hom,hom,het,hom,hom)	0.77	7.06	0.60	3.00	0.21	1.34
(hom,hom,het,hom,het)	0.81	7.87	0.61	3.19	0.29	1.80
(a_0,a_1,b_0,b_1,q_i)	*Low Portfolio*		*Low Portfolio*		*Low Portfolio*	
(0,0,het,0,0)	0.59	5.15	1.20	7.63	-0.08	-0.30
(hom,0,het,hom,0)	0.43	3.40	1.30	5.67	-0.12	-0.30
(hom,hom,het,0,0)	0.57	5.08	1.18	7.54	-0.12	-0.51
(hom,hom,het,hom,0)	0.44	3.45	1.24	5.16	-0.11	-0.18
(het,0,het,0,0)	0.55	4.78	1.18	7.61	-0.16	-0.59
(het,hom,het,hom,0)	0.42	3.28	1.22	5.17	-0.16	-0.38
(het,het,het,het,0)	0.35	0.80	1.42	1.38	0.65	1.13
(hom,hom,het,hom,hom)	0.44	4.49	1.28	6.79	-0.07	0.06
(hom,hom,het,hom,het)	0.56	3.40	1.52	7.06	0.11	0.66
(a_0,a_1,b_0,b_1,q_i)	*Middle Portfolio*		*Middle Portfolio*		*Middle Portfolio*	
(0,0,het,0,0)	0.90	20.97	0.52	8.64	0.25	3.80
(hom,0,het,hom,0)	0.73	7.87	0.61	3.32	0.21	1.42
(hom,hom,het,0,0)	0.88	20.25	0.49	8.11	0.21	3.23
(hom,hom,het,hom,0)	0.75	7.46	0.55	2.71	0.22	1.53
(het,0,het,0,0)	0.89	20.60	0.51	8.47	0.24	3.66
(het,hom,het,hom,0)	0.76	7.64	0.55	2.77	0.24	1.64
(het,het,het,het,0)	0.87	3.51	0.39	0.56	0.09	0.35
(hom,hom,het,hom,hom)	0.76	6.96	0.55	2.58	0.24	1.52
(hom,hom,het,hom,het)	0.79	8.17	0.53	2.81	0.32	1.99
(a_0,a_1,b_0,b_1,q_i)	*High Portfolio*		*High Portfolio*		*High Portfolio*	
(0,0,het,0,0)	1.05	14.41	0.01	0.36	0.69	7.10
(hom,0,het,hom,0)	0.88	8.18	0.11	0.70	0.65	3.46
(hom,hom,het,0,0)	1.03	14.85	-0.01	0.12	0.65	6.91
(hom,hom,het,hom,0)	0.90	7.78	0.05	0.28	0.66	3.45
(het,0,het,0,0)	1.05	14.01	0.02	0.35	0.69	6.76
(het,hom,het,hom,0)	0.91	7.89	0.05	0.30	0.68	3.57
(het,het,het,het,0)	1.06	2.56	0.31	0.01	0.39	0.53
(hom,hom,het,hom,hom)	0.91	8.12	0.03	0.16	0.67	3.49
(hom,hom,het,hom,het)	0.95	8.87	0.02	0.00	0.74	4.05

Graph 1: LR-statistic for Null of No Randomness in Beta Against Alternative of Homogeneous Randomness in Beta

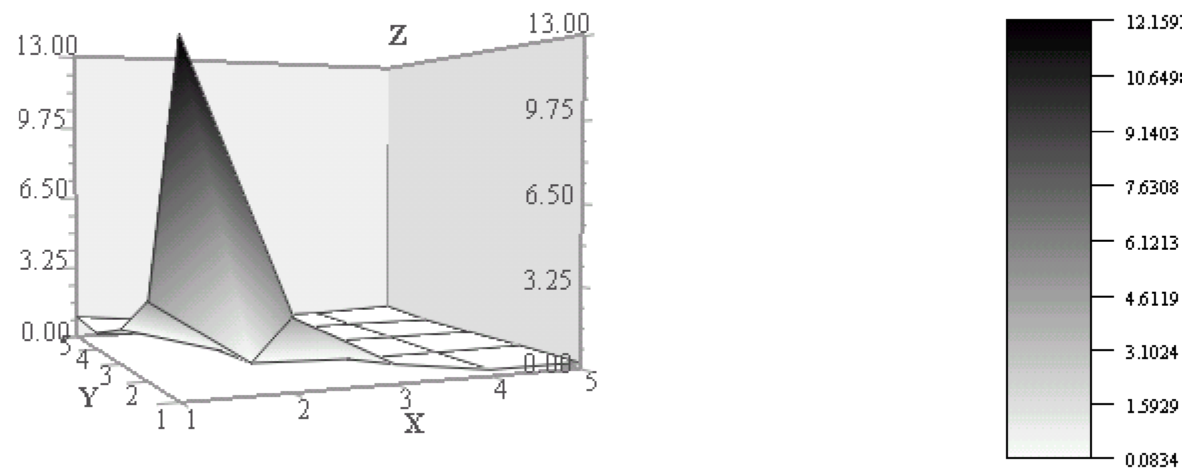


Graph 2: LR-statistic for Null of No Randomness in Beta Against Alternative of Heterogeneous Randomness in Beta





Graph 3: *Average Estimates of θ_i for the 25 Portfolios, Excluding the Portfolios that Were Based on the First Quintile of Size or Book-to-market Values. Z Dimension Represents θ_i, X Dimension Represents Portfolio Number Based on Size; and Y Dimension Represents Portfolio Number Based on Book-to-Market Values.*



Graph 4: *Average Estimates of θ_i for the 25 Portfolios. Z Dimension Represents θ_i, X Dimension Represents Portfolio Number Based on Size; and Y Dimension Represents Portfolio Number Based on Book-to-Market Values.*