The Growth-Inequality Relationship in a Neoclassical Model[1]

Debasis Bandyopadhyay

Department of Economics,
The University of Auckland,
Private Bag 92019,
Auckland, New Zealand.
E-mail: debasis@auckland.ac.nz.

Parantap Basu

Department of Economics,
Fordham University, Bronx,
New York 10458, U.S.A.
E-mail: basu@fordham.edu.

March 2001

Abstract

Barro (2000) reports how the growth-inequality relationship varies between the rich and poor countries. We provide an economic explanation for that finding in a purely neo-classical growth model with discrete occupational choice and redistributive taxes. In our model a fiscal redistributive tax program directly impacts the steady state distribution of human capital, which in turn determine income inequality and the growth rate by influencing the occupational choice of the agents. The proportion of innovators in the economy and the redistributive tax rate determine the steady state growth rate and income inequality. There are multiple steady states. Across those steady states, the correlation between growth rates and income inequality depends crucially on the degree of redistribution, output elasticity of unskilled labor and the degree of institutional barrier to knowledge diffusion.

(Prepared for presentation at the World Congress of the Econometric Society, Seattle and the CentER at The Netherlands. Preliminary, Comments welcome.)

---

[1] We gratefully acknowledge the Marsden Grant # 96-UOA_SOC_0018 from the Royal Society of New Zealand for partially supporting this research. We thank Ian King for helpful comments and Gail Pacheco and Sol Roberts for research assistance. The usual disclaimer applies.

1. Introduction

Whether inequality retards or promotes growth is a long-standing theoretical and empirical issue. Recent pioneering work of Torsten Persson and Guido Tabellini (1994) report a strong negative relationship between growth and inequality. They rationalize this relationship as a politico-economic equilibrium outcome of a model economy. Although Persson and Tabellini's theoretical model provides important insight about the link between distribution and growth, their empirical cross-country growth-inequality regression has sparked further debate. Kristin Forbes (2000) argues that there is an omitted variable bias in Persson and Tabellini's cross-country regressions. After correcting for this bias, she finds a positive association between growth and inequality. In fact, finding an empirical relation between growth and inequality over a cross section of countries is problematic when countries differ widely in terms of structural characteristics. Barro (2000) reports that a broad panel of countries shows little overall relation between income inequality and rates of growth. Abhijit Banerjee and Esther Duflo (2000) questions the validity of the empirical results in a provocative paper and ask, "What can the data say?" Until now there is no clear consensus about the empirical relationship between growth and inequality.

Even at a theoretical level, economists differ about the relationship between growth and inequality. Woojn Lee and John. E. Roemer (1998) report in terms of their model that the relationship between private investment and inequality does not necessarily show a monotonic negative relationship. They argue, in terms of a theoretical model, that the relationship between inequality and private investment is not necessarily monotonic. Chien Chou and Gabriel Talmain (1996) consider the effect of distribution of wealth on endogenous rate of growth. They argue that the relationship between inequality and growth depends on the curvature of the labor Engel curve. Peter Orazem and Leigh Testfastion (1997) develop a model where income redistribution can result in sub-optimal choices that offset the beneficial effects of income transfer. However, because of the assumption of diminishing returns, their model has steady state in level and thus does not provide insight about long-run growth.

Most of these growth models dealing with fiscal redistributive policy focus mainly on the effect of redistributive policy on either growth or inequality. Little efforts have been directed, towards understanding how a redistributive tax policy could influence the growth-inequality relationship. Understanding the endogenous dynamics of growth, distribution and fiscal policy is a major challenge facing the growth theorists nowadays, as emphasized by the following quote from Persson and Tabellini (1992):

"....Future theoretical research should try to study the joint dynamics of growth, income distribution and policy. "

A few recent papers take this issue of endogeneity of growth-inequality relationship seriously. For example, Mervyn King (1992) attempts a synthesis of the models developed by Persson and Tabellini (1994) and Phillipe Aghion and Patrick Bolton (1992). While Aghion and Bolton (1992) focus on the link from growth to distribution, Persson and Tabellini (1994) stress more on the link from distribution to growth. However, none of these models explicitly focus on the two-way link between growth and income-inequality in terms of a redistributive fiscal policy. The issue is important because the nature of the relationship between growth and inequality might depend in an important way on the type of redistributive policy chosen by the government.

In this paper we explore the link between income distribution and growth when the fiscal authority is involved in redistributive taxation of capital income. We propose a theoretical framework here to deal with this issue. As in Glenn Loury (1981), we invoke the extreme form of capital market incompleteness in the sense that the credit market is absent. Human capital is the only device for consumption smoothing and the sole engine of growth. This incompleteness of the credit market is crucial in preserving the dynastic heterogeneity in our model. Although individuals are ex-ante identical in terms of preference over dated consumption, due to past investment in human capital, they differ in terms of the endowment of human capital which evolves endogenously in the model. As a result, income inequality arises as an equilibrium outcome and it persists across generations. This happens without bringing any element of uncertainty in the production technology as in Abhijit Banerjee and Andrew Newman (1991). [2]

Our model also involves the externalities associated with human capital similar to Robert E. Lucas, Jr (1988) and abstract knowledge as in Paul M. Romer (1990). However, unlike Romer (1990), the total factor productivity in our model depends on the intensity of innovative activity represented by the proportion of innovators (who we call managers) in the economy. The fraction of people choosing a managerial occupation is the driving force behind innovation in our model.[3]

Why is the proportion of innovators an important component of the total factor productivity? It is well known that growth is significantly determined by total factor productivity. Recent literature on endogenous growth models following Lucas (1988) and Romer (1990) stress the importance of non-rival knowledge in determining total factor productivity. In a recent paper, Edward Prescott (1998) argues that it is the extent of non-rival knowledge that a country exploits rather than the available stock of knowledge itself that accounts for the cross country disparity in income.[4] How much

[2] In a similar spirit, Scott Freeman (1996) develops a discrete occupational choice model and demonstrates that income inequality persists across generations. Freeman (1996), however, does not address the issue of growth-inequality relationship in the context of redistributive taxation.

[3] Our model is an extension of Debasis Bandyopadhyay (1993). In our model the educated elite choose a managerial occupation and undertake all the investment in human capital. The motivation for including the proportion of managers in the total factor productivity function stems from the study of Bandyopadhyay (1997) who finds that the proportion of educated people significantly explains cross-country disparity in growth rates.

[4] In a very influential article Oded Galor and Danile Tsiddon (1997) highlight the importance of high-ability individuals in determining economic growth.

available knowledge would be exploited in the economy depends on the intensity of innovations, which is traced back in the model to the proportion of innovators or managers in the economy.

In our model, a redistributive capital income taxation impacts the long-run growth through two distinct channels. First, it directly influences the steady state rate of investment by impacting the post tax return to capital. Second, it indirectly influences growth by impacting a fundamental state variable in our model, which is the occupational distribution. A change in the redistributive tax rate, by altering the steady state proportion of managers, may influence growth as well as the post-tax shares of human capital in output. The relationship between growth and income inequality is thus endogenous in our model. It is driven in this model by the interaction between the initial distribution of human capital and the redistributive tax rate chosen by the government.

The model generates various steady state relationships between growth and income inequality depending on the interaction between initial distribution of human capital and the redistributive tax rate across countries. If countries differ widely in terms of their fiscal redistributive taxation, the model predicts a robust positive association between growth and income inequality as in Forbes (2000). A higher redistributive capital income tax rate lowers the steady state growth rate through the usual distortionary effect of driving a wedge between the marginal product of capital and return to capital. As the redistributive tax rate increases, the steady state post tax factor share goes in favor of the worker thus making growth and inequality correlate positively.

On the other hand, if we compare countries that are heterogeneous in initial distribution of skills, correlation between growth and income inequality does not show any robust pattern. A calibration exercise with model's steady state property indicates that the correlation between growth and inequality depends critically on technology parameters involving the skill intensity and the degree of institutional barriers to knowledge.

The model thus suggests that the cross-country correlation between growth and income inequality depends on the complex interaction between the tax policy environment, initial distribution of human capital and the economy-wide technology. Different countries may be in different steady states because of different redistributive tax regimes, different initial history of human capital distribution and different technology. The choice of the sample of countries may, therefore, make a crucial difference in determining the correlation between growth and income inequality. An econometrician, without controlling for these structural differences across countries, may bias a cross-country regression of growth on income inequality.

The rest of the paper is organized as follows. In the following section, we lay out the model and its comparative statics. Section 3 derives the steady state properties of the model. Section 4 examines the growth-inequality relationship under alternative environments regarding tax policy and initial distribution of human capital and reports some calibration results regarding the growth-inequality correlation. Section 5 ends with a few concluding remarks regarding the cross-country correlation between income inequality and growth rates.

2. The Model

We consider an environment consisting of a single perishable consumption good, variable human capital, manual labor, and technology that is partly determined by the distribution of human capital. An agent lives two periods, one as a child attached to an adult and one as an adult with a child of her own. There is a continuum of dynasties with measure one and at each date t, a typical dynasty consists of an adult and a child. The adult has one unit of labor and h units of human capital. She earns her income by choosing between the occupations of manager and worker and then divides her income between current consumption and investment in her child's education. Investment in human capital is the only means of transferring consumption in our model. We assume that human capital cannot be used as collateral for loans and there is no separate tangible capital in the economy. This rules out a viable credit market in the model, which is crucial in terms of preserving dynastic heterogeneity.

Preferences display intergenerational altruism, and so the adult maximizes the present discounted value of consumption of her dynasty. Dynasties differ only in terms of the adult's endowment of human capital at date 0. At date t, Ψ_t denotes the cumulative distribution of human capital among the date t adults. The history specifies the initial distribution Ψ_0. For simplicity, we consider a two-point distribution for Ψ_0, where h can take two possible values, 0 and $h_0>0$, such that

$$\Psi_0(h) = 1-m, \text{ for } 0 \le h < h_0; \; \Psi_0(h)=1, \text{ and for all } h \ge h_0. \quad (1)$$

In other words, initially a fraction m of adults are skilled adults possessing nonzero units of human capital and a fraction $1-m$ of adults have no skill, or equivalently, have zero units of human capital.

Groups of adults carry out production. Each group consists of a manager and one or more workers. The output q of a group depends on the manager's human capital h, the number n^d of workers she employs and the total factor productivity level $A > 0$ such that $q = Ah^{1-a}(n^d)^a$, where $0 < a < 1$ measures the output elasticity of a worker. We assume that the total factor productivity A depends on the stock A_0 of non-rival knowledge, following Romer (1990), and on the external spillover of knowledge that increases with the economy's average human capital stock H, following Lucas (1988).

Moreover, we assume that the total factor productivity A also depends on the intensity of the innovative activity measured by the relative proportion m of adults who search for technology and innovate as managers. In other words, unlike Lucas (1988) two countries may have the same stock of average human capital but could still experience different growth rates because of the differences in the proportion m of adults who are innovators. In particular, we assume that $A = A_0 m^{\theta} H^b$, where $b > 0$ is a parameter measuring the degree of Lucasian externality and $\theta \ge 0$ is a parameter representing a special aspect of the total factor productivity, which we discuss next.

Note that for a given θ, a higher intensity of innovations represented by a higher proportion of managers, m enhances the total factor productivity A in the economy. On the other hand, for a given m, a higher value of θ lowers the value of the total factor productivity. The parameter θ thus captures a measure of institutional barriers to positive external effects of innovative activities of the managers. If θ equals 0, there is no such barriers and a manager can exploit[5] 100% of the technology A_0H^b available to the economy where she operates. In such a case, the production function reduces to the Lucas-Romer technology. To summarize, at each date $t \geq 0$ the output q_t of a manager is given by

$$q(h, n_t^d; H_t, m_t) = A_0 m_t^{\theta} H_t^b h^{1-a} (n_t^d)^a, \qquad t=0, 1, 2.... \qquad (2)$$

The above specification of total factor productivity has also been used earlier in Bandyopadhyay (1993). Since our central interest in this paper is to understand the interaction between tax policy, growth and distribution, we assume that $b=a$. This assumption makes the aggregate production function linear in the reproducible input H, which is necessary although not sufficient to ensure endogenous growth in the model.[6] Observe that there are m firms per capita in this economy, each being run by one manager. On average, there are $(1-m)/m$ workers per firm. Note that $\theta > 0$ implies that the total factor productivity in the economy decreases as number of workers per manager increases. At any given point in time, a manager's human capital is given. Also, an increase in the number of workers per manager gives rise to overcrowding and that results in diseconomies of scale.

At each date $t \geq 0$ given the wage rate w_t , and the two external factors H_t and m_t, a manager with h units of human capital employs n_t^d number of workers so as to

$$\underset{n^d > 0}{Maximize} \left| q\left(H_t, m_t, n_t^d, h\right) - w_t n_t^d \right| \quad t=0, 1, 2.... \qquad (3)$$

[5] These institutional barriers may be related to stage of economic development characterized by the volume of transport and the communication per capita. It could also depend on the per capita volume of patent laws, which prevent instant diffusion of new technology or knowledge from one firm to another.

[6] In section 3, we demonstrate that a steady state equilibrium is characterized by the following two properties: (i) labour market clears with the property that the workers and managers do not switch their respective occupations; (ii) only managers invest in human capital. Such a steady state ensures that the aggregation of the production function (2) across all managers yields a linear relationship between per capita output and the economy's average human capital H making it an "AK" type growth model à la Rebelo (1991).

The first order condition of (3) yields $w_t = aA_0 H_t^a m_t^{\theta} h^{1-a} (n_t^d)^{a-1}$, or, equivalently, the optimal number $n_t^d(h)$ of workers employed by a manager with *h* units of human capital is

$$n_t^d(h) = \left(\frac{aA_0 m_t^{\theta} H_t^a}{w_t} \right)^{\frac{1}{1-a}} h, \quad t=0,\ 1,\ 2.... \qquad (4)$$

By (3) and (4), at each date *t*, the indirect profit of a manager is proportional to her human capital stock *h* and is given by $r_t h$, where,

$$r_t = (1-a) A_0 m_t^{\theta} H_t^a (aA_0 m_t^{\theta} H_t^a / w_t)^{a/(1-a)}, t=0,1,\ 2,.... \qquad (5)$$

The Government

The government in this economy undertakes a redistributive tax-subsidy program. At each date, the government levies a constant proportional tax (τ_1) on the income of the managers and provides a lump sum transfer of z_t units to each worker.[7] In addition, the managers receive as an educational subsidy, or equivalently, as a reimbursement for their educational expenses incurred by their parents. The amount of subsidy for acquired education is, however, in constant proportion (τ_2) to the manager's income level.[8] Let the adult's occupation $n_t(.)$ be an indicator function of her human capital stock *h* such that if she is a worker, $n_t(.) = 1$, otherwise, if she is a manager, $n_t(.) = 0$. Let H_{mt} denote the total human capital of all managers at date t. Then the budget constraint of the government can thus be written as:

$$\tau_2 r_t H_{mt} + (1-m_t) z_t = \tau_1 r_t H_{mt}, \text{ where, } H_{mt} = \int_{\{h: n_t(h)=0\}} h d\Psi_t(h). \qquad (6)$$

[7] In the steady state, the level of tax on workers grows over time at a balanced rate and hence, in the steady state, a lump-sum tax is equivalent to taxing worker's income proportionally.

[8] Notice that in the present setting, the educational subsidy is proportional to the level of rental income. The implication is that adults with higher rental income from human capital receive greater subsidy. The net subsidy determined by the fraction τ is financed by wage income taxation. Rogerson and Fernandez (1995) find that a similar pattern of financing educational subsidy is an equilibrium outcome in a median voter setting.

Defining $\tau = \tau_1 - \tau_2$ as the effective tax rate for the managers such that the budget constraint (6) can be rewritten as:

$$z_t = \frac{\tau r_t H_{mt}}{(1 - m_t)} \qquad (7)$$

Note that in principle we do not impose any restriction on the sign of the effective redistributive tax rate τ, meaning τ may be negative, in which case we have an effective investment tax credit or an educational subsidy financed by taxing worker's income. The sign of τ depends on the equilibrium property of the model.

The Breakeven Skill Level

Let $\overline{w}_t$ and $\overline{r}_t$ respectively denote the post subsidy wage rate and the after-tax price of human capital at each date t. In other words, $\overline{w}_t = w_t + z_t$ and $\overline{r}_t = (1-\tau)r_t$. At each date t, let x_t denote the level of *breakeven skill* such that an adult with x_t units of human capital earns an equal amount net of tax and subsidy either as a manager or as a worker. By (5), it follows, therefore, that x_t satisfies

$$\overline{w}_t = \overline{r}_t\, x_t, \qquad t=0,\, 1,\, 2, \qquad (8)$$

At each date $t \geq 0$, her occupational choice $n_t(.)$ and the resulting income $y_t(.)$ as functions $h \geq 0$ are

$$n_t(h,\tau) = 1, \text{ if } h < x_t; \qquad n_t(h,\tau) = 0 \text{ if } h > x_t; \qquad (9)$$

$$n_t(h,\tau) = 1 \text{ or } 0, \quad \text{if } h = x_t,$$

and

$$y_t(h,\tau) = n_t(h,\tau)\cdot \overline{w}_t + (1 - n_t(h,\tau))\cdot \overline{r}_t\, h. \qquad (10)$$

Figure 1 illustrates how the breakeven skill level divides the adults into two occupational groups, workers and managers, according to their individual stock of human capital.

<Figure 1 comes here>

At date t+1, an adult's human capital h_{t+1} is positively related to her parent's human capital h_t and the investment s_t in her schooling made by her parent at date t. In particular,

$$h_{t+1} = (1-\delta)h_t + s_t, \quad 0<\delta<1 \quad t=0, 1, 2, \qquad (11)$$

The above formulation presumes a positive externality $\delta < 1$ associated with family upbringing in the tradition of Benabou (1996). It also assumes $\delta > 0$ such that without a positive investment in schooling the current generation can transfer only a fraction $(1-\delta)$ of existing knowledge to the future generation. Consequently, knowledge is maintained or accumulated only if a generation acquires them through investment in schooling. This feature is similar to Mankiw et al. (1992).

Following Barro (1974) we assume intergenerational altruism. At each date t, the utility v_t of the adult is a function of her family's consumption c_t and her child's utility v_{t+1} as a grown-up adult. In other words,

$$v_t = V(c_t, v_{t+1}) = u(c_t) + \beta v_{t+1}, \qquad t=0, 1, 2, ... \qquad (12)$$

We assume that $u(.)$ is strictly concave, bounded above, $u(0)=-\infty$, $u'(0)=\infty$, $0<\beta<1$, such that $v_0 = \sum_{t=0}^{\infty} \beta^t u(c_t)$.

The adult with h units of human capital chooses a suitable occupation $n_t(h)$ following (9) and divides her income $y_t(h)$, given by (10), between consumption c_t and investment s_t such that

$$c_t + s_t \leq y_t(h,\tau) \qquad t=0, 1, 2,\ldots \tag{13}$$

Note the absence of a viable credit market is implicit in the above budget constraint. At *t= 0* the optimization problem of the adult with $h \geq 0$ units of human capital is to choose a sequence $\{c_t(h,\tau) \geq 0, s_t(h,\tau) \geq 0, n_t(h,\tau) \in \{0,1\}\}_{t=0,1,2......}$, so as to

$$\textit{Maximize} \sum_{t=0}^{\infty} \beta^t u(c_t) \text{ subject to (9), (10)(11) and (13).} \quad t=0, 1, 2,.... \tag{14}$$

Characteristics of Equilibrium

The set of sequences $\{c_t(h, \tau), s_t(h,\tau), n_t(h, \tau), n_t^d(h, \tau) : h \geq 0 ; x_t, r_t, m_t, H_t, w_t, \tau\}_{t=0,1,2,..}$ and the initial distribution Ψ_0 describe the model's *equilibrium* such that at each $t \geq 0$, the labor demand $n_t^d(.)$ satisfies (4), the implicit rental price r_t of human capital satisfies (5), the breakeven skill x_t satisfies (8), the sequence $\{c_t(h, \tau), s_t(h,\tau), n_t(h,\tau)\}_{t=0,1,2,..}$, satisfies (14), and $\{H_t, m_t\}_{t \geq 0}$ coincides with the same generated by the optimal sequence $\{s_t(h,\tau), n_t(h,\tau)\}_{t=0,1,2,..}$, such that

$$m_t = \int_{\{h:n_t(h,\tau)=0\}} d\Psi_t(h,\tau), \tag{15}$$

$$H_{t+1} = (1-\delta)\int hd\Psi_t(h,\tau) + \int s_t(h,\tau)d\Psi_t(h,\tau), \; H_0 = \int hd\Psi_0(h,\tau), \tag{16}$$

and the labour market clears such that at each date *t=0, 1, 2,…*,

$$\int_{\{h:n_t(h,\tau)=0\}} n_t^d(h, w_t; H_t, m_t) d\Psi_t(h,\tau) = 1 - m_t \tag{17}$$

Notice that the labor demand function does not depend on the redistributive tax rate because the tax is based on indirect profit and not on the output of the firms. On the other hand, the labor supply or, equivalently, occupational choice as characterized in (8) and (9) depends on the after tax wage rate. Nevertheless, the market clearing wage does not depend on τ because the profit maximizing firm equates the before tax real wage to the marginal product of labor. Figure 2 illustrates the labor market equilibrium in a situation where subsidy z_t is negative. Note that because of the discrete occupational choice, the labor supply curve (called L^s schedule) is a step function. At $\bar{w}_t=0$, the breakeven skill, x_t equals zero which means L^s equals zero, because everybody chooses to be a manager. At $\bar{w}_t^*$ which is the post tax wage when $x_t=h_0$, an adult is indifferent between the two occupations. This explains horizontal segment BC of the labor supply function over the range $1\text{-}m \leq L^s \leq 1$. The labor market equilibrium condition (17) holds at the point where the MPL schedule intersects the labor supply schedule corresponding to $L^s=1\text{-}m_t$ as shown in (17).

<Figure 2 comes here>

The goods market clears such that at each date $t=0, 1, 2..$,

$$\int_{h\geq 0}(c_t(h,\tau)+s_t(h,\tau))d\Psi_t(h,\tau)=\int_{\{h:n_t(h,\tau)=0\}}q(h,n_t^d(h);H_t,m_t)d\Psi_t(h,\tau). \qquad (18)$$

The distribution of human capital evolves as

$$\Psi_{t+1}((1-\delta)h+s_t(h,\tau))=\Psi_t(h). \qquad (19)$$

This completes the definition of the equilibrium.

III. STEADY STATE

The steady state equilibrium of our model describes a balanced growth path from its initial state. In particular the steady state equilibrium with the initial distribution Ψ_0 is such that at each date t, $m_t = m$, 0<m<1, $r_t = r$ >0 and there is a stationary growth rate $\gamma \geq 0$ such that

$$h_t = h_0(1+\gamma)^t,\ \Psi_t((1+\gamma)^t h) = \Psi_0(h),\ h \geq 0,\ z_t = z(1+\gamma)^t,\ w_t = w(1+\gamma)^t, \quad (20)$$

where w and z denote the respective initial states of the equilibrium wage rate and subsidy

We now explore the necessary and sufficient conditions for the existence of such a steady state in this model. First we examine the optimal rules for occupational choice and investment in schooling for each dynasty for the given sequence of steady state prices $r_t = r$, and $w_t = w(1+\gamma)^t$ and the redistributive tax rate τ. Later we find the values of r and w such that given the above optimal choice rules, the steady state prices clear all markets.

Lemma 1*: If the utility function is logarithmic*[9] *and at each date t,* $r_t = r$ *and* $w_t = w(1+\gamma)^t$ *then there are two constants 1>i>0 and h*>0 such that, at each date the adult member of a dynasty with the initial human capital h finds it optimal to be a manager and to invest a constant fraction i of her human capital in her child's education if h>h* and only if h≥h*, where,*

$$i = \beta[(1-\tau)r + 1 - \delta]) - 1 + \delta, \quad (21)$$

and

$$h^* = \frac{w+z}{(1-\tau)r - i}\mathrm{B}^{10},\ \mathrm{B} > 0. \quad (22)$$

Proof: Appendix.

---

[9] Lemma 1 can be extended for a strictly concave function with a minor assumption on u'''.

[10] The value of B depends on r and τ in a way that involves intricate formulas. To secure readability of the paper we describe them in appendix where we provide the proof of Lemma 1.

Lemma 2: *If a dynasty's initial human capital stock* $h < x = \frac{\overline{w}}{\overline{r}}$ *then the optimal choice rule on a balanced growth path such that* $r_t = r$ *and* $w_t = w(1+\gamma)^t$ *for the adult members of that dynasty is to be a worker, investing zero units in their child's education at all dates.*

Proof: Appendix.

By Lemma 2, all adult members of a dynasty with zero units of initial stock of human capital chooses to be workers and do not invest in their child's education. Consider now a redistributive tax regime τ and an initial distribution Ψ_0 of human capital such that the initial adult of a fraction *m* of all dynasties possesses $h_0 > 0$ units of human capital while the initial adult of the remaining *1-m* fraction of dynasties have no human capital. Then the following lemma holds:

Lemma 3: *If at each date t, the adult members of a fraction m of all dynasties with* $h_0 > 0$ *units of initial human capital choose to be managers investing a constant fraction* $i > \delta$ *of their human capital on their child's schooling while the adult members of the remaining fraction 1-m of all dynasties with zero units of initial human capital choose to be workers investing zero units in their child's schooling then in equilibrium:* $\gamma = (i - \delta) > 0$, $r_t = r(m)$, $z_t = z(m, h_0, \tau)(1+\gamma)^t$, *and* $w_t = w(m, h_0)(1+\gamma)^t$.

Proof: By assumption, at each date *t*, each of the fraction *m* of all adults invests a constant fraction *i* of her human capital. Denote $i - \delta$ by γ. By (11), it follows, therefore, $h_t = h_0(1+\gamma)^t$. The remaining (*1-m*) fraction of adults has zero units of human capital at each date *t*. Consequently, the total human capital of all managers, H_{mt} as well as the economy's average human capital H_t grows at a rate γ from its initial state mh_0 such that

$$H_t = H_{mt} = mh_0(1+\gamma)^t, \qquad t=0, 1, 2,\ldots \tag{23}$$

It follows, therefore, by (4), (15), (17) and (23), that $w_t = w(1+\gamma)^t$ such that

$$w = aA_0h_0m^{1+\theta}(1-m)^{a-1} = w(m,h_0)\,, \qquad (24)$$

and by (5) and (24) r_t=r such that

$$r = (1-a)A_0m^{\theta}(1-m\ \ )^{a} = r(m). \qquad (25)$$

By (7) and (24) the equilibrium subsidy is given by $z_t = z(1+\gamma)^t$ such that

$$z = \frac{\tau r(m)mh_0}{(1-m)} = z(m,h_0,\tau)\ . \qquad (26)$$

By (23)-(26), we completely characterize the market clearing wage rate w_t , the implicit rental price r_t of human capital and the equilibrium subsidy z_t that balances the government's budget constraint as described in Lemma 3.

Given the initial distribution Ψ_0 such that $\Psi_0(0) = 1-m$, $0 < m < 1$, and a tax policy τ, by Lemmas 1-3, the steady state investment rate and the balanced growth rate are respectively functions *i(.)* and $\gamma(.)$ of m and τ such that

$$i(m,\tau) = \beta[(1-\tau)r(m) + 1 - \delta]) - 1 + \delta \qquad (27)$$

and

$$\gamma(m,\tau) = i(m,\tau) - \delta\ . \qquad (28)$$

Note a few special features of a steady state of this model. First, the wage rate as shown in (24) increases with the economy's average human capital and the relative proportion of managers. Second, both the implicit rental price r of human capital in (25), and the available subsidy per worker in (26) are inverted-*U* shaped functions of the steady proportion m of adults who are managers. A new manager generates external benefits to other managers with her innovative activities. She, however, adds to the relative scarcity of workers and hence boosts the wage rate or, equivalently, the cost of production for all managers. For a low value of *m*, additional benefits are higher than additional costs and, therefore, returns to schooling increases with additional managers in the economy. A high value of *m*, however, turns the balance in the opposite direction. At $m = m^* = \theta/(\theta+a)$, the return to schooling reaches its maximum. Third, the growth rate γ is directly related to the steady state rate of investment, which in turn depends positively on the after tax return on schooling. It follows, therefore, that for a given tax policy τ the growth rate γ also attains its maximum at the same m^*. Figure 3 illustrates how the growth rate varies with the relative proportion *m* of managers by drawing $i(m,\tau)$ and δ schedules. The steady state growth rate is the difference between $i(m,\tau)$ and δ, which reaches its maximum at $m^*=\theta/(\theta+a)$.

<Figure 3 comes here>

Clearly, it follows from Figure 3 that if $i(m^*,\tau) > \delta$ then there are two roots $m_L^1(\tau)>0$ and $m_L^2(\tau)>0$ that solve the equation $i(m,\tau)=\delta$ for any given $0<\tau<1$. Consequently, we can ensure a non-negative balanced growth state in this model only if $i(m^*,\tau) > \delta$ and $m_L^1(\tau) \le m \le m_L^2(\tau)$.

By Lemmas 1 and 2 we can characterize a steady state with two distinct dynasties, workers and managers. In particular, to preserve the occupational segregation across dynasties and to have endogenous growth the initial human capital stock of the dynasty of managers must be higher than a threshold stock h^* and the same for the dynasty of workers must be zero. By (24)-(26), the constant B in (22) is a function $B(.,.)$ of *m* and τ and, therefore, the threshold stock h^* is also a function $h^*(.)$ of *m*, h_0 and τ such that

$$h^*(m,\tau,h_0) = \frac{w(m,h_0) + \tau r(m).\left(\frac{m}{1-m}\right)h_0}{(1-\tau)r(m) - i(m,\tau)} B(m,\tau). \qquad (29)$$

Note that we can express the right hand side of (29) as a product between h_0 and two other functions $\xi(.,.)$ and $B(.,.)$ of m and τ such that

$$\xi(m,\tau) = \frac{\left[\dfrac{am}{(1-a)(1-m)} + \dfrac{\tau m}{1-m}\right]}{\left[(1-\beta)(1-\tau)+(1-\beta)(1-\delta)r(m)^{-1}\right]}, \qquad (30)$$

and

$$B(m,\tau) = \left(\frac{1-\delta+i(m,\tau)}{1-\delta}\right)^{\left(k(m,\tau)-\frac{\beta(1-\beta^{k(m,\tau)})}{1-\beta}\right)}, \qquad (31)$$

where

$$k(m,\tau) = \frac{\ln\left[1+\dfrac{(1-\beta)\ln((1-i(m,\tau))/(1-\tau)r(m))}{\beta[\ln(1+i(m,\tau)-\delta)-\ln(1-\delta)]}\right]}{\ln\beta}. \qquad (32)$$

The following lemma helps us characterize the admissible range of m over which such a dynastic steady state as specified in Lemmas 1 through Lemma 3 holds.

Lemma 4: $h_0 \geq h^*$ *if and only if* $\xi(m,\tau) \leq 1/B(m,\tau)$.

Proof: By (30), (31) and (32), $h^*(m,\tau, h_0)$ is linear and homogenous in h_0 such that

$$h^*(m, \tau, h_0)=\xi(m,\tau)B(m,\tau)h_0. \qquad (33)$$

Q.E.D.

The issue arises whether the admissible set of steady states of m implied by Lemmas 1 through 4 is connected. It is straightforward to verify that $\xi(m,\tau)$ in (30) is monotonically increasing in *m*.[11] However, no such monotonicity could be established for the function $B(m,\tau)$ specified in (31). Even though $B(m,\tau)$ may not be monotonic in *m*, for connectedness of the set of *m*, we only need that $B(m,\tau)^{-1}$ and $\xi(m,\tau)$ satisfy a single crossing property. More specifically, it is important that $\xi(m,\tau)$ intersects $B(m,\tau)^{-1}$ from below.

Define $m_c(\tau)$ as the value of *m* that solves the root of the equation $\xi(m,\tau)=1/B(m,\tau)$ for a given τ. Because of the intricate nature of the $B(m,\tau)$ function, a formal proof of the existence of a unique $m_c(\tau)$ and the analysis of its comparative statics properties lie beyond the scope of this paper. We have extensively simulated $B(m,\tau)^{-1}$ and $\xi(m,\tau)$ and found that for a wide range of parameter values, the aforementioned single crossing property holds. Figure 4 provides an illustration.[12] Note that $B(m,\tau)^{-1}$ shows very little variation with respect to m. On the other hand, $\xi(m,\tau)$ is monotonically increasing in *m*. The value of $m_c(\tau)$ at which these two schedules intersect is around 0.34 for this simulation.

<Figure 4 comes here>

Recall our interest here is to characterize the set of path dependent steady state equilibria that preserve some special features of the initial distribution of human capital. Our next task is, therefore, to characterize the restrictions on the space of the initial distributions of human capital $\Psi_0(.)$ defined by (1) in a way such that the equilibrium with the initial distribution $\Psi_0(.)$ describes a balanced growth path as defined by (20), which we call a steady state. Based on Lemmas 1 through 4, we are now ready to state the central proposition as follows.

---

[11] To see this note that the partial derivative of $\xi(m,\tau)$ with respect to *m* is:

$$\xi_m = \frac{\xi(m)}{m(1-m)}\left[1-(1-\delta)\frac{am}{(1-\tau)r(m)+1-\delta}+(1-\delta)\frac{\theta(1-m)}{(1-\tau)r(m)+1-\delta}\right].$$

Note that since $0 < a < 1$ and $0 < m < 1$, the second term in the square bracket is less than unity. Since $\theta > 0, a > 0,\ 0 < m < 1,\ r(m) > 0$ and $\xi(m) > 0$ it follows that $\xi_m > 0$.

Proposition 1*: The equilibrium with the initial distribution* Ψ_O *of human capital such that* $\Psi_0(0) = \Psi_0(h^*) = 1 - m$, $h_0 > h^*$ *and* $\Psi_0(h_0) = 1$ *describes a balanced growth path with a non-negative rate of growth as defined above in (20), if and only if the following conditions hold:*

(i) $i(m^*, \tau) \geq \delta$; *and* (ii) $m_L{}^1(\tau) \leq m \leq \min(m_c(\tau), m_L^2(\tau))$.

In other words, we characterize the admissible set of steady states by the above restrictions on the space of *m* such that the steady state equilibrium preserves the inequality present in the initial distribution of human capital. In this sense, the steady state equilibrium is path dependent.

4. Growth-Inequality Relationship

Growth and income inequality are endogenous in the present setting and are determined by the interaction between two fundamental state variables, the redistributive tax rate (τ), and the distribution of human capital parameterized by the proportion of managers (*m*). A change in the redistributive tax rate (τ) has a direct effect on the steady state growth rate and factor share via its effect on the post tax return to capital. On the other hand, it also has an indirect effect on growth and income inequality via its effect on the steady state occupational distribution.

To see it clearly, denote the post tax factor share as $\omega(m,\tau)$, which is the measure of income inequality in the present context. Note that $\omega(m,\tau)$ is the steady state ratio of the income of managers to the income of workers. In other words,

(34) $$\omega(m,\tau) = \frac{(1-\tau)r(m)h_0}{w_0 + z_0}$$

[12] For the purpose of this simulation we set, a=.1, δ=.1, A_0=1, θ=.5, β=.9.

Notice that $\omega(m,\tau)$ is simply the relative income of the mangers, which can as well be interpreted as the post tax skill premium for the managers.

Plugging (7), (24), (25), (26), one obtains the following expression for the steady state factor share:

$$(35) \quad \omega(m,\tau) = \frac{(1-\tau)(1-a)(1-m)}{\left(a+\tau(1-a)\right)m}$$

Based on this factor share the Gini coefficient of the income distribution (call it Gini) is given by:

$$(36) \quad Gini = (1\text{-}a)(1\text{-}\tau) - m$$

The appendix provides a derivation of the Gini coefficient. Notice that the model's income Gini coefficient is state dependent. It depends on the country's initial distribution of human capital parameterized by m and the redistributive tax regime measured by τ.[13] Everything else remaining the same, a country with a higher initial inequality in the distribution of human capital (meaning lower m) will end up with greater income inequality. Steady state growth (equation 27) and income-inequality (equation 36) experiences differ across countries because of differences in m and τ.

To see this clearly, consider a CES utility function for which the growth rate as a function of the steady state proportion m of managers and the effective redistributive tax rate τ satisfies

$$(37) \quad \gamma(m,\tau) = (\beta[(1-\tau)(1-a)A_0 m^{\theta}(1-m)^{a} + 1\text{-}\delta])^{1/\lambda} - 1,$$

and by (35)-(36),

$$(38) \quad m=(1\text{-}a)(1\text{-}\tau)\text{-}gini; \text{ or } \qquad m = \frac{(1-\tau)(1-a)}{\left(a+\tau(1-a)\right)(1+\omega)}.$$

[13] It is straightforward to verify that as long as $\omega(m,\tau)>1$, the Gini coefficient is positive.

Equations (37)-(38) together represent the model's endogenous relationship between growth and income inequality. This relationship between growth and factor shares is driven by an important fundamental, which is the initial distribution of human capital. Notice that the above growth-inequality relationship is valid only in the model's steady states. In other words, given a specific redistributive tax regime, τ, this relationship is defined over the set D(τ) of initial proportion *m* of managers such that

(39) $D(\tau) = \{Gini=(1-a)(1-\tau)-m:\ m_L^{1}(\tau) \leq m \leq \min(m_c(\tau), m_L^2(\tau))\}$

An important feature of the model is that the sign of the growth-gini correlation critically depends on the relative magnitudes of m_L^1, m_c and m^*, which depend on the values of the structural parameters and, in particular, on the values of three crucial parameters τ, a and θ. By (36), clearly, the model's gini coefficient decreases with m. However, one may discern from Figure 3 that the growth rate, $\gamma(m) = i(m) - \delta$, increases with m if $m<m^*$ and it decreases with m if $m>m^*$. It follows, therefore, that the growth-gini correlation would be negative if $m<m^*$ and it would be positive if $m>m^*$. In our empirical exercise we classify countries into different groups defined by a unique set of values for the three structural parameters: (i) τ, the net redistributive tax rate, (ii) a, the output elasticity of unskilled labor and, (iii) θ, the parameter indicating institutional barriers to external spillover of information. A higher value of τ, by (30)-(32), corresponds to a lower value of m_c and, by (27) and Figure 3, corresponds to a higher value of m_L^1. Also, a higher value of either a or θ corresponds to a higher value of m_L^1. Note that $m^* = \frac{\theta}{a+\theta}$ and, therefore, is independent of τ. Clearly, m^* decreases with a and increases with θ. How the value of m_c changes with respect to changes in a or θ is ambiguous. By manipulating the equations (30)-(32) we note, however, that under a unique but non-trivial restrictions on the structural parameters, a higher value of a or θ corresponds to a higher value of m_c. Under that special restriction on the parameter space, we have the following proposition for characterizing how the growth-inequality relationship may vary between different groups of countries.

Proposition 2: Between two groups of countries, the group with a larger redistributive tax rate, or with a smaller output elasticity of unskilled labor, or with a greater institutional barrier to spillover of knowledge would be more likely to have a negative growth-gini relationship. In other words, the cross-country correlation coefficient between the growth rate and the gini-coefficient would be lower for a group of countries that share a larger redistributive tax rate, or a smaller output elasticity of unskilled labor, or a greater institutional barrier to spillover of knowledge.

Tables 1 through 3 plot growth-inequality correlation for different groups of countries defined by different parameter values. Values of the parameters, A_0, β and δ are chosen to match the world average growth rate between 1960-90 which equals approximately 2% in our sample of countries. Table 1 reports the correlation for four admissible tax rates, τ.[14] The growth-inequality correlation turns negative for a tax rates higher than 30% for these model economies while it is positive for τ less than 30%. The intuition for this shift in the correlation can be found by looking at the columns where the model's $m_c(\tau)$ and m^* are reported. In all the cases reported in Table 1, $m_c(\tau)$ exceeds m^*. In such a situation, when m decreases from m_c, which is the maximum sustainable steady state proportion of managers, the growth rate and inequality are positively related. As m falls below the threshold m^*, the growth-inequality relationship turns negative. Consequently, the relationship between the model's gini coefficient and the growth rate is inverted U shaped as illustrated in Figures 5-6. The sign of the correlation depends on how close $m_c(\tau)$ is to m^*.

<Figures 5 and 6 come here>

Table 1: Growth-Gini Correlation under alternative tax regimes

Tax Regimes under $D(\tau)$	Growth-Gini Correlation	$m_c(\tau)$	m^*	Average Growth Rate (%)	Average Gini (%)
0.1	0.58	0.28	0.09	7.70%	57.30%
0.2	0.10	0.22	0.09	5.28%	52.10%
0.3	-0.34	0.18	0.09	2.83%	46.05%
0.4	-0.61	0.15	0.09	0.36%	39.55%

Other Parameter Values: a=.2, θ=.02, δ=.1, A_0=.35, β=.95

Table 2 reports how the correlation between growth and inequality increases as the value of a, which measure the output elasticity of unskilled labor, increases.

[14] It is difficult to find a published estimate of the cross-country average capital income tax rate. Mendoza, Razin and Tesar (1994) find that the effective capital income tax rates range from 25% to 60% across major

Table 2: Growth-Gini Correlation for Various a values

	a=.3	a=.35	a=.4	a=.45	a=.5	a=.55
Growth-Gini Correlation	-.26	-.25	-.06	.12	.008	.37
Average Growth Rate	3.17%	2.87%	2.25%	1.78%	1.57%	1.14%

The other parameter values are θ=.7, A_0=.3, δ=.04, β=.96, τ=0.3.

There are two noteworthy features in Table 2: First, countries with a more unskilled labor-intensive technology (a higher a) experience lower growth. Second, the correlation coefficient increases and, in particular, changes sign from negative to positive as the value of a increases. Notice also that the association between a and the growth-gini correlation coefficient is non-linear. Countries with a more skilled labor-intensive technology (low a) display negative correlation between growth and inequality; the pattern is reversed for countries with more unskilled labor-intensive technology. Everything else equal, a low value of the parameter a lowers the steady state proportion of managers in the economy. On the other hand, a low value of a also raises the growth maximizing proportion of managers, m^* (see Figure 3). The economy will be, therefore, operating in the segment where $m<m^*$ where growth and inequality are inversely related, discussed as a special case earlier.

We now turn to the comparative static properties of the growth-inequality correlation with respect to θ. Table 3 reports how the correlation between growth and inequality changes as the value of θ increases.

OECD countries. In the context of our model, not all tax rates are admissible for the steady state equilibrium. Table 1 reports the tax rates which belong to the set D(τ) defined in (38).

Table 3**:** *Growth-Gini Correlation for Various θ Values*

	θ=.5	θ=.6	θ=.7	θ=.8
Growth-Gini Correlation	.15	.006	-.06	-.21
Average Growth Rate	3.22%	2.64%	2.24%	2.13%

a=.4, A_0=.3, δ=.04, β=.96, τ=0.3.

Countries with higher θ tend to have a negative growth inequality correlation. In the model, following (2), we interpret a larger value of θ as a greater degree of institutional barrier to knowledge-diffusion. A sufficiently large barrier pushes the growth maximizing proportion of managers m^* beyond m_c, the upper bound for all steady states. Consequently, within the range $m<m^*$ countries tend to have negative correlation between growth and inequality.

Some Calibration Results

We looked at a sample of 88 countries and examined the growth-gini correlations for sub-sample of countries that differ in terms of the proportion of educated people. The variable m is proxied by the proportion of educated people (with secondary and higher secondary education) obtained from the data set created by Barro and Lee (1993). The series for per capita growth rates came from the World Bank Development indicators and the gini coefficients are obtained from Deininger and Squire (1998) and several other sources. There are two available series for the gini coefficients for a sufficiently large number of countries necessary for our experiment; one for the year 1985 and the other for the year 1990. Table 2 reports the growth-gini correlations for the bottom and top third of countries in terms of their proportion of educated people.

Table 4

Data	Countries at the Bottom Quartile of Educated People (low *m* group)	Countries at the Top Quartile of Educated People (high *m* group)
Growth-Gini85 Correlation	-0.05	0.13
Growth-Gini90 Correlation	-0.15	0.07
Average Per Capita Growth Rate	0.82%	2.12%
Average Gini for 1985	49.43%	31.66%
Average Gini for 1990	49.15%	32.84%

Notice that the growth-gini correlation reverses its sign from negative to positive as the sample of countries changes from low to high *m* groups. This is consistent with model's prediction. While looking at the model's steady state property (summarized in Table 1), note that the correlation reverses its sign when τ changes from 0.2 to 0.3. In order to calibrate the model's growth-Gini correlation with the data, we, therefore, compute the model's growth-gini correlation at finer grids of τ in the range of [0.2, 0.3].

Table 5

Tax rate: τ	0.20	0.21	0.23	0.25	0.27
Model's Growth - Gini Correlation	0.10	0.05	-0.05	-0.14	-0.23

Other Parameter Values: a=.2, θ=.02, δ=.1, A_0=.35, β=.95

Table 5 closely matches the sample correlation in the proposed range of τ. An important testable hypothesis emanating from this calibration exercise is that, countries with negative growth-gini correlation may possibly have higher redistributive tax rates.

The data for the share of agriculture in GDP were obtained from the World Development Indicators. Table 6 reports the growth-gini correlations for the bottom and top quartile of countries sorted by their share of agriculture. We argue that the institutional barrier to knowledge spillover directly varies with the share of agriculture in the GDP. Presumably, knowledge spillover would be faster in countries which are more industrialized, have larger volume of transport and communication and, therefore, a smaller share of primary sector output in GDP.

Table 6

	Countries at the bottom quartile of the share of agriculture (low θ)	Countries at the top quartile of the share of agriculture (high θ)
Growth-Gini85 Correlation	.37	-.21
Growth-Gini90 Correlation	.33	-.20
Average Growth Rate	2.49%	0.58%
Average Gini85	34.78%	45.33%
Average Gini90	34.76%	45.73%

The correlation reverses its sign from positive to negative as the sample of countries changes from low to high agricultural share, while the average growth rate is lower for countries with larger share of agriculture. In the next step, we searched for model's parameter values that match the correlation coefficients in Table 6. As reported in Table 3, the model reproduces the reversal of sign of the correlation with a change in the parameter θ alone. The growth rates do not match well unless we also control for the tax rate simultaneously. This suggests that countries in these two sub groups possibly differ also in terms of other structural characteristics.

5. Concluding Remarks

Countries may differ widely in terms of fiscal regimes, the initial distribution of human capital as well as technology. [15] Because of the state dependent nature of the relationship between growth and inequality, no robust correlation between growth and income inequality may be obtained when countries widely differ in terms of structural characteristics in the sense described in the model. Barro (2000) reports that higher inequality tends to retard growth in poor countries and encourage growth in richer countries. We present a general equilibrium model whose steady state growth-inequality calculations help us to understand an economic argument to explain Barro's findings. The poorer countries in Barro's sample are likely to be countries with high level of redistributive tax rates. The poorer countries possibly also use a technology with a higher output elasticity of unskilled labor and have a greater institutional barrier to knowledge spillover. Our model implies that in Barro's ample of poor countries the negative effect of high redistributive taxes and large institutional barriers to knowledge spillovers on the growth-gini correlation must have outweighed the positive effect of large output elasticity of unskilled labor on the same. In general, the model provides a purely neoclassical explanation for how various structural differences may imply a qualitatively different growth-inequality relationship between two groups of countries.

---

[15] Mendoza, Razin and Tesar (1994) find that even major OECD countries with similar infrastructure differ widely in terms of the effective tax rates on capital income. Robert J. Barro and J. Lee (1995) document that the proportion of educated people differs widely across countries.

Appendix

Proof of Lemma 1:

In a balanced growth state given a stationary growth rate $\gamma \geq 0$, the time-consistent optimisation problem (14) of the initial adult member of a dynasty is given by

$$\underset{\{s_t,h_{t+1}\}}{Max} \sum_{t=0}^{\infty} \beta^t [u(n_t \overline{w}(1+\gamma)^t + (1-n_t)\overline{r}h_t - s_t) + \lambda_t (h_{t+1} - (1-\delta)h_t - s_t)],$$

where $\overline{w} = w + z$, $\overline{r} = (1-\tau)r$ are initially given values, while n_t is an indicator function that describes the occupational choice rule as a function of the adult's human capital given, by (8) and (9).

Define $i_t = s_t / h_t$. First we show that a dynasty with h^* units of initial stock of human capital would derive equal utility from either of the above stationary sequences and, therefore, would be indifferent between the two rules. Then we establish that if a managerial dynasty possesses $h_0 > h^*$ units of initial human capital then it would prefer $\{ i_t = i \}$ to $\{ i_t = 0 \}$ while if it possesses $h_0 < h^*$ then it would have the opposite preference.

If $i_t = 0$ there is an integer k such that $h^*(1-\delta)^k = x(1+\gamma)^k$. By (10) and (13), it follows that at each date $t=0, 1, 2,..,k-1$, $c_t = rh^*(1-\delta)^t$ and then for $t \geq k$, $c_t = w(1+\gamma)^t$. If, however, at date t, $i_t = i = \gamma + \delta$ then by (10) and (13), $c_t = (r-i)h^*(1+i-\delta)^t$. Note that h^* must be such that it equates the steady state utility for a dynasty of managers which chooses either $\{i_t=i\}$ or $\{i_t=0\}$. For a log utility function, h^* must solve

$$\sum_{t=0}^{\infty} \beta^t Ln[(\overline{r}-i)h^*](1+i-\delta)^t = \sum_{t=0}^{k-1} \beta^t Ln(\overline{r}h^*)(1-\delta)^t + \sum_{t=k}^{\infty} \beta^t Lnw(1+\gamma)^t \qquad \text{(A.1)}$$

which means h^* must satisfy

$$h^* = \frac{\overline{w}}{\overline{r}-i}\left[\frac{1+i-\delta}{1-\delta}\right]^{\alpha(k)}, \text{ where } \alpha(k) = (\beta^{k+1} + k(1-\beta) - \beta)/(1-\beta). \qquad \text{(A.2)}$$

[Details of the algebra for the derivation (A.2) are omitted for brevity but available from the authors upon request.] It follows from the definition of the numbers *k* and *i* that *h** satisfies

$$h^* = x\left(\frac{1+\gamma}{1-\delta}\right) = \frac{\overline{w}}{\overline{r}}\left[\frac{1+i-\delta}{1-\delta}\right]^{k}. \qquad \text{(A.3)}$$

By solving equations (A.2) and (A.3) simultaneously we can determine the unique values of the two unknowns *k* and *h** as follows:

$$h^* = \frac{\overline{w}}{\overline{r}-i}\mathrm{B}, \text{ where } \mathrm{B} = \left(\frac{1-\delta+i}{1-\delta}\right)^{\left(k - \frac{\beta(1-\beta^k)}{1-\beta}\right)}, \text{ and}$$

$$k = \frac{\ln\left[1 + \frac{(1-\beta)\ln(1-i/(1-\tau)r)}{\beta[\ln(1+i-\delta) - \ln(1-\delta)]}\right]}{\ln\beta}.$$

In the following steps we first establish that if a dynasty's initial human capital $h_0 > h^* = \frac{\overline{w}}{\overline{r}-i}\mathrm{B}$ then the pair of sequences {$n_t=0$} and {$i_t = i = \gamma + \delta$} constitute an optimal policy rule for that dynasty. We also establish that the corresponding optimal consumption sequence of the dynasty is smooth in the sense that it grows at a constant rate γ from its initial state $(\overline{r}-i)h_0$. The concavity of the utility function would then imply that the above optimal policy rule is unique. Next we show that if $h_0 = h^*$ then the initial adult of the dynasty would be indifferent between choosing either {$n_t=0$, $i_t=0$} or {$n_t=0$, $i_t=i$}. In other words, both policy rules would yield the same value of the indirect utility for the initial adult of the dynasty.

Step 1: If $h_0>h^*$ and for all $t=0, 1, 2, \ldots, i_t=i=\gamma+\delta$ then, by (11), it follows that at each date t, $h_t=(1+\gamma)^t h_0 \geq (1+\gamma)^t h^* > \frac{\overline{w}}{\overline{r}-i}(1+\gamma)^t > \frac{\overline{w}}{\overline{r}}(1+\gamma)^t = x_t$.

Step 2: If, at all t, $h_t > x_t$ then all adults of the dynasty specialize in the managerial occupation by choosing $n_t=0$. Consequently, $i>0$ implies the following FOC:

$$u'((\overline{r}-i)h_t)=\beta(\overline{r}+1-\delta)u'((\overline{r}-i)(1+i-\delta)h_t). \qquad \text{(A.4)}$$

If u is logarithmic[16] then there exists a unique i that solves (A.4), where

$$i=\beta[(1-\tau)r+1-\delta])-1+\delta \text{ such that } \gamma=\beta[(1-\tau)r+1-\delta])-1. \qquad \text{(A.5)}$$

Consequently, by Step 1, if $h_0>h^*$ then $\{n_t=0,\ i_t=i\}$ is optimal.

Step 3: Next we show that if, at all date t, $h_t > x_t$ then any investment policy such that $i_t \neq i_{t+1}$ for some date t is sub optimal. In particular, if $i_t \neq i_{t+1}$ for some date t then the FOC of optimization implies

$$u'((\overline{r}-i_t)h_t)=\beta(\overline{r}+1-\delta)u'((\overline{r}-i_{t+1})(1+i_t-\delta)h_t) \qquad \text{(A.6)}$$

It is straightforward to verify that if $u(c)=\text{Ln}c$ then we can combine the values of i_{t+1} as a function $f(.)$ of i_t that solves (A.6) to generate a first order difference equation having the following properties:

(i) If $i_t<i$ then $f(i_t)<i_t$.

(ii) If $i_t>i$ then $f(i_t)>i_t$

[16] If u is not logarithmic but strictly concave then the sufficient condition for the existence of a solution for i is: $\frac{u'(\overline{r}h_t)}{u'((1-\delta)\overline{r}h_t)} < \beta(\overline{r}+1-\delta) < \frac{u'(\overline{r}-\delta)h_t/2)}{u'((\overline{r}-\delta)h_t/2)(1+(\overline{r}-\delta)/2))}$.

(iii) $f' > 0$ and $f'' > 0$.

We can generalise this result for a strictly concave utility function with an additional assumption that $u''' > 0$. It follows, therefore, that the function *f(.)* crosses the 45^0 line from below. Consequently, any sequence $\{i_t\}$ that solves (A.6) other than the stationary sequence $\{i_t=i\}$ must be either monotonically increasing such that $i_t < i_{t+1} < i_{t+2} \ldots$, if $i_t < i$, or monotonically decreasing such that $i_t > i_{t+1} > i_{t+2} \ldots$, if $i_t > i$. Since $f'' > 0$, if $i_t > i$ then the increasing sequence would violate the non-negativity constraint on consumption $c_{t+k}=(r-i_{t+k})h_{t+k}$ after a finite period k and if $i_t < i$ then the decreasing sequence would violate the non-negativity constraint on investment $i_{t+v}>0$ after a finite period v. Consequently, for a dynasty of managers the only sequence that satisfies the individual optimisation problem is $\{i_t=i\}$ where i satisfies (A.5).

Step 4: The policy rule $\{n_t=0, i_t=i>0\}$ is optimal if and only if $h_0 > h^*$. Step 3 rules out all non-stationary investment sequences as possible candidates for an optimal policy rule for a dynasty of managers. We, therefore, consider only two possible stationary sequences $\{ i_t = i \}$ and $\{ i_t = 0 \}$. Note from (A.1) that the investment rule $\{i_t=i>0\}$ is preferable to $\{i_t=0\}$ if $h_0 > h^*$ and only if $h_0 \le h^*$. We conclude, therefore, the policy rule $\{n_t=0, i_t=i>0\}$ such that i satisfies (21) is optimal, if $h_0 > h^*$ and only if $h_0 \ge h^*$ where h^* satisfies (22).

Proof of Lemma 2:

Suppose that the adult member of a dynasty with the initial human capital stock $h<x=\frac{\overline{w}}{\overline{r}}$ chooses $i_t>0$ at some date t. To ensure a non-zero return from that investment at some future date (say, $t+k$) the adult member of the dynasty must be a manager, or equivalently, must possess human capital greater than the level of basic skill x_{t+k} at date $t+k$. In particular, the value of h and her income should be such that the discounted present value of future benefit from that investment must exceed the opportunity cost of that investment. Note that a concave utility function implies that (i) as the wage rate increases the opportunity cost of investment decreases over time and (ii) a positive investment in schooling at any date raises the discounted present value of future benefit of all subsequent dates. Consequently, along a balanced growth path, the investment in schooling must be non-decreasing over time such that $s_{t+1} \geq s_t$. In other words, if the adult member of a dynasty invests at date *t*, it is optimal for all future adult members in her dynasty to invest.

By Lemma 1 it follows, therefore, that at some future date $t+\nu$ such that ν>k the adult members of the dynasty must possess human capital above the threshold stock h_{t+v}* for making non-zero investment in schooling at that date. In other words, s_t>0 implies that there must be ν>k such that $h_{t+k} > x_{t+k}$ and $h_{t+\nu} > h_{t+v}$*. The time-consistent optimization problem of the initial adult member of a dynasty is given by

$$\underset{\{s_t, h_{t+1}\}}{Max} \sum_{t=0}^{\infty} \beta^t [u(n_t \overline{w}(1+\gamma)^t + (1-n_t)\overline{r}h_t - s_t) + \lambda_t (h_{t+1} - (1-\delta)h_t - s_t)],$$

where n_t denotes the indicator function given by (9).

FOC: s_t>0 implies

$$u'(\overline{w}_t - s_t) = \lambda_t . \qquad \text{(A.7)}$$

h_{t+1}>0 implies that

$$\lambda_t = \beta(1-\delta)\lambda_{t+1}, \text{ if } h_{t+1} < x_{t+1}; \qquad \text{(A.8)}$$

or, $\lambda_t = \beta u'(rh_{t+1} - s_{t+1})\overline{r} + (1-\delta)\lambda_{t+1}$, if $h_{t+1} > x_{t+1}$.

which upon the substitution of (A.7) implies

$$u'(\overline{w}_t - s_t) = \beta(\overline{r} + 1 - \delta)u'(\overline{r}h_{t+1} - s_{t+1}), \ h_{t+1} > x_{t+1}. \qquad \text{(A.8a)}$$

Since k denotes the number of periods after which $h_{t+k}>x_{t+k}$, it follows, by repeated substitution of (A.8) and (A.8a):

$$\lambda_t = \beta(1-\delta)\lambda_{t+1} = \beta(1-\delta)(\beta(1-\delta)\lambda_{t+2}) = .. = \beta^k(1-\delta)^k\lambda_{t+k} = \beta^k(1-\delta)^k\beta(r+1-\delta)\lambda_{t+k+1}$$

$$= \beta^{k+1}(1-\delta)^k(r+1-\delta)\lambda_{t+k+1} = ... = \beta^v(1-\delta)^k(r+1-\delta)^{v-k}\lambda_{t+v}.$$

By (A.7) for the date $t+v$, it follows:

$$u'(\overline{w}_t - s_t) = \beta^v(1-\delta)^k(\overline{r}+1-\delta)^{v-k}u'((\overline{r}-i_{t+v})h_{t+v}). \qquad \text{(A.9)}$$

Define $h_{mt} = \dfrac{h_{t+v}}{(1+i-\delta)^v}$. Note that $h_{mt} > h_t^*$, since $h_{t+v} > h_{t+v}^* = (1+i-\delta)^v h_t^*$.

By Lemma 1, the optimal rule for investment by all adult members of a dynasty from period $t+v$ onwards must be $i_t=i$ and by repeated substitution in (A.4) yields:

$$u'((\overline{r}-i)h_{mt}) = \beta^v(\overline{r}+1-\delta)^v u'((\overline{r}-i)h_{t+v}). \qquad \text{(A.10)}$$

Note that the strict concavity of u implies that

$$u'(\overline{w}_t - s_t) > u'(\overline{w}_t) > u'((\overline{r}-i)h_{mt}) \text{, since, } h_{mt}>h_t^*>\frac{\overline{w}_t}{(\overline{r}-i)}.$$

We use (A.10) to substitute the R.H.S. and note that $r>0$ implies,

$$u'(\overline{w}_t - s_t) > \beta^v(\overline{r}+1-\delta)^v u'((\overline{r}-i)h_{t+v}) > \beta^v(1-\delta)^k(\overline{r}+1-\delta)^{v-k}u'((\overline{r}-i)h_{t+v}),$$

which obviously contradicts (A.9). It is, therefore, not optimal for a dynasty with initial human capital $h<x=\frac{\bar{w}}{\bar{r}}$ to invest in schooling on a balanced growth path.

Derivation of equation (36), Gini coefficient: Define $\bar{a}$ as the worker's steady state post tax share in income. In other words,

$$\text{(A.11)} \qquad \bar{a} = \frac{(1-m)(w_0+z_0)}{(1-m)(w_0+z_0)+(1-\tau)r(m)h_0 m} \ ,$$

which upon the use of (35), reduces to

$$\text{(A.12)} \quad \bar{a} = \frac{1}{1+m(1-m)^{-1}\omega(m,\tau)} .$$

Next, note that in the steady state, the initial distribution of human capital (1) is preserved, which means (1 – m) fraction of the population have $\bar{a}$ fraction of total income and m fraction the population have (1 – $\bar{a}$) fraction of total income. The Lorenz ratio for income (*Gini*) is therefore given by:

(A.13) $Gini = 1-\bar{a}-m$, which after simplification yields (35).

References

Atkinson, Andrew., Chari, V., Kehoe, P. (1999). Taxing Capital Income: A Bad Idea. *Federal Reserve Bank of Minneapolis, Quarterly Review, Summer 1999*.

Aghion, Phillipe. and Bolton, Patrick. (1992). Distribution and Growth in Models of Imperfect Capital Markets. *European Economic Review. Vol. 36 (2-3). p 603-11.*

Barro, Robert J. (2000). Inequality and Growth in a Panel of Countries. *Journal of Economic Growth*. Vol. 5 (5-32), p 5-30.

Benabou, Roland (1996). Unequal Societies*. Centre for Economic Policy Research Discussion Paper*: 1419*, p 44*

Bandyopadhyay, Debasis (1993), Distribution of Human Capital, Income Inequality and the Rate of Growth, *Ph.D thesis, University of Minnesota*.

Bandyopadhyay, Debasis (1997). Distribution of Human Capital and Economic Growth. *Department of Economics Working paper Series, the University of Auckland.*

Bandyopadhyay, Debasis and Basu Parantap (1997), Optimal Redistributive Tax in a Growth Model with Discrete Occupational Choice, *Department of Economics Working paper Series, the University of Auckland.*

Banerjee, Abhijit, V and Duflo, Esthar. (2000). "Inequality and Growth: What Can the Data Say?" *mimeo,* **M.I.T.**

Banerjee, Abhijit, V and Newman, Andrew F. (1994). Poverty, Incentives, and Development. *American Economic Review, Vol, 84 (2),* p 211-15.

Barro, Robert. J. (1974). Are Government Bonds Net Wealth? *Journal of Political Economy 82,* p 1095–1117.

Barro, R. J. and J. Lee, (1993), International Comparisons of Educational Attainment, *Journal of Monetary Economics* 32, 363-94.

Chamley, Christophe (1986). Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives. *Econometrica. Vol. 54 (3),* p 607-22.

Chou, Chien. , Talmain, Gabriel. (1996), "Redistribution and Growth: Pareto Improvements," *Journal of Economic Growth*, Vol 1(4), p 505-23, Dec 1996.

Deininger, K., & Squire, L. (1998). *A New Data Set Measuring Income Inequality,* **World Bank Economic Review**, 10(3), 565-591.

Freeman, Scott (1996), "Equilibrium Income Inequality among Identical Agents," *Journal of Political Economy*, 104, 5, 1047-1064.

Forbes, Kristin. (2000), A Reassessment of the Relationship between Inequality and Growth, *American Economic Review, Forthcoming.*

Galor, Oded and Danile Tsiddon (1997). Technical Progress, Mobility, and Economic Growth, *American Economic Review*, 87, 3, p. 363-381.

Judd, Kenneth.L (1985), Redistributive Taxation in a Simple Perfect Foresight Model, *Journal of Public Economics*, 28, p. 59-83.

King, M (1992), Growth and Distribution, *European Economic Review,* 36, 585-592.

Lee, W and J. E. Roemer (1998), Income Distribution, Redistributive Politics and Economic Growth*, Journal of Economic Growth,* 3(3), 217-240.

Loury, Glenn. (1981), Intergenerational Transfers and the Distribution of Earnings, *Econometrica. Vol. 49 (4). p 843-67.*

Lucas, Robert. E., (1988), On the Mechanics of Economic Development. *Journal of Monetary Economics 22*, p 3-42.

Mankiw, N. Gregory., Romer, David. and Weil, D. N., (1992). A Contribution to the Empirics of Economic Growth. *Quarterly Journal of Economics*, *107,* p 407-37.

Mendoza, Enrique. G, A. Razin, and L.L. Tesar, (1994), Effective Tax Rates in Macroeconomics: Cross Country Estimates of Tax Rates on Factor Incomes and Consumption, *Journal of Monetary Economics*, 34, 297-323.

Orazem, Peter. and Tesfatsion, Leigh. (1997) Macrodynamic Implications of Income -Transfer Policies for Human Capital Investment and School Effort. *Journal of Economic Growth. Vol. 2 (3). p 305-29.*

Persson, T. and G. Tabellini (1994), Is Inequality Harmful for Growth?, *American Economic Review* 84, 600-621.

Persson, T. and G. Tabellini (1992), Growth, Distribution and Politics, *European Economic Review*, 36, 593-602.

Prescott, E.C. (1998), Needed: A Theory of Total Factor Productivity, *International Economic Review*, 39, 3, 525-551.

Rogerson, R and R. Fernandez (1995). On the Political Economy of Education Subsidy, *Review of Economic Studies*, 62, 249-262.

Rebelo, Sergio J. (1991) Long -Run Policy Analysis and Long-Run Growth, *Journal of Political Economy, 94, 5, p. 500-521*.

Romer, Paul., (1990). Endogenous Technological Change. *Journal of Political Economy,* 98, p. 71-102.

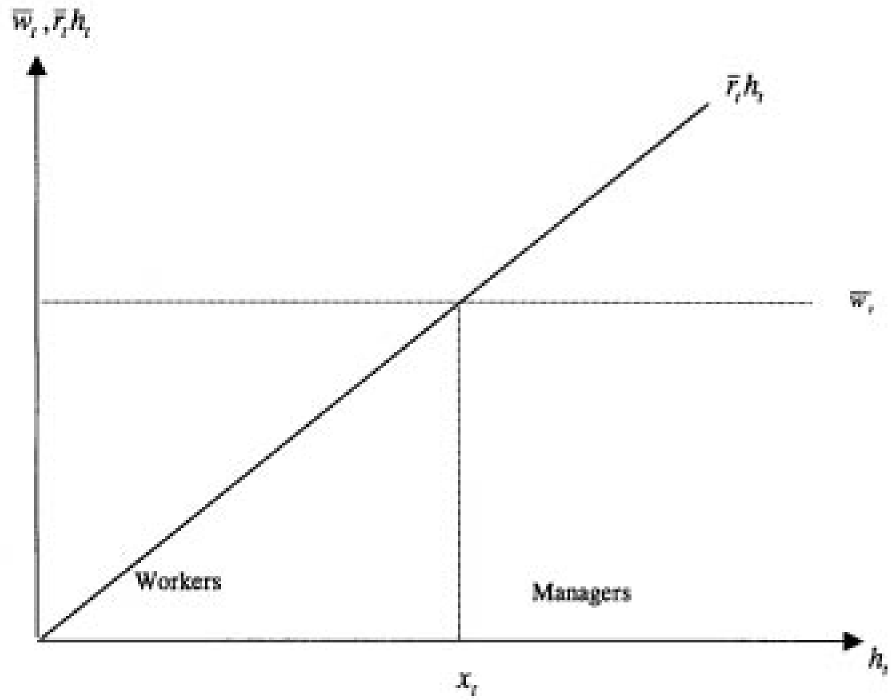
$\overline{w}_t, \overline{r}_t h_t$
$\overline{r}_t h_t$
$\overline{w}_t$
Workers
Managers
x_t
h_t

Figure 2

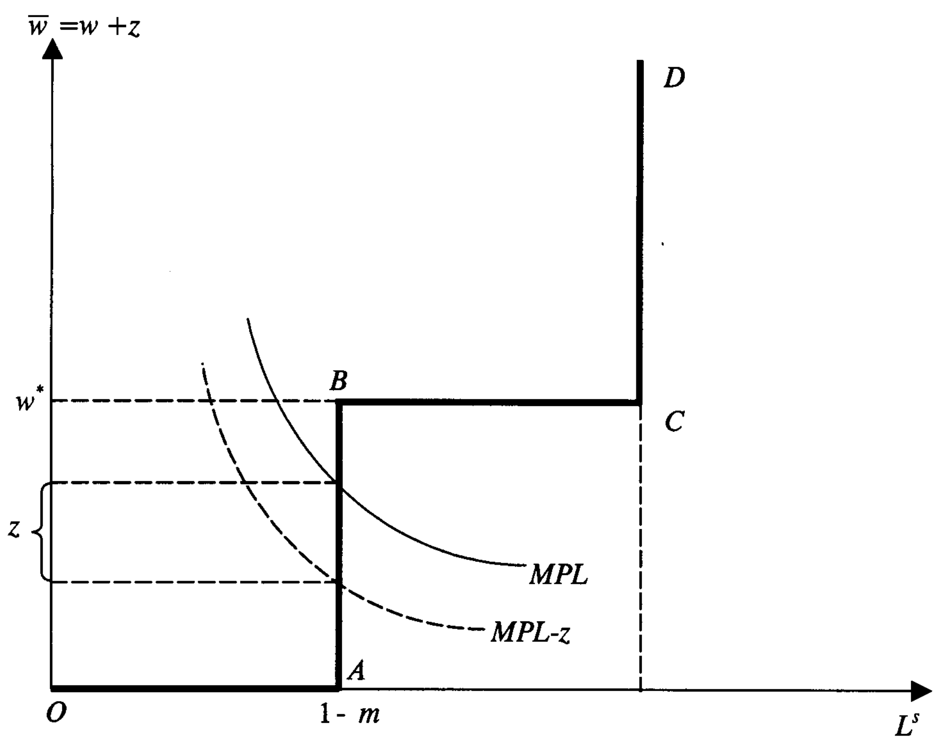
$\overline{w} = w + z$
D
B
w^*
C
z
MPL
MPL-z
A
O
1- m
L^s

Figure 3

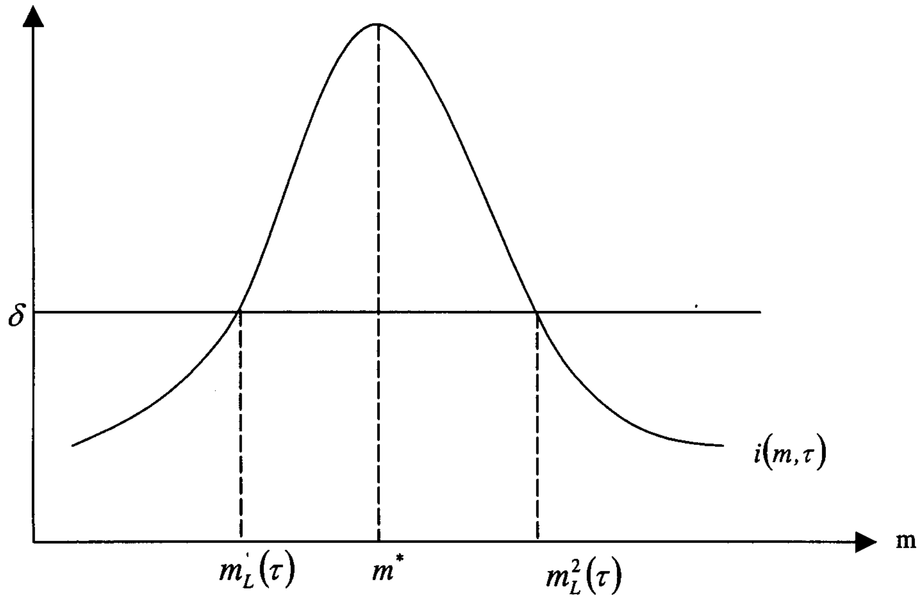

Figure 4: Plots of ξ(m,τ) and B(m,τ)$^{-1}$ for alternative alternative τ values

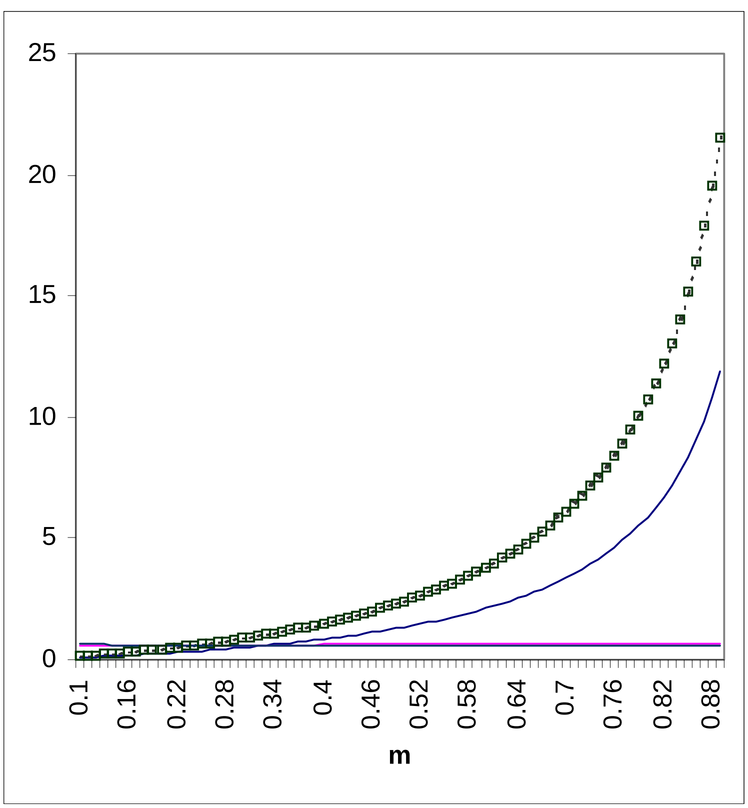

Figure 5: Growth-Gini Relationship for a Steady State Scenario when m< $m_c(\tau)$ and τ=.3

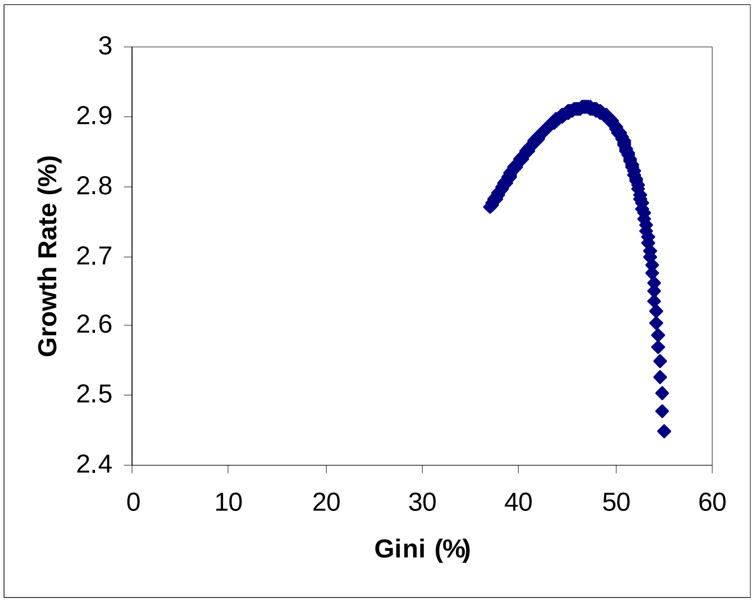

Figure 6: Growth-Gini Relationship for a Steady State Scenario when m< $m_c(\tau)$ and τ=.1

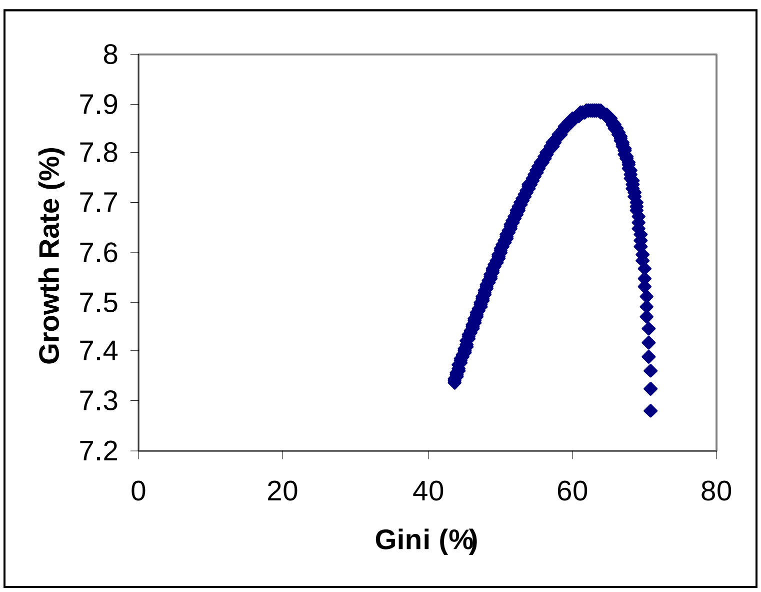

Figure 7: Steady State Growth-Gini Relationship when $m=m_c(\tau)$

Note: a=.1, δ=.1, A_0=5.5, θ=.2, β=.95