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Capturing the Shape of the Business Cycle with Autoregressive Leading Indicator Models

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1. Introduction

The motivation and the basis of this paper is the 1999 working paper written by Harding and Pagan titled, "Dissecting the Cycle". In their paper Harding and Pagan conclude that non-linear models do not capture the shape of the Business Cycle. The aim of this paper is to use the Harding and Pagan non-parametric procedure, to determine whether leading indicator variable models capture the business cycle. We use the same modelling strategy as in Anderson and Vahid (2000) to develop bivariate non-linear autoregressive models of GDP growth and interest rate spread for the US. These models were re-estimated for an updated data sample. We conclude that bivariate non-linear leading indicator models have a superior performance to univariate non-linear models, and that they can capture the shape of the US Business Cycle. The paper starts in section two with a brief introduction to the old graphical approach of observing the business cycle, and in particular the Burns and Mitchell methodology is described. We compare this to the new methodology where parametric models are developed to capture the business cycle. The Harding and Pagan BBQ algorithm is presented in section 2.2. Having established the algorithm for extracting the business cycle, the non-parametric measures of Harding and Pagan for describing the business cycle are presented in sections 2.3 and 2.4. In particular the Harding and Pagan excess index is discussed in section 2.3 and the tests for parametric models are described in section 2.4. A summary and discussion of the results from the Harding and Pagan "Dissecting the Cycle" paper follow in section 2.5. In sections 3.1 to 3.4 the linear and nonlinear bivariate autoregressive leading indicator models are developed and presented, and the simulation procedure for the purpose of evaluating these models via the non-parametric Harding and Pagan methodology is described. Results and discussion of the empirical application to US data follows in section 4, and the paper concludes in section 5 with general conclusions and findings.

2.1. Graphical versus Parametric Approach

The study of the business cycle from macroeconomic data has been a long lasting laborious procedure. Macroeconomists at the beginning of this field of study, were fascinated with the cyclical patterns observed in graphed data, which they referred to as the business cycle. Burns and Mitchell at the NBER[1] were the pioneers of the business cycle analysis. They developed their methodology based on a graphical approach. They identified that peaks and troughs of many time series seemed to be clustered together, and they referred to the dates of when the clustering of peaks and troughs occurred as the "reference cycles"[2]. Researchers tended to move away from the graphical approach, as there was considerable criticism of the Burns and Mitchell approach for lacking a statistical foundation[3]. Koopmans (1947) was the first to expose this, and to lead researchers into the field of parametric modelling of time series. Since then the majority of business cycle researchers have diverted their efforts towards this avenue. There are numerous methods for detrending time series, and extracting the business cycle. Boschan and Ebanks (1978) describe how NBER type researchers use "phase averaging". The ABS use Henderson filters[4] for smoothing time series in an effort to obtain estimates of the trend, and academics apply Hodrick-Prescott filters to remove trend and Band-Pass filters to remove both the trend and irregular component. Canova (1998) shows that the cyclical component produced from various time series are sensitive to the type of detrending procedure used.

2.2. BBQ algorithm

In their paper "Dissecting the Cycle", Harding and Pagan revert back to the graphical approach of extracting the business cycle. Their central aim is to enhance a graphical procedure for observing the business cycle, with the previously missing statistical foundation. Harding and Pagan identify four items of interest when considering the business cycle. They claim that the researcher should be most interested in the duration of the cycle and the cycle phases, i.e., how long it takes for the cycle and its phases to be completed. The second highlighted item of interest is the amplitude of the cycle and its phases. This is the total change (increase or decrease) in the indicator of economic activity[5], usually GDP, over the phases of the cycle and at the completion of the actual cycle. The third item of interest is the asymmetric behaviour of the cycle between its phases. The contraction phase of the cycle is usually of shorter duration than the expansion phase. The fourth and last item of interest is the cumulative movement within phases of the cycle i.e., the cumulative loss (gain) through a contraction (expansion) phase.

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[1] Burns and Mitchell (1946). It should be noted that Simkins (1994) refers to Burns and Mitchell as pioneers.
[2] There was difficulty faced as many time series do not have the same phase movement.
[3] Stock and Watson (1998) outline this point.
[4] ABS (1987)
[5] The indicator of economic activity used in the Harding and Pagan paper is the GDP (referred to often as output). Harding and Pagan refer to Burns and Mitchell who argue that you cannot go past the GDP as a single measure of economic activity. See Harding and Pagan (1999) footnote 14, page 15.

When using a graphical approach as a means to extract the business cycle, an algorithm which is able to perform the following, is needed. First and foremost the algorithm should be able to identify turning points. The identified turning points should be alternating, ensuring the chance of capturing the alternating phases of the business cycle ie. peak to trough and trough to peak. Identifying alternating turning points is not, however, enough to be able to extract the business cycle. The first two steps need to be combined with censoring criteria to ensure that the extracted business cycle is of some desirable form. Harding and Pagan describe the BB algorithm set out by Bry and Boschan (1971), in association with the NBER, as the best known algorithm. The BB algorithm was designed for monthly data. Turning points were identified when a local maximum (minimum) occurred within a time frame given by:

$$y_t > (<) y_{t\pm k} \qquad \text{for k} = 1, \ldots\text{K} \quad \text{where K} = 5 \text{ months} \tag{1}$$

This mathematical rule was combined with the following censoring criteria. Each phase of a cycle should have minimum duration of six months and the duration of the whole cycle should be of minimum fifteen months. Harding and Pagan adjusted these rules with the aim of applying them to quarterly data, and the BBQ[6] algorithm was created. In the BBQ algorithm turning points are identified when a local maximum (minimum) is observed around two quarters:

$$y_t > (<) y_{t\pm k} \qquad \text{for k} = 1, \ldots\text{K} \quad \text{where K} = 2 \text{ quarters} \tag{2}$$

Each phase of a cycle should have a minimum duration of 2 quarters, and the whole cycle should have a minimum duration of five quarters[7].

2.3 Harding and Pagan's Non-parametric measure of the Business Cycle characteristics

Having created the algorithm for extracting the turning points, Harding and Pagan develop a non-parametric procedure in a means for testing how well various parametric models capture the business cycle. Consider a typical recession phase of output shown graphically in figure 1.

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6 Bry and Broschan for Quarterly data

7 It should be noted that this is a basic outline of the Harding and Pagan censoring criteria. For more details the reader should refer to the Harding and Pagan paper.

The recession phase can be thought of as the triangle ABC. The height of the triangle shows the amplitude A_i of the expansion ie. the total loss in output from peak A to trough C. The base of the triangle shows the duration D_i of the phase ie. how long it takes for this recession phase to be completed from peak A to trough C. The area of the triangle ABC,

$$C_{Ti} = \frac{1}{2} D_i A_i \tag{3}$$

can approximate the cumulated losses in output from peak to trough. Harding and Pagan refer to this as the "*triangle approximation*" to the cumulative losses. A better approximation of the total area under the actual path can be given by the summation of the area of each of the rectangles r_t, shown in figure 2.

Let the total area covered by the rectangles equal to the actual cumulated loss C_i:

$$C_i = \sum_{t=1}^{T} r_t \tag{4}$$

An even better approximation to the total area under the path would be the rectangular approximation minus the area of the small triangles shown in figure 3. The area of each of the small triangles is $s_t = \alpha_t/2$[8]. Thus the total area of phase i covered by the small triangles is

$$S_t = \frac{1}{2} \sum_{t=1}^{T} \alpha_t = \frac{1}{2} A_i . \tag{5}$$

Therefore the best approximation to the area under the actual path will be

$$\overset{\frown}{ABC} = \mathrm{C_i} - \frac{1}{2} \mathrm{A_i} . \tag{6}$$

From the above-calculated areas we can calculate what we refer to as the "Excess" area shown in figure 4. Mathematically this area is calculated by,

$$- Excess = C_i - \frac{1}{2} A_i - \frac{1}{2} D_i A_i . \tag{7}$$

[8] The base of each small triangle is equal to 1 unit of duration.

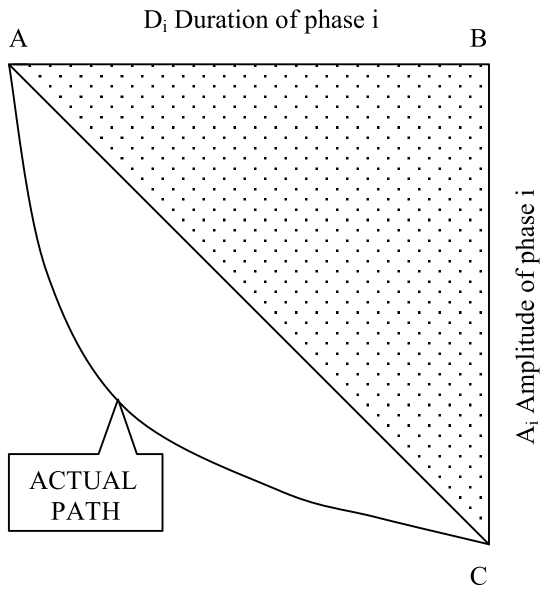

Figure 1:

Typical recession phase. Triangle approximation of cumulated losses shown by the shaded area.

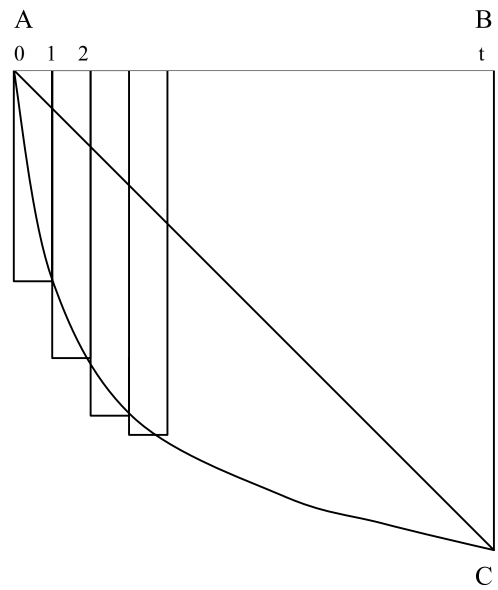

Figure 2:

Rectangle approximation of the cumulative losses.

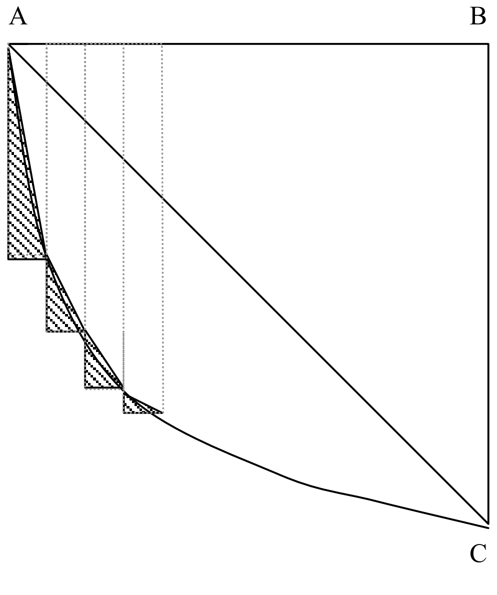

Figure 3:

Small triangles shown by the shaded area.

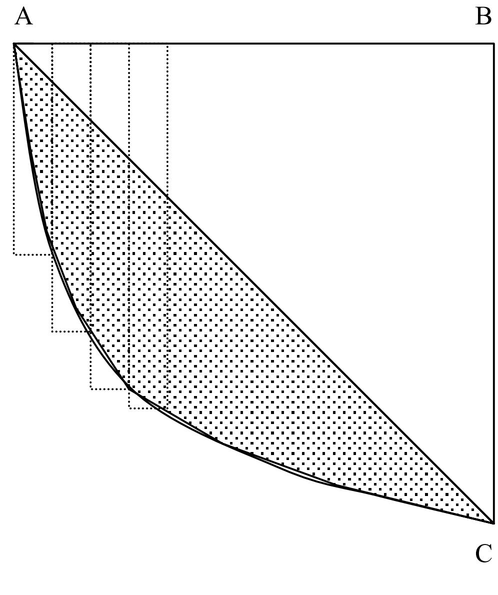

Figure 4:

Shaded area shows the excess area.

From this area Harding and Pagan created the following "*excess index*"

$$E_i = \frac{\frac{1}{2} D_i A_i + \frac{1}{2} A_i - C_i}{D_i}, \qquad (8)$$

which is what Harding and Pagan use to graphically evaluate parametric models of the business cycle. The excess index describes the shape of the actual business cycle relative to the triangle approximation. Harding and Pagan show that the models considered in their paper are able to capture the triangle approximation well i.e. they do well in capturing the duration and amplitude of a cycle. However they do not perform any better than the random walk in capturing the actual shape of the cycle which is described by the excess index. The ability of models to capture the excess index is put to the test via simulating GDP data from the models and passing it through the Harding and Pagan "Dissecting the Cycle" Gauss program[9].

2.4. Tests of parametric models

Harding and Pagan created a Gauss program, which passes GDP data through the BBQ algorithm, and returns the average measures for each phase of the business cycle (ie PT-peak to trough and TP-trough to peak) presented in table 1[10]. They recommend testing alternative parametric time series models of GDP by examining if the models can generate business cycles which are not statistically significantly different from the business cycles in the actual real GDP series. They simulate data from the estimated statistical model, form the empirical density function of the four shape measures and observe where the shape measure of the actual data lies on this empirical density. If the actual measurements are in the 5% upper or lower tails of the density, then it would be unlikely that the actual data was generated by that statistical model.

[9] We would like to thank Don Harding for making this Gauss program available to us.
[10] It should be noted that the measures for each cycle present in the data are obtained and then they are averaged over the number of cycles present.

Measure	Description
Mean Duration (quarters)	How many quarters it takes on average to complete each phase of the cycle.
Mean Amplitude (%)	Total percentage decrease or increase in output per phase of the cycle on average.
Cumulation (%)	Cumulative percentage loss or gain in output per phase of the cycle on average.
Excess (%)	The average excess percentage value per quarter relative to the triangle approximation.

Table 1: Average measures returned by the Harding and Pagan Gauss program.

2.5. Summary of Models considered by Harding and Pagan and Results

The first model that Harding and Pagan consider, is the random walk with drift model

$$y_t = y_{t-1} + \mu_y + \sigma e_t \qquad \text{where } e_t \sim \mathrm{N}(0,1), \tag{8}$$

with μ_y and σ set to their sample estimates for the period of 1947:1-1997:1 for US GDP. The results are presented in column 2 of table 2. The values marked with an asterisk are of most interest. The asterisk indicates that in the simulated distribution, less than 5% of simulated values were found to be further out in the tail of the distribution than the sample estimate. This rejects the hypothesis that the model can reproduce the sample estimate. Observing the results, one can see that the random walk with drift does well in capturing the average length of the business cycle for both phases. It also captures the mean amplitude and cumulation for the expansion phase of the business cycle. However the important statistic here, is the inability of the model to capture the excess value from trough to peak.

After simulating from the random walk Harding and Pagan make a few more alterations in an attempt to improve this linear model[11]. They examine whether the inclusion of some non-linearity in models, due to the asymmetry between expansion and contraction phases in the cycle, improves the overall results. They consider the Hamilton non-linear univariate model

[11] For more details see Harding and Pagan (1999) pages 19-26.

and the duration dependant modification of the Hamilton model by Durland and McCurdy[12]. They generate 15,000 GDP observations from these models and their results are presented in table 2. Also Harding and Pagan consider the linear bivariate VAR(2) model with the two variables being real GDP and Investment.

	US Data	RW + drift	Hamilton	Dur Dep	VAR(2)
Mean Duration (qrts)					
PT	3	2.3	4.4	4.8*	3.2
TP	17.8	16.4	20	16.9	23.2
Mean Amplitude (%)					
PT	-2.5	-1.5*	-2.8	-3.3*	-1.8
TP	20.2	16.6	27.3	25.0	23.3
Cumulation (%)					
PT	-4.1	-1.8*	-8.2	-8.5*	-3.8
TP	256	254	496	293	549
Ecxess (%)					
PT	-0.1	0.0	0.0	0.0	0.0
TP	1.1	-0.0*	-0.0*	0.0*	-0.0*

Table 2: Harding and Pagan (1999) simulated Business Cycle characteristics.

One can observe from table 2 that the nonlinear Hamilton model and the linear VAR(2) are improvements to the Random Walk with respect to capturing the mean duration, amplitude and cumulation. When duration dependence is added to the Hamilton model, this improvement disappears. The important observation from this table and is that none of the models do better than the Random Walk with Drift in capturing the Excess Index from Trough to Peak. All of these Excess values are rejected at an overall 10% significance level. Thus Harding and Pagan conclude that, "there is little evidence that nonlinear effects are important to the nature of the business cycle"[13].

3.1. Models considered in this paper

The Harding and Pagan findings provide clear motivation for further investigation into alternative models that might capture the "shape" of the business cycle. Initially two main directions for research are pursued in this ongoing study. First and foremost, having recognised

[12] For extensive details on the nature of these models see Hamilton (1989) for the Hamilton model and Durland and McCurdy (1994) for the Hamilton model with duration dependence.
[13] Harding and Pagan (1999) page 31.

the asymmetric nature of the phases of the business cycle, there is motivation for further study of the performance of non-linear models. The Harding and Pagan paper considered univariate non-linear models. In this paper we explore the performance of *multivariate* non-linear models. Secondly, the linear bivariate model employed by Harding and Pagan made use of GDP and a real variable (investment). As discussed in the next paragraph there are arguments supporting the use of financial or monetary variables in place of real variables. This led to "Inspecting the US Business Cycle"[14] (Athanasopoulos 2000), an initial study of the non-linear models presented by Anderson and Vahid in their paper "Predicting the Probability of a Recession with Autoregressive Non-Linear Leading Indicator Models" (2000).

In their paper Anderson and Vahid develop linear and non-linear indicator models for US output (GDP) growth. The spread between short and long-term interest rates is used as the leading indicator of output in their bivariate models, and both the interest rate spread and M2 are used as leading indicators of output in their trivariate models. The aim of these models, as can be gathered from the title of the paper, is to predict the probability of a recession. The use of financial or monetary variables as leading indicators comes from the fact that a large body of historical literature has shown that such variables Granger-cause output.

Anderson and Vahid use the spread between short and long-term interest rates[15] as the main leading indicator variable, and then explore the contribution of M2, as an additional leading indicator variable. The use of spread as the main leading indicator is based on recent research literature, which recognises the spread as a good leading indicator of output[16]. They then explore the use of M2 based on literature that has used M2 as a leading indicator variable[17]. It should be noted here that the conclusion that M2 does not enhance the performance of their bivariate models is reached. Hence in this paper we only consider the bivariate models using spread as a leading indicator of output growth.

As can be gathered, the models developed by Anderson and Vahid comply with the two directions of development stated above. Firstly the models use more than one variable and

[14] It should be noted that in this initial study the models of Anderson and Vahid were employed as they we presented. In this paper these models have been re-estimated due to the availability of updated data for the US.
[15] Hence in their bivariate models spread is used rather than M2.
[16] See Estrella and Mishkin (1998) and Karunaratne (1999)
[17] See Stock and Watson (1989). Also Anderson and Vahid refer to Alan Greenspan the Federal Reserve Chairman, who in 1993 informed the Congress that M2 has been "downgraded as a reliable indicator of financial conditions in the economy", and that "the historical relationship between money and income had broken down".

secondly they make use of financial or monetary rather than real variables. In the following sections we re-estimate the bivariate linear and non-linear leading indicator models with spread being the leading indicator of output growth and evaluate their performance based on the Harding and Pagan BBQ algorithm and non-parametric measures outlined in sections 2.2 and 2.3.

3.2. The Data

The data used in our simulation procedure is for the period of 1960:1 to 2000:1. It consists of seasonally adjusted US real GDP quarterly observations and a spread between long term (10-year Treasury Bonds) and short term (3-month Treasury Bills) interest rates[18]. Output growth y_t (= 100 x Δln real GDP_t) is referred to as output and s_t (the difference between the interest rates above) is referred to as the spread.

3.3. Model Development

The basis of the development of the bivariate models was a VAR for output and spread. AIC criterion selected a VAR(5). However after omitting the insignificant lags and testing for serial correlation present in the residuals[19] of each equation the highest lag order is three. The models were firstly estimated by employing equation by equation OLS and then by using Seemingly Unrelated Regression. The resulting models of these estimations are presented in Appendix 1. It should be noted that the final model specification of these two estimation procedures is the same, with very similar parameter estimates. Therefore the only linear model used in the simulation procedure which follows is the ARLI (linear Autoregressive Linear Indicator Model) estimated via equation by equation using OLS.

Having developed the VAR(3) as the linear base of our US data study we then examine whether the data presents any non-linearities. There is strong evidence of an omitted CDR term in the output equation with a p-value of zero. The null hypothesis of linearity is rejected multiple times in favour of STAR non-linearities (Luukkonen et al. (1988)) as can be observed in table 3. This test is based on the joint significance of non-linear terms being present in the

[18] All US data was drawn from the US Federal Reserve Data Base (FRED).

linear VAR equations. The non-linear terms consist of products of the possible transition variable with the lagged regressors and the third power of the possible transition variable Anderson and Vahid (2000)).

Test	***Transition variable***	***Output (y) equation***	***Spread (s) equation***
Luukkonen et al.	y_{t-1}	0.5888	0.0034
	y_{t-2}	0.0449	0
	s_{t-1}	0.0112	0.0019
	s_{t-2}	0.0099	0.0571
	s_{t-3}	0.0235	0

Table 3: p-values for non-linearity tests.

Based on the minimum p-value observed for the STAR non-linearities, the second lag of spread was selected as the transition variable in the output equation. The other two lags of spread were also trialed as transition variables (since they were found to be significant in the test), but the second lag of spread returned the highest likelihood value. In the same manner, the third lag of spread was first selected as the transition variable in the spread equation and then the first lag was trialed as well. In this case the maximised likelihood value returned for the spread equation was higher for the first lag of spread than the third. Thus the first lag of spread was used as the transition variable in the spread equation. The resulting bivariate non-linear, autoregressive leading indicator model (Bi-NARLI) was estimated equation by equation and the resulting model is presented in Appendix 1. We also estimated a model in which both the output and the spread equations are restricted to have the same non-linear factor. This model, which has the second lag of spread as the transition variable, is referred to as the common non-linear autoregressive leading indicator model (Com-NARLI) and it is also presented in Appendix 1.

3.4. Simulating Output

In order to study the performance of the models developed in this paper, in "capturing" the business cycle, output data was simulated from these models. The models were fitted to the period for which they were estimated and the covariance structure of the residuals was

[19] It should be noted that the tests performed for the presence of non-linearities to enhance the development of the non-linear models highly depend on the presence of no serial correlation thus tests where performed to ensure no serial correlation was present in our residuals.

calculated. By providing the models with the three initial observations[20], plus a random error term with the covariance structure of the residuals, we were able to simulate output data. 10,000 output series of 163 observations were generated and each one was passed through the Harding and Pagan program to obtain the statistics of interest outlined in section 2.3. This enabled us to view the simulated distribution for each of these statistics. These results are presented and discussed in the following section.

4.1. Empirical Application and Results

The first model considered, as in the Harding and Pagan paper was the random walk with drift,

$$\Delta y_t = \mu_y + \sigma e_t \qquad \text{where } e_t \sim \text{N}(0,1), \tag{17}$$

with μ_y and σ set to their sample estimates for the period of 1960:1-2000:4 for US GDP growth (output). The simulation results for this model (see table 4, column 2) were very similar to the Harding and Pagan results. Table 4 shows the average over the 10,000 simulated series, of all the statistics of interest in the Harding and Pagan methodology. The boundaries of the 90% intervals of the simulated distributions are presented in the brackets. Values with asterisks signify that more than 5% of the simulated values were found to be further in the tail of the distribution than the sample estimate, thus making these significantly different from the sample estimate. The mean duration, mean amplitude and cumulation from peak to trough were found to be significantly different to the sample estimates, as was the excess index from trough to peak. Harding and Pagan (1999) have shown that generally other models were improvements on the random walk in capturing most statistics. However the random walk performed as well as all the others in capturing the excess statistic from trough to peak. This can be interpreted as the models not being able to capture the actual shape of the business cycle[21]. The results of the random walk can be used as the basis for evaluating the performance of the linear and non-linear models developed in this paper.

When simulating output from the bivariate linear model (ARLI) we find that it has performed better than the random walk in that the mean duration, mean amplitude and cumulation

[20] Due to the highest lag being of 3rd order.

[21] As shown section 2.3 this means that models do well in capturing the triangle approximation of the phases of the business cycle. However they do not perform well in capturing the better approximation which is the triangle plus the excess area.

statistics were statistically the same as the sample estimates. However, as was the case with the random walk, the excess statistic from trough to peak was found to be significantly different from the sample estimate. An improvement in capturing the excess statistic comes when the bivariate non-linear models are considered. From table 4 it is clear that both specifications of the non-linear leading indicator models perform very well in capturing the shape of the business cycle via the excess statistic.

	US Data	RW + DRIFT	ARLI	Bi-NARLI	Com-NARLI
Mean Duration (qrts)					
PT	3.8	2.51* (2,3.55)	3.28 (2,5)	2.8765 (2,4)	3.08 (2,4.4)
TP	20.4	35.12 (16.5,70.5)	28.51 (14.37,54.5)	36.99 (17.2,72.5)	32.61 (15.8,66)
Mean Amplitude (%)					
PT	-2.14	-1.1057* (-1.79,-0.54)	-1.52 (-2.58,-0.74)	-1.7119 (-3.11,-0.63)	-1.84 (-3.22,-0.79)
TP	22.86	32.44 (15.1,64.41)	27.99 (13.4,54.19)	35.51 (16.43,69.2)	34.38 (16.2,69.43)
Cumulation (%)					
PT	-4.23	-1.5* (-3.12,-0.53)	-3.15 (-7.57,-0.85)	-2.8081 (-6.39,-0.67)	-3.35 (-7.48,-0.91)
TP	342	977 (173,2839)	704 (138,2084)	1101 (194,3301)	954 (181,2851)
Excess (%)					
PT	-0.0979	-0.0015 (-0.15,0.15)	0 (-0.16,0.16)	0.01 (-0.22,0.26)	0.017 (-0.19,0.23)
TP	1.3636	0.023* (-1.19,1.25)	0.02* (-1.21,1.31)	0.0788 (-1.2,1.399)	0.0632 (-1.3,1.398)

Table 4: US Business Cycle and Simulated Business Cycle characteristics. The values in the brackets show the 90% bounds of the simulated distributions for each statistic. The values with asterisks highlight the sample statistics for the US being outside the 90% bounds.

4. Conclusion

There are various conclusions that can be drawn from this paper. First of all this paper has confirmed that univariate nonlinear models cannot adequately capture the business cycle. Univariate models, as presented by Harding and Pagan, fail to capture the actual shape of the business cycle for the US. This is not necessarily an argument against the non-linearity of these models, but more an argument against the univariate representation of them. The bivariate linear and non-linear models seem to be more appropriate than univariate ones. In particular the non-linear autoregressive leading indicator models (Bi-ARLI and Com-NARLI) seem to have a superior performance to the univariate models considered in the Harding and Pagan paper. The natural extension to this study, which is under way, is to examine the performance of such models for countries other than the US.

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APPENDIX 1

BIVARIATE MODELS OF OUTPUT AND THE SPREAD (1960:1-2000:4)
(standard errors are presented in brackets)

ARLI-OLS model for output and spread:

$$\hat{y}_t = \underset{(0.11)}{0.29} + \underset{(0.08)}{0.20}\, y_{t-1} + \underset{(0.08)}{0.14}\, y_{t-2} + \underset{(0.06)}{0.2}\, s_{t-2}$$

$$\hat{s}_t = \underset{(0.08)}{0.26} - \underset{(0.05)}{0.15}\, y_{t-2} + \underset{(0.08)}{1.06}\, s_{t-1} - \underset{(0.11)}{0.36}\, s_{t-2} + \underset{(0.08)}{0.19}\, s_{t-3}$$

ARLI-SUR model for output and spread:

$$\hat{y}_t = \underset{(0.11)}{0.29} + \underset{(0.08)}{0.19}\, y_{t-1} + \underset{(0.08)}{0.14}\, y_{t-2} + \underset{(0.05)}{0.2}\, s_{t-2}$$

$$\hat{s}_t = \underset{(0.08)}{0.26} - \underset{(0.05)}{0.15}\, y_{t-2} + \underset{(0.08)}{1.06}\, s_{t-1} - \underset{(0.11)}{0.35}\, s_{t-2} + \underset{(0.08)}{0.18}\, s_{t-3}$$

Bi-NARLI model for output and spread:

$$\hat{y}_t = \underset{(0.23)}{-0.52}\, y_{t-1} + \underset{(0.24)}{0.49}\, y_{t-2} + \underset{(0.29)}{0.50}\, y_{t-3} - \underset{(0.26)}{0.66}\, s_{t-1} \underset{(0.36)}{-1.37}\, s_{t-2} +$$
$$f_{yt} \text{ X } (\underset{(0.16)}{0.81} + \underset{(0.24)}{0.71}\, y_{t-1} - \underset{(0.25)}{0.43}\, y_{t-2} - \underset{(0.29)}{0.57}\, y_{t-3} + \underset{(0.29)}{0.73}\, s_{t-1} - \underset{(0.37)}{1.40}\, s_{t-2})$$
$$f_{yt} = (\,1 + \exp\{-14\,(s_{t-2} - 0.024)\}\,)^{-1}$$

$$\hat{s}_t = \underset{(0.09)}{0.45} - \underset{(0.08)}{0.24}\, y_{t-2} + \underset{(0.13)}{1.19}\, s_{t-1} \underset{(0.14)}{-0.56}\, s_{t-2} +$$
$$f_{st} \text{ X } (\,\underset{(0.25)}{0.21}\, y_{t-2} - \underset{(0.07)}{0.11}\, y_{t-3} - \underset{(0.17)}{0.47}\, s_{t-1} + \underset{(0.20)}{0.35}\, s_{t-2} + \underset{(0.09)}{0.34}\, s_{t-2})$$
$$f_{st} = (\,1 + \exp\{-6.79\,(s_{t-1} - 1.24)\}\,)^{-1}$$

Com-NARLI model for output and spread:

$$\hat{y}_t = \underset{(0.43)}{-1.45} - \underset{(0.24)}{0.39}\, y_{t-1} + \underset{(0.27)}{0.76}\, y_{t-2} + \underset{(0.18)}{0.29}\, s_{t-3} - \underset{(0.53)}{1.89}\, com_t$$

$$\hat{s}_t = \underset{(0.29)}{1.25} + \underset{(0.13)}{0.23}\, y_{t-1} - \underset{(0.16)}{0.46}\, y_{t-2} + \underset{(0.07)}{1.03}\, s_{t-1} - \underset{(0.11)}{0.19}\, s_{t-2} + com_t$$

$$com_t = (\,1 + \exp\{-1.19\,(s_{t-2} - 0.555)\}\,)^{-1} \text{X } (-\underset{(0.4)}{1.27} - \underset{(0.13)}{0.32}\, y_{t-1} + \underset{(0.17)}{0.39}\, y_{t-2} + \underset{(0.08)}{0.19}\, s_{t-3})$$