

# Optimal Exchange Market Intervention with Heteroscedastic Exchange Rates

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## Abstract

In a floating exchange rate regime one of the most important variables is the degree to which the monetary authorities intervene in the foreign exchange market, and the extent to which the authorities intervene often depends on their policy motives. In Australia, there has been a distinct policy shift towards inflation targeting in the early 1990s. In this paper, we model the optimal intervention and examine its nature of dependence on the exchange rate and its volatility. Further, we investigate whether the optimal intervention rule is consistent with the policy shift in the 90s. We define large and “normal” optimal interventions conditional on the quantiles of the distribution of optimal intervention series. The mean of the optimal intervention was shown to follow a nonlinear two-state regime switching threshold model with threshold parameters being defined as the quantile estimates. We find that the optimal intervention is affected by its short-term dynamics and the exchange rate volatility, the latter only when the optimal intervention is large, and that the variance of the optimal intervention is time varying and follows an EGARCH process. The policy shift towards inflation targeting is apparent in the historical decomposition of the target variables, output growth and inflation. The results also indicates that the greater the relative weights on inflation to output, the higher the degree of optimal intervention, which is consistent with the theoretical results discussed in the literature.

Keywords: Foreign exchange intervention, GARCH, VAR, Historical Decomposition  
JEL Classification: C52, F31

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## 1.0 Introduction

Since the collapse of the Bretton Woods system most countries have moved towards either a managed floating or a pure floating regime. In the case of a managed float the main concern is the degree to which the monetary authorities intervene in the foreign exchange market. The extent to which the authorities intervene in the market largely depends on their policy motives. Several studies have shown that the optimal intervention policy would be the one that minimises the exchange rate volatility and achieves macroeconomic targets, such as output growth and inflation (e.g. Roper and Turnovsky (1980) and Weymark (1999))<sup>1</sup>. In Australia, there has been a distinct policy shift towards inflation targeting in the early 1990s.<sup>2</sup>

The primary objective of the paper is to derive and model the optimum intervention rule for Australia and examine whether the policy change is consistent with the theoretical model over the post-float period. Using the quantile estimates of the distribution of optimal intervention, we define the large and “normal” levels of intervention series. A nonlinear threshold model with two regimes will be used to model the optimal intervention, and investigate the effects of the exchange rate and its volatility on optimal intervention, in particular, when it is “normal” or large. Using a bivariate VAR, historical decompositions of output growth and inflation would be estimated to examine whether a policy shift towards inflation targeting in the early 1990’s is consistent with the optimal intervention rule.

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<sup>1</sup> Related studies examining the optimal level of intervention include Boyer (1979), Roper and Turnovsky (1980), Turnovsky (1983,1985), Jones (1984) and Cadenillas and Zapatero (1999).

<sup>2</sup> See Grenville (1997) for a detailed discussion of the Australia monetary policy changes over the post-float period.

Deriving an optimal level of intervention has been given considerable attention in the literature. In a small open economy model of intervention, the optimal level of intervention derived by optimizing a loss function, which is a weighted average of policy target variables, was shown to be constant over time; see, for example, Roper and Turnovsky (1980). Recently, Weymark (1999) has shown that the level of intervention is not constant as the disturbances of economic variables such as the exchange rate, output and inflation are generally highly volatile yielding time varying optimal intervention. Incorporating the volatilities of economic variables into the model, Weymark has shown that the optimal intervention can have profound policy implications, depending on the weights on output growth and inflation. Theoretical results reveal that the greater the relative weighting on inflation, the higher the degree of ‘optimal’ intervention.

The remainder of this paper is organised as follows. Section 1 presents the theoretical model of the optimal intervention rule with heteroscedastic exchange rates. Section 2 provides a description and the features of the data series used in this study. The empirical results of the models of optimal intervention for Australia and their analyses are reported in section 3. The results of historical decompositions of the output and inflation are also reported in this section. The final section gives some concluding comments and suggestions for further research.

## 2.0 The Model

In this section we consider the Weymark’s (1999) model to derive a time varying optimal intervention coefficient. The small open economy model is given as follows:

$$y_t = \mathbf{a} \{p_t - E[p_t | t-1]\} + u_t \quad (2.1)$$

$$m_t = p_t + b_1 y_t - b_2 \{E[e_{t+1} | t]\} - e_t + v_t \quad (2.2)$$

$$p_t = p_t^* + e_t \quad (2.3)$$

where  $y_t$  is the real domestic output,  $p_t$  is the domestic price level,  $p_t^*$  is the foreign price level,  $i_t$  is the domestic interest rate,  $m_t$  is the money,  $e_t$  is the exchange rates per unit of foreign currency, and  $u_t$  and  $v_t$  are random error terms. All variables are expressed in logarithms. Equation (2.1) describes the domestic supply of output. Equation (2.2) describes the domestic money market equilibrium in which money supply is assumed to be equal to money demand, while the equation (2.3) describes the purchasing power parity condition.

By substitution, the following semi-reduced form for  $e_t$ ,  $p_t$  and  $y_t$  can be obtained.

$$e_t = \frac{1}{1+b_2+b_1a} \left[ -(1+ab_1)p_t^* + b_1aE[p_t|t-1] - b_1u_t + b_2E[e_{t+1}|t] - v_t + m_t \right] \quad (2.4)$$

$$p_t = \mathbf{b} \{ p_t^* - p_t^* + b_2p_t^* + ab_1E[p_t|t-1] - b_1u_t + b_2E[e_{t+1}|t] - v_t + m_t \} \quad (2.5)$$

$$y_t = \{ ab_2p_t^* + ab_2E[e_t|t-1] - av_t + am_t - a(1+b_2)E[p_t|t-1] + u_t(1+b_2) \} \mathbf{b}^{-1} \quad (2.6)$$

In order to compute the optimal intervention coefficient, policy authorities generally either minimizes a loss function or maximizes a profit function. Following Waymark (1999), this paper considers the loss function defined in terms of  $y_t$  and  $p_t$  given as,

$$L_t = y_t^2 + sp_t^2 \quad (2.7)$$

where  $s$  is the weighing parameter representing the price stabilisation relative to output stabilisation; see Weymark (1999) for details. The optimal monetary policy can be obtained by minimising the loss function with respect to  $m_t$  as follows:

$$m_t(opt) = -b_2E[p_t^*|e_t] + \frac{[a(a(1+b_2) - sb_1)]}{a^2 + s} E[p_t|t-1] + b_2E[e_{t+1}|t] + \frac{[b_1s - a(1+b_2)]}{a^2 + s} E[u_t|e_t] + E[v_t|e_t] \quad (2.8)$$

The model (2.8) assumes that the private agents know what the policy authorities' objective function is and that policy authorities use observed exchange rates to guide its policy actions. However,  $e_t$  is unknown at the time price forecasts are made. It is also assumed that foreign prices are independent of its error term and output disturbances are white noise.

In the optimal intervention literature, economic disturbances have been assumed to be white noise, although it is contrary to empirical evidence.<sup>3</sup> Relaxing this assumption, the error term of the money demand function of equation (2.2) is specified to follow an ARCH process. That is,

$$v_t = \mathbf{e}_t [a_0 + \sum_{i=1}^q a_i \mathbf{n}_{t-i}^2]^{\frac{1}{2}} \quad (2.9)$$

where  $\mathbf{e}_t$  is white noise and independent of  $v_{t-1}$ . The optimal monetary policy where the money demand disturbances are characterised by an ARCH process is,

$$m_t = -b_2 E[p_t^* | e_t] + \frac{[(sb_2 - \mathbf{a}(1 + b_2))]}{(\mathbf{a}^2 + s)} E[u_t | e_t] + b_2 E[e_{t+1} | t] \\ + [\mathbf{a}_0 + \sum_{i=1}^q \mathbf{a}_i \mathbf{n}_{t-i}^2] + E[\mathbf{e}_t | e_t] \quad (2.10)$$

Following Buiter and Eaton's (1985), equation (2.10) can be expressed as

$$m_t(opt) = -r_t e_t \quad (2.11)$$

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<sup>3</sup> See Weymark (1999)

$$\mathbf{r}_t = \{b_2 c \text{cov}(\mathbf{r}_t^*, e_t) - \frac{[sb_1 - \mathbf{a}(1+b_2)]}{(\mathbf{a}^2 + s)} \text{cov}(u_t, e_t) - [\mathbf{a}_0 + \sum_{i=1}^q \mathbf{a}_i \mathbf{n}_{t-i}^2] \text{cov}(\mathbf{e}, e_t)\} [\text{var}(e_t)]^{-1} \quad (2.12)$$

The optimal intervention coefficient,  $\mathbf{r}_t$  in equation (2.12), characterises the monetary authority's exchange rate policy. If  $\mathbf{r}_t = 0$ , then it implies that the monetary authority does not intervene and let the exchange rate floats freely. If  $\mathbf{r}_t = \infty$ , then the authority intervenes heavily and adopts a fixed exchange rate policy. If  $0 < \mathbf{r}_t < \infty$ , then the authority operates under a managed exchange rate system.

From equation (2.12), it is evident that for the optimal intervention coefficient to be constant over time, the variance of exchange rate must also be constant. This is inconsistent with mounting empirical evidence of heteroscedastic properties of exchange rates, (eg. Bollerslev, Chou and Kroner (1986)). The variance and covariances of equation (2.12), can be used to obtain the optimal level of intervention,

$$\mathbf{r}_t = -b_2 + \frac{[sb_1 - \mathbf{a}] b_1 \mathbf{s}_u^2 + (a_0 + \sum_{i=1}^q a_i v_{t-i}^2) (\mathbf{a}^2 + s) \mathbf{s}_e^2}{\mathbf{a} b_1 \mathbf{s}_u^2 + (1 + \mathbf{a} b_1) (\mathbf{a}^2 + s) \mathbf{s}_{p^*}^2} \quad (2.13)$$

It is evident from equation (2.13) that by incorporating ARCH characteristics to the disturbances of the domestic money equation (2.2) the optimal intervention rule can be shown to be time varying.

The differentiation of  $\mathbf{r}_t$  with respect to  $s$ , where  $s$  is the relative weighting the policy authorities place on inflation to output, we obtain

$$\frac{\partial \mathbf{r}_t}{\partial s} = \frac{\mathbf{a} b_1 (1 + \mathbf{a} b_1)^2 \mathbf{s}_u^2 \mathbf{s}_p^2 + \mathbf{a} b_1^3 \mathbf{s}_u^4 + \mathbf{a} b_1 [a_0 + \sum_{i=1}^q a_i v_{t-i}^2] \mathbf{s}_e^2 \mathbf{s}_u^2}{[\mathbf{a} b_1 \mathbf{s}_u^2 + (1 + \mathbf{a} b_1) (\mathbf{a}^2 + s) \mathbf{s}_{p^*}^2]^2} > 0 \quad (2.14)$$

Clearly, the optimal intervention is positively related to  $s$  and hence the degree of intervention increases with the relative weight that the central bank places on inflation. That is, the more emphasis the policy authorities place on price stability, the higher the optimal degree of exchange rate intervention.

In section 4, the optimal intervention rule for Australia will be modeled and examined whether its movements correspond to those of output growth and inflation. Since Australia has inflation targeting as one the main objectives of macroeconomic policy, we should expect a larger optimal degree of exchange rate intervention when the policy authority places more emphasis on price stability relative to output stability.

### 3.0 Data Description

Monthly data series were collected from dX Data Base for the period 1984:1 to 2000:1.<sup>4</sup> The optimal intervention model was estimated for the Australian dollar exchange rate against the US dollar ( $usd_t$ ). The Australian money stock (M1-monetary aggregates) ( $m_t$ ) represents the domestic level of money. The Australian consumer price index ( $p_t$ ) represents the domestic price level. The US CD rate ( $i_t^{us}$ ) represents the US interest rates. The US consumer price index ( $p_{us}$ ) represents a proxy for the foreign price level. The Australian gross domestic product (GDP) ( $y_t$ ) represents the domestic level of output and the US GDP ( $y_t^{us}$ ) represents the foreign level of output. All series denoted by lower case letters, except the US interest rates, indicate that these variables are measured in logarithms. See Appendix 1 for further details on data construction. The data series are plotted in Appendix 2. Time series properties of each of the series are reported in Appendix 3. The results indicate that all series are integrated of order one.

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<sup>4</sup> Monthly series for Australian GDP growth and inflation rate are not available and therefore quarterly series were used.

We note that the optimal intervention series was computed using monthly money supply and exchange rate series, while the historical components of output and inflation series were estimated using quarterly data.

The net foreign exchange purchases by the Reserve Bank is used as a proxy for the actual level of intervention by the RBA. The series were obtained from the RBA. The graphical representation of this series is given in Appendix 2. Negative values represent purchases of foreign exchange (lending support to the Australian currency), while the positive values represent the sales of foreign exchange. This intervention series is not a pure intervention series as it also includes purchases and sales by the RBA on behalf of the Government. However, this series is used to represent intervention in many previous studies; see, for example, Andrew and Broadbent (1994).

## **4.0 Empirical Results**

In this section, an empirical analysis of modeling Australian optimal intervention is given. Section 4.1 provides the results of modeling the optimal intervention rule described in Section 2. Moreover, Section 4.2 investigates the RBA policy implications using historical decompositions of some target variables.

### **4.1 Modeling Optimal Intervention**

Theoretical models of the optimal intervention rule predict that in the presence of ARCH-type disturbances of exchange rates, output growth and inflation rates, optimal intervention should possess time varying properties. Therefore, LM procedures were first used to test for the presence of ARCH or its extensions in the US dollar ( $usd_t$ ), Australian output growth ( $y_t$ ) and the Australian inflation rate ( $p_t$ ). The equations to which the tests were performed and the empirical results are reported in Appendix 4. The results indicate the null of no ARCH disturbances is rejected for the US dollar and the Australian inflation rate. In equation (2.11),  $r_t$  is computed as the ratio of change in money to change in exchange rates multiplied by  $-1$ . The optimal intervention

coefficient for Australia is expected to be influenced by the US dollar, the volatility of the US dollar and the US interest rate. The best model specification of  $r_t$  was selected based on AIC (Akaike Information Criterion).

After some preliminary analysis, we consider the following model for the optimal intervention coefficient:

$$r_t = a_0 + a_1 r_{t-1} + a_2 r_{t-2} + a_3 r_{t-3} + a_4 \Delta usd_t + a_5 h_{usd,t} + a_6 i_t^{us} + e_{1,t} \quad (4.1)$$

Where EGARCH (Exponential GARCH) specification for the conditional variance for the error term is given by

$$\log(h_{e_{1,t}}^2) = k_0 + k_1 \log(h_{e_{1,t-1}}^2) + k_2 \left| \frac{e_{1,t-1}}{h_{e_{1,t-1}}} \right| + k_3 \frac{e_{1,t-1}}{h_{e_{1,t-1}}} + k_4 h_{usd,t} \quad (4.2)$$

$\Delta$  is the difference operator,  $usd$  is the Australian bilateral exchange against the US dollar and  $i^{us}$  is the US interest rate and  $h_{usd}$  is the variance series of  $usd$ . Australia is a small open economy, and hence the US interest rate is known to affect its economy, and therefore it is included in the model. The results of equation (4.1) and (4.2) are tabulated in Table 1. The empirical result shows that neither the US dollar exchanges rate nor the US interest rate is significant. However, there is strong evidence to suggest that the exchange rate volatility plays major role in explaining the optimal intervention model.

From Weymark (1999), we believe that the effects of short-term dynamics, exchange rate volatility depend on whether or not the optimal intervention is large or “normal”; that is, these effects are asymmetric. The model (4.1) and (4.2) do not capture such asymmetric effects. Therefore, we consider a two-state threshold model, which can capture the asymmetric effects of the variables on the optimal intervention. We first define the threshold parameters as the 5 per cent lower ( $Q_L$ ) and upper ( $Q_U$ ) quantiles of the estimated distribution of the optimal intervention series using the simple ordered statistics.

Now, to define the large and “normal” optimal intervention, we define the dummy variable,  $I_t$  as follows:

$$I_t = \begin{cases} 1 & \text{if } r_t < Q_L \text{ or } r_t > Q_U \\ 0 & \text{if } Q_L \leq r_t \leq Q_U \end{cases}$$

Assuming the response of optimal intervention coefficient to the US interest rate, exchange rate and its volatility depends on whether the optimum intervention is large (in absolute values) or “normal”, we consider the following threshold model for  $\rho_t$

$$\begin{aligned} r_t = & \mathbf{l}_0 + \mathbf{l}_1 I_t r_{t-1} + \mathbf{l}_2 I_t r_{t-2} + \mathbf{l}_3 I_t r_{t-3} + \mathbf{l}_4 I_t h_{usd,t} + \mathbf{l}_5 (1 - I_{t-1}) r_{t-1} + \mathbf{l}_6 (1 - I_{t-2}) r_{t-2} \\ & + \mathbf{l}_7 (1 - I_{t-3}) r_{t-3} + \mathbf{l}_8 (1 - I_t) h_{usd,t} + \mathbf{e}_{2,t} \end{aligned} \quad (4.3)$$

Where the following EGARCH specification for the conditional variance is given by

$$\log(h_{e_{2,t}}^2) = c_0 + c_1 \log(h_{e_{2,t-1}}^2) + c_2 \left| \frac{\mathbf{e}_{2,t-1}}{h_{e_{2,t-1}}} \right| + c_3 \frac{\mathbf{e}_{2,t-1}}{h_{e_{2,t-1}}} \quad (4.4)$$

where  $I_{t-i} r_{t-i}$ ,  $i = 1, 2$ , are the short-term dynamics,  $I_t \mathbf{D}^{us}$  is the US interest rates,  $I_t h_{usd}$  is the  $usd_t$  GARCH volatility and  $I_t \mathbf{D}usd_t$  is the US dollar exchange rate corresponding to large optimal interventions, while the terms  $(1 - I_{t-i}) r_{t-i}$ ,  $(1 - I_t) \mathbf{D}^{us}$  and  $(1 - I_t) \mathbf{D}usd_t$  are the same variables respectively when the optimal intervention is “normal”. The estimates of equations (4.3) and (4.4) are tabulated in Table 2. The results indicate that

the threshold model plays a significant role in explaining the nature of the relationship between the optimal intervention rule and the other variables in the model.

A complete set of results is not reported to save space but are available from the author on request. The results indicates that the short-term dynamics have significant effects on the optimal intervention in that the effect is quicker when the optimal intervention is large, and it is slower when it is “normal”. Further, we find that the mean of  $r_t$  is affected by the exchange rate volatility when the optimal intervention is large only. The variance of an optimal intervention follows an EGARCH process and no evidence of any economic variables having significant effects on the variance.

## **4.2 Historical Decomposition of target variables**

The theoretical model described in section 2 predicts that the optimal intervention rule depends on the relative importance the central bank places on price stability and output stability. In particular, the model predicts that the optimal degree of intervention increases with the weights assigned by the central bank to price stability. In this empirical analysis, we examine whether movements of the optimal intervention rule consistent with those of output growth and inflation that are estimated historical decompositions using a bivariate VAR model.

An index for the actual level of intervention and the optimal level of intervention has been represented in Figure 1. The actual level of intervention by the RBA is greater than the optimal level of intervention for the period 1985 to 1994, with exceptions in 1988:1, 1989:3 and 1991:3. According to the optimal intervention rule, the Central Bank has been putting too much emphasis on price stability during this period. Between 1994 and 2000 the situation is reversed, as the optimal level of intervention appears to be larger than the actual level of intervention except in periods 1996:4, 1998:4 and 1999:4.

Since the commonly used variance decomposition does not allow us to identify the components in levels at any particular time, output growth and inflation rate series are decomposed using Sims' (1980) Vector Autoregression (VAR) methodology. Consider the following reduced form of VAR:

$$x_t = A_1 x_{t-1} + A_2 x_{t-2} + A_3 x_{t-3} + \dots + A_j x_{t-j} + e_t \quad (4.13)$$

where  $x_t$  comprises a vector of variables of interest. In our case  $x_t$  is a 2 x 1 matrix comprising Australian output growth  $y_t$  and the Australian inflation  $p_t$ . By recursive substitution for  $x_{t-i}$  we can obtain an expression for the historical decomposition in terms of initial conditions and a reduced form error term given as,

$$x_t = \text{initial conditions} + \sum_{i=0}^T C_i e_{t-i} \quad (4.14)$$

where  $C_i = 0$  for  $i < 0$

$$C_0 = I_n$$

$$C_1 = A_1$$

$$C_i = C_{i-1}A_1 + C_{i-2}A_2 + C_{i-3}A_3 + \dots + C_j A_j \quad i \geq 1$$

The initial conditions will depend on the number of lags,  $j$ , and the number of observations,  $T$ . The historical contributions of lagged output growth and inflation on output growth are represented in Figure 2. An 'eye ball' testing of Figure 2 shows that during the pre-1991 period the Australian GDP growth was primarily driven by inflation. The main driving force during the post-1991 period appears to be GDP growth. A historical decomposition of the inflation rate (Figure 2) shows inflation rate was major driving force of itself during the earlier years and its role decreases dramatically over the post-1993 period. The model (4.14) enables us to examine whether or not the movements of optimal intervention are consistent with those of the

target variables, namely, output growth and inflation, used to derive the optimal intervention coefficient.

An inspection of the historical decompositions of output growth (Figure 1), those of inflation rate (Figure 2) and the optimal intervention rule (Figure 3) shows that when the target variables are primarily driven by inflation rate (pre-1991), the optimal intervention is much higher. These findings support the theoretical predictions of the optimal intervention model that the greater the relative weight the authority put on inflation to output growth the larger the optimal intervention rule.

Actual level of intervention by the RBA (net foreign exchange purchases) has been plotted with the optimal level of intervention in Figure 4. There is no apparent relationship between the two series. However, it is evident that before 1994, actual intervention is much higher than the optimal intervention level and it decreases thereafter.

## **5.0 CONCLUSION**

Several studies have shown that exchange rate arrangement should depend on the source and strength of shocks on macroeconomic targets. It has been suggested that policy authorities should consider the effects of shocks through the goods market and financial market integration on the deviations from long run equilibrium targets. For example, Weymark (1999) forms a loss function comprising two target variables, namely, output growth and inflation rates, to derive an optimal intervention rule. Incorporating the volatility of the exchange rates into the model, it has been shown that the optimal intervention rule is not constant over time.

This paper provide empirical evidence of modelling the optimal intervention rule for the Australian case over the post-float period. The results indicate that the Australian dollar exchange rate and inflation rate exhibit conditional heteroscedasticity. Evidence in the literature suggests that the effects of economic variables on optimal intervention depend on whether or not it is large or “normal”. Capturing such asymmetric effects, a non-

linear two-state threshold model was used for modelling the intervention with threshold parameters being defined as the 5 per cent lower and upper quantiles of the distribution of the optimal intervention. The mean of the optimal intervention is significantly affected by the short-term dynamics and the exchange rate volatility, particularly when the optimal intervention is large, and its variance follows an EGARCH process.

Historical decompositions of the target variables, output growth and inflation rates, using a bivariate VAR were used to analyse the relative weights the authorities place in inflation. The results reveal that the policy shift towards inflation targeting is apparent in the data. A comparison of the historical contributions of the inflation and output growth with the optimal level of intervention supports the theoretical predictions of the optimal intervention model, that, the greater the relative weighting on inflation to output growth the larger the optimal intervention rule. Further research is required to examine the relationship between the optimal level of intervention and the target variables, output growth and inflation. The relationship between the optimal and the actual intervention by the RBA need to be examined, which may provide possible exchange rate policy implications.

Table 1: Estimates of the Symmetric Response Models

Models	Variables	Estimates	t-ratio
Mean Equation (4.1)*	Constant	5.986	2.034
	$\mathbf{r}_{t-1}$	0.009	0.136
	$\mathbf{r}_{t-2}$	0.205	2.938
	$\mathbf{r}_{t-3}$	-0.236	-3.294
	$\Delta usd_t$	17.243	0.568
	$h_{usd,t}$	-241.641	-1.871
	$\Delta i_t^{us}$	-2.279	-0.877
Mean Equation (4.1)	Constant	-1.204	-2.124
	$\mathbf{r}_{t-1}$	-0.129	-10.598
	$\mathbf{r}_{t-2}$	0.029	1.751
	$\mathbf{r}_{t-3}$	-0.040	-1.469
	$\Delta usd_t$	9.587	0.874
	$h_{usd,t}$	-125.769	-6.188
	$\Delta i_t^{us}$	0.952	1.019
	$h_{1,t}$	0.939	7.220
Variance Equation (4.2)	Constant	6.067	25.798
	$\frac{\mathbf{e}_{1,t-1}}{h_{e1,t-1}}$	-0.536	-12.093
	$\frac{\mathbf{e}_{1,t-1}}{h_{e1,t-1}}$	0.798	22.328
	$\log(h_{e1,t-1}^2)$	-0.644	-14.423
	$h_{usd,t}$	-725.932	-1.822

\* Assuming the variance of the error term in model (1) is constant

Table 2: Estimates of the Asymmetric Response Models

Models	Variables	Estimates	t-ratio
Mean Equation (4.3)*	Constant	4.650	1.687
	$I_{t-1} \mathbf{r}_{t-1}$	-0.047	-0.605
	$I_{t-2} \mathbf{r}_{t-2}$	0.308	3.935
	$I_{t-3} \mathbf{r}_{t-3}$	-0.030	-0.339
	$I_t h_{usd,t}$	-154.068	-0.920
	$(1-I_{t-1}) \mathbf{r}_{t-1}$	-0.039	-0.291
	$(1-I_{t-2}) \mathbf{r}_{t-2}$	-0.012	-0.097
	$(1-I_{t-3}) \mathbf{r}_{t-3}$	-0.711	-5.930
	$(1-I_t) h_{usd,t}$	-182.898	-1.532
Mean Equation (4.3)	Constant	4.359	2.775
	$I_{t-1} \mathbf{r}_{t-1}$	-0.041	-7.703
	$I_{t-2} \mathbf{r}_{t-2}$	0.306	1.829
	$I_{t-3} \mathbf{r}_{t-3}$	-0.138	-2.850
	$I_t h_{usd,t}$	-154.030	-1.4226
	$(1-I_{t-1}) \mathbf{r}_{t-1}$	-0.043	1.162
	$(1-I_{t-2}) \mathbf{r}_{t-2}$	0.008	-0.175
	$(1-I_{t-3}) \mathbf{r}_{t-3}$	-0.670	-7.756
	$(1-I_t) h_{usd,t}$	-183.050	2.483
Variance Equation (4.4)	Constant	4.126	4.866
	$\left  \frac{\mathbf{e}_{2,t-1}}{h_{e2,t-1}} \right $	-0.855	-3.571
	$\frac{\mathbf{e}_{2,t-1}}{h_{e2,t-1}}$	-0.932	-6.709
	$\log(h_{e2,t-1}^2)$	0.085	0.466

\* Assuming the variance of the error term in model (3) is constant

Figure 1: Historical Decomposition of Output Growth

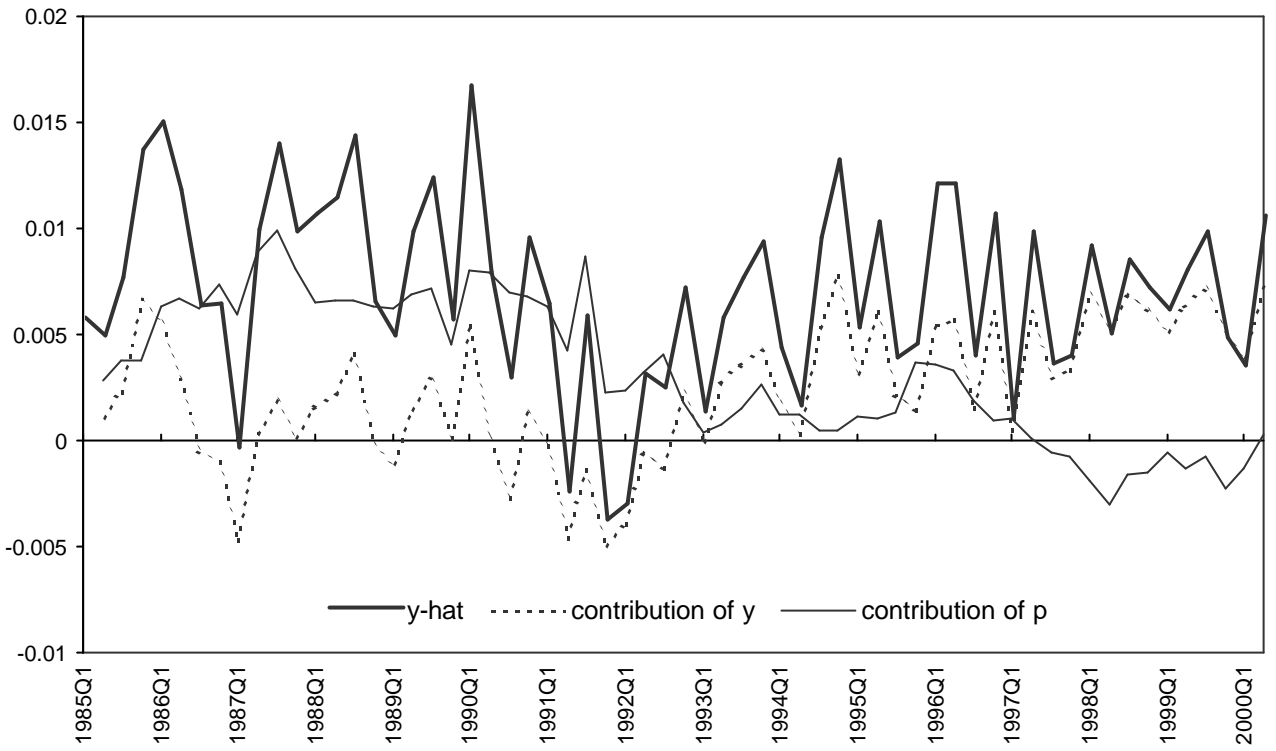


Figure 2: Historical Decomposition of Inflation

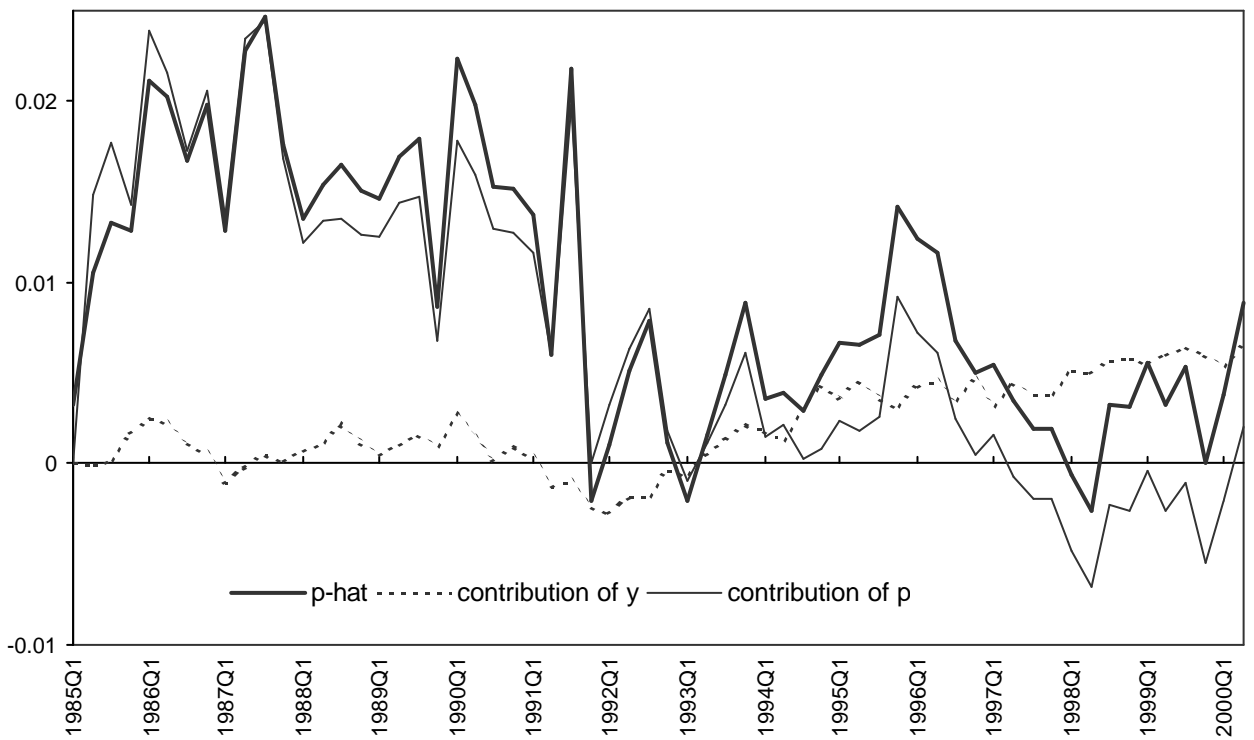


Figure 3: Optimal Intervention Rule

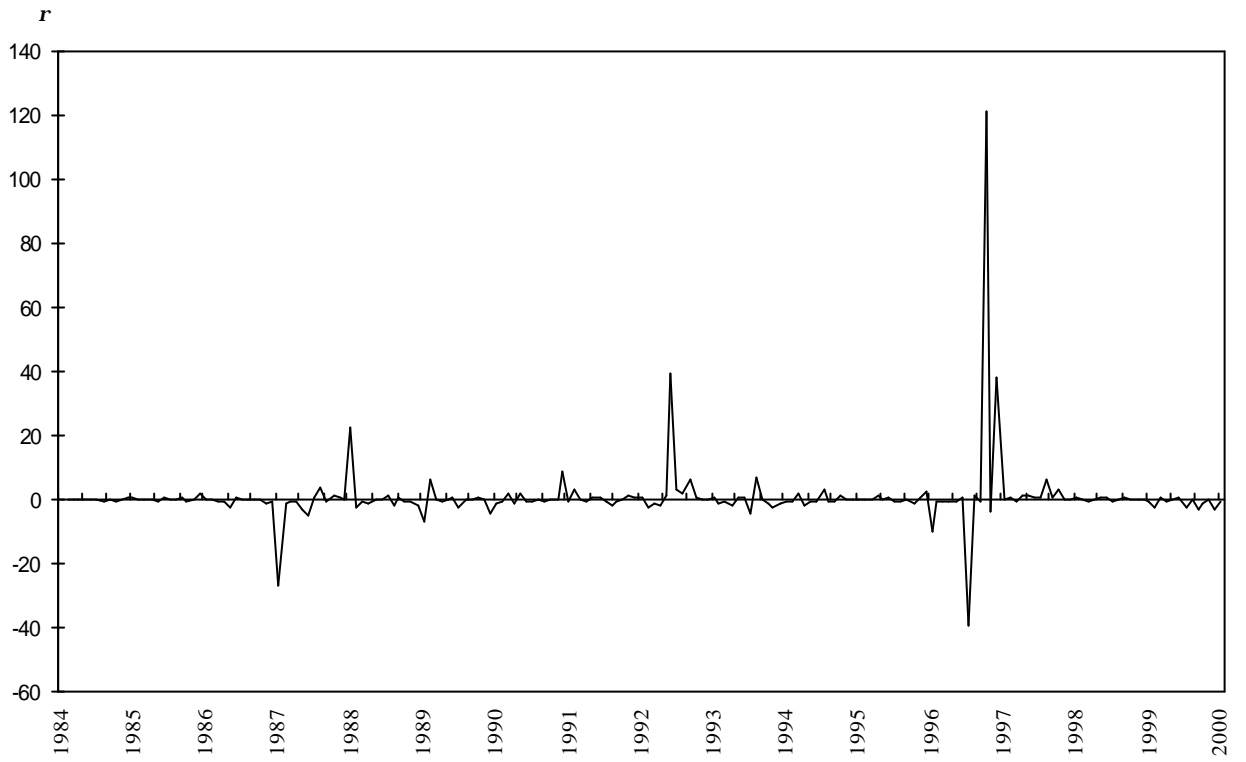
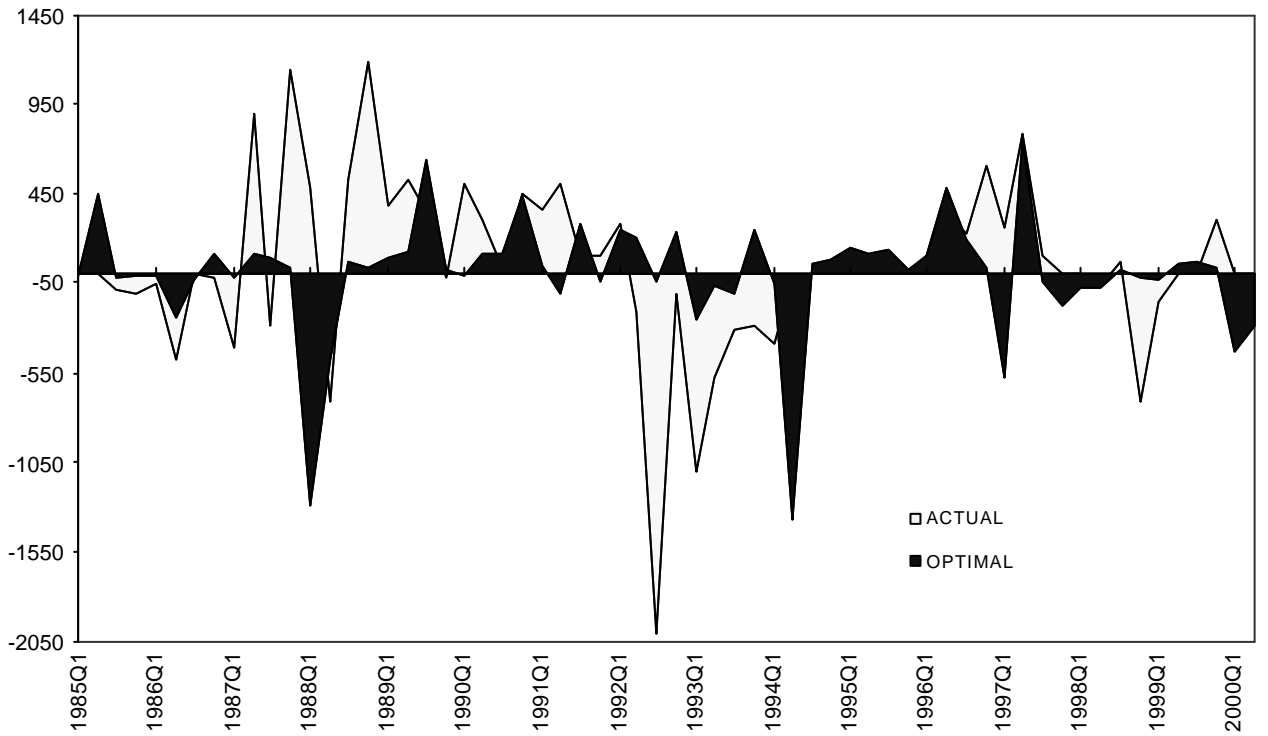


Figure 4: Actual and Optimal Intervention



## 5.0 References

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## Appendix 1: Data Appendix

The following monthly measures of the data series were collected from dX Data over the period covering 1984:1 to 2000:3

$(AUD/USD)_t = \Delta \ln(E_t)$  where  $E_t$  is the logarithm of the Australian dollar exchange rate against the US dollar [OECD MEI] and  $\Delta$  is the first difference operator.

$m_t = \Delta \ln(M_t)$ , where  $M_t$  is the logarithm of the Australian money stock (M1-monetary aggregates) [RBA Bulletin]

$i_t^{us} = \Delta I_t^{us}$  where  $I_t$  is the US CDs [OECD MEI]

Quarterly measures of the data series were collected from dX Data over the period covering 1984:1 to 2000:1.

$p_t = \ln(P_t) - \ln(P_{t-1})$ , where  $P_t$  is the Australian Consumer Price Index [OECD MEI]

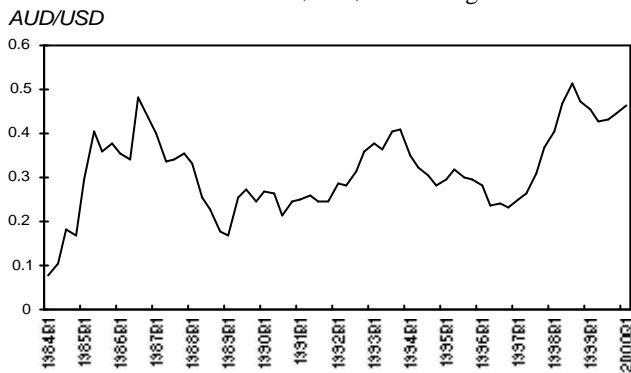
$p_t^{us} = \ln(P_t^{us}) - \ln(P_{t-1}^{us})$ , where  $P_t^{us}$  is the US Consumer Price Index [OECD MEI]

$y_t = \ln(Y_t) - \ln(Y_{t-1})$ , where  $Y_t$  is the Australian GDP [OECD MEI]

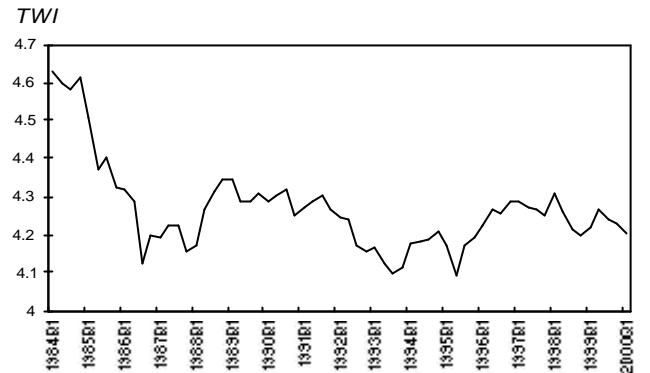
$y_t^{us} = \ln(Y_t^{us}) - \ln(Y_{t-1}^{us})$ , where  $Y_t^{us}$  is the US GDP [OECD MEI]

## Appendix 2: Graphs of the Data Series

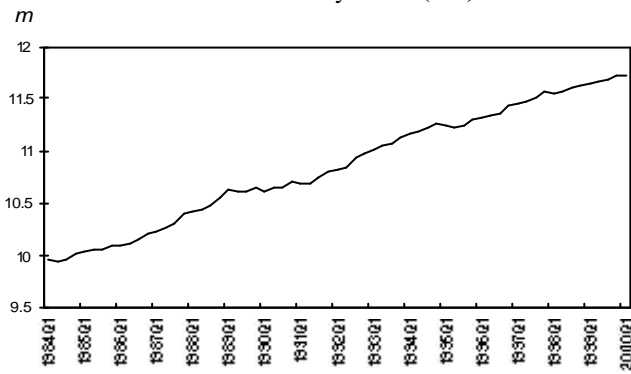
Nominal \$US/\$A Exchange Rate



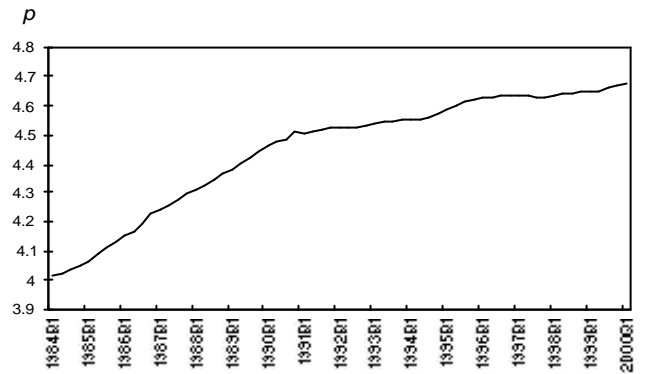
Australian Nominal Trade Weighted Index



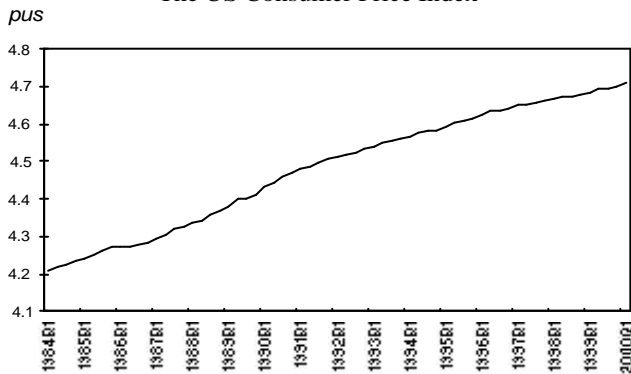
Australian Money Stock (M1)



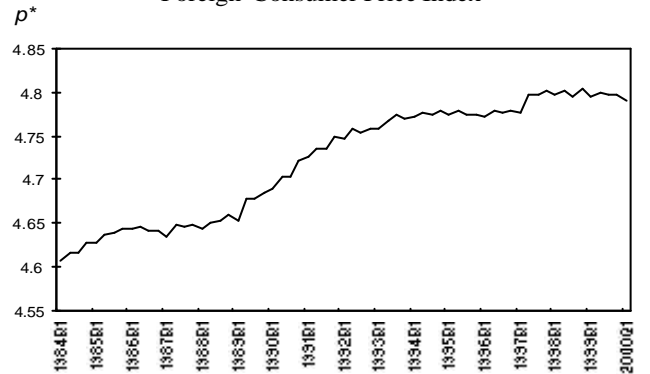
Australian Consumer Price Index



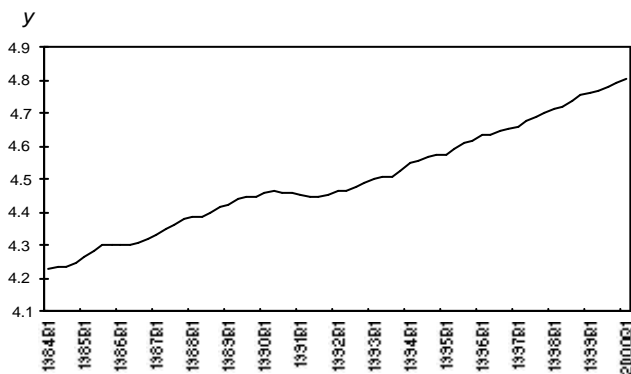
The US Consumer Price Index



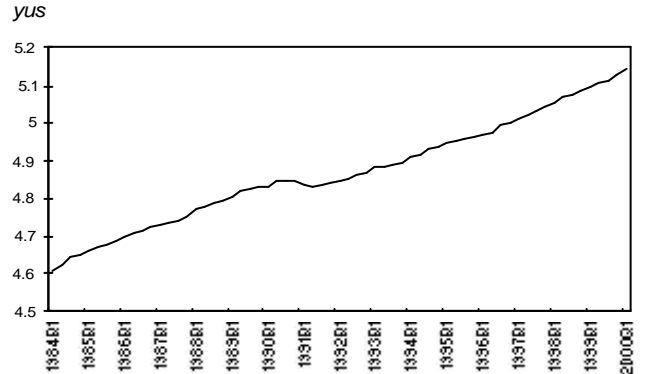
'Foreign' Consumer Price Index



Australian Gross Domestic Product



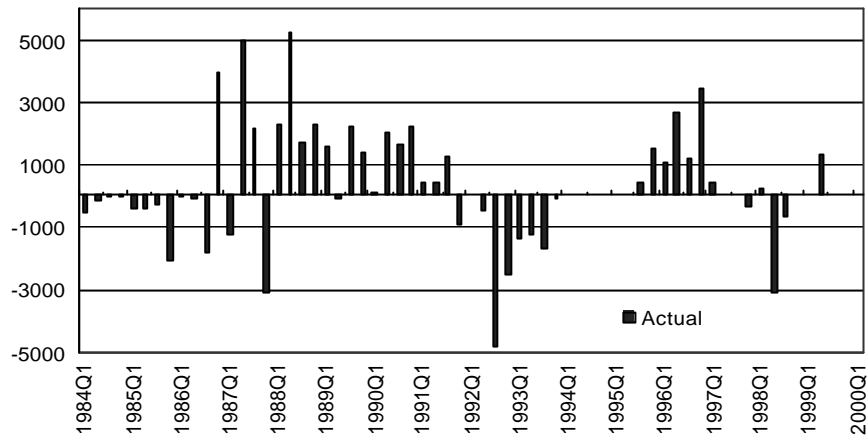
The US Gross Domestic Product



## Appendix 2 (cont.):

### Net Foreign Exchange Purchases by the Reserve Bank of Australia

(in Millions of Australian Dollars)



Note: Positive Values Note: Positive values indicate the sale of the Australian dollar and negative values represent the purchases of the Australian dollar

### Appendix 3: Unit Root Tests

<i>SERIES</i>	<i>m</i>		<i>p</i>		<i>p<sup>us</sup></i>		<i>y</i>	
<i>84:1-99:3</i>								
	ADF	Test Stat	ADF	Test Stat	ADF	Test Stat	ADF	Test Stat
Level	-0.290	-3.306 {4}	-0.026	-2.380 {2}	-0.026	-1.142 {3}	-0.047	1.122*** {1}
First Difference	-1.155	-6.765*** {1}	-0.492	-4.149*** {1}	-0.594	-5.028*** {0}	-0.819	-6.948*** {0}
	<i>y<sup>us</sup></i>		<i>P*</i>		<i>usd</i>			
	ADF	Test Stat	ADF	Test Stat	ADF	Test Stat		
Level	0.007	1.395 {2}	-0.027	-0.604 {2}	-0.271	-3.155* {6}		
First Difference	-0.579	-3.801*** {0}	-0.828	-4.099*** {1}	-0.845	-6.699 {0}		

The number of lags used in the ADF test are in {}. \*\*\*, \*\* and \* denote the rejection of the null of a unit root at a 10,5 and a 1 percent significance level respectively.

## Appendix 4: GARCH Specification Equations

$$usd_t = a_0 + a_1 usd_t + \mathbf{e}_{usd,t} \quad (A1)$$

$$usd_t = \mathbf{e}_{usd,t} + b_1 \mathbf{e}_{usd,t-4} \quad (A2)$$

$$h_{usd,t} = b_2 \mathbf{e}_{usd,t-1}^2 + b_3 h_{usd,t-1}$$

$$p_t = a_2 p_{t-2} + a_3 p_{t-3} + \mathbf{e}_{p,t} + a_4 \mathbf{e}_{p,t-1} \quad (A3)$$

$$p_{b,t} = b_4 p_{b,t-2} + b_5 p_{b,t-3} + \mathbf{e}_{pb,t} + b_6 \mathbf{e}_{pb,t-1} \quad (A4)$$

$$h_{pb,t} = b_4 \mathbf{e}_{pb,t-1}^2 + b_5 h_{pb,t-1} + b_6 h_{pb,t-2}$$

$$y_{m,t} = a_5 y_{m,t-1} + \mathbf{e}_{my,t} \quad (A5)$$

Table A1: GARCH Properties of the US Dollar Exchange Rate

	$usd_t$ (A1)	With GARCH (A2)
<i>Constant</i>	0.002 (0.001)	-0.264 (0.098)
$\mathbf{D}usd_{t-1}$	0.314 (0.072)	-0.067 (0.025)
Variance Equations		
<i>Constant</i>	-0.155 (0.120)	
$\frac{ \mathbf{\varepsilon}_{usd,t-1} }{h_{usd,t-1}}$	-	0.016 (0.071)
$\frac{\mathbf{\varepsilon}_{usd,t-1}}{h_{usd,t-1}}$	-	0.125 (0.036)
$\log(h_{usd,t-1}^2)$	-	0.981 (0.011)
Log-Likelihood	455.573	486.981
BG-LM (4)	6.560	-
ARCH-LM (4)	6.610	-

Table A2: GARCH Properties of the Australian Inflation Rate

	$p_{b,t}$ (A3)	With GARCH (A4)
Mean Equations		
$p_{b,t-2}$	0.621 (0.105)	0.612 (0.124)
$p_{b,t-3}$	0.273 (0.101)	0.268 (0.108)
$e_{pb,t-1}$	0.541 (0.121)	0.697 (0.115)
Variance Equations		
$e_{pb,t-1}^2$	-	0.591 (0.211)
$h_{pb,t-1}$	-	-0.173 (0.064)
$h_{pb,t-2}$	-	0.377 (0.106)
Log-Likelihood	232.098	241.177
BG-LM	5.025	-
ARCH-LM (4)	15.038	-

Table A3: GARCH Properties of the Australian Output Growth

	$y_t$ (A5)
Mean Equation	
$y_{t-1}$	0.667 (0.087)
Log-LH	216.740
BG-LM	1.649
ARCH-LM (4)	4.132