The Law of Consumer Demand in Japan: A Macroscopic Microeconomic View

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Abstract. In economics, there have been found few fundamental theories as complying with statistical laws, really examined by observation. Econometrics still clings to a very special structure, patronized with favoritism by a traditional economic theory. In 1990's, Hidenbrand, Grandmont, Grodal and others have tried to formulate consumer demand by a procedure definitely different from the traditional way of using an individualistic utility function. An alternative approach is to make assumptions on the population of the households as a whole—*the macroscopic microeconomic approach*. If the households' demand functions are not identical then one needs a certain form of *heterogeneity* of the population of households. This approach really also requires empirical tests. The method of nonparametric test on income distribution to estimate covariances on households spending may be applied. In this article, Japanese Family Expenditure Data is used for estimation. The law of consumer demand is one of the most important topics since a birth of economics. Our *statiscal economics* should, first of all, demand innovation in this area.

1 The Law of Consumer Demand

1.1 Slutsky Equation

If we should derive the law of consumer demand in terms of the individual demand functions (or individual preferences), we could be forced to employ some restrictive assumptions such as that individual demand functions are linear in income, i. e.,

$$f^i(p, x^i) = x^i f^i(p, 1).$$

Otherwise, we contented ourselves with an *ado hoc* assumption of gross substitutability to be imposed on the Slutsky equation. An individual demand function is usually expressed as the form of Slutsky equation :

$$\frac{\partial f_j^i(p, x^i)}{\partial p_k} = \frac{\partial h_j^i(p, x^i)}{\partial p_k} - \frac{\partial f_j^i(p, x^i)}{\partial x^i} f_k^i(p, x^i) \quad (1)$$

Total Demand Effects = Substitution Effects *plus* Income Effects.

Compensated income h[1]is defined as the original demand $f(p, x(p))$ after a footnotesize prices variation from p to q

$$x(q) = x(p) - (p - q)f(p, x(p)). \tag{2}$$

The assumption of gross substitutablity implies that a footnote size proce variation is always held positive.

Without an *ad hoc* such as gross substitutability hypothesis, Hildenbrand has been successful to introduce the form of *heterogeneity* of the population of households by constructing a certain set of macroscopic hypotheses. We can only give a brief summary on them here. We denote the population of households by I. Households can be classified into groups of different income x. Households with income x_i are called x_i-household. x_i-household may also be called x_i-cloud, because each member in x_i-household has a various vector of demand $f^i(p, x^i)_{i \in I}$ where $f^i(p, x^i)$ is demand of each household i. The market demand $F(p)$ thus is an average idea of households demand, the sum of household demands divided by the total number of households. That is,

$$F(p) = \frac{1}{\#I} \sum_{i \in I} \left[f_j^i(p, x^i) \right], \tag{3a}$$

$$S(p) = \frac{1}{\#I} \sum_{i \in I} \left[\frac{\partial h_j^i(p, x^i)}{\partial p_k} \right], \tag{3b}$$

$$A(p) = \frac{1}{\#I} \sum_{i \in I} \left[\frac{\partial f_j^i(p, x^i)}{\partial x^i} f_k^i(p, x^i) \right]. \tag{3c}$$

A Jacobian matrix of the market demand function $F(p)$, considering the so-called Slutsky decomposition, can be written in the form of

$$\frac{\partial F(p)}{\partial p} = S(p) - A(p) \tag{4}$$

by which it must hold the condition

$$\frac{\partial F(p)}{\partial p} < 0 \tag{5}$$

in order to assure the *Law of Demand* in average.

[1] In view of an individualistic utility function, h can be derived from the problem

$$h^i(p, x^i) : = arg \quad min_{u(z)=\xi} \quad pz :$$
$$h(q, x(q)) : q \to f(q, x(q))$$

1.2 An Alternative Derivation of the Law of Demand by Hildenbrand

The following hypotheses supporting the above condition are smartly proposed by Hildenbrand(1994 pp.19-21).

Hypothesis 1.The average Slutsky substitution effect matrix $S(p)$ is negative semidefinite.

Hildenbrand is not interested in empirical testing on the Hypothesis 1. He instead discussed the assumptions being compatible with *generic function* to be defined. See Hildenbrand(1994, Chap.2).

Hypothesis 2.The average income effect matrix A is positive semidefinite.

We can test **Hypothesis 2** empirically by calculating the matrix of second moments of each clouds of households demand underlying *the Family Expenditure Data* officially published. Second moments can be utilized an index of *spread*. In other words, *spread* may be interpreted a measure of heterogeneity of households. The cloud $\{y^i\}_{i \in I}$ is judged *more spread*, if $m^2\{y^i\}_{i \in I} > m^2\{z^i\}_{i \in I}$.

Hildenbrand then proved quite ingeniously that **Hypothesis 2** can be equivalent to **Hypothesis 3**, provided the demand functions satisfy the budget identity. See Hildenbrand(1994 Appendix 5).

Hypothesis 3[Increasing spread and expanding dispersion of household's demand] For every sufficiently small $\triangle > 0$, the cloud $\{f^i(p, x^i + \triangle)\}_{i \in I}$ is *more spread* than $\{f^i(p, x^i)\}_{i \in I}$

Dispersion on the clouds can be defined to measure heterogeneity of households. To remove the so-called Giffen effects, the average income effect matrix A must be positive semi-definite. According to a new theory of Hildenbrand, this corresponds to the fact that "dispersion in each incmoe to measure heterogenity of households may increase, as income size increase". In short, it then holds:

$$cov\{y^i\}_{i \in I} - cov\{y^j\}_{j \in I} > 0, \tag{6a}$$

$$Ell(cov\ \nu_2) \supset Ell(cov\ \nu_2). \tag{6b}$$

[2] Our empirical test thus comes down to calculate the *matrix of covariance* around the mean on the clouds.

1.3 Illustration in the One-Commodity Economy

The one commodity example is a short way for us to understand a new perspective of Hildenbrand's theory. Lewbel(1994, Appendix) gives a compact view on it in terms of the one commodity world. The *Slutzky Equation* in one commodity case is the slope of an individualistic demand curve:

$$\frac{\partial f}{\partial p} = \frac{\partial f}{\partial p} + \frac{\partial f}{\partial x} f - \frac{\partial f}{\partial x} f = s - (\frac{1}{2}\frac{\partial f^2}{\partial x})' \tag{7}$$

[2] *Ell* means ellipsoid of dispersion in (19) of Section 2.5.

Applying operator E to averaging among individual agents,

$$\frac{\partial E(f)}{\partial p} = E(s) - \frac{1}{2}E(\frac{\partial f^2}{\partial x}) = E(s) - \frac{1}{2}E[R'(x)] \tag{8}$$

1) Hypothesis 1 implies $E(s) < 0$. 2) $R(x)$ is $E(f^2)$, the square mean of various demands of income level x, since $R(x) = E(f^2)$. [3]

$$R(x) = \underbrace{(E(f^2) - \{E(f)\}^2)}_{\text{variance}} + \underbrace{\{E(f)\}^2 = E(f^2)}_{\text{square mean of } f.} \tag{9}$$

The Larger the value of $R(x)$, the diversity or heterogeneity of household demands at the similar income levels will be more. We can easily know the effects of variance in this model. $R(x)$ therefore measures the *spread* of household demands.Given a density function of income distribution ρ, in general,

$$E[R'(x)] = \int R'(x)\rho(x)dx = \int E(\frac{\partial f^2}{\partial x} \mid x)\rho(x)dx \tag{10}$$

$E[R'(x)]$ depends on the rank of demand and also on the configuration of income distribution. These features are really demanding the empirical analysis.

2 Statistical Tests

2.1 Estimation of the Income Distribution in Japan

We are faced to a limited source of data in income distribution. In Japan, we can only obtain a data of at most 18 classes *ranked* in family income. Officially, nothing more precise than this. Even in this case, we must collect these from each annual report since 1979 to make out the annual sequential data. Furthermore, it is to be noted that there is a discontinuity in classification size of 18 classes because how to measure income bandwidth has changed since 1991. In fact, income size among individuals has increased much more in the late 80's due to the final stage of the so-called bubble economy. Thus it must be noted that we are forced to employ another classification in size to the income data to the periods since 1991.

2.2 The Covariance Matrix under the Average Derivative Method

This task is closely connected with an empirical testing of **Hypothesis 3**. In fact, Hildenbrand in cooperation with Haerdle, and Kneip working in the field of nonparametric testing, estimated the covariance matrices derived from

[3] $R'(x) = \frac{\partial R(x)}{\partial x} = \frac{\partial E(f^2)}{\partial x}$. Then $R(x) = E(f^2)$.

spending structures of individual households which are available in *UK Family Expenditure Survey (FES)*,*French Enquete Budget de Famille (EBF)* and the surveys in other developed countries. His essence of estimation has been the use of *the average derivative method* of Haerdle and Stoker(1989). In order to use this method, he has used *Rosenblatt-Parzen kernel density estimator*

$$\frac{15}{16}(1-u^2)^2 \quad for \quad |u| \leq 1 \tag{11}$$

More precisely for our practical purpose, this can be specified by the use of *bandwidth* h in the following manner:

$$\frac{1}{h}\frac{15}{16}(1-(\frac{x-X}{h})^2)^2 \tag{12}$$

Here x denotes income, X an average income. Incidentally, it is noted that the quardratic form after differentiation can leave a cubic form

Our data source comes from the volumes of the *Annual Report on the Family Income and Expenditure* issued by the Statistics Bureau, Management and Coordination Agency, Government of Japan. In each volume, we can find the pages of table 4 in the recent editions of*Yearly Average of Monthly Receipts and Disbursements per Household by Yearly Income Group*(All Household) of 18 income rank classifications.Table 1 is a set of each average income in each income class and its number of households in the same cloud in 1998.

Table 1. Income Clouds in 1998

class[a]	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.	16.	17.	18.
income[b]	153	226	275	324	373	422	473	523	573	622	674	722	772	843	946	1110	1358	2011
households	162	198	322	448	493	491	504	506	483	477	441	402	349	633	477	780	380	395

[a] i. implies the i-th ranked income class from the bottom.
[b] income is measured in ¥10^4.

It is noted that *income* is an average income $X[i]$ in each income class I for $i \in I$, while *households* is a total number $\sum q_I^i$ in each income class I also for $i \in I$. We have replicated a figure of x by the number of $\sum q_I^i$ as if there really existed the average households. q is the total number of households in the concerned year.

$$q = \sum_I \sum_i q_I^i \tag{13}$$

2.3 Cross Validation

We select h, bandwidth of kernel function to minimize $W(i,j)$, according to Haerdle(1990):

$f(x)$:the sum of *kernel function* $K(x)$ all over the samples –in our case, a so-called samples, namely replicating each average by the number of households in each income group— when income x is taken a ratio to the total income of the sample. This is to be discounted by $q-1$, where q is the total households.

$W(i,j)$:the (i,j) element of $K(x)/f(x)$. Here i shows each item of spending object, j each income size.

In many simulations of

$$\textit{minimizing } W(i,j)$$

on our sample data of Japan for the periods of 1979 to 1998, it seems us better to choose $h = 1.5$[4]

Then the kernel density kernel density will be, if X is 1000th household's income in terms of the total average income,

$$k(x, X[1000]) = 0.000079(1 - 0.444444(-0.447446 + x^2))^2.$$

Thus the kernel density distribution for 1998 will be expressed in the following manner:

$$\rho = \sum_i^q k(x, X[i]) \tag{14}$$

We can then give the derivatives of ρ with respect to x. Hence it follows:

$$L = \frac{D(\rho, x)}{\rho} \tag{15}$$

In our annual report of family expenditure data, there are the 10 consumption aggregates categories of Living Expenditure $j = 1, ..., 10$ as follows: $j = 1$. Food; 2. Housing; 3. Fuel, light, and water charges; 4. Furniture and household utensils; 5. Clothes and footwear; 6. Medical care; 7. Transportation and communication; 8. Education; 9. Reading and recreation; 10. Other living expenditure.

Table 2. Spending structure by the representative household in each income class

income class	1. Food	2. Housing	3. Fuel	4. Furniture	5. Clothes	6. Medical	7. Transport	8. Education	9. Reading	10. Other
$I_{j \in 1to18}$	*	*	*	*	*	*	*	*	*	*

[4] As chosen $h = 1.5$, the eigenvalues of covariance matrices remarkably holds all positive almost the periods of 1979-1998. See Aruka(2000).

The spending structure table 2 consists of 18 rows multiplied by 10 columns. Underlying this table,

$$v = v(I, j) \tag{16}$$

thus, *the desired covariance matrix* can be generated. By *the average derivative method* average derivative method of Haerdle und Stoker(1989), we identify the distribution of the x- households income $\nu(p \mid x)$ with

$$v \textit{ appreciated by } L = \frac{D(\rho, x)}{\rho},$$

to attain *the covariance matrix* for any smooth income distribution ν. That is to say, this may be identical to the

$$C_\rho(p) = \int \partial_x cov\ \nu(p|x)\rho(x)dx. \tag{17}$$

2.4 Empirical Tests for Growing Variances

The two kinds of the estimation results are summarized in terms of the diagonal elements of the above matrices to the average total expenditures, namely, the *second moment curves* and *variance curves* respectively. In this paper, we only refer to *variance curves* since the property of *increasing dispersion* is concerned here. Viewing roughly, at the first sight, it may be satisfied that **Hypothesis 3** may be empirically variable in the concerned periods of the Japan economy. See Aruka(2000). First of all, we can investigate empirically how estimates of *the conditional variance curve* behave, as we expected them to be increasing as expenditures of households increase.

Growing variances easily checked without resort to Rosenblatt-Parzen kernel estimator. I have applied *the normal kernel estimator* to smooth our spending structure table and then calculated the covariant matrix in each year for 1979-1998. See Aruka(2000). The diagonal elements of covariance matrix, given the correspondence of variances to total expenditures, may construct *a variance curve* as Hildenbrand called. It may be easily verified that *variance* of each commodity aggregate is inclined to *grow* with respect to an increase of size of total expenditures in the almost intervals under consideration. Due to the restriction of allocated space, I only show the result of 1998 in Fig. 1.

2.5 Empirical Tests for Expanding Dispersion

Secondly, we may estimate *the ellipsoid of dispersion*. Hildenbrand virtually suggested the use of *average increasing dispersion of x-Households' Demand*.

Property 1: $\int\{cov\ \nu(x + \triangle) - cov\ \nu(p \mid x)\}\rho dx$ is *positive semidefinite* if $\triangle > 0$.

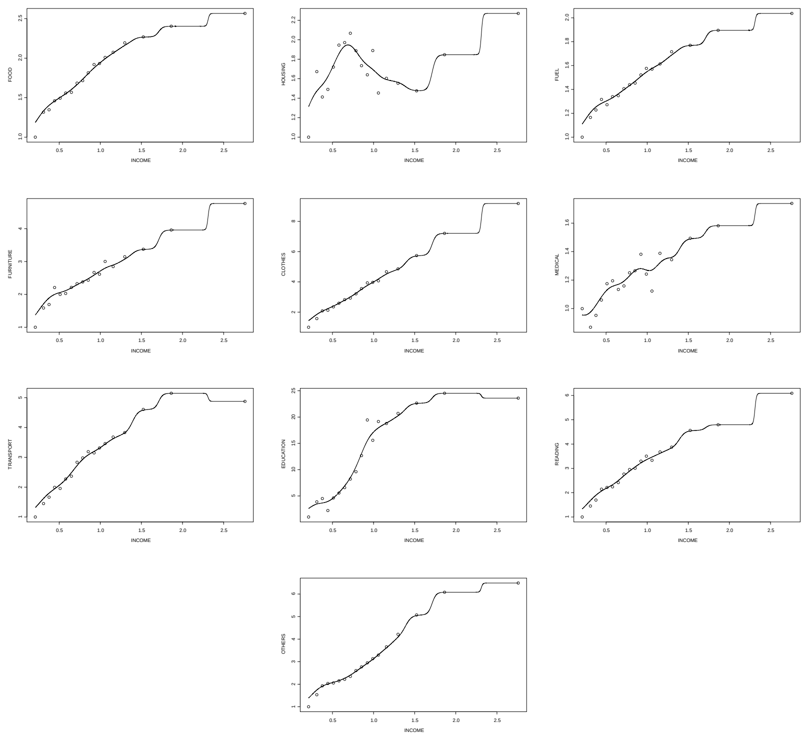

Fig. 1. Variance curves in 1998

I believe that **Property 1** can be empirically verified because all the eigenvalues may be positive in many successive years, although they decisively depends on how to select a bandwidth. The confidence on **Property 1**may be also demonstrated by the following empirical confirmation *growing dispersion*.[5]

In Hildnebrand(1994,pp.78-79), we have a concise illustration on *the ellipsoid of dispersion*. Let s_i for $i = 1, 2$ be *standard deviation*, and r be the

[5] Since this may be considered $C_\rho = \int cov\ \nu(x,p)\rho dx$ by identifying ∂_x with $cov\ \nu(x + \triangle) - cov\ \nu(p|x)$, **Property1** virtually is equivalent to say that C_ρ is *positive semidefinite*. Thus the Statistical Tests of **Property 1** requires to check whether the eigenvalues of C_ρ are *all semipositive* or not. According to Hildenbrand(1994), and Hildenbrand and Kneip(1993), these eigenvalues should be subject to *bootstrap test*. Our matrix of C_ρ derived by the *average derivative method* has produced all strictly positive eignvalues of 10 distinct roots all in 90's, if a bandwidth h is specified 1.5. Over a half of eigenvalues, we may have eigenvalues of nearly zero. If we should apply bootstrap test to our covariance matrices C_ρ, there could appear many negative eigenvalues. See Aruka(2000).

correlation coefficient which is defined as $r = \frac{cov(1,2)}{s_1 s_2}$.[6] It then holds the covariance matrix

$$cov\ \nu = \begin{bmatrix} s_1^2 & rs_1s_2 \\ rs_1s_2 & s_2^2 \end{bmatrix}. \tag{18}$$

Thus we have an expression of *ellipsoid* in terms of variance:

$$q = x\ cov^{-1}\ \nu\ x = \frac{s_2x_1^2 - 2rs_1s_2x_1x_2 + s_1x_2^2}{(1-r^2)s_1^2s_2^2}. \tag{19}$$

Here we rather used the covariance matrix derived from the average derivative method. Instead of observing all x on the domain of all the households in the economy, let all x be *fixed* at 1, while $\triangle = 0$ [7]. It then follows the covariance matrix:

$$\partial_{\triangle} cov\ \nu(\triangle, p)\ |_{\triangle=0} =: C(p) \tag{20}$$

Now suppose x to be fixed at 1:$\bar{x} = 1$. We arrange two different cases against $\bar{x} = 1$: $x = 0.5\bar{x}; x = 1.5\bar{x}$. It may normally be guessed that dispersion expands as x increases. Taking any two consumption aggregates, I here can show the results of ellipsoids both in 1998 in Fig.2. See Aruka(2000) for another year.In the concerned framework, three ellipsoids at the levels of $\bar{x} = 0.5, 1, 1.5$ can be depicted. We only notice the fuel-medical ellipsoid in 1998 violating our property in that the inner two circles are crossing.

2.6 The Statistical Test of Property 2

Finally, we are ready to verify the other property proposed by Hildenbrand to assure the dispersion of variances:

Property 2: $\partial_{\triangle} cov\ \nu(\triangle, p)\ |_{\triangle=0}$ is *positive semidefinite* on the hyperplane $F(p)$.

In order to check this property, let Γ be a projection matrix which is orthogonal to C. Applying Gram-Schmidt orthogonirization to C, for instance, we can obtain Γ. It is seen that"the matrix C is positive semidefinite if and only if eignevalues of $\Gamma C \Gamma$ is positive semidefinite." Hildenbrand(1994, p.109). Given the covariance matrix C in 1998, then, we can easily calculate eigenvalues of C.

Table 3. Egienvalues in 1998

$\Gamma C \Gamma$	6	0	0	0	0	0	0	0	0	0
C	5.7355	0.329683	0.11656	0.01638	0.00695	0.00356	0.00144	0.00022	0.00009	0.00002

Thus **Property 2** has completely been proven in our case.

[6] $cov(1,2)$ denotes $(1,2)$th component of covariance matrix.

[7] This may be regarded as the form C_ρ appreciated at all $x = \bar{x}$.

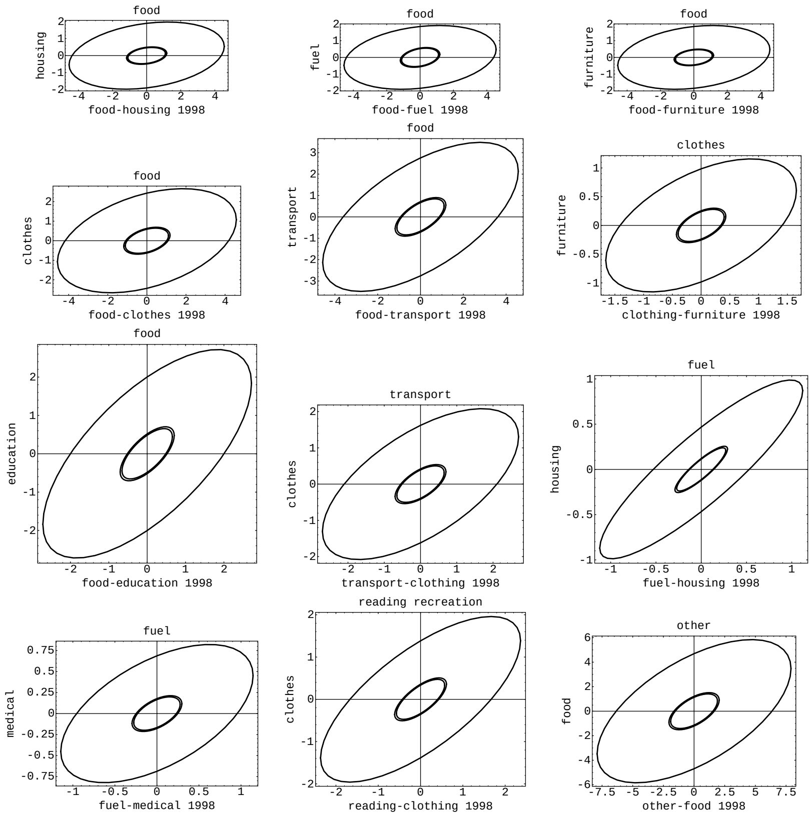

Fig. 2. Ellipsoid of dispersion in 1998

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