Distributional Consequences of Monetary Policy: A Continuous Time Heterogeneous Agent Model for India*

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Abstract

This paper evaluates the redistributive properties of a monetary policy shock in a continuous time heterogeneous agent model designed for India. In contrast to the existing literature on India that features limited heterogeneity, my model features a continuum of households indexed by the joint distribution of assets and idiosyncratic labor productivity. A number of model parameters are calibrated using numerous data sources. The steady state moments of household consumption and assets are matched with the moments from survey data to examine the reliability of the model. A one percentage point reduction in monetary policy in the span of one year leads to, on an average, 4% to 8.5% increase in household consumption, with the highest magnitude of increase observed in the households at the 25^{th} to 75^{th} percentile of the asset distribution.

Keywords: Monetary policy; Redistribution; India; Heterogeneous agent

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1 Introduction

Redistribution and inequality have been the central question in the macroeconomic literature in the past few decades. One of the many ways that this question has been addressed is by analyzing the general equilibrium outcomes of a monetary policy intervention on household consumption, income, and assets. The introduction of household heterogeneity in such models has significantly improved our understanding of the uneven impact of policy interventions (e.g., Gornemann et al. (2016); Bilbiie (2017); Kaplan et al. (2018); Auclert (2019); Bhandari et al. (2021); Acharya et al. (2023)[1]).

This evolution in the standard literature is majorly restricted to developed economies. The literature that examines the monetary policy propagation into the real economic variables in India predominantly uses time series econometric techniques on aggregate variables[2]. Gabriel et al. (2016) and Ghate et al. (2018) undertake a more comprehensive overview that analyzes the relative price movements and monetary policy responses in India using a general equilibrium framework. These models, however, feature limited heterogeneity, such as two or three agents in a discrete time formulation.

I extend this strand of literature to a continuum of heterogeneous households by building upon the seminal work by Aiyagari (1994)[3]. And I follow the contentious time recast of Aiyagari (1994) by Achdou et al. (2022), who provides a portable numerical solution method for the three pioneering heterogeneous agent models of income and asset distribution by Aiyagari (1994), Bewley (1986) and Huggett (1993). In addition to the original theoretical framework featuring a continuum of heterogenous households indexed in their asset and productivity, I incorporate some of the features of the Indian economy, such as different levels of price stickiness for different categories of consumer goods and different skilled labor transitioning between two production sectors.

A continuous time approach is advantageous in two broad aspects: the computational efficiency and the handling of the borrowing constraint. The numerical solution employs a finite difference method that uses only consecutive values of the value function in the successive iteration. Due to the extremely sparse nature of the transition

[1]See McKay and Wolf (2023) for a summary of this literature.

[2]e.g., Ahmed and Dua (2001); Bhattacharyya and Ray (2007); Aleem (2010); Singh (2011); Khundrakpam and Jain (2012); Mohanty (2012); Sengupta (2014); Das (2015); Mishra et al. (2016); Ghosh (2019); Goyal and Parab (2021)

[3]Aiyagari (1994) is one of the pioneering studies that incorporate "a considerable amount of individual dynamics, uncertainty, and asset trading" as the main mechanism of individual consumption smoothening. The general equilibrium model features endogenous heterogeneity, infinite horizons, and borrowing constraint.

matrix, a continuous time solution to the heterogeneous agent model is more efficient than the discrete-time solution[4]. On the other hand, in a continuous time, the borrowing constraint binds only at the borrowing limit and nowhere else in the state space. This provides more realistic values of the variables in the steady state.

My theoretical model features a continuum of households indexed by their asset holding and idiosyncratic labor productivity. At any time, the state of the economy is, therefore, defined as the joint distribution of these two variables. Labor productivity follows an exogenous stochastic Poisson process with two states: high and low. Low-productive labor is employed in a traditional production sector and high-productive labor is employed in a relatively modern production sector. They produce two distinct kinds of consumer goods: traditional goods are sold in a perfectly competitive market where prices adjust quickly, whereas modern goods are sold in a monopolistically competitive market where the prices are sticky due to the cost associated with price adjustments. I evaluate the effect of a change in the monetary policy arising from a one-time shock (that phases out at an exogenous rate) on household consumption, saving, and asset holding. The assumption that high and low-productive labor is hired in two different sectors whose production technology and price movements are different leads to a dramatic difference in the steady-state consumption, saving behavior, and asset holding.

I calibrate a bunch of parameters on the Indian economy using numerous data sources. The transition probabilities between the two types of skilled labor are calculated using the Indian Human Development Survey (IHDS) round I and II. Historical data on the rate of labor income tax and government lump-sum transfer to households are obtained from the Reserve Bank of India database. The Consumer Expenditure Survey provides data on the household budget allocation on different goods and services, from which I calculate the expenditure shares on traditional and modern goods. The household borrowing limit is obtained from the Debt & Investment Survey. I also estimate the mean revision rate of the monetary policy shock using historical data on the short-term interest rate from the Reserve Bank of India. I match different moments of the steady state consumption and assets with moments from the household surveys

[4]This approach follows the mathematical theory of "Mean Field Games" (MFG) introduced in economic literature by Lasry and Lions (2007). The "backward–forward MFG system" solves the household optimization problem and derives the evolution of the state.

to analyze whether the model realistically fits the economy.

I observe the response of the economy to a one-time monetary policy shock of magnitude -1% which mean-reverts at a yearly constant rate of 0.50. Due to the heterogeneous framework, I am able to show how this shock propagates to each household in the entire joint distribution of assets and productivity. I also calculate the percentage deviations of household consumption from steady state at different moments for different skills. The average consumption response varies between 5.22% to 7.16% for low-productive households, and it varies between 3.96% to 8.46% for high-productive households. The average consumption responses at the bottom of the distribution are relatively higher than at the top. I also show that the economy returns to the steady state as the shock mitigates.

The remainder of the paper is organized as follows. Section 2 lays out the model featuring household heterogeneity and multiple sectors in production, and discusses some of the theoretical results derived from this model. The numerical solution, calibration techniques and the detailed discussion of the results are presented in section 3. And section 4 provides concluding remarks.

2 A Heterogeneous Agent Model with Two Sectors of Production

There are four agents in the economy: households, firms, monetary policy authority, and government. Households are heterogeneous in asset holding and productivity. There are two production sectors that produce two different goods: the traditional representative firm produces identical traditional consumer goods, and modern monopolistically competitive firms produce a continuum of modern consumer goods. The government operates through tax and transfer. Monetary policy authority sets the short-term nominal interest rate.

2.1 Households

Households are heterogeneous in their asset $b_{i,t}$ and labor productivity $z_{i,t}$. Labor productivity follows an exogenous stochastic Poisson process with two states: high (H) and low (L), with respective probabilities λ_H and λ_L. Time t is continuous. Households receive a flow of utility from consumption $c_t^i \geq 0$ and a flow of dis-utility from supplying labor $l_{i,t} \in [0,1]$. $l_{i,t}$ is the hours of work out of total time endowment equals unity. Households maximize lifetime utility

$$\mathbb{E}_0 \int_0^\infty e^{-\rho t} \left[\frac{(c_t^i)^{1-\sigma}}{1-\sigma} - \frac{(l_{i,t})^{1+\psi}}{1+\psi} \right] dt, \quad i \in L, H \tag{1}$$

ρ is discount rate for future, $1/\sigma$ is intertemporal elasticity of substitution, $\sigma \geq 0$ and $1/\psi$ is Frisch elasticity of labour supply, $\psi \geq 0$. Households can save and borrow asset $b_{i,t}$ at real interest rate $r_{b,t}$ and $r_{-b,t}$ respectively and $r_{-b,t} = r_{b,t}$. Households can borrow liquid assets up to an exogenous limit $\underline{b}_i$. Asset holding evolves according to,

$$\dot{b}_{i,t} = (1-\tau)\, w_{i,t}\, z_{i,t}\, l_{i,t} + r_{b,t}\, b_{i,t} + T - c_t^i, \quad i \in L, H \tag{2}$$

$\dot{b}_{i,t}$ is the changes in the asset of households, τ is the rate of tax on labor income imposed by the government, $w_{i,t}$ is real wage rate, T is a lump-sum transfer from the government to the household, c_t^i is real consumption expenditure of the household. All the variables are expressed in real terms.

Households consume a homogeneous good produced in the traditional sector and a continuum of differentiated goods produced in the modern sector. Purchase of household i is c_t^i, an index of the traditional good $c_{A,t}^i$ and all of the continuum of modern goods $c_{M,t}^i$. Following Aoki (2001),

$$c_t^i = \frac{(c_{M,t}^i)^\gamma\, (c_{A,t}^i)^{1-\gamma}}{(\gamma)^\gamma\, (1-\gamma)^{1-\gamma}} \tag{3}$$

γ is the share of modern goods, and the elasticity of substitution between traditional and modern goods is one. Household i's consumption from the modern sector is a Dixit-Stiglitz index of demand for all of the continuum of differentiated goods j.

$$c_{M,t}^i = \left[\int_0^1 \left[c_{M,t}^i(j) \right]^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}} \tag{4}$$

$\epsilon > 0$ is the elasticity of substitution between differentiated modern goods. Optimal allocation of consumption spending on traditional goods and modern goods by household i provides standard Dixit-Stiglitz result of demand as a function of relative price.

$$c_{A,t}^i = (1-\gamma) \left(\frac{P_{A,t}}{P_t} \right)^{-1} c_t^i \tag{5}$$

$$c_{M,t}^i = \gamma \left(\frac{P_{M,t}}{P_t} \right)^{-1} c_t^i \tag{6}$$

$$c^i_{M,t}(j) = \left(\frac{P^j_{M,t}}{P_{M,t}}\right)^{-\epsilon} c^i_{M,t} = \gamma \left(\frac{P_{M,t}}{P_t}\right)^{-1} \left(\frac{P^j_{M,t}}{P_{M,t}}\right)^{-\epsilon} c^i_t \tag{7}$$

where $P_{A,t}$ is the price of traditional good, $P^j_{M,t}$ is the price of modern good j, $P_{M,t}$ is a Dixit-Stiglitz price index of the continuum of modern goods and P_t is the index of traditional good price and modern goods price index.

$$P_{M,t} = \left[\int_0^1 \left(P^j_{M,t}\right)^{1-\epsilon} dj\right]^{\frac{1}{1-\epsilon}} \tag{8}$$

$$P_t = (P_{M,t})^{\gamma}(P_{A,t})^{1-\gamma} \tag{9}$$

Heterogeneity among households leads to total consumption demand from each sector as a function of the sequence of equilibrium asset holding, productivity state, the return on asset, wage rate, tax rate, and transfers.

$$C_{A,t} = \sum_{i=L,H} \left[\int c^i_{A,t} \left(b_{i,t}, z_{i,t}\, ; \{\Gamma_t\}_{t\geq 0}\right) d\mu_t\right] \tag{10}$$

$$C_{M,t}(j) = \sum_{i=L,H} \left[\int c^i_{M,t}(j \left(b_{i,t}, z_{i,t},\ ; \{\Gamma_t\}_{t\geq 0}\right)) d\mu_t\right] \tag{11}$$

where $\Gamma_t = \{r_{b,t}, w_{L,t}, w_{H,t}, \tau, T\}$ is the sequence of the tax rate, transfer, equilibrium returns, and wage rate, μ_t is the joint distribution of asset and productivity $\mu_t\left(db_t, dz_t; \{\Gamma_t\}_{t\geq 0}\right)$. Household i maximizes lifetime utility subject to equations of motion for asset taking equilibrium path of $\{w_{i,t}\}_{t\geq 0}$, $\{r_{b,t}\}_{t\geq 0}$, and $\{\tau, T\}_{t\geq 0}$ as given. This optimization provides decision rules for consumption (c^i_t), and labor supply ($l_{L,t}, l_{H,t}$). As will be explained later, $\{\tau,\ T\}_{t\geq 0}$ are determined by the government, $\{r_{b,t}\}_{t\geq 0}$ and $\{w_{i,t}\}_{t\geq 0}$ are determined by market clearing conditions for asset and labor under the assumption that factor markets are perfectly competitive. The household Euler equation for inter-temporal consumption choice takes the form (derivation in Appendix 5.1),

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\sigma}(r_{b,t} - \rho). \tag{12}$$

2.2 Firms

There are two types of firms in the economy, both producing final consumption goods. However, their production technology, nature of the product, and price stickiness differ.

2.2.1 Traditional Firms

Traditional sector representative firms produce output $Y_{A,t}$ of homogeneous goods using only low productive labor $L_{L,t}$.

$$Y_{A,t} = L_{L,t} \tag{13}$$

Traditional goods are sold in a perfectly competitive market where prices adjust immediately. The wage rate equals the marginal revenue product of labor because labor is hired from a perfectly competitive labor market, and price $P_{A,t}$ is determined where demand and supply match for traditional goods.

$$Y_{A,t} = C_{A,t} \tag{14}$$

2.2.2 Modern Firms

Modern sector firm j produces output $y^j_{M,t}$ of differentiated good j using high productive labor $l^j_{H,t}$ and capital k^j_t. Modern firms hire capital from households. Households invest their asset into productive capital at a given interest rate $r_{b,t}$, and modern firms hire this capital at a given interest rate $r_{k,t}$ as in Aiyagari (1994). At equilibrium $r_{b,t} = r_{k,t} - \delta$, where δ is the rate of depreciation of capital. The production function of the modern good j is,

$$y^j_{M,t} = (k^j_t)^\alpha \; (l^j_{H,t})^{1-\alpha} \tag{15}$$

α is the share of capital. Modern goods are sold in a monopolistically competitive market where price $P^j_{M,t}$ adjust slowly. The wage rate and return on capital equal their marginal revenue product under the assumption of perfectly competitive factor markets. At equilibrium demand and supply match for each modern good j.

$$y^j_{M,t} = c^j_{M,t} \tag{16}$$

Cost minimization by firm j at any time period t solves for the marginal cost which is identical for all firms in modern sector and profit for firm j is

$$Q^j_t = \left(P^j_{M,t} - MC_t\right) \; y^j_{M,t} \tag{17}$$

following Rotemberg (1982) modern firms face cost ${\Theta_t}^j$ every time they adjust price. This cost is a quadratic function of the rate of inflation for good j denoted by $\Pi^j_{M,t}$ due to price adjustment, a parameter $\theta > 0$, and an index of output from all firms in the modern sector.

$$\Theta_t{}^j = \frac{\theta}{2}\left(\Pi^j_{M,t}\right)^2 \left[\int_0^1 \left(y^j_{M,t}\right)^{\frac{\epsilon-1}{\epsilon}} \, dj\right]^{\frac{\epsilon}{\epsilon-1}} \tag{18}$$

where $\Pi^j_{M,t} = \frac{\dot{P}^j_{M,t}}{P^j_{M,t}}$. Firm j when it adjusts price chooses $\{P^j_{M,t}\}_{t\geq 0}$ so as to maximize lifetime profit subject to price adjustment cost.

$$\int_0^\infty e^{-\int_0^t r_{k,s} ds} \left(Q^j_t - \Theta_t{}^j\right) dt \tag{19}$$

2.3 Monetary Policy Authority

Monetary policy authority sets nominal interest rate on asset $i_{b,t}$ following Taylor rule

$$i_{b,t} = \bar{r}_{b,t} + \beta \Pi_t + \zeta_t \tag{20}$$

where $\bar{r}_{b,t}$ is steady state real interest rate on asset, $\beta > 0$ and $\zeta_t = 0$ at steady state. Given $i_{b,t}$ and Π_t real interest rate is determined by Fisher equation $r_{b,t} = i_{b,t} - \Pi_t$ and $\bar{r}_{b,t}$ is the value of $r_{b,t}$ consistent with the equilibrium condition of asset market as described below. This paper studies the economy's adjustment after a one-time shock ζ_t.

2.4 Government

Government operates through proportional income tax at rate τ, lump-sum transfer T where $\tau, T > 0$. The government budget always balances, therefore,

$$T = \sum_{i=L,H} \tau \, w_{i,t} \, z_{i,t} \int l_{i,t}\left(b_{i,t}, z_{i,t} \, ; \{\Gamma_t\}_{t\geq 0}\right) d\mu_t \quad i \in L, H \tag{21}$$

2.5 Equilibrium

Given the stochastic process of productivity shocks, the equilibrium of the economy is characterized by a sequence of consumption, hours worked, asset holding, and the prices of goods, wage rates, return on asset, return on capital, and measure μ_t such that households and firms objective functions are satisfied, the decisions satisfy aggregate consistency condition, govt budget balances and all markets clear.

The asset market clears when,

$$A_t = K_t \tag{22}$$

where $A_t = \sum_{i=L,H}\{\int b_{i,t}\, d\mu_t\}$ is total household asset and $K_t = \int k_t^j\, dj$ is total capital input in modern sector.

Labor markets for low- and high-productivity labor clear when, respectively,

$$\int l_{L,t}\left(b_{L,t}, z_{L,t}\,; \{\Gamma_t\}_{t\geq 0}\right)\, d\mu_t = L_{L,t} \tag{23}$$

$$\int l_{H,t}\left(b_{H,t}, z_{H,t}\,; \{\Gamma_t\}_{t\geq 0}\right)\, d\mu_t = \int l_{H,t}^j\, dj \tag{24}$$

Goods market clear when (14) and (16) holds.

2.6 Inflation in the Modern Sector

Lemma 1: The solution to the optimization problem of firm j provides the evolution of inflation in the modern sector (*proof* in Appendix 5.2).

$$\dot{\Pi}_{M,t}^j + \frac{\epsilon}{\theta}\left(MC_t - MC_t^*\right) = \left(\Pi_{M,t}^j - r_{b,t} + \frac{\dot{Y}_{M,t}}{Y_{M,t}} - \frac{1}{P_{M,t}}\right)\Pi_{M,t}^j \tag{25}$$

this is the New Keynesian Philips curve of the modern sector.

2.7 Aggregate Inflation in the Economy

Lemma 2: The aggregate inflation in the economy is determined jointly by the inflation in the traditional and modern sectors.

Proof: Taking the log of equation 9 and first order derivative with respect to time t, then combining with equation 25, I obtain,

$$\Pi_t = (1-\gamma)\,\Pi_{A,t} + \gamma\,\left(P_{M,t}^j\right)^{\frac{1-\epsilon}{\epsilon}} \int_0^1 \left[\Pi_{M,t}^j\,\left(P_{M,t}^j\right)^{\frac{\epsilon-1}{\epsilon}}\right]\, dj\,. \tag{26}$$

3 Taking the Model to Data

The solution method for continuous-time heterogeneous agent models is based on a finite difference method. The method is based on the solution of two paired equations: the Hamilton–Jacobi–Bellman (HJB) and the Kolmogorov Forward (KF) or Fokker–Planck equation. First one needs to solve the HJB equation for a given time path of prices. Then, solve the KF equation for the evolution of the joint distribution of productivity and assets. Having solved the HJB equation, obtaining the time path of the distribution becomes easier. That is because the KF equation is the "transpose problem" of the HJB equation. The next step is to iterate and repeat these steps until an equilibrium time path of prices is found. In computation, the continuous time approach imparts a number of advantages over discrete time. One of the important advantages is that it handles the borrowing constraints internally, so there is no need to manipulate the solution around them externally. The continuous-time solution with discretized state space is, extremely sparse in nature, which gives computational advantages and the algorithm becomes simple and efficient. Implementing it requires only some basic understanding of matrix algebra and software packages that can solve sparse linear systems (e.g., Python, Matlab). The HJB is,

$$\begin{aligned}\rho V(b_{i,t}, z_{i,t}) = max\ \ & u(c^i_t, l_{i,t}) \\ & + V_b\big[(1-\tau)\, w_{i,t}\, z_{i,t}\, l_{i,t} + r_{b,t}\, b_{i,t} + T - c^i_t\big] \\ & + \lambda_i(V_{-i}(b_{i,t}, z_{i,t}) - V_i(b_{i,t}, z_{i,t}))\end{aligned} \tag{27}$$

KF,

$$0 = -\frac{\partial}{\partial b_{i,t}}[s(b_{i,t}, z_{i,t})\; g(b_{i,t}, z_{i,t})] - \lambda_i g_i(b_{i,t}, z_{i,t}) + \lambda_{-i} g_{-i}(b_{i,t}, z_{i,t}) \tag{28}$$

where,

$$s(b_{i,t}, z_{i,t}) = (1-\tau)\, w_{i,t}\, z_{i,t}\, l_{i,t} + r_{b,t}\, b_{i,t} + T - c^i_t$$

$-i = L$ when $i = H$ and vice versa, $g(b_{i,t}, z_{i,t})$ is the density function corresponding to the state $\mu(b_{i,t}, z_{i,t})$. These two ordinary differential equations, along with the boundary conditions, characterize the steady-state equilibrium of the economy. HJB provides optimal consumption and saving, while KF provides the evolution of the joint distribution of asset and productivity.

3.1 Calibration

In this section, I describe the calibration strategies and the calibrated parameters for the structural parameters in this model.

3.1.1 Household Assets

I use the definition of household financial assets following the National Sample Survey Office (NSSO)[5]. I use their latest household Debt & Investment Survey conducted from

[5]NSSO, under the Ministry of Statistics and Programme Implementation, the Government of India, conducts national level surveys on a wide range of socio-economic variables of households in India.

Jan-Dec 2019. This definition of financial assets includes all household deposits in financial institutions (checking, saving, call, and money market accounts), government bonds, and corporate bonds net of revolving consumer credit, etc., and all household debts. To mention some of the stylized facts from data: around 84.4% and 85.2% of the population aged 18 years and above have some financial assets in banks in rural and urban India, around 35% households in Rural India and around 22.4% households in Urban India has some kind of financial debts. On average, the amount of debt is Rs. 59,748 among rural households and Rs. 1,20,336 among urban households. The net (asset minus debt) household assets as a proportion of per capita GDP varies between -2.98 and 3.26. However, the distribution is extremely skewed. Therefore, the 10^{th} and 90^{th} percentile shows the net asset holding to be, respectively, -0.32 and 0.60. I set the borrowing limit at -0.32. I only target the mean of household asset holding in the numerical solution. My model produces realistic values of the asset, which I show by comparing different moments generated by the model at the steady state with those in the data.

3.1.2 Earning Dynamics

The probabilities attached to the two-state productivity process are also estimated using household survey data. The frequency of productivity shocks contributes significantly to the determination of asset holding. For example, a lower probability of transitioning from a low to high productivity state may lead to higher borrowing, whereas low probability of transitioning from a high to low productivity state may lead to a higher asset accumulation. Indian Human Development Survey (IHDS), I and II, provides panel data on household wage earnings between 2005 and $2011-12$. I obtain the real wage earnings (2011-12 prices) of households and divide them into five quantiles (say $Q1-Q5$). I consider $Q1$ and $Q2$ to be low wage earners and $Q3-Q5$ to be high wage earners. I calculate the proportion of households moving from low to high wage earning and high to low wage earning between IHDS I and II. This provides me with the probability of households transitioning between low and high productivity (I assume wage-earning shifts are productivity shifts). The probability of transition from high to low productivity is 0.22, and the probability of transitioning from low to high is 0.30.

3.1.3 Tax and Transfer

The rate of income tax varies widely over income slabs, age, and occupation. Therefore, I take the average income tax collection as a percentage of total taxable income for the last three years. The data is obtained from the RBI bulletin on the Union budget 2020-21 and 2022-23. The three-year average income tax rate is around 11%. I obtain the data on government direct benefit transfers to households from the Ministry of Finance press release. The total direct benefit transfers from the state government and the central government to households as a percentage of the GDP in three consecutive pre-pandemic years ($2017-18$, $2018-19$, and $2019-20$) are 0.88%, 0.97%, and 1.33% respectively. Therefore, $T = 1.06\%$.

3.1.4 Share of Modern Goods in the Consumer Basket

I use the latest Consumer Expenditure Survey by NSSO conducted in $2011-12$ to obtain the share of household consumer expenditure dedicated to food and other consumer items. To make an association between the goods consumed by households and goods in the theoretical model, I categorize all food items as traditional goods and everything else as modern goods. The average share of consumer expenditure on food alone by agricultural households is around 0.53, and that of non-agricultural households is 0.50. Therefore, $\gamma_L = 0.47$ for agricultural households (low-productivity households in my model) and $\gamma_H = 0.50$ for non-agricultural households (high-productivity households in my model). However, for now, I use only one value of γ in the numerical solution; therefore, $\gamma = 0.48$, a simple average of γ_L and γ_H.

3.1.5 Mean Reversion Rate of the Monetary Policy Shock

I calibrate η using the weighted average call rate WACR, which is the short-term interest rate and also the operating target of monetary policy in India. The data on annual WACR from $2000-2001$ to $2023-24$ is obtained from the *Handbook of Statistics on Indian Economy* by the RBI. An ARIMA model estimation on the log of WACR values reveals that the auto-correlation coefficient is 0.50, which leads to $\eta = 0.69$.

3.1.6 Others

For the rest of the parameters, I stay as close as possible to the parameterizations that are well-accepted in the New Keynesian literature. The parameter values, their descriptions, and the sources can be found in the table 1.

Table 1: Parameter Values, Description and Sources

Parameter	Value	Description	Source
Households			
ρ	0.98	Discount factor	Gabriel et al. (2016)
σ	1.99	Inverse of inter-temporal elasticity of substitution	Gabriel et al. (2016)
ψ	3	Inverse of Frisch elasticity of labour supply	Anand and Prasad (2010)
λ_1	0.30	Probability of low to high productivity	Calculated by Author
λ_2	0.22	Probability of high to low productivity	Calculated by Author
γ	0.48	Share of modern goods in consumer basket	Calculated by Author
$\underline{b}$	-0.32	Borrowing limit	Calculated by Author
Government			
τ	0.11	Rate of tax on labor income	Calculated by Author
T	0.0106	Lump-sum transfer by the government	Calculated by Author
Firms			
δ	0.069	Rate of depreciation of capital	Gabriel et al. (2016)
ϵ	10	Elasticity of substitution between modern goods	Anand and Prasad (2010)
α	0.32	Share of capital	Gabriel et al. (2016)
θ	100	Cost of adjustment parameter	Kaplan et al. (2018)
Monetary Policy			
β	1.25	Taylor rule parameter	Kaplan et al. (2018)
η	0.69	Rate of mean reversion of the shock	Calculated by Author

3.2 Steady State Policy Functions

I derive the steady state path of household consumption and savings following the numerical solution method described before. Figure 1 depicts the steady state consumption of households over different values of the assets. Steady state consumption of high-productivity households is higher than that of the low-productivity households at all levels of household asset holdings. The slope of the policy functions indicates that, at the steady state, households with higher asset holdings consume more. $\underline{b}$ is the borrowing limit; therefore, households lying between $\underline{b}$ and 0 are the household who are borrowing at steady state. Consumption and asset holding exhibit a linear relationship, except for the low productivity households at extremely high levels of borrowing. Figure 2 illustrates the steady-state distribution of assets. The density at different levels of assets for low productivity households lies above the high productivity households up until the 90^{th} percentile. This behavior is influenced by the high probability of the low-productivity household being stuck at the low-productivity state (probability value $1 - 0.30 = 0.70$). This encourages them to save more for consumption smoothing. The high density of the high-productivity households at the extremely high values of the asset is also influenced by the high probability of the high-productivity household remaining in the high-productivity state (probability $1 - 0.22 = 0.78$). The density of low-productivity households declines beyond the 90^{th} percentile of the asset. The shape of the asset distribution is influenced by the assumption that different productivity labor is employed in different sectors. While obtaining the steady state distribution of assets, through my choice of parameters, I target the overall mean; table 2 shows that the model-generated mean is exactly the same as the mean in the data. Similar to the parameterization, I use the NSSO Household Debt & Investments Survey conducted in 2019 to calculate different moments. It shows that the model performs well except for the extreme values of the asset. I refrain from matching the moments separately for high and low-skilled households because, in the data, the skill differentiation is not precise. To test if the model-generated values for consumption are realistic, I match different moments with the consumption surveys. The latest available data on nationwide household consumption is NSSO 68^{th} round Household Consumer Expenditure Survey that was conducted in 2011-12. I obtain the real per-capita consumption by

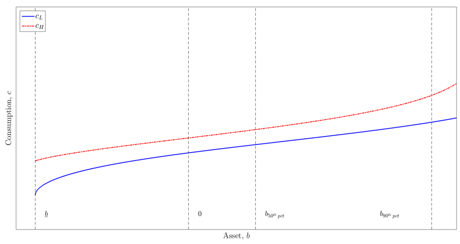

Figure 1: Consumption policy function

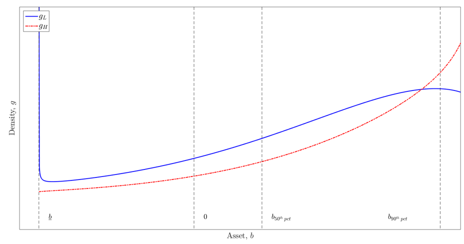

Figure 2: Distribution of Asset

Table 2: Moments of Per-capita Asset Holding

	Data	Model
Targeted		
Mean	0.14	0.14
Non-Targeted		
1^{st} Percentile	-0.31	-1.94
10^{th} Percentile	-0.23	-0.18
90^{th} Percentile	0.51	0.62
99^{th} Percentile	0.60	3.60

Table 3: Moments of Per-capita Consumption

	Data (2011-12 prices)	Model
Mean	0.13	0.14
10^{th} Percentile	0.04	0.11
25^{th} Percentile	0.06	0.12
50^{th} Percentile	0.09	0.14
75^{th} Percentile	0.15	0.16
90^{th} Percentile	0.23	0.18

using 2011-12 consumer prices. Table 3 shows that the model generates a realistically close distribution. Due to the limitation on data availability, I can not match these moments exactly; however, the persistently higher values of the moments indicate that the model is performing well. I do not match the moments separately for high and low-skilled households for the same reason.

3.3 Economy's Response to a Monetary Policy Shock

Now, I evaluate the response of the economy to a one-time unexpected shock on monetary policy. At the time $t = 0$, the shock hits the economy with a magnitude of $\zeta_0 = -1$ percent. This magnitude is consistent with the historical monetary policy stance in India. The shock mean reverts at rate η, i.e., $\zeta_t = e^{-\eta t}\zeta_0$. Figure 3 depicts the effect of

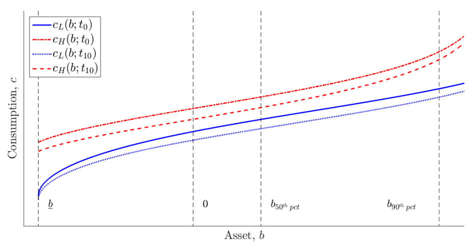

Figure 3: Transition of consumption policy function

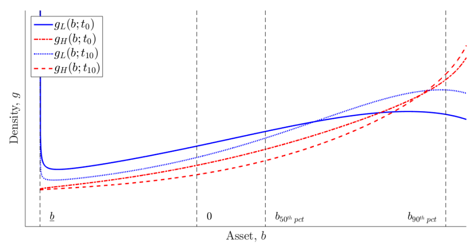

Figure 4: Transition of asset distribution

shocks on steady-state consumption. As the interest rate on assets decreases due to the shock, consumption increases multifold for both high- and low-productivity households.

This is simply the substitution effect. The rise in consumption is much higher for low-productivity households up until almost the 99^{th} percentile. Beyond that, the negative income effect takes dominance, and the consumption falls. Figure 4 illustrates the effect of the shock on the distribution of assets. The density of low-productivity households decreases sharply at the high percentile of the asset. This is simply because there is now a lower incentive to save. However, extremely rich, high-productivity households remain almost unaffected by this negative shock on the interest rate. Table 4 reveals the

Table 4: Consumption Redistribution After the Shock

	Steady-state c_L	c_L at year 1	% increase	Steady-state c_H	c_H at year 1	% increase
Mean	0.1189	0.1265	6.41	0.1446	0.1536	6.27
10^{th} Percentile	0.0854	0.0911	6.66	0.1108	0.1202	8.46
25^{th} Percentile	0.1020	0.1093	7.16	0.1225	0.1324	8.12
50^{th} Percentile	0.1206	0.1292	7.12	0.1402	0.1501	7.05
75^{th} Percentile	0.1378	0.1463	6.19	0.1625	0.1712	5.38
90^{th} Percentile	0.1499	0.1577	5.22	0.1848	0.1921	3.96

moment-to-moment consumption redistribution from the steady-state due to the shock for high and low-skilled households. The decrease in the interest rate leads to a higher consumption demand for both skilled types and throughout the distribution. And the percentage increase is relatively higher at the lower percentiles. As the shock mitigates, the economy returns to the steady state (see figure 9 in Appendix).

4 Conclusion and Discussion

I evaluate the redistributive effects of monetary policy shocks on household consumption, saving, and asset distribution in India by developing a general equilibrium model with continuous time heterogeneity. I introduce an idiosyncratic labor productivity process with the assumption that at each state of this process, labor is hired in different sectors of the economy. This structure of the theoretical model produces realistic stationary policy functions of household consumption-saving behavior and generates a realistic distribution of assets. A number of parameters are calibrated using household surveys and aggregate-level data on the Indian economy. The paper focuses on the economy's response to a one-time deterministic monetary policy shock.

Throughout model calibration, evaluation of the steady state, and the impulse response to the shock, I match the moments with survey data to examine the performance of the model. The heterogeneous structure enables me to quantify the effect for each household and display the redistributive effect over the entire joint distribution of assets and productivity. The results suggest that household consumption and asset holding respond substantially to a monetary policy easing.

Moving forward, I would like to explore this model's reaction to other types of shock (e.g., sector-specific shocks) as well. I intend to extend this model to the introduction of non-homotheticity in demand for traditional goods and then compare the redistribution with this baseline model results.

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5 Appendix

5.1 Derivation of Euler Equation

Households maximize lifetime utility

$$\int_0^\infty e^{-\rho t}\left[\frac{(c_t^i)^{1-\sigma}}{1-\sigma} - \frac{(l_{i,t})^{1+\psi}}{1+\psi}\right] dt, \quad i \in L, H$$

subject to

$$\dot{b}_{i,t} = (1-\tau)\, w_{i,t}\, z_{i,t}\, l_{i,t} + r_{b,t}\, b_{i,t} + T - c_t^i$$

$$b_{i,t} \geq -\underline{\mathrm{b}}$$

household's Hamilton Jacobi Bellman (HJB) equation therefore is,

$$\begin{aligned}\rho V(b_{i,t}, z_{i,t}) = max \;\; & u(c_t^i, l_{i,t}) \\ & + V_b\big[(1-\tau)\, w_{i,t}\, z_{i,t}\, l_{i,t} + r_{b,t}\, b_{i,t} + T - c_t^i\big] \\ & + \lambda_i(V_{-i}(b_{i,t}, z_{i,t}) - V_i(b_{i,t}, z_{i,t}))\end{aligned}$$

and the Kolmogorov Forward equation is,

$$0 = -\frac{\partial}{\partial b_{i,t}}[s(b_{i,t}, z_{i,t})\, g(b_{i,t}, z_{i,t})] - \lambda_i g_i(b_{i,t}, z_{i,t}) + \lambda_{-i} g_{-i}(b_{i,t}, z_{i,t})$$

where,

$$s(b_{i,t}, z_{i,t}) = (1-\tau)\, w_{i,t}\, z_{i,t}\, l_{i,t} + r_{b,t}\, b_{i,t} + T - c_t^i$$

$-i = L$ when $i = H$ and vice versa, $g(b_{i,t}, z_{i,t})$ is the density function corresponding to the state $\mu(b_{i,t}, z_{i,t})$. From the first order conditions of the optimization problem 27, I obtain,

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\sigma}(r_{b,t} - \rho)$$

this is the consumption Euler equation.

5.2 Derivation of Philips Curve

Modern firms set prices so as to maximize lifetime profit minus adjustment cost,

$$\int_0^\infty e^{-\int_0^t r_{k,s} ds}\, (Q_t^j - {\Theta_t}^j)\, dt$$

where,

$${\Theta_t}^j = \frac{\theta}{2}\big(\Pi_{M,t}^j\big)^2\, Y_{M,t}$$

and

$$Q_t^j = \big(P_{M,t}^j - MC_t\big) \left(\frac{P_{M,t}^j}{P_{M,t}}\right)^{-\epsilon} Y_{M,t}$$

the optimization problem in recursive form,

$$\begin{aligned} r_{b,t} J = max \;\; & \big(P_{M,t}^j - MC_t\big) \left(\frac{P_{M,t}^j}{P_{M,t}}\right)^{-\epsilon} Y_{M,t} \\ & - \frac{\theta}{2}\big(\Pi_{M,t}^j\big)^2 Y_{M,t} + J_p P_{M,t}^j \Pi_{M,t}^j + J_t \end{aligned} \tag{29}$$

first-order condition,

$$J_p = \frac{\theta \Pi^j_{M,t} Y_{M,t}}{P^j_{M,t}} \tag{30}$$

and envelop condition is,

$$\begin{aligned}(r_{b,t} - \Pi^j_{M,t}) J_p = &\left(\frac{P^j_{M,t}}{P_{M,t}}\right)^{-\epsilon} \frac{Y_{M,t}}{P_{M,t}} \\ &- \epsilon \left(\frac{P^j_{M,t}}{P_{M,t}} - MC_t\right) \left(\frac{P^j_{M,t}}{P_{M,t}}\right)^{-\epsilon} \frac{Y_{M,t}}{P_{M,t}} \\ &+ J_{pp} P^j_{M,t} \Pi^j_{M,t} + J_{tp}\end{aligned} \tag{31}$$

differentiating 30 with respect to time, substituting into 31 and simplifying I obtain the New Keynesian Philips Curve,

$$\dot{\Pi}^j_{M,t} + \left(\Pi^j_{M,t} - r_{b,t} + \frac{\dot{Y}_{M,t}}{Y_{M,t}} - \frac{1}{P_{M,t}}\right) \Pi^j_{M,t} = \frac{\epsilon}{\theta}(1 - MC_t) - \frac{1}{\theta} \tag{32}$$

substituting the flexible price markup $\frac{1}{MC^*_t} = \frac{1-\epsilon}{\epsilon}$,

$$\dot{\Pi}^j_{M,t} + \frac{\epsilon}{\theta}(MC_t - MC^*_t) = \left(\Pi^j_{M,t} - r_{b,t} + \frac{\dot{Y}_{M,t}}{Y_{M,t}} - \frac{1}{P_{M,t}}\right) \Pi^j_{M,t}\,.$$

5.3 Relative Consumer Demand and Relative Wage

Given the consumption share γ and the elasticity ϵ, a monetary policy expansion will increase the relative demand for modern goods.

Combining equation 5, 6 and 7 I obtain,

$$\frac{C_{M,t}(j)}{C_{A,t}} = \left(\frac{\gamma}{1-\gamma}\right) \left(\frac{P_{A,t}}{P_{M,t}}\right) \left(\frac{P^j_{M,t}}{P_{M,t}}\right)^{\epsilon} \tag{33}$$

Given the values of γ and ϵ, a monetary policy expansion leads to higher prices; traditional good prices adjust immediately, whereas modern good prices are sticky, leading to an increase in the relative prices of traditional goods and a higher relative demand for modern goods.

It is also interesting to observe the sensitivity of relative consumption demand to the share and the elasticity of substitution parameters. Figure 5 and 6 plots $\frac{C_{M,t}(j)}{C_{A,t}}$ across $P_{A,t}$ and $P^j_{M,t}$ for different combinations of γ and ϵ. They exhibit high sensitivity to both structural parameters. The non-linearity in the relationship between relative demand

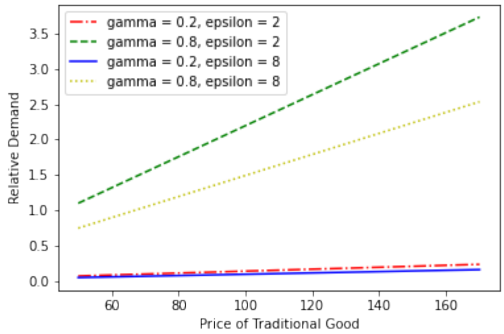

Figure 5: Relative Demand Across $P_{A,t}$

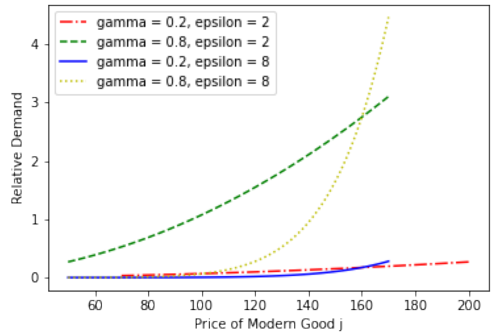

Figure 6: Relative Demand Across $P^j_{M,t}$

and prices in modern goods comes from product differentiation in the monopolistically competitive modern goods market.

Given the capital-labor ratio and the share of capital in the modern sector, a monetary policy expansion decreases the relative wage rate of the modern sector.

The relative wage rate is a function of relative prices, and given the parameter value α and the capital-labor ratio used in firm j, a monetary policy expansion reduces the relative price $\frac{P^j_{M,t}}{P_{A,t}}$ because traditional goods prices adjust immediately while modern goods prices are sticky. Therefore, the relative wage rate of the high skilled labor decreases due to the monetary policy expansion.

$$\frac{w^j_{H,t}}{w_{L,t}} = (1-\alpha)\left(\frac{P^j_{M,t}}{P_{A,t}}\right)\left(\frac{k^j_t}{l^j_{H,t}}\right) \tag{34}$$

Figure 7 and 8 display that this relationship between relative wage and relative prices

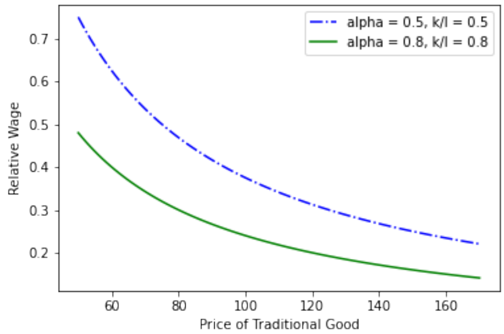

Figure 7: Relative Wage Across $P_{A,t}$

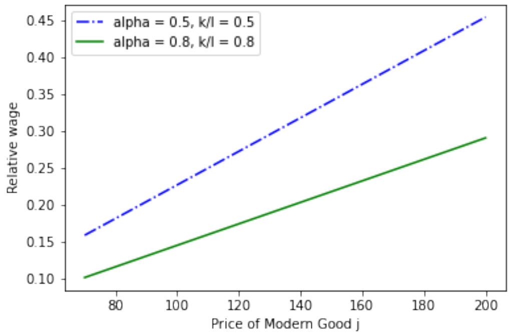

Figure 8: Relative Wage Across $P^j_{M,t}$

is highly sensitive to the structural parameter α.

5.4 Impulse Response

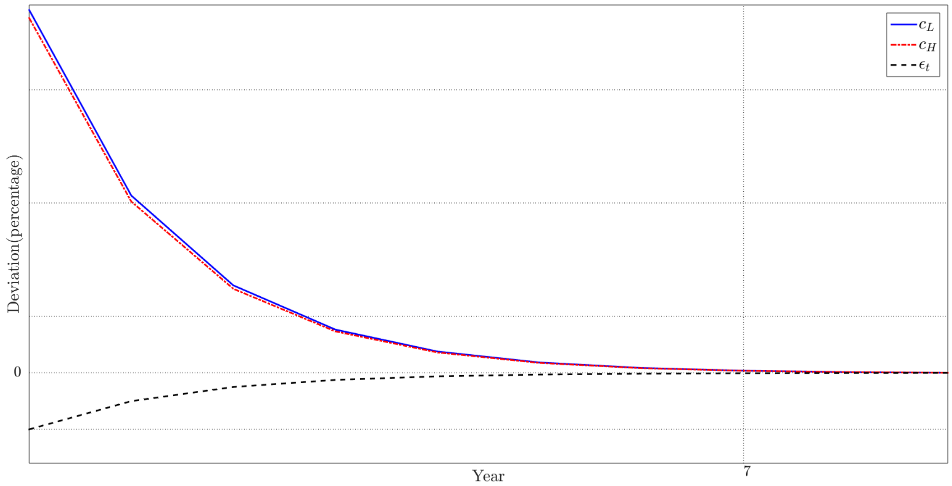

Figure 9: Impulse Response of Consumption

The economy returns to the steady state after the shock is mitigated. Figure 9 shows that the percentage deviation from the steady state consumption systematically falls throughout the years after the one-time shock, and it converges to the steady state.