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Inequality and Technological Change

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We study how technological change affects between- and within-education-group inequality in the United States. We develop a model with heterogeneous workers and firms in which the demand for skills is characterized by firms' recruiting behavior. We use the model to quantify the relative contribution of two types of technological change that affect the relative demand for skilled labor: technological change in firm-specific productivity and technological change in labor productivity. We find that technological change in labor productivity, in the form of higher returns to skill in production, is the main driver of the increase in between- and within-group inequality. Technological change in firm productivity, in the form of higher firm productivity dispersion, plays a less important role in explaining rising inequality, except for the increase in within-group inequality for workers without a college degree.

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1. INTRODUCTION

A striking feature of the trend in inequality in the United States in recent decades has been its increase both across and within education groups (between- and within-group inequality).[1] A major driver of rising inequality is skill-biased technological change (SBTC), broadly understood as a change in production technology that increases the relative demand for skilled labor. While these changes are generally difficult to measure empirically, a recent paper by Decker et al. (2020) rigorously estimates one particular form of SBTC, i.e., rising firm productivity dispersion.[2] This paper studies the

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[1] See, e.g., Goldin and Katz (2007).

[2] They use the US Census Bureau's Longitudinal Business Database, which covers the universe of private non-farm establishments in the US, linked to manufacturing data from the Census of Manufacturers and

importance of this particular technological change in explaining changes in inequality across different groups of workers with different skill levels.

Technological change has both direct and indirect effects on inequality. The direct effects operate through marginal products. When firm productivity and worker productivity are complements in production, technological change in the form of higher returns to skill or higher firm productivity dispersion naturally translates into greater inequality. The indirect effects emerge when these changes alter firms' recruiting behavior and wage policies, affecting not only the demand for skills but also how workers are matched with firms in the presence of labor market frictions.

To study these effects, we develop a tractable equilibrium model with heterogeneous workers and firms in which the demand for skills is characterized by firms' recruiting behavior. In the model, workers are exogenously given a level of schooling, either college or non-college, and within each schooling level, they differ in skill. On average, college workers are more skilled. Firms also differ by productivity level. Labor markets are frictional and segmented by educational attainment. Matching occurs randomly, and workers can search on the job. Technology-skill complementarity in production implies that higher-productivity firms benefit more from hiring high-skilled workers than low-productivity firms.

The model features sorting of firms in equilibrium. High-productivity firms recruit more intensively in the college submarket, where workers tend to be more skilled, whereas low-productivity firms focus on the non-college submarket to avoid competing with more productive rivals. This results in the assortative matching of high-skilled workers with high-productivity firms.

In this framework, we consider two types of SBTC: technological change in firm productivity, in the form of higher firm productivity dispersion, and technological change in labor productivity, in the form of higher returns to skill in production. The former increases wage inequality within and between groups because firms that hire college graduates become relatively more productive. The sorting mechanism is also at play; more high-productivity firms recruit and bid more aggressively for college workers—stronger firm sorting. The latter technological change raises the relative productivity of college workers and skilled workers.

Using the IPUMS-CPS, we calibrate the model to the United States in 1980 and 2015. The main drivers of increasing earnings inequality are technological changes in firm productivity and labor productivity, both of which constitute SBTC. We rely on an external estimate from Decker et al. (2020) for the change in firm productivity dispersion and residually calibrate the change in the returns to skill. The model matches nicely between- and within-group inequality measures. The calibrated model also matches untargeted moments such as distributions of firm size and higher order wages moments.

To understand the channels through which technological change affects inequality, we perform the following counterfactual exercise. We take the 1980 economy and

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the Annual Survey of Manufacturers. This allows them to estimate firm productivity dispersion for 1981-2013. There are studies documenting the link between rising inequality and increasing dispersion in firm productivity, e.g., Faggio et al. (2010), Card et al. (2013), Dunne et al. (2004), Barth et al. (2016), and Song et al. (2019).

separately add higher firm productivity dispersion and an increase in the returns to skill, letting firms adjust their skill demand through recruiting behavior, but holding fixed the schooling attainment distribution to that in 1980. This exercise allows us to calculate the relative contribution of each technological change that constitutes SBTC. We then add these changes together to see how SBTC as a whole affects the change in inequality.

Our main finding is that technological change in labor productivity, in the form of higher returns to skill in production, is the main driver of the increase in between- and within-group inequality. In contrast, technological change in firm productivity, in the form of higher firm productivity dispersion, plays a less important role in explaining rising inequality, except for the increase in within-group inequality for workers without a college degree. Specifically, technological change in labor productivity explains 87% of the increase in between-group inequality and 66% and 52% of the increase in within-group inequality for college and non-college workers, respectively. The contribution of technological change in firm productivity to the change in these inequality measures is 21%, 23%, and 50%, respectively. Total SBTC, which combines these two technological changes, explains almost all of the observed increase in inequality, while non-SBTC changes, such as changes in the distribution of educational attainment, play a limited role.

To elucidate the economic forces behind these results, we decompose the measures of inequality. In the case of between-group inequality, we derive a novel equation that decomposes it into differences in average firms' wages across education groups (*firm pay difference*) and differences in average labor productivity in production across education groups (*labor productivity difference*). Higher firm productivity dispersion raises between-group inequality through an increase in the firm pay difference. There are more high-productivity firms that have a high demand for skills and intensively recruit college workers as a result of firm sorting. This leads to stiffer competition in the college labor market and thus a larger firm pay difference. On the other hand, an increase in the returns to skill in production rises between-group inequality mainly through an increase in the labor productivity difference between college and non-college workers. Since college workers are more skilled, this technological change rises their marginal product more.

For within-group inequality, following Postel-Vinay and Robin (2002), we decompose it into labor productivity variation (*person effect*), wage policy variation across firms (*firm effect*), and wage policy variation within each firm (*friction effect*). We find that the increase in within-group inequality is largely explained by an increase in the person effect for both education groups. The increase in the person effect is driven by the increase in the returns to skill, which generates greater variation in workers' productivity, especially for college workers, as they are more skilled than non-college workers. The increase in the firm effect and the friction effect is mainly driven by higher firm productivity dispersion, as it increases not only the variation of wages across firms but also that of outside offers.

Overall, we find that technological change in labor productivity explains the lion's share of rising inequality because our calibration implies a substantial change in the returns to skill in production. On the other hand, technological change in firm productivity plays a less important role in explaining rising inequality because the change in firm productivity dispersion is relatively small and the model implies only limited amplification through labor market frictions.

Related Literature

It is well-known that SBTC is a major driving force for the increasing inequality (e.g., Acemoglu, 2002). We consider two forms of SBTC that may affect inequality. One is the standard technological change in labor productivity such as an increase in the returns to skill in production. The other is increasing dispersion in firm productivity, for which there is an independent measure by Decker et al. (2020), who use the US Census Bureau's Longitudinal Business Database linked to manufacturing data from the Census of Manufacturers and the Annual Survey of Manufacturers and estimate establishment-level productivity from 1981 to 2013. Haltiwanger and Spletzer (2020) show that rising between-firm inequality is mainly due to rising dispersion across industries.

The presence of assortative matching has been widely documented in recent literature (e.g., Lise et al. 2016, Hagedorn et al. 2017, Lopes de Melo 2018, Bagger and Lentz 2018, Song et al. 2019). On the theoretical side, assortative matching results from factors such as production complementarity between worker and job characteristics (Lise et al. 2016, Lopes de Melo 2018), heterogeneity in search technology (Bagger et al. 2014, Bagger and Lentz 2018) or firms' screening (Helpman et al. 2010). In our model, assortative matching occurs by a different mechanism: labor market segmentation and sorting of firms. Using employer-employee matched data, Engbom and Moser (2017) show that higher education degrees help sorting towards high-wage firms and that this sorting explains a substantial part of the returns to college.

Empirical studies typically decompose wage variation through the estimation of Mincerian wage functions that include worker and firm fixed effects (see e.g., Abowd et al. 1999, Card et al. 2013). Relative to this literature, wage decomposition based on structural models has the advantage of being able to address the potential biases of a reduced-form wage decomposition. For example, by considering the dynamics of worker mobility, Postel-Vinay and Robin (2002) find much less variation in worker fixed effects than do static error-component models that typically attribute historical wage differences to worker fixed effects. Our paper contributes to the decomposition literature based on structural models by explicitly modeling the selection of firms that create jobs for different education levels and the systematic variation in labor market risk with education, each of which potentially causes systematic bias in reduced-form wage estimation.

Our results are broadly consistent with the findings of several empirical studies. Our findings suggest that the change in the labor productivity difference is important for explaining the trends in both between-group inequality (Hendricks and Schoellman 2014) and within-group inequality (Taber 2001, Lemieux 2006). By explicitly considering employer heterogeneity, from which the previous papers abstract, we also find that changes in firm characteristics are also important in explaining the trends in inequality, especially for non-college workers. This echoes the results of recent papers that emphasize the role of firms in accounting for earnings inequality (Card et al. 2013, Song et al. 2019).

2. The Model

2.1 *Environment*

Time is continuous and infinite. There is measure one of heterogeneous workers and measure one of heterogeneous firms. Workers face a constant birth/death rate μ, whereas firms live forever. Both types of agents are risk neutral and discount the future at a common discount rate r. We use $\rho = r + \mu$ to denote the worker's effective discount rate.

Workers are characterized by exogenous states (z, s), where $z \in \mathcal{Z}$ denotes the skill level, and s denotes one of two levels of schooling: non-college (NC) and college (CL). Denote by Ψ the joint distribution of worker type and by Ψ^s the marginal distribution of skills conditional on educational level. Firms are heterogeneous in their productivity level $p \in \mathcal{P} \equiv [b, \overline{p}]$, and Γ denotes the distribution of firms' type.

Labor Market. The labor market is segmented by schooling levels. In each submarket $s \in \{NC, CL\}$, workers and firms are matched randomly, and production takes place. Firms can post vacancies in both submarkets, but workers can only participate in the submarket corresponding to their educational level. Jobs created in submarket s are destroyed at an exogenous rate δ_s.

Production. The marginal product of a firm with productivity p (henceforth, p-firm) of hiring an additional worker with type (z, s) (henceforth, (z, s)-worker) is given by $p \cdot A(z, s)$, where $A(z, s)$ are efficiency units of labor.[3] The total output of a p-firm is equal to p times the sum of its employees' efficiency units of labor across both submarkets.

Wage Determination. A worker participating in submarket s contacts a firm at rate λ_s. The type of the firm is drawn from the sampling distribution F^s. Upon matching, productivities are revealed to both parties, and the worker and the firm bargain over the wage under complete information.

We briefly describe our wage bargaining framework, which follows closely Cahuc et al. (2006). Without loss of generality, we assume that wages are set in terms of efficiency units of labor. Consider a (z, s)-worker who is contacted by a p-firm. If the worker is unemployed, bargaining results in a wage $\phi_0^s(p)$. If the worker is employed by a firm with productivity $p' < p$, the new firm poaches the worker from the incumbent, and bargaining results in a wage $\phi^s(p', p)$, where the first argument denotes the productivity of the last employer.[4] If the worker is employed by a firm with productivity $p' \geq p$, the

[3] Note that A is a function of s because we will also consider education-specific technological change, e.g., a change in the role of the social network in production, access to which may depend on the level of schooling. However, it is difficult to distinguish such technological changes from a change in returns to college (see, e.g., Krusell et al., 2000). We will return to this point in Section 4.1.

[4] In the class of labor search models with on-the-job search and wage renegotiation, wages typically depend on the individual history of past offers. In the present model, however, p' is a sufficient statistic for this history.

incumbent can successfully deter poaching. In this case, however, bargaining resumes and results in a wage raise from the current wage w to $\phi^s(p,p')$ if and only if $p \in (g^s(w,p'),p']$, where $g^s(w,p')$ is the productivity level satisfying $\phi^s(g^s(w,p'),p') = w$.

Firms. Let $\pi^s(w,p,z)$ denote the profit for a p-firm hiring a (z,s)-worker at wage w. This profit must satisfy:

$$\rho\pi^s(w,p,z) = \rho(p-w)\cdot A(z,s) - [\delta_s + \lambda_s(1-F^s(p))]\,\pi^s(w,p,z) \\ +\lambda_s \int_{g^s(w,p)}^{p} [\pi^s(\phi^s(x,p),p,z) - \pi^s(w,p,z)]\,dF^s(x). \quad (1)$$

The first term on the right-hand side is the (normalized) flow profit. The second term is the expected capital loss that stems from either separation or poaching. The third term is the expected capital loss caused by the wage raise necessary to deter poaching.

To hire a worker with schooling level s, a firm must contact her in the corresponding submarket. Once a contact is made, there are three possible scenarios: the firm hires an unemployed worker, it hires a worker previously employed at a firm with lower productivity, or it fails to hire a worker because she is already employed at a firm with higher productivity. The expected profit per worker contacted $\Pi^s(p)$ thus satisfies

$$r\Pi^s(p) = u_s \int_{z\in\mathcal{Z}} \pi^s(\phi_0^s(p),p,z)d\Psi^s(z) \\ +(1-u_s)\int_{z\in\mathcal{Z}} \left\{\int_b^p \pi^s(\phi^s(x,p),p,z)l^s(x)dx\right\} d\Psi^s(z), \quad (2)$$

where u_s is the unemployment rate in submarket s, and $l^s(p)$ is the mass of workers employed by a generic p-firm. The first (second) term on the right side represents the expected profit, conditional on an unemployed (employed) worker being hired.

Since the production technology displays constant returns to scale, the recruiting decision is made separately for each submarket. Denote the contact frequency in submarket s as $\eta_s v_s$, where v_s is a measure of recruiting effort, call it "vacancies" as in Mortensen (2003), and η_s is the aggregate efficiency of recruiting effort.[5] For each s, a p-firm chooses a recruiting effort policy $v_s(p)$ to solve:

$$\max_{v_s}\left[\eta_s v_s \Pi^s(p) - \frac{\chi}{2}v_s^2\right], \quad (3)$$

where the second term represents recruiting costs.

Sampling Distribution. The probability of being contacted with a p-firm in submarket s is proportional to the number of vacancies these firms create. Thus, the sampling

[5]Since v_s is a measure of recruiting effort made at a point in time, an unfilled "vacancy" is immediately destroyed.

distribution function is given by:

$$F^s(p) = \frac{\displaystyle\int_{x \leq p} v_s(x) d\Gamma(x)}{\displaystyle\int_{x \in \mathcal{P}} v_s(x) d\Gamma(x)}. \tag{4}$$

Contact Rates and Recruiting Efficiency. Following Mortensen (2003), we assume that the aggregate flow of contacts per period in submarket s is proportional to the product of aggregate recruiting effort and workers' aggregate contact rate, i.e., $M_s = \eta_s \lambda_s$. Hence, in our model, the following must hold for $s \in \{NC, CL\}$:[6]

$$\lambda_s = \frac{\displaystyle\int_{x \in \mathcal{P}} v_s(x) d\Gamma}{\displaystyle\int_{z \in \mathcal{Z}} d\Psi(z,s)}, \tag{5}$$

$$\eta_s = 1. \tag{6}$$

Equilibrium. A *stationary equilibrium* consists of wage functions $\{\phi_0^s, \phi^s\}$, policies and value functions for the firms $\{v_s, \pi^s, \Pi^s\}$, contact rates and sampling distributions $\{\lambda_s, F^s\}$, with a distribution of workers Ψ such that:
(i) $\{\phi_0^s, \phi^s\}$ solve the wage bargaining problem;
(ii)$\{v_s\}$ solves the recruiting decision problems;
(iii) $\{\pi^s, \Pi^s\}$ satisfies the respective recursive equations; and
(iv) $\{\lambda_s, F^s\}$ are consistent with individual choices.

Skill-Biased Technological Change (SBTC). The literature defines SBTC broadly as a general term for any change in production technology that increases the relative demand for skilled labor. We consider two types of technological change that constitute SBTC: technological change in firm productivity corresponding to a change in Γ, and technological change in labor productivity corresponding to a change in the function $A(z,s)$.

2.2 *Inequality Measures and Decomposition*

Our goal is to study how technological change affects between- and within-group inequality. To understand the underlying economic forces, we propose two analytical decomposition equations for these inequality moments.

We use $\mathbf{w}(z,s,p,p')$ to denote the actual wage paid to a (z,s)-worker at a p-firm with the best outside offer made by a p'-firm. We can write:[7]

$$\mathbf{w}(z,s,p,p') = A(z,s)\phi^s(p',p) \tag{7}$$

[6]To see this, notice that since each p-firm contacts a worker at rate $\eta_s v_s(p)$, the aggregate flow of vacancies must also satisfy $M_s = m_s \eta_s \int_{x \in \mathcal{P}} v_s(x) d\Gamma(x)$, where m_s is the ratio of the measure of firms to that of workers searching a job in submarket s, i.e., $m_s = \left(\int_{z \in \mathcal{Z}} d\Psi(z,s)\right)^{-1}$. For this to be consistent with the expression in the main text, equations (5) and (6) must hold.

[7]This equation also holds for the newly employed by setting $p' = b$.

Between-Group Inequality. We measure between-group inequality as the difference in average log wages between CL and NC. Applying this definition to equation (7) we obtain our first decomposition equation:

$$\underbrace{\mathbb{E}[\log \mathbf{w} \mid CL] - \mathbb{E}[\log \mathbf{w} \mid NC]}_{\text{BG inequality}} = \underbrace{\mathbb{E}[\log \phi^s(p',p)|CL] - \mathbb{E}[\log \phi^s(p',p)|NC]}_{\text{Firm pay difference}} + \underbrace{\mathbb{E}[\log A(z,s)|CL] - \mathbb{E}[\log A(z,s)|NC]}_{\text{Labor productivity difference}}. \tag{8}$$

According to this equation, between-group wage inequality equals the sum of two components. The first is the difference in the conditional mean of log wages measured in efficiency units, which we label *firm pay difference*. The second is the difference in the conditional mean of log efficiency units of labor, which we label *labor productivity difference*.

This decomposition clarifies how the change in between-group inequality is shaped by the two types of SBTC. Technological change in firm productivity (change in Γ) affects the firm pay difference, but not the labor productivity difference if skill formation is exogenous.[8] In contrast, technological change in labor productivity (change in A) has not only a direct effect on the labor productivity difference between the two education groups, but also an indirect effect on the firm pay difference via a change in firms' recruiting behavior.

Within-Group Inequality. We measure within-group wage inequality as the conditional variance of log wages. Applying this definition to equation (7) and using the law of total variance, we obtain our second decomposition equation:

$$\underbrace{\text{Var}\left[\log \mathbf{w}|s\right]}_{\text{WG inequality}} = \underbrace{\text{Var}[\log A(z,s)|s]}_{\text{Person effect}} + \underbrace{\text{Var}\left[\mathbb{E}[\log \phi^s(p',p) \mid p]|s\right]}_{\text{Firm effect}} + \underbrace{\mathbb{E}\left[\text{Var}[\log \phi^s(p',p) \mid p]|s\right]}_{\text{Friction effect}}, \tag{9}$$

where the labels of each component on the right-hand side follow Postel-Vinay and Robin (2002). The person effect measures heterogeneity in labor productivity. The firm effect measures variation in average log wages across firms. The friction effect measures average within-firm log wage variation, independent of the person effect. The latter effect arises purely because two identical workers in the same firm may have different wage offer histories.

This equation illustrates how the two types of SBTC affect the change in within-group inequality. Technological change in firm productivity (change in Γ) affects directly the firm effect and indirectly the friction effect, but leaves the person effect unchanged. In contrast, technological change in labor productivity (change in A) affects directly the person effect and indirectly the other two effects via a change in firms' recruiting behavior.

[8]In the working paper version of this paper, which is available upon request, we endogenize the schooling decisions. The main results are materially unchanged.

2.3 *Analytical Characterization of Equilibrium*

We characterize the stationary equilibrium in nearly closed form, which is essential to address the computation of equilibrium when there is heterogeneity on both sides of the labor market.

Unemployment Rates. Worker flows determine the unemployment rate u_s in each submarket s. Inflows to unemployment are given by $(1-u_s)\delta_s+\mu$, which accounts for workers who either lose their jobs or enter the labor market. Outflows are given by $(\lambda_s+\mu)\,u_s$, which accounts for workers who either find a job or die. In a stationary equilibrium, inflows and outflows must be equal. Imposing this condition yields the following expression for the unemployment rate:[9]

$$u_s=\frac{\delta_s+\mu}{\delta_s+\mu+\lambda_s}. \tag{10}$$

The unemployment rate is increasing in the separation rate and decreasing in the job finding rate.

Wage Equations and Worker Distribution. The optimal wage equations derived in Cahuc et al. (2006) can be easily applied to our segmented labor market structure with multiworker firms. For a worker employed at a p-firm with the best outside option $p'\le p$, the solution to the bargaining problem is given by

$$\phi^s(p',p)=p-(1-\beta)\int_{p'}^{p}\frac{\rho+\delta_s+\lambda_s(1-F^s(x))}{\rho+\delta_s+\lambda_s\beta(1-F^s(x))}\mathrm{x}, \tag{11}$$

where β denotes the worker's bargaining power.[10]

The fraction of workers employed by a p-firm is given by

$$l^s(p)=\frac{\delta_s+\mu+\lambda_s}{[\delta_s+\mu+\lambda_s(1-F^s(p))]^2}(\delta_s+\mu)f^s(p),$$

where f^s is the density of F^s.

Profit per Worker and Optimal Vacancies. We derive profit per worker that is linear in efficiency units of labor, i.e., $\pi^s(w,p,y)=\pi^s(w,p)\cdot A(z,s)$, where

$$\pi^s(w,p)=(1-\beta)\int_{g^s(w,p)}^{p}\frac{\rho}{\rho+\delta_s+\lambda_s\beta(1-F^s(x))}dx. \tag{12}$$

[9]For each s, let E_s denote employed workers, U_s denote unemployed workers, and N_s denote new entrants. Inflows into unemployment are $\delta_sE_s+N_s$. Outflows are $(\lambda_s+\mu)U_s$. Stationarity requires not only inflows and outflows to be equal but also that new entrants make up for the workers that die, i.e., $N_s=\mu(E_s+U_s)$. Thus, in a stationary equilibrium, $(\lambda_s+\mu)U_s=\delta_sE_s+\mu(E_s+U_s)$. Using $u_s=U_s/(E_s+U_s)$ and rearranging yields equation (10).

[10]For an unemployed worker matched with a p-firm, the wage function is given by $\phi_0^s(p)=\phi^s(b,p)$ for any $p\in\mathcal{P}$.

See Appendix A for derivation. We can then write the expected profit per worker contacted as follows:

$$r\Pi^s(p) = \int_{z\in\mathcal{Z}} A(z,s)d\Psi^s(z)\left[u_s\pi^s(\phi_0^s(p),p) + (1-u_s)\int_{x\leq p}\pi^s(\phi^s(x,p),p)l^s(x)dx\right]. \quad (13)$$

This equation makes clear that $\Pi^s(\cdot)$ does not depend on the entire distribution of workers' efficiency units of labor but only on the average across workers participating in submarket s.

Using this result, we have the measure of recruiting effort made by a p-firm:

$$v_s(p) = \frac{\Pi^s(p)}{\chi}. \quad (14)$$

Thus, the optimal recruiting effort decision is linear in the conditional mean of efficiency units of labor in submarket s. This linear property of the optimal policy proves very useful in the computation of the equilibrium.

2.4 *Discussions*

In this section, we discuss some of the model assumptions and implications.

Segmented Labor Markets. Labor markets are assumed to be segmented by schooling levels. This assumption aligns with signaling models à la Spence (1973) in which educational attainment partially reveals workers' productivity.

We rule out the possibility that college graduates take jobs that do not require college degrees (see, e.g., Lee et al. 2017), although the empirical relevance of this option is not conclusive in the literature. In the context of labor search, the presence of such an option implies a shorter unemployment duration for college graduates than for non-college graduates since the former have a higher job-finding rate; however, the observed durations are quite similar for these two groups.

Sorting of Firms and Assortative Matching. Our economy exhibits positive firm sorting—high-productivity firms tend to recruit more and are thus more active in submarkets with higher education. To understand this, we use equation (13) to write the expected profit per worker contacted as $\Pi^s(p) = r^{-1}\mathbb{E}\left[\pi^s(w,p)\mid p,s\right] \times \mathbb{E}[A(z,s)\mid s]$. The first term is the expected profit per worker expressed in efficiency units of labor, and the second term is the average efficiency units of labor in submarket s. According to equation (14), this equation determines in which submarket a firm creates more vacancies and is thus more active. Since the second term is independent of p, the degree of firm sorting depends on how $\mathbb{E}\left[\pi^s(w,p)\mid p,s\right]$ varies with p and s. The expected profit per worker is decreasing in the likelihood that prospective employees are contacted by a firm that would either poach the worker away or trigger a wage increase. High-productivity firms find it relatively easy to hire and retain employees since there are few competitors. They thus post more vacancies in submarkets with more productive workers, i.e., those with higher educational attainment. In contrast, low-productivity firms have difficulty retaining workers in submarkets in which high-productivity firms are more active. Therefore, they participate more actively in submarkets with lower educational attainment, where the likelihood that their employees are poached away is low. As a result, positive firm sorting occurs in equilibrium.[11]

In the calibration, we will assume that CL workers tend to be more skilled than NC workers. Together with sorting of firms, this implies assortative matching of high-skilled workers with high-productivity firms, i.e., a positive correlation between z and p across matches. This feature of the equilibrium stems from the segmented labor market structure.[12]

Determinants of College Premium. The macroeconomic literature often distinguishes three channels through which the supply and demand for skills can affect the college premium (see, e.g., Krusell et al. 2000). The first is the relative quantity effect in which faster growth in skilled labor, measured as the number of college graduates, relative to that of unskilled labor, measured as the number of high school graduates, reduces the skill premium. The second is the relative efficiency effect in which faster growth in the labor efficiency of college graduates relative to that in the labor efficiency of high school graduates increases the college premium. The third is the technology-skill complementarity effect.[13]

We also have these three channels in the model. First, the relative quantity effect operates through frictional labor markets. In particular, when there is a relative increase in college graduates, it makes relatively harder for them to be contacted by a firm, which pushes down their wages and results in a decrease in the college premium. We also have a relative efficiency effect, as faster growth of productivity of skilled labor widens the gap between the average productivities of college and non-college workers. Finally, firm productivity and worker skills are complements in production.

3. Data and Calibration

We calibrate the model to the U.S. economy in 1980 and 2015, assuming that the economy is in a steady state.

[11]Using data on male workers from the Quarterly Workforce Indicators from 1994 to 2017, we find two pieces of indirect evidence of firm sorting. First, small firms hire more high school (HS) workers than college (CL) workers, but large firms do the opposite. In particular, the shares of HS and CL workers working at firms with fewer than 50 workers are 36% and 30%, respectively, whereas those working at firms with at least 500 workers are 31% and 35%, respectively. Second, HS workers are more likely to work at small firms than CL workers, and CL workers are more likely to work at large firms than HS workers. Specifically, among HS workers, 32% work at firms with fewer than 50 workers, and 45% work at firms with at least 500 workers. The corresponding numbers for CL workers are 27% and 53%.

[12]Note that assortative matching arises only if there are frictions in the labor market. If there are no labor market frictions in our model, all workers would work for the most productive firm, and measured assortative matching would thus be zero. Using monotone comparative statics, Gola (2021) theoretically characterizes conditions under which assortative matching occurs in a frictionless matching framework.

[13]Krusell et al. (2000) consider a production function that exhibits decreasing marginal returns in the labor inputs. Technology-skill complementarity emerges from capital-skill complementarity in their model because skilled labor is more complementary to equipment capital than is unskilled labor.

3.1 *Data*

An ideal data set to estimate the model with two-sided heterogeneity would be an employer-employee matched data set of representative samples of workers and firms both in 1980 and 2015 that includes information on educational attainment of workers. To the best of our knowledge, it does not exist for the United States.[14] We thus target inequality measures, using data from the Current Population Survey (CPS), and then argue why these wage moments are informative to identify key structural parameters.

We restrict attention to white males aged 25-55 who are working full time.[15] To calculate the inequality measures, we first estimate Mincerian regressions:

$$\log w_{i,t} = \alpha + \sum_{s} \beta_{s,t} D_{s,t} + \delta X_{i,t} + \epsilon_{i,t},$$

where $w_{i,t}$ represents the earnings of worker i at time t, $D_{s,t}$ are dummies for each educational level s, and $X_{i,t}$ are controls for experience and industry (Card 1999; Haltiwanger and Spletzer 2020).

We measure between-group inequality (college premium) as the difference in average log earnings between workers with a college degree and those without. To calculate the average earnings of each group, we use the output of the Mincerian regressions to control for the effect of experience and industry on earnings. We let $\log w_{s,t} = \mathbb{E}\left[\log w_{i,t} - \delta X_{i,t} \mid s\right]$ denote average log earnings of educational group s and calculate between-group inequality simply as $\log w_{CL,t} - \log w_{NC,t} = \beta_{CL,t} - \beta_{NC,t}$.

We measure within-group inequality as the conditional variance of log earnings for each educational group. This is simply the variance of the predicted residuals of the Mincerian regression, i.e., Var $\left[\log w_{i,t} - \log \hat{w}_{i,t} | s\right]$ = Var $\left[e_{i,t} | s\right]$, where $\hat{w}_{i,t}$ are predicted earnings and $e_{i,t}$ are predicted residuals.

3.2 *Calibration*

Pre-Determined and Exactly Identified Parameters. The continuous-time economy is discretized such that one time interval corresponds to one month. The interest rate r is assumed to be time-invariant and set such that the annual rate is 2%. The birth/death rate μ reflects 40 years of expected working life. These parameters imply an annual effective discount rate of 4.5%. The value of leisure b is normalized to 1. The distribution of educational attainment for each point in time is estimated directly from the data.

[14] As far as we know, at least two employer-employee matched data sets exist for the United States, both of which have highly restricted access. One is the Master Earnings File (MEF) of the Social Security Administration. The MEF contains annual labor earnings information from 1978 for almost all workers in the U.S., but it does not contain information about their educational attainment. The other is the Longitudinal Employer-Household Dynamics (LEHD) of the U.S. Census Bureau. The LEHD program contains detailed information about workers including their educational attainment. However, states participate in the program at different times and the majority of them participated only in late 90s.

[15] Appendix B contains a detailed data description. We restrict attention to a somewhat homogeneous group. We drop women from the sample because the educational attainment and labor force participation of women have changed dramatically in recent decades for various reasons, not only those on which we focus in this paper. Figure B.1 displays the trends in the inequality measures.

We identify some parameters by the corresponding empirical moments. The expected unemployment duration in submarket s is given by $1/(\lambda_s + \mu)$.[16] Given μ, the unemployment duration of each education group in the data thus identifies the contact rate λ_s, while λ_s is endogenously pinned down by equation (5). The denominator on the right-hand side of that equation is calculated from the data on unemployment rates and educational attainment distribution. Thus, the unemployment duration effectively identifies the determinants of the firms' recruiting policies in the numerator. In particular, the recruiting cost parameter χ in equation (14) is calibrated to match the average unemployment duration in the data. Given μ and λ_s, we use the unemployment rate of each education group and equation (10) to identify the separation hazard rate δ_s.

Calibration. The firm productivity distribution is Pareto, $\Gamma(p) = 1 - p^{-\gamma}$, with the Pareto parameter γ. The bargaining parameter β is common across submarkets, as in Flinn and Mullins (2015). Assume that the log skill of NC workers follows a normal distribution $N(\mu_z, \sigma^2)$ and that the log skill of CL workers follows an exponentially modified Gaussian (EMG) distribution $EMG(\mu_z, \sigma^2, \alpha)$. The EMG distribution is known to be a good representation of the skill distribution, especially for its thick Pareto right tail (see, e.g., Heathcote and Tsujiyama, 2021). The exponential part of the distribution, captured by α, can be interpreted as the returns to college, which combines the productivity enhancement of college education and selection into college based on ability. We set μ_z to normalize the average skill of NC workers to 1. Finally, we assume that efficiency units are given by $A(z,s) = \varphi_s z^\theta$, where φ_s is an education-specific parameter with $\varphi_{NC} = 1$.[17]

We jointly calibrate the model to the economy in 1980 and 2015. We assume that the only time-dependent parameters are those related to SBTC $\{\gamma, \varphi_{CL}, \theta\}$. Assuming that SBTC started after 1980 (Card and Dinardo, 2002), $\varphi_{CL,1980}$ and θ_{1980} are normalized to 1. Technological change in labor productivity is governed by $\{\varphi_{CL,2015}, \theta_{2015}\}$. The Pareto parameter of the firm productivity distribution γ_{1980} is calibrated. For its change, we rely on an external estimate from Decker et al. (2020), who estimate firm productivity in the United States since 1981 using census data. We set γ_{2015} so that the standard deviation of log firm productivity increases by 0.05.[18] We use the absolute rather than the relative change from that paper because what matters for the absolute increase in inequality is the level change in productivity dispersion. Decker et al. (2020) estimate three measures of productivity.[19] While these productivity measures differ substantially in construction, they show a similar absolute increase in dispersion over time.

[16] Unemployment can be interrupted either because the worker finds a job or because she dies. Since both events behave as Poisson processes, interruption is also Poisson with arrival rate $\lambda_s + \mu$.

[17] As we discussed in Section 2.1, φ_s is indistinguishable from the scale parameter of the exponential part of the skill distribution, which is implicitly normalized to 1 here. In Section 4.1, we will consider another specification that is isomorphic to this calibration and show that our results are qualitatively similar.

[18] This corresponds to a 59% increase in the standard deviation of log firm productivity. If we instead use their relative change (11%), our finding that most of the increase in inequality is due to technological change in labor productivity rather than to technological change in firm productivity is robust, and, not surprisingly, the latter change plays an even smaller role. The results are available upon request.

[19] Two of them are based on extracting residuals from the revenue function using different approaches, TFPS and TFPP, and the other is standard revenue-based productivity, TFPR.

TABLE 1. **Calibrated Parameters and Targeted Moments**

Parameter	Description	Value 1980	Value 2015	Target
Exactly Identified				
$\begin{bmatrix}\delta_{NC}\\ \delta_{CL}\end{bmatrix}$	Separation rates	$\begin{bmatrix}0.013\\ 0.004\end{bmatrix}$	$\begin{bmatrix}0.007\\ 0.002\end{bmatrix}$	Unemployment rates
χ	Vacancy creation cost	0.5	3.4	Unemployment duration
Calibrated/Normalized				
γ	Firm productivity	**13.7**	9.3	
θ	Returns to skill	1	**1.20**	Between- and within-group inequality
φ_{CL}		1	**1.09**	
$\begin{bmatrix}\mu_z\\ \sigma\\ \alpha\end{bmatrix}$	Labor productivity	$\begin{bmatrix}-0.06\\ \mathbf{0.35}\\ \mathbf{4.03}\end{bmatrix}$		
β	Bargaining power	**0.31**		

Note: We calibrate $\{\gamma_{1980}, \theta_{2015}, \varphi_{CL,2015}, \sigma, \alpha, \beta\}$, which are in bold, by targeting inequality moments in 1980 and 2015. γ_{2015} is set so that the standard deviation of log firm productivity increases by 0.05 (Decker et al., 2020). μ_z is set to normalize the average skill of NC workers. θ_{1980} and $\varphi_{CL,1980}$ are normalized to 1.

We calibrate 6 parameters $\vartheta \equiv \{\gamma_{1980}, \theta_{2015}, \varphi_{CL,2015}, \sigma, \alpha, \beta\}$ by targeting inequality measures in 1980 and 2015, i.e., between-group inequality, and within-group inequality for each NC and CL for each point in time. We solve

$$\min_{\vartheta \in \Theta} \left[\mathcal{M}(\vartheta) - \mathcal{M}_{\text{data}}\right]^{\text{T}} \left[\mathcal{M}(\vartheta) - \mathcal{M}_{\text{data}}\right], \tag{15}$$

where $\mathcal{M}_{\text{data}}$ is a vector of the targeted moments and $\mathcal{M}(\vartheta)$ is a vector of the corresponding model moments given ϑ.

The three SBTC-related parameters $\{\gamma, \theta, \varphi_{CL}\}$ are all responsible for increasing inequality. While it is difficult to provide a formal identification, the calibration structure makes it clear which moments are informative for which parameters. Since θ_{1980} and $\varphi_{CL,1980}$ are normalized, the inequality moments in 1980 are useful to identify γ_{1980}. For its value in 2015, we rely on the external estimate. While θ_{2015} and $\varphi_{CL,2015}$ are both related to the inequality moments in 2015, $\varphi_{CL,2015}$ is independent of the within-group inequality of NC workers by construction, which is demonstrated by numerical comparative statics in Appendix C. Thus, this moment is helpful in identifying θ_{2015}.

Table 1 presents the calibrated parameters and the targeted moments. We find $\theta_{2015} = 1.20$. This means, for example, that due to technological change, workers with twice the average skill are 30% more productive in 2015 than they were in 1980. We get $\varphi_{CL,2015} = 1.09$, which implies that the returns to college increase by 9%. Finally, we obtain $\beta = 0.31$. Cahuc et al. (2006) estimate the bargaining parameter for different occupations and industries using French data. Excluding the exceptionally high estimate of 0.98 for executives, managers, and engineers in the construction sector,

TABLE 2. **Model Performance**

Targeted Moments	**Data**		**Model**	
	1980	**2015**	**1980**	**2015**
Labor market conditions				
Unemployment rate: NC	5.0	5.5	5.0	5.5
Unemployment rate: CL	1.8	2.4	1.8	2.4
Unemployment duration (months)	3.5	6.4	3.4	6.3
Inequality measures				
Between-group: CL/NC	34.5	52.2	34.5	52.1
Within-group: NC	15.2	27.1	16.0	26.4
Within-group: CL	21.1	33.7	20.6	34.2

Note: We use five-year averages for empirical moments to mitigate cyclical fluctuations. Between-group inequality is measured as the difference in average log earnings (log point). Within-group inequality is defined as the conditional variance of log earnings multiplied by 100.

the remaining 15 estimates range from 0.00 to 0.38. In their preferred specification, Flinn and Mullins (2015) estimate the bargaining parameter of 0.25, which is close to our estimate, although the setups are not fully comparable.

3.3 *Model Performance*

Table 2 displays the fit of the model for both 1980 and 2015. The model well replicates the increasing trends in between- and within-group inequality measures. Our model thus successfully captures key aspects of earnings distributions for different groups of the population.

Our calibration is also externally validated by empirical moments that are not targeted. The success in matching these moments lends credence to the model's predictions on the wage inequality trend. Figure 1 plots the distribution of firm size in 1980 and in 2015. The distribution in the data is constructed using the Business Dynamics Statistics of the U.S. Census Bureau. The model does a good job in replicating the overall shape of the distribution, especially, in capturing the significant fraction of small firms. The distribution in 2015 is hardly different from that in 1980 in the data, which is also captured by the model.

For the wage distribution, in the data, the wage ratio of 50th percentile to 10th percentile (P50/P10) changes from 1.90 to 2.27 and P90/P50 from 1.70 to 2.40 between 1980 and 2015. Our model captures this increasing trend: P50/P10 changes from 1.71 to 2.05 and P90/P50 from 1.68 to 1.95, although the magnitude of the increase is somewhat smaller.

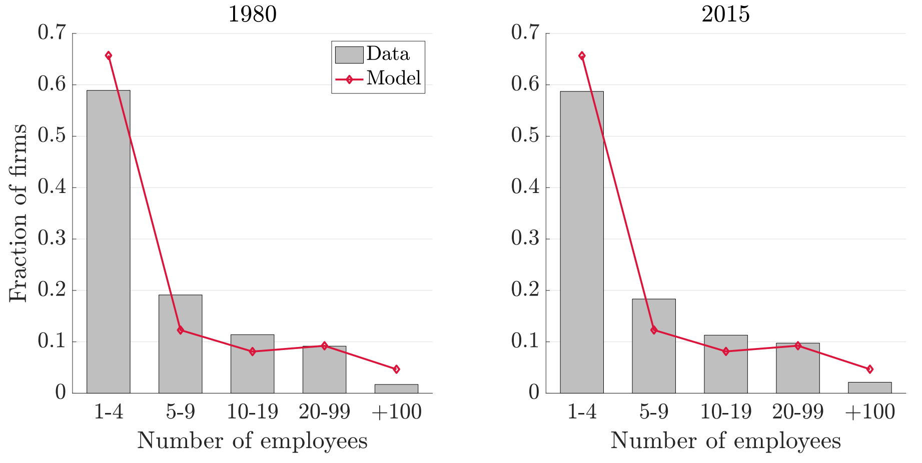

FIGURE 1. Firm Size Distribution. The figure plots the firm size distribution in the model against that in the data from Business Dynamics Statistics for 1980 and 2015. We group large firms with more than 99 employees into one category as their density is tiny. The firm size in the model is obtained by normalizing the average size to that in the data (19.8 in 1980, 23.9 in 2015).

4. Quantitative Analysis

In our calibration, SBTC drives the change in inequality over time. As discussed in the introduction and Section 2.1, our notion of SBTC consists of two parts: technological change in firm productivity, in the form of higher firm productivity dispersion (a decrease in γ), and technological change in labor productivity, in the form of an increase in the returns to skill (an increase in θ and φ_{CL}). In this section, we assume that these technological changes are exogenous shocks to the economy and analyze how they affect between- and within-group inequality.

To understand the channels through which technological change affects inequality, we conduct two counterfactual experiments. First, we take the 1980 economy and separately add the change in firm productivity dispersion or in the returns to skill, holding the schooling attainment distribution fixed at that of 1980. This exercise allows us to calculate the relative contribution of each technological change that constitutes SBTC. Second, we add these changes together to see how SBTC as a whole affects the change in inequality and whether there is any interaction between the two types of technological change. Note that simply adding SBTC to the 1980 economy does not result in the 2015 economy, because we still keep the education attainment distribution and the labor market parameters (δ_s, χ) at their 1980 values. Thus, this exercise also allows us to calculate the contribution of these non-SBTC changes, which turns out to be small.

4.1 *Changes in Inequality*

Table 3 reports the changes in between- and within-group inequality measures between 1980 and 2015. The row labeled “SBTC”presents the results for the

TABLE 3. **Counterfactual Analysis.**

	ΔBG		ΔWG: NC		ΔWG: CL	
	Level	**Pct.**	**Level**	**Pct.**	**Level**	**Pct.**
SBTC	**19.2**	**109%**	**10.6**	**101%**	**11.9**	**88%**
Firm Productivity	3.7	21%	5.2	50%	3.1	23%
Labor Productivity	15.4	87%	5.4	52%	9.0	66%
Model: 1980-2015	**17.6**		**10.4**		**13.6**	

Note: ΔBG denotes the change in between-group inequality of college workers (CL) relative to non-college workers (NC) between 1980 and 2015. ΔWG corresponds to the change in within-group inequality for each education group. The last row shows the change in each inequality moment predicted by the model between 1980 and 2015.

second counterfactual experiment, and the rows "Firm Productivity"and "Labor Productivity"show the results for the first counterfactual experiment.

Between-Group Inequality. With SBTC, between-group inequality increases by 19.2 log points, which is 9% larger than the predicted increase of 17.6 log points between 1980 and 2015. This result confirms that SBTC is the main driver of the increase in between-group inequality. The non-SBTC changes mitigate the effect of SBTC. Both the increasing dispersion of firm productivity and the increase in the returns to skill contribute to the higher between-group inequality, accounting for 21% and 87% of the total change, respectively. This means that the latter is the main technological change responsible for the higher between-group inequality. There is little interaction between the two technological changes.

Within-Group Inequality. With SBTC, within-group inequality increases by 10.6 for NC workers and 11.9 for CL workers, accounting for 101% and 88% of the predicted increase between 1980 and 2015, respectively. Thus, SBTC is also the main driver of the increase in within-group inequality. The increasing dispersion of firm productivity explains 50% of the total change for NC workers and 23% for CL workers. For the increase in the returns to skill, the contribution increases to 52% and 66%, respectively. Thus, this technological change is again most responsible for the higher within-group inequality, although the change in the distribution of firm productivity is equally important for the increase in within-group inequality for NC workers.

Robustness. As we discussed in Section 2.1, it is difficult to distinguish the change in φ_{CL} from a change in returns to college (see, e.g., Krusell et al., 2000). Our result in Table 3 is based on the somewhat strong assumption that the education-specific change (i.e., the 9% increase in φ_{CL}) is all related to production technology and thus considered part of SBTC.

TABLE 4. **Decomposition: Between-Group Inequality**

	△BG	Firm Pay Diff.	Labor Prod. Diff.
SBTC	**19.2**	**4.3**	**15.0**
Firm Productivity	3.7	3.7	0.0
Labor Productivity	15.4	0.4	15.0
Model: 1980-2015	**17.6**	**2.6**	**15.0**

Note: The decomposition is based on equation (8).

One can imagine the other extreme, where the education-specific change is entirely due to an improvement in the productivity-enhancing effect of college.[20] In this case, the change in the returns to college is attributed to the non-SBTC change, reducing the contribution of the technological change in labor productivity. However, even in this extreme case, we find qualitatively the same results. Technological change in labor productivity, now due only to the change in θ, explains 26% of the predicted change in between-group inequality, more than the contribution of technological change in firm productivity. Total SBTC accounts for 51%, so it is again the main driver of change in between-group inequality. For within-group inequality for CL workers, the contribution of technological change in labor productivity falls only slightly from 66% to 59%, and SBTC explains 82% of the observed change, compared to 88% before. The result for within-group inequality for NC workers is unchanged by construction.

4.2 *Decomposition*

To elucidate the economic forces behind these changes in the inequality measures, we use our decomposition equations (8-9).

Between-group inequality. Table 4 shows the decomposition of the change in between-group inequality based on equation (8), in which we decompose the change in between-group inequality into the firm pay difference and the labor productivity difference. They increase by 2.6 and 15.0 log points, respectively, accounting for 15% and 85% of the overall increase between 1980 and 2015. Thus, the college premium increases mainly because the labor productivity gap between CL workers and NC workers widens.

The increase in the labor productivity difference is entirely due to the technological change in labor productivity. That is, the increase in the returns to skill widens the labor productivity gap between CL workers and NC workers. In contrast, increasing firm productivity dispersion does not affect the labor productivity difference.

With SBTC, the firm pay difference increases by 4.3 log points. Most of the increase comes from increasing productivity dispersion. In 2015, there are more high-productivity firms that have high demand for skills. Wages increase in both submarkets

[20]This interpretation corresponds to another calibration that is isomorphic to our baseline calibration. In that calibration, similar to our baseline calibration, we assume that the skill of CL workers follows a Pareto log-normal distribution, $z = z_1 \cdot z_2$, where $z_1 \sim LN(\mu_z, \sigma^2)$ and $z_2 \sim P(x, \alpha)$. Here, x is a scale parameter of the Pareto distribution, and in our baseline calibration, we implicitly assumed $x = 1$ throughout the time. In the isomorphic calibration, we instead assume that $\varphi_{CL} = 1$ but x increases to 1.09 in 2015.

TABLE 5. **Decomposition: Within-Group Inequality**

	$\triangle$WG: NC	**Person**	**Firm**	**Friction**
SBTC	**10.6**	**5.4**	**2.3**	**2.9**
Firm Productivity	5.2	0.0	2.3	2.9
Labor Productivity	5.4	5.4	-0.0	-0.0
Model: 1980-2015	**10.4**	**5.4**	**2.4**	**2.6**
	$\triangle$WG: CL	**Person**	**Firm**	**Friction**
SBTC	**11.9**	**9.1**	**1.1**	**1.8**
Firm Productivity	3.1	0.0	1.2	1.9
Labor Productivity	9.0	9.1	-0.1	-0.1
Model: 1980-2015	**13.6**	**9.1**	**2.1**	**2.4**

Note: The decomposition is based on equation (9).

but disproportionately more in the CL market, because these high-productivity firms intensively recruit CL workers, resulting in stronger firm sorting and a larger firm pay difference. The increase in the returns to skill also strengthens firm sorting as it increases the size of the match surplus more in the CL market.

The non-SBTC changes attenuate the effect of SBTC, reducing the firm pay difference by 1.7 log points. Since a larger (smaller) supply of CL (NC) workers makes it easier (harder) for firms to contact these workers, the logic of competitive labor markets suggests that the firm pay difference should fall in response. This is the relative quantity effect at work.

Within-Group Inequality. Table 5 shows the decomposition of the change in within-group inequality based on equation (9), in which we decompose the increase in within-group inequality into the person, firm, and friction effects. They explain 52%, 23%, and 25% of the predicted change in within-group inequality for NC workers and 67%, 15%, and 18% for CL workers, respectively. Thus, the increase in within-group inequality is largely driven by the increase in variation in labor productivity within educational groups.

The increase in the person effect is entirely driven by the increase in the returns to skill. Larger returns translate into a larger person effect (equation 9), and the effect of higher θ is stronger for CL workers because they are more skilled than NC workers. The increase in the firm and friction effect is mainly driven by the increase in the firm productivity dispersion. Wages per efficiency unit of labor depend on the productivity of the employer (equation 11), and thus, wage variation across firms hinges on the properties of the productivity distribution. In addition, the appearance of many high-productivity firms increases the variation of outside offers and thus the likelihood of climbing the ladder within less productive firms, which leads to higher frictional wage variation. Hence, the value of the firm and the frictional effects are higher.

5. Conclusions

In this paper, we develop an equilibrium model in which skill demand is characterized by firms' recruiting behavior. The model features firm sorting and assortative matching of high-skilled workers with high-productivity firms. It provides novel decomposition equations that can be used to study how different forms of SBTC shape between- and within-group inequality over time in the United States.

The main result of our analysis is that the increase in between- and within-group inequality is largely driven by the change in the returns to skill in production, while higher firm productivity dispersion plays an important role in explaining the increase in within-group inequality of non-college workers.

Appendix A: Derivation of Equation (12)

We derive the equation by guess and verify. We suppress the schooling level s. Rearranging equation (1) and applying integration by parts, we obtain

$$(\rho+\delta+\lambda)\pi(w,p)=\rho(p-w)-\lambda\int_{g(w,p)}^{p}\pi_w(\phi(x,p),p)\phi_1(x,p)F(x)dx. \tag{16}$$

Differentiating both sides with respect to w and applying Leibniz's rule yields

$$(\rho+\delta+\lambda)\pi_w(w,p)=-\rho+\lambda\phi_1(g(w,p),p)g_w(w,p)\pi_w(w,p)F(g(w,p)).$$

Since, by the definition of $g(w,p)$, we have $\phi_1(g(w,p),p)g_w(w,p)=1$, we can write

$$\pi_w(w,p)=-\frac{\rho}{\rho+\delta+\lambda\left[1-F(g(w,p))\right]}.$$

Plugging this expression back into equation (16) and noting

$$\rho(p-w)=\rho\int_{g(w,p)}^{p}\phi_1(x,p)dx,$$

we have

$$\pi(w,p)=\rho\int_{g(w,p)}^{p}\frac{\phi_1(x,p)}{\rho+\delta+\lambda(1-F(x))}dx. \tag{17}$$

From equation (11), the derivative of the wage equation with respect to the first argument is:

$$\phi_1(p',p)=(1-\beta)\frac{\rho+\delta+\lambda(1-F(p'))}{\rho+\delta+\lambda\beta(1-F(p'))}.$$

Plugging this into equation (17) yields equation (12).

Appendix B: Data: Current Population Survey (CPS)

We use the Current Population Survey (CPS), relying on IPUMS-CPS. The CPS is a nationally representative data set that provides important demographic and employment information. Our sample is composed of white males aged 25-55. We drop women from the sample because the educational attainment and labor force participation of women have changed dramatically in recent decades for various reasons, not only those on which we focus in this paper. We also drop nonparticipants in the labor force and samples with missing observations. The sample weights provided are used in computing the empirical moments.

We first define the education categories. To do so, we use a variable for educational attainment provided by IPUMS-CPS.[21] We define high school dropouts as those with fewer than 12 years of completed schooling or those without a high school diploma; high school graduates as those having 12 years of completed schooling and not reporting no diploma; some college attendees as those with any schooling beyond 12 years and less than 4 years of college; and college graduates as those with 4 or more years of completed schooling. We define non-college workers as a sum of high school graduates and some college attendees. We do not use high school dropouts in the analyses. We measure experience as years of schooling subtracted from age minus 5. In the Mincerian regressions, we include cubic controls for experience and controls for industry codes.

For earnings inequality measures, we use those working full-time (40+ weeks and 35+ usual hours per week) for wages and salary in the private labor force. Self-employed workers are excluded. All amounts are adjusted to 2015 U.S. dollars using the CPI. We impute average hourly wages for each observation using reported work weeks and usual hours per week. We then drop those with imputed hourly wages falling below one-half of the federal minimum wage.

We follow Autor et al. (2008) for top coding. Prior to 1988, wage and salary incomes were collected in a single variable. After 1988, they were reported as two separate variables, corresponding to primary and secondary earnings. For each of these variables, top-coded values are simply reported at the top-code maximum, except for the primary earnings variable in 1996 or later. For those, top-coded values are assigned the mean of all top-coded earners, and we reassign the top-coded value. We then multiply the top-coded earnings value by 1.5. After 1988, we simply sum the two earnings values to calculate total wage and salary earnings.

Figure B.1 displays the trends in the between- and within-group inequality.

Appendix C: Numerical Comparative Statics

Figure C.1 shows the numerical comparative statics for the inequality moments in 2015 with respect to the parameters related to the returns to skill, θ_{2015} and $\varphi_{CL,2015}$. In the figure, we take the calibrated parameters and change the value of only one parameter at a time. The red vertical lines indicate the calibrated values for the parameters, and the red horizontal lines indicate the empirical moments. The right panel shows that the within-group inequality for NC workers is independent of $\varphi_{CL,2015}$, thus identifying θ_{2015}.

[21] This variable, called EDUC, is constructed from two other variables, HIGRADE and EDUC99. HIGRADE, available prior to 1992, gives only the respondent's highest grade completed, whereas EDUC99, available since 1992, also provides data on highest degree or diploma attained. In EDUC, the categories of HIGRADE are given the same codes as their equivalents in EDUC99.

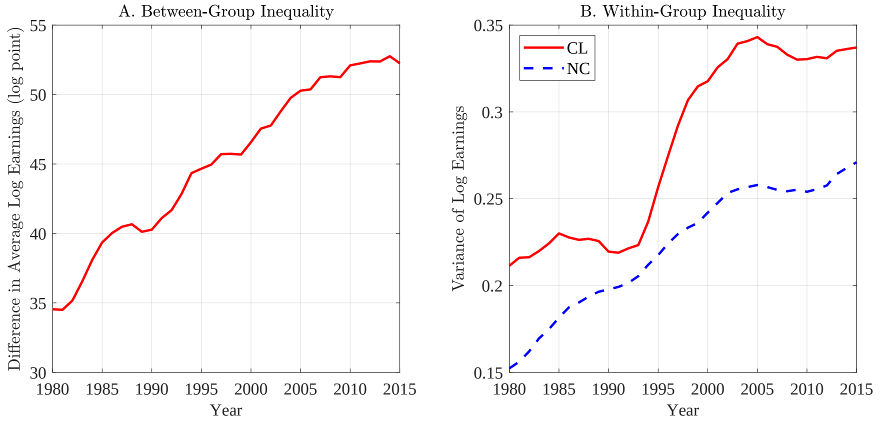

FIGURE B.1. Between- and within-group earnings inequality. Panel A plots the between-group inequality measured as the difference in average log earnings between college graduates (CL) and non-college workers school graduates (NC) (log point). Panel B plots the within-group inequality of CL and NC measured as the variance of log earnings. All values are five-year-centered moving averages.

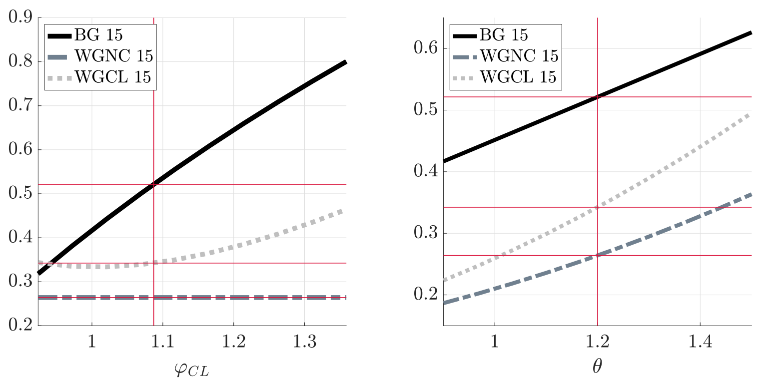

FIGURE C.1. Numerical Comparative Statics: The figure shows the numerical comparative statics for the inequality moments in 2015 with respect to θ_{2015} and $\varphi_{CL,2015}$. BG 15 is the between-group inequality in 2015, WGNC 15 is the within-group inequality for non-college workers in 2015, and WGCL 15 is the within-group inequality for college workers in 2015.

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