SUPPLEMENT TO "THE MACROECONOMIC IMPACT OF MICROECONOMIC SHOCKS: BEYOND HULTEN'S THEOREM"

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APPENDIX B: GENERALIZATION OF SECTION 3.2 TO MULTIPLE GOODS

FOR THE EXAMPLE in Section 3.2, the economy with extreme complementarity $\theta = 0$ has $Y = A/a$, where $1/a$ is the sales to output ratio in steady state. Therefore, in this example, although Hulten's approximation fails in log terms, Hulten's theorem is globally accurate in *linear* terms. In other words, our examples so far may suggest that extreme complementarities can only have outsized effects, in linear terms, if we restrict the movement of labor across industries.

However, this impression is false. To see this, consider a slightly more complex example where we generalize the example above by allowing multiple industries. Aggregate consumption is Cobb–Douglas across goods with equal weights ($b_i = 1/N$). Each good is produced using labor and the good itself as an intermediate input. We assume full labor reallocation/constant returns to scale. We have

$$Y = \prod_i c_i^{1/N},$$

and

$$y_i = \overline{y}_i A_i \left(\omega_{il} \left(\frac{l_i}{\overline{l}_i} \right)^{\frac{\theta_i - 1}{\theta_i}} + (1 \omega_{il}) \left(\frac{x_i}{\overline{x}_i} \right)^{\frac{\theta_i - 1}{\theta_i}} \right)^{\frac{\theta_i}{\theta_i - 1}},$$

with

$$y_i = c_i + x_i,$$

and perfect reallocation of labor. Then we have the following.

PROPOSITION 1: *Consider the model described above. Then*

$$1 - \frac{1}{\rho_{ji}} = (\theta_i - 1)\left(\frac{1}{\omega_{il}} - 1 \right),$$

and

$$\frac{d \log \xi}{d \log A_i} = \frac{1}{N}(\theta_i - 1)\left(\frac{1}{\omega_{il}} - 1 \right).$$

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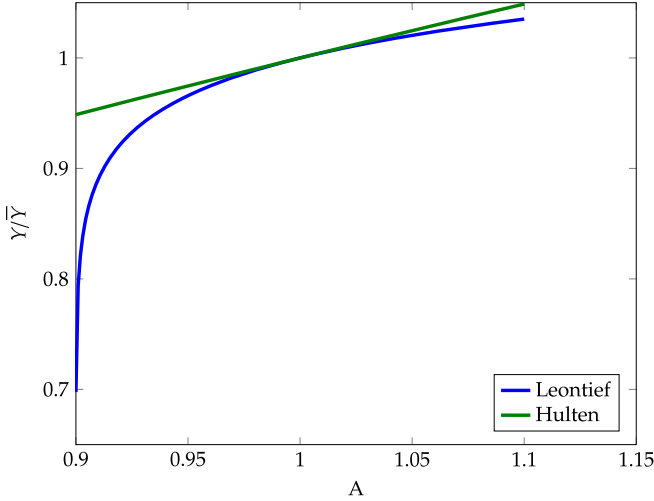Emmanuel Farhi: efarhi@harvard.edu

FIGURE S1.—Aggregate output for the Leontief case $\theta_i \approx 0$ with two industries.

In Figure S1, we plot output as a function of TFP shocks in linear terms. As promised, this economy features strong aggregate complementarities in the sense that a negative TFP shock can cause a drastic reduction in output even in linear terms, despite the fact that labor can be costlessly reallocated across sectors. This happens because, in equilibrium, a negative shock to industry i does not result in more labor being allocated to production in industry i. This follows from the fact that consumption has a Cobb–Douglas form, and so the income and substitution effects from a shock to i offset each other. Since no new labor is allocated to i, if i faces a low structural elasticity of substitution $\theta_i \approx 0$, its output falls dramatically in response to a negative shock. This can then have a large effect on aggregate consumption. Of course, Cobb–Douglas consumption is simply a clean way to illustrate this intuition. If the structural elasticity of substitution in consumption were less than unity ($\theta_0 < 1$), then these effects would be even further amplified.

PROOF OF PROPOSITION 1: First, consider

$$\max_{x_i} y_i - x_i,$$

which has the first-order condition

$$x_i = y_i(1 - \omega_{il})^{\theta_i} \left(\frac{A_i \overline{y}_i}{\overline{x}_i} \right)^{\theta_i - 1} = y_i(1 - \omega_{il}) A_i^{\theta_i - 1},$$

where we use the fact that $\overline{X}_i = \overline{y}_i(1 - \omega_{il})$. Substitute this into the production function for y_i to get

$$y_i = \frac{A_i \overline{y}_i \overline{a}^{\theta_i/(\theta_i - 1)} l_i / \overline{l}_i}{\left(1 - (1 - \overline{a}) A_i^{\theta_i - 1} \right)^{\frac{\theta_i}{\theta_i - 1}}}.$$

Substitute this into $c_i = y_i - x_i$ to get

$$c_i = \frac{A_i \overline{y}_i \overline{a}^{\theta_i/(\theta_i-1)} l_i/\overline{l}_i}{\left(1 - (1-\overline{a}) A_i^{\theta_i-1}\right)^{\frac{1}{\theta_i-1}}}.$$

Substitute these into the utility function to get aggregate consumption when labor cannot be reallocated. To get aggregate consumption when labor is reallocated, maximize aggregate consumption for the non-reallocative solution with respect to l_i:

$$\frac{Y}{\overline{Y}} = \left(\sum_i^N \overline{b}_i^{\theta_0} \left(\frac{A_i \overline{y}_i \overline{a}_i^{\frac{\theta_i}{\theta_i-1}}/\overline{l}_i}{\left(1 - (1-\omega_{il}) A_i^{\theta_i-1}\right)^{\frac{1}{\theta_i-1}}} \right)^{\theta_0-1} \right)^{\frac{1}{\theta_0-1}} \overline{l}. \qquad Q.E.D.$$

APPENDIX C: ADJUSTMENT COSTS IN THE QUANTITATIVE MODEL

In this section, we explain how to extend the quantitative model of Section 6.1 to allow for adjustment costs. For each composite intermediate input, we allow for the possibility that there are adjustment costs, indexed by $\kappa \geq 0$, in adjusting the quantity of the input compared to its steady-state value:

$$\hat{X}_i = X_i \left(1 - \frac{\kappa}{2} \left(\frac{X_i}{\overline{X}_i} - 1 \right)^2 \right),$$

where X_i are units of good i purchased and $\hat{X}_i$ are the units of good i actually used. When $\kappa = 0$, there are no adjustment costs.

Introducing adjustment costs increases the volatility of the Domar weights. For the model with adjustment costs, we choose the value of the adjustment cost parameter κ so that, given the microeconomic elasticities of substitution, the model matches the volatility of the Domar weights at an annual frequency (when the model already overshoots without adjustment costs, we set them to zero). We then keep the same value of κ when we move to quadrennial frequency. By picking a suitable value for κ, even the model with fully mobile labor can match the volatility of the Domar weights. We report these results in Table II. Interestingly, once we pick κ to match the volatility of Domar weights at annual frequency, the model also roughly matches the volatility of the Domar weights at a quadrennial frequency. The results are consistent with what we found in Table I of the paper. In the final column of Table II, we also report the value of resources destroyed by the adjustment cost directly:

$$\Delta = E \left(\frac{\sum_i p_i (X_i - \hat{X}_i)}{GDP} \right).$$

In all cases, the amount of resources destroyed directly by the adjustment costs is not large enough to mechanically drive the reductions in average aggregate output. For example, whereas at quadrennial frequency, the reduction in expected log aggregate output is around $1.5\% - 0.5\% \approx 1.0\%$, the value of the resources destroyed by the adjustment costs is less than 0.5%.

APPENDIX D: ADDITIONAL TABLES AND FIGURES

TABLE II

SIMULATED AND ESTIMATED MOMENTS FOR THE MODEL WITH ADJUSTMENT COSTS[a]

$(\sigma, \theta, \epsilon, \kappa, t)$	Mean	Std	Skewness	Ex-Kurtosis	σ_λ	Δ
No reallocation						
(0.9, 0.5, 0.001, 0, 1)	−0.0034	0.012	−0.18	0.1	0.115	0
(0.9, 0.5, 0.001, 0, 4)	−0.0187	0.030	−1.11	3.6	0.267	0
(0.9, 0.6, 0.2, 2, 1)	−0.0033	0.011	−0.27	0.21	0.124	0.0007
(0.9, 0.6, 0.2, 2, 4)	−0.0152	0.028	−0.63	1.57	0.286	0.0046
Full reallocation						
(0.9, 0.5, 0.001, 3, 1)	−0.0031	0.012	−0.25	0.26	0.124	0.0006
(0.9, 0.5, 0.001, 3, 4)	−0.0166	0.030	−0.98	2.47	0.279	0.0046
(0.9, 0.6, 0.2, 4, 1)	−0.0026	0.011	−0.23	0.23	0.129	0.0004
(0.9, 0.6, 0.2, 4, 4)	−0.0140	0.029	−0.75	1.05	0.291	0.0028

[a]The simulated moments are calculated from 10,000 draws. The parameter t measures the length of the time interval for the shocks: annual and quadrennial. Finally, the column Δ is the share of lost resources.

TABLE III

MOMENTS OF LOG OUTPUT ESTIMATED FROM 50,000 DRAWS USING THE SECOND-ORDER TAYLOR APPROXIMATION WITH THE BENCHMARK ELASTICITIES $(\sigma, \theta, \varepsilon) = (0.9, 0.5, 0.001)$[a]

	Mean	Std	Skewness	Ex-Kurtosis
No reallocation, Annual	−0.0031	0.011	−0.16	0.1
No reallocation, Quadrennial	−0.0173	0.027	−0.60	1.0
Full Reallocation, Annual	−0.0021	0.011	−0.09	0.0
Full Reallocation, Quadrennial	−0.0110	0.026	−0.25	0.1

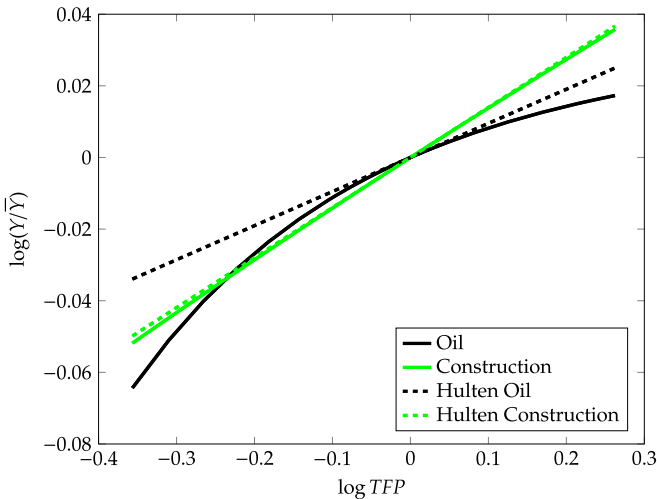[a]This is the version of the model with no adjustment costs $\kappa = 0$.

FIGURE S2.—The effect of TFP shocks to the "oil and gas" industry and the construction industry. Construction has a bigger sales share, but "oil and gas" is more important for large negative shocks. This graph shows that the ranking of which industry is more important is not monotonic in the size of the shock.

TABLE IV

ANNUAL SHOCKS, MODEL WITH FULL REALLOCATION AND NO ADJUSTMENT COSTS

$(\sigma, \theta, \epsilon)$	Mean	Std	Skew	Ex-Kurtosis	σ_λ
(0.8, 0.5, 0.001)	−0.0023	0.011	−0.06	0.0	0.074
(0.9, 0.5, 0.001)	−0.0022	0.011	−0.08	0.0	0.069
(0.99, 0.5, 0.001)	−0.0021	0.011	−0.07	0.0	0.065
(0.8, 0.5, 0.2)	−0.0020	0.011	−0.07	0.0	0.066
(0.9, 0.5, 0.2)	−0.0020	0.011	−0.08	0.0	0.062
(0.99, 0.5, 0.2)	−0.0019	0.011	−0.06	0.0	0.058
(0.8, 0.5, 0.99)	−0.0014	0.011	−0.02	0.0	0.044
(0.9, 0.5, 0.99)	−0.0013	0.011	−0.03	0.0	0.040
(0.99, 0.5, 0.99)	−0.0013	0.011	−0.02	0.0	0.036
(0.8, 0.4, 0.001)	−0.0023	0.011	−0.08	0.0	0.079
(0.9, 0.4, 0.001)	−0.0022	0.011	−0.06	0.0	0.075
(0.99, 0.4, 0.001)	−0.0022	0.011	−0.07	0.0	0.071
(0.8, 0.4, 0.2)	−0.0021	0.011	−0.06	0.0	0.073
(0.9, 0.4, 0.2)	−0.0021	0.011	−0.08	0.0	0.068
(0.99, 0.4, 0.2)	−0.0020	0.011	−0.07	0.0	0.064
(0.8, 0.4, 0.99)	−0.0013	0.011	−0.04	0.0	0.052
(0.9, 0.4, 0.99)	−0.0014	0.011	−0.04	0.0	0.047
(0.99, 0.4, 0.99)	−0.0013	0.011	−0.01	0.0	0.044
(0.8, 0.6, 0.001)	−0.0022	0.011	−0.06	0.0	0.068
(0.9, 0.6, 0.001)	−0.0021	0.011	−0.08	0.0	0.063
(0.99, 0.6, 0.001)	−0.0020	0.011	−0.07	0.0	0.059
(0.8, 0.6, 0.2)	−0.0021	0.011	−0.05	0.0	0.061
(0.9, 0.6, 0.2)	−0.0020	0.011	−0.05	0.0	0.056
(0.99, 0.6, 0.2)	−0.0020	0.011	−0.04	0.0	0.052
(0.8, 0.6, 0.99)	−0.0014	0.011	−0.02	0.0	0.037
(0.9, 0.6, 0.99)	−0.0013	0.011	−0.02	0.0	0.033
(0.99, 0.6, 0.99)	−0.0013	0.011	−0.01	0.0	0.029
(0.8, 0.99, 0.001)	−0.0022	0.011	−0.09	0.0	0.052
(0.9, 0.99, 0.001)	−0.0020	0.011	−0.05	0.0	0.047
(0.99, 0.99, 0.001)	−0.0021	0.011	−0.06	0.0	0.044
(0.8, 0.99, 0.2)	−0.0021	0.011	−0.04	0.0	0.043
(0.9, 0.99, 0.2)	−0.0019	0.011	−0.05	0.0	0.039
(0.99, 0.99, 0.2)	−0.0018	0.011	−0.04	0.0	0.035
(0.8, 0.99, 0.99)	−0.0013	0.011	−0.03	0.0	0.011
(0.9, 0.99, 0.99)	−0.0013	0.011	−0.02	0.0	0.006
(0.99, 0.99, 0.99)	−0.0013	0.011	0.01	0.0	0.001

TABLE V

QUADRENNIAL SHOCKS, MODEL WITH FULL REALLOCATION AND NO ADJUSTMENT COSTS

$(\sigma, \theta, \epsilon)$	Mean	Std	Skew	Ex-Kurtosis	σ_λ
(0.8, 0.5, 0.001)	−0.0112	0.026	−0.31	0.3	0.178
(0.9, 0.5, 0.001)	−0.0113	0.026	−0.28	0.4	0.176
(0.99, 0.5, 0.001)	−0.0107	0.026	−0.27	0.3	0.163
(0.8, 0.5, 0.2)	−0.0102	0.026	−0.25	0.2	0.162
(0.9, 0.5, 0.2)	−0.0101	0.026	−0.23	0.1	0.152
(0.99, 0.5, 0.2)	−0.0098	0.026	−0.22	0.2	0.144
(0.8, 0.5, 0.99)	−0.0070	0.025	−0.09	0.0	0.113
(0.9, 0.5, 0.99)	−0.0066	0.025	−0.09	0.1	0.103
(0.99, 0.5, 0.99)	−0.0064	0.025	−0.09	0.0	0.095
(0.8, 0.4, 0.001)	−0.0116	0.026	−0.32	0.3	0.228
(0.9, 0.4, 0.001)	−0.0110	0.026	−0.32	0.3	0.228
(0.99, 0.4, 0.001)	−0.0107	0.026	−0.27	0.3	0.212
(0.8, 0.4, 0.2)	−0.0106	0.026	−0.27	0.3	0.201
(0.9, 0.4, 0.2)	−0.0104	0.026	−0.24	0.2	0.195
(0.99, 0.4, 0.2)	−0.0097	0.026	−0.25	0.2	0.173
(0.8, 0.4, 0.99)	−0.0072	0.025	−0.09	0.0	0.134
(0.9, 0.4, 0.99)	−0.0070	0.025	−0.09	0.0	0.125
(0.99, 0.4, 0.99)	−0.0067	0.025	−0.08	0.0	0.117
(0.8, 0.6, 0.001)	−0.0112	0.026	−0.26	0.2	0.159
(0.9, 0.6, 0.001)	−0.0108	0.026	−0.27	0.3	0.149
(0.99, 0.6, 0.001)	−0.0105	0.026	−0.26	0.2	0.140
(0.8, 0.6, 0.2)	−0.0102	0.026	−0.23	0.2	0.143
(0.9, 0.6, 0.2)	−0.0100	0.026	−0.23	0.2	0.133
(0.99, 0.6, 0.2)	−0.0096	0.026	−0.20	0.1	0.123
(0.8, 0.6, 0.99)	−0.0071	0.025	−0.07	0.0	0.093
(0.9, 0.6, 0.99)	−0.0066	0.025	−0.06	0.0	0.083
(0.99, 0.6, 0.99)	−0.0064	0.025	−0.06	0.0	0.075
(0.8, 0.99, 0.001)	−0.0106	0.026	−0.20	0.1	0.112
(0.9, 0.99, 0.001)	−0.0104	0.026	−0.19	0.1	0.103
(0.99, 0.99, 0.001)	−0.0101	0.026	−0.19	0.1	0.096
(0.8, 0.99, 0.2)	−0.0100	0.025	−0.15	0.1	0.093
(0.9, 0.99, 0.2)	−0.0095	0.026	−0.14	0.1	0.085
(0.99, 0.99, 0.2)	−0.0091	0.026	−0.13	0.1	0.078
(0.8, 0.99, 0.99)	−0.0064	0.025	−0.02	0.0	0.024
(0.9, 0.99, 0.99)	−0.0062	0.025	−0.01	0.0	0.013
(0.99, 0.99, 0.99)	−0.0058	0.025	0.01	0.0	0.003

TABLE VI

ANNUAL SHOCKS, MODEL WITH NO LABOR REALLOCATION AND NO ADJUSTMENT COSTS

$(\sigma, \theta, \epsilon)$	Mean	Std	Skew	Ex-Kurtosis	σ_λ
(0.8, 0.5, 0.001)	−0.0036	0.011	−0.23	0.2	0.128
(0.9, 0.5, 0.001)	−0.0034	0.012	−0.18	0.1	0.115
(0.99, 0.5, 0.001)	−0.0032	0.011	−0.20	0.1	0.104
(0.8, 0.5, 0.2)	−0.0026	0.011	−0.13	0.1	0.079
(0.9, 0.5, 0.2)	−0.0026	0.011	−0.11	0.0	0.070
(0.99, 0.5, 0.2)	−0.0025	0.011	−0.13	0.0	0.063
(0.8, 0.5, 0.99)	−0.0014	0.011	−0.01	0.0	0.018
(0.9, 0.5, 0.99)	−0.0014	0.011	−0.01	0.0	0.014
(0.99, 0.5, 0.99)	−0.0012	0.011	−0.01	0.0	0.011
(0.8, 0.4, 0.001)	−0.0039	0.012	−0.23	0.2	0.137
(0.9, 0.4, 0.001)	−0.0035	0.011	−0.21	0.2	0.123
(0.8, 0.4, 0.2)	−0.0028	0.011	−0.14	0.1	0.082
(0.9, 0.4, 0.2)	−0.0026	0.011	−0.12	0.1	0.073
(0.99, 0.4, 0.2)	−0.0030	0.011	0.15	5.9	0.065
(0.8, 0.4, 0.99)	−0.0015	0.011	−0.05	0.0	0.020
(0.9, 0.4, 0.99)	−0.0013	0.011	−0.05	0.0	0.016
(0.99, 0.4, 0.99)	−0.0014	0.011	−0.04	0.0	0.014
(0.8, 0.6, 0.001)	−0.0034	0.011	−0.20	0.1	0.122
(0.9, 0.6, 0.001)	−0.0032	0.011	−0.20	0.1	0.109
(0.99, 0.6, 0.001)	−0.0030	0.011	−0.14	0.1	0.098
(0.8, 0.6, 0.2)	−0.0026	0.011	−0.12	0.0	0.077
(0.9, 0.6, 0.2)	−0.0024	0.011	−0.11	0.1	0.068
(0.99, 0.6, 0.2)	−0.0023	0.011	−0.10	0.0	0.061
(0.8, 0.6, 0.99)	−0.0015	0.011	−0.05	0.0	0.016
(0.9, 0.6, 0.99)	−0.0013	0.011	0.00	0.0	0.011
(0.99, 0.6, 0.99)	−0.0013	0.011	−0.02	0.0	0.009
(0.8, 0.99, 0.001)	−0.0030	0.011	−0.15	0.1	0.107
(0.9, 0.99, 0.001)	−0.0028	0.011	−0.13	0.1	0.095
(0.99, 0.99, 0.001)	−0.0027	0.011	−0.11	0.1	0.086
(0.8, 0.99, 0.2)	−0.0026	0.011	−0.11	0.0	0.072
(0.9, 0.99, 0.2)	−0.0024	0.011	−0.09	0.0	0.063
(0.99, 0.99, 0.2)	−0.0022	0.011	−0.07	0.0	0.056
(0.8, 0.99, 0.99)	−0.0014	0.011	−0.01	0.0	0.010
(0.9, 0.99, 0.99)	−0.0013	0.011	−0.01	0.0	0.005
(0.99, 0.99, 0.99)	−0.0011	0.011	0.00	0.0	0.001

TABLE VII

QUADRENNIAL SHOCKS, MODEL WITH NO REALLOCATION AND NO ADJUSTMENT COSTS

$(\sigma, \theta, \epsilon)$	Mean	Std	Skew	Ex-Kurtosis	σ_λ
(0.8, 0.5, 0.001)	−0.0202	0.031	−1.26	4.5	0.297
(0.9, 0.5, 0.001)	−0.0187	0.030	−1.11	3.6	0.267
(0.8, 0.5, 0.2)	−0.0139	0.028	−0.58	1.1	0.180
(0.9, 0.5, 0.2)	−0.0133	0.027	−0.52	0.9	0.160
(0.99, 0.5, 0.2)	−0.0176	0.024	−0.66	1.3	0.138
(0.8, 0.5, 0.99)	−0.0073	0.025	−0.09	0.0	0.041
(0.9, 0.5, 0.99)	−0.0068	0.025	−0.07	0.0	0.033
(0.99, 0.5, 0.99)	−0.0068	0.025	−0.09	0.0	0.027
(0.8, 0.4, 0.001)	−0.0217	0.032	−1.40	5.3	0.320
(0.9, 0.4, 0.001)	−0.0201	0.031	−1.30	4.8	0.287
(0.8, 0.4, 0.2)	−0.0146	0.028	−0.67	1.4	0.187
(0.9, 0.4, 0.2)	−0.0137	0.028	−0.59	1.1	0.167
(0.8, 0.4, 0.99)	−0.0075	0.025	−0.13	0.0	0.048
(0.9, 0.4, 0.99)	−0.0069	0.025	−0.10	0.0	0.039
(0.99, 0.4, 0.99)	−0.0069	0.025	−0.09	0.0	0.034
(0.8, 0.6, 0.001)	−0.0188	0.030	−0.99	2.5	0.281
(0.9, 0.6, 0.001)	−0.0176	0.029	−0.90	2.2	0.253
(0.99, 0.6, 0.001)	−0.0163	0.028	−0.65	1.0	0.229
(0.8, 0.6, 0.2)	−0.0136	0.027	−0.49	0.7	0.175
(0.9, 0.6, 0.2)	−0.0129	0.027	−0.44	0.7	0.154
(0.99, 0.6, 0.2)	−0.0128	0.026	−0.50	0.6	0.138
(0.8, 0.6, 0.99)	−0.0070	0.025	−0.08	0.0	0.036
(0.9, 0.6, 0.99)	−0.0066	0.025	−0.08	0.0	0.027
(0.99, 0.6, 0.99)	−0.0067	0.025	−0.09	0.1	0.021
(0.8, 0.99, 0.001)	−0.0163	0.028	−0.64	1.1	0.246
(0.9, 0.99, 0.001)	−0.0153	0.028	−0.58	0.9	0.221
(0.99, 0.99, 0.001)	−0.0145	0.027	−0.53	0.8	0.200
(0.8, 0.99, 0.2)	−0.0128	0.026	−0.40	0.4	0.162
(0.9, 0.99, 0.2)	−0.0120	0.026	−0.35	0.3	0.143
(0.99, 0.99, 0.2)	−0.0114	0.026	−0.29	0.3	0.127
(0.8, 0.99, 0.99)	−0.0066	0.025	−0.01	0.0	0.021
(0.9, 0.99, 0.99)	−0.0061	0.025	−0.03	0.0	0.011
(0.99, 0.99, 0.99)	−0.0057	0.025	0.00	0.0	0.002

APPENDIX E: MACRO MOMENT APPROXIMATIONS

The notes in this section were prepared with the assistance of research assistant Chang He. Let output be $Y(A)$, where A is the $N \times 1$ vector of productivity parameters. Suppose that A is distributed according to a multivariate normal distribution, and that the elements of A are independent. Let $Y^*(A)$ be the second-order Taylor approximation of Y around the mean vector of A.

Second-Order Taylor Approximation

Let $\boldsymbol{\mu}_A$ denote the mean vector of A. The second-order Taylor expansion of $Y(A)$ is

$$Y^*(A) = Y(\boldsymbol{\mu}_A) + \sum_{i=1}^{N} \frac{\partial Y(\boldsymbol{\mu}_A)}{\partial A_i}(A_i - \mu_{A_i}) + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} \frac{\partial^2 Y(\boldsymbol{\mu}_A)}{\partial A_i \partial A_j}(A_i - \mu_{A_i})(A_j - \mu_{A_j}).$$

We introduce the following abbreviations:

$$Y_i = \frac{\partial Y(\boldsymbol{\mu}_A)}{\partial A_i}, \qquad Y_{ij} = \frac{\partial^2 Y(\boldsymbol{\mu}_A)}{\partial A_i \partial A_j},$$

$$\mu_{A_i} = \int_{-\infty}^{\infty} A_i f_A(A_i)\, dA_i, \qquad \mu_{A_{i,k}} = \int_{-\infty}^{\infty} (A_i - \mu_{A_i})^k f_A(A_i)\, dA_i,$$

$$\mu_{A_i, A_j} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (A_i - \mu_{A_i})(A_j - \mu_{A_j}) f_A(A_i, A_j)\, dA_i\, dA_j,$$

where f_A is the density function of $\boldsymbol{A}$.

Mean Value Approximation

Let $\boldsymbol{\mu}_{Y*}$ be the mean value approximation of $Y(\boldsymbol{A})$. We have

$$\boldsymbol{\mu}_{Y*} = E\big[Y^*(\boldsymbol{A})\big] = \int_{-\infty}^{\infty} Y^*(\boldsymbol{A}) f_A(\boldsymbol{A})\, d\boldsymbol{A}$$

$$= \int_{-\infty}^{\infty} \Bigg[Y(\boldsymbol{\mu}_A) + \sum_{i=1}^{N} Y_i(A_i - \mu_{A_i})$$

$$+ \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} Y_{ij}(A_i - \mu_{A_i})(A_j - \mu_{A_j}) \Bigg] f_A(\boldsymbol{A})\, d\boldsymbol{A}$$

$$= Y(\boldsymbol{\mu}_A) + \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} Y_{ij} \mu_{A_i, A_j}.$$

Expanding the quadratic and since elements of $\boldsymbol{A}$ are independent, we get

$$\boldsymbol{\mu}_{Y*} = Y(\boldsymbol{\mu}_A) + \frac{1}{2} \sum_{i=1}^{N} Y_{ii} \mu_{A_i, 2}.$$

Variance Approximation

Let $\boldsymbol{\sigma}_{Y*}^2$ be the variance approximation of $Y(\boldsymbol{A})$:

$$\boldsymbol{\sigma}_{Y*}^2 = E\big(\big[Y^*(\boldsymbol{A}) - Y(\boldsymbol{\mu}_A) \big]^2 \big) = E\big(Y^{*2}(\boldsymbol{A}) \big) - Y^2(\boldsymbol{\mu}_A)$$

$$= \int_{-\infty}^{\infty} \Bigg[Y(\boldsymbol{\mu}_A) + \sum_{i=1}^{N} Y_i(A_i - \mu_{A_i})$$

$$+ \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} Y_{ij}(A_i - \mu_{A_i})(A_j - \mu_{A_j}) \Bigg]^2 f_A(\boldsymbol{A})\, d\boldsymbol{A} - \boldsymbol{\mu}_{Y*}^2.$$

Since elements of A are independent, we get

$$\sigma_{Y*}^2 = \sum_{i=1}^N Y_i^2 \mu_{A_i,2} + Y^2(\mu_A) - \mu_{Y*}^2 + Y(\mu_A)\sum_{i=1}^N Y_{ii}\mu_{A_i,2} + \sum_{i=1}^N Y_i Y_{ii}\mu_{A_i,3}$$

$$+ \frac{1}{4}\sum_{i=1}^N Y_{ii}^2 \mu_{A_i,4} + \frac{1}{2}\sum_{i=1}^N\sum_{j=i+1}^N Y_{ii}Y_{jj}\mu_{A_i,2}\mu_{A_j,2} + \sum_{i=1}^N\sum_{j=i+1}^N Y_{ij}^2\mu_{A_i,2}\mu_{A_j,2}.$$

Skewness Approximation

Let ν_{Y*} be the skewness approximation of $Y(A)$. By definition, $\nu_{Y*} = \mu_{Y*,3}/\sigma_{Y*}^3$.
Using the definition of skewness, and that $\int_{-\infty}^\infty Y^{*2}(A)f_A(A)\,dA = \sigma_{Y*}^2 + \mu_{Y*}^2$, we have

$$\mu_{Y*,3} = E\left(\left[Y^*(A) - Y(\mu_A)\right]^3\right)$$

$$= \int_{-\infty}^\infty \left[Y^*(A) - Y(\mu_A)\right]^3 f_A(A)\,dA$$

$$= \int_{-\infty}^\infty Y^{*3}(A)f_A(A)\,dA - 3\mu_{Y*}\sigma_{Y*}^2 - \mu_{Y*}^3$$

$$= \int_{-\infty}^\infty \left[Y(\mu_A) + \sum_{i=1}^N Y_i(A_i - \mu_{A_i})\right.$$

$$\left. + \frac{1}{2}\sum_{i=1}^N\sum_{j=1}^N Y_{ij}(A_i - \mu_{A_i})(A_j - \mu_{A_j})\right]^3 f_A(A)\,dA - 3\mu_{Y*}\sigma_{Y*}^2 - \mu_{Y*}^3.$$

Simplifying the equation above and using the fact that the elements of A are independent, we have

$$\mu_{Y*,3} = \sum_{i=1}^N Y_i^3 \mu_{A_i,3} + Y^3(\mu_A) + \frac{3}{2}Y^2(\mu_A)\sum_{i=1}^N Y_{ii}\mu_{A_i,2} + 3Y(\mu_A)\sum_{i=1}^N Y_i^2\mu_{A_i,2}$$

$$+ 3Y(\mu_A)\sum_{i=1}^N Y_i Y_{ii}\mu_{A_i,3} + \frac{3}{4}\sum_{i=1}^{N-2}\sum_{j=i+1}^{N-1}\sum_{k=j+1}^N Y_{ii}Y_{jj}Y_{kk}\mu_{A_i,2}\mu_{A_j,2}\mu_{A_k,2}$$

$$+ \frac{3}{2}\sum_{i=1}^{N-1}\sum_{j=i+1}^N Y_{ii}Y_{ij}Y_{jj}\mu_{A_i,3}\mu_{A_j,3} + \frac{3}{8}\sum_{i=1}^N\sum_{\substack{j=1\\j\neq i}}^N Y_{ii}Y_{jj}^2\mu_{A_i,2}\mu_{A_j,4} + \frac{1}{8}\sum_{i=1}^N Y_{ii}^2\mu_{A_i,6}$$

$$+ \frac{3}{2}\sum_{i=1}^N\sum_{\substack{j=1\\j\neq i}}^N Y_i^2 Y_{ij}\mu_{A_i,2}\mu_{A_j,2} + \frac{3}{2}\sum_{i=1}^N Y_i^2 Y_{ii}\mu_{A_i,4}$$

$$+ \frac{3}{2}Y(\mu_A)\sum_{i=1}^{N-1}\sum_{j=i+1}^N Y_{ii}Y_{jj}\mu_{A_i,2}\mu_{A_j,2} + \frac{3}{4}Y(\mu_A)\sum_{i=1}^N Y_{ii}^2\mu_{A_i,4}$$

$$+ \frac{3}{2} \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} Y_i Y_{ii} Y_{jj} \mu_{A_i,3} \mu_{A_j,2} + \frac{3}{4} \sum_{i=1}^{N} Y_i Y_{ii}^2 \mu_{A_i,5}$$

$$+ \frac{3}{2} \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N-1} \sum_{\substack{k=j+1 \\ k \neq i}}^{N} Y_{ii} Y_{jk}^2 \mu_{A_i,2} \mu_{A_j,2} \mu_{A_k,2} + \frac{9}{4} \sum_{i=1}^{N-2} \sum_{j=i+1}^{N-1} \sum_{k=j+1}^{N} Y_{ij} Y_{ik} Y_{jk} \mu_{A_i,2} \mu_{A_j,2} \mu_{A_k,2}$$

$$+ \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} Y_{ij}^3 \mu_{A_i,3} \mu_{A_j,3} + \frac{3}{2} \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} Y_{ij}^2 Y_{jj} \mu_{A_i,2} \mu_{A_j,4}$$

$$+ 6 \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} Y_i Y_j Y_{ij} \mu_{A_i,2} \mu_{A_j,2} + 3 Y(\mu_A) \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} Y_{ij}^2 \mu_{A_i,2} \mu_{A_j,2}$$

$$+ 3 \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} Y_i Y_{ij} Y_{jj} \mu_{A_i,2} \mu_{A_j,3} + 3 \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} Y_i Y_{ij}^2 \mu_{A_i,3} \mu_{A_j,2} - 3 \boldsymbol{\mu}_{Y^*} \boldsymbol{\sigma}_{Y^*}^2 - \boldsymbol{\mu}_{Y^*}^3.$$

We can then use the expression of $\boldsymbol{\sigma}_{Y^*}^2$ from before to compute $\boldsymbol{\nu}_{Y^*} = \boldsymbol{\mu}_{Y^*,3}/\boldsymbol{\sigma}_{Y^*}^3$.

APPENDIX F: RELATION TO ACR

Arkolakis, Costinot, and Rodríguez-Clare (2012), henceforth ACR, considered an open-economy model with no intermediate inputs and a single factor of production per country. They imposed some macro-level restrictions, and proved a powerful characterization of the gains from trade. Namely, they assumed that (1) trade is balanced, (2) profits are a constant share of revenues, and (3) import demand system is CES. Using these assumptions, they showed that the gains from trade, as measured by the change in real income associated with going to autarky, is given by the reciprocal of the domestic expenditure share raised to the reciprocal of the trade elasticity. The ACR result, and its generalizations (summarized in Costinot and Rodriguez-Clare (2014)), suggest that one can quantify the gains from trade without needing to directly estimate the size of the trade shock.

In Baqaee and Farhi (2019), we showed how under certain conditions, changes in iceberg trade costs in an open-economy model can be recast as productivity shocks in an associated closed-economy model. This then allowed us to use our results to study the second-order effects of trade shocks. For simplicity, we work with a one-factor model (like ACR), but these results can be extended to the case of multiple factors. We also restrict ourselves to nested-CES economies in standard form.

We start by associating a fictitious nested-CES domestic closed-economy model to the true nested-CES open-economy model, both in standard form. The closed economy has the same set C of domestic producers as the open economy and the same elasticities of substitution, but its input-output matrix $\Omega_{ij}^c \equiv \Omega_{ij}/(\sum_{k \in C} \Omega_{ik})$ is different because each domestic producer only sources from other domestic producers, and not from foreign producers, where C denotes the set of domestic producers.

We show that effects on domestic welfare of a change in trade costs in the true open-economy model are identical to the effects on aggregate output of a set of productivity shocks $(\lambda_{ic}/\overline{\lambda}_{ic})^{1/(1-\theta_i)}$, where λ_{ic} is the domestic cost share of producer i and $\overline{\lambda}_{ic}$ is its

steady-state value. It is straightforward to leverage our results to characterize the effects of these shocks up to the second order. Only in some special cases resembling those underpinning our network-irrelevance result in Corollary 1 can a global expression be derived. The baseline ACR specification falls in this category: it has a single sector and no intermediate goods. In this case, $\lambda^c = 1$ and we get $Y^c/\overline{Y}^c = (\lambda_c/\overline{\lambda}_c)^{1/(1-\theta)}$, where λ_c is the domestic cost share and $\overline{\lambda}_c$ is its steady-state value.

REFERENCES

ARKOLAKIS, C., A. COSTINOT, AND A. RODRÍGUEZ-CLARE (2012): "New Trade Models, Same Old Gains?" *American Economic Review*, 102 (1), 94–130. [11]

BAQAEE, D. R., AND E. FARHI (2019): "Trade Theory With Global Production Networks," Technical report. [11]

COSTINOT, A., AND A. RODRIGUEZ-CLARE (2014): "Trade Theory With Numbers: Quantifying the Consequences of Globalization," *Handbook of International Economics*, 4, 197. [11]