

SUPPLEMENT TO “RISK PREFERENCES AND THE MACROECONOMIC  
ANNOUNCEMENT PREMIUM”  
(*Econometrica*, Vol. 86, No. 4, July 2018, 1383–1430)

HENGJIE AI

Carlson School of Management, University of Minnesota

RAVI BANSAL

Fuqua School of Business, Duke University and NBER

IN THIS SUPPLEMENT, we provide details of the proofs omitted in the main text and the appendices of the paper. In Section S.1, we prove the equivalence between the Arrow–Debreu setup and the sequential market setup in the two-period model. Section S.2 contains details of the certainty equivalent functionals of dynamic preferences that Theorems 1 and 2 allow for and the associated A-SDF. Section S.3 provides details of the continuous-time model in Section 5 of the paper.

S.1. THE TWO-PERIOD MODEL

In this section, we provide a formal derivation of the A-SDF in the two-period model. We also establish the equivalence between Arrow–Debreu markets and sequential markets in the context of our model. We show that both formulations lead to the same set of asset pricing equations. Section S.1.3 discusses the A-SDF of the recursive utility in the two-period model.

S.1.1. *The Arrow–Debreu Market*

We use  $\{\bar{C}_0, \{\bar{C}_1(s)\}_{s=1}^N\}$  to denote aggregate endowment in our two-period model and use  $\{C_0(s), C_1(s)\}_{s=1}^N$  for the consumption choice of the agent. From an individual agent’s perspective, the decision for  $C_0$  is made after the announcement, and therefore is allowed to depend on  $s$ . At the aggregate level,  $\bar{C}_0$  does not depend on  $s$ .

Trading on the Arrow–Debreu market happens in period  $0^-$ . Let  $q_0(s)$  be the period  $0^-$  price of an Arrow–Debreu security that delivers one unit of consumption good in period  $0^+$  and state  $s$ , for  $s = 1, 2, \dots, N$ . Similarly, let  $q_1(s)$  be the Arrow–Debreu price of one unit of consumption good in period one and state  $s$ . Because markets are complete, the utility maximization problem of the representative agent can be written as

$$\begin{aligned} & \max \mathcal{I}[u(C_0(s)) + \beta u(C_1(s))] \\ & \text{subject to: } \sum_{s=1}^N [q_0(s)C_0(s) + q_1(s)C_1(s)] \leq \sum_{s=1}^N [q_0(s)\bar{C}_0 + q_1(s)\bar{C}_1(s)]. \end{aligned}$$

In the above setup, because the announcement is made at time  $0^+$ , from the agent’s perspective, consumption at time  $0^+$  is allowed to depend on  $s$ , which we write as  $C_0(s)$ . To

---

Hengjie Ai: [hengjie@umn.edu](mailto:hengjie@umn.edu)  
Ravi Bansal: [ravi.bansal@duke.edu](mailto:ravi.bansal@duke.edu)

save notation, as in the paper, we denote  $V_s = u(C_0(s)) + \beta u(C_1(s))$ . Optimality implies

$$q_0(s) = \lambda \frac{\partial \mathcal{I}[V]}{\partial V_s} u'(C_0(s)), \quad q_1(s) = \lambda \frac{\partial \mathcal{I}[V]}{\partial V_s} \beta u'(C_1(s)),$$

where  $\lambda$  is the Lagrangian multiplier of the budget constraint. In equilibrium, market clearing implies that  $C_0(s) = \bar{C}_0$  for all  $s$ . If we normalize the price of one unit of state-non-contingent consumption at time  $0^+$  to be 1, that is,  $\sum_{s=1}^N q_0(s) = 1$ , then, for all  $s$ ,

$$q_0(s) = \frac{\frac{\partial \mathcal{I}[V]}{\partial V_s}}{\sum_{s=1}^N \frac{\partial \mathcal{I}[V]}{\partial V_s}}, \quad (\text{S.1.1})$$

and  $\frac{q_1(s)}{q_0(s)} = \beta \frac{u'(\bar{C}_1(s))}{u'(\bar{C}_0)}$ . That is, we can simply use ratios of marginal utilities to compute Arrow–Debreu prices. Clearly, equation (S.1.1) implies the expression of the A-SDF in equation (12) of the paper.

### S.1.2. The Sequential Market

Here, we show that the two-period version of the sequential market setup described in Section 4 leads to the same asset pricing equation, (12). In period  $0^-$ , there is no consumption decision and the agent chooses investment in a vector of announcement returns to maximize:

$$\begin{aligned} & \max_{\{\xi_j\}_{j=1}^J} \mathcal{I}[V(W')] \\ & \text{subject to: } W'_s = W - \sum_{j=1}^J \xi_j + \sum_{j=1}^J \xi_j R_{A,j}(s), \quad \text{all } s, \end{aligned} \quad (\text{S.1.2})$$

where  $W' = \{W'_s\}_{s=1}^N$  is the realizations of wealth in the next period, and  $V(W') = \{V_s(W'_s)\}_{s=1}^N$  is a vector of value functions. For each  $s$ , the value function  $V_s(W)$  is defined by the optimal portfolio choice problem on the post-announcement market:

$$\begin{aligned} & V_s(W) = \max_{C_0(s), C_1(s)} u(C_0(s)) + \beta u(C_1(s)) \\ & \text{subject to: } C_1(s) = (W - C_0(s))R_{P,s}. \end{aligned} \quad (\text{S.1.3})$$

Note that  $R_{P,s}$  is the return from period  $0^+$  to period 1 after announcement  $s$ . Because the announcement fully reveals the true state of the world,  $R_{P,s}$  is a risk-free return.

The first-order condition for (S.1.2) with respect to  $\xi_j$  implies that for any announcement returns  $R_{A,j}$ ,

$$\sum_{s=1}^N \frac{\partial}{\partial V_s} \mathcal{I}[V(W')] \frac{\partial V_s(W'_s)}{\partial W'_s} [R_{A,j}(s) - 1] = 0, \quad (\text{S.1.4})$$

where  $W'_s$  denotes the equilibrium wealth of the agent in period  $0^+$  after announcement  $s$ . The envelope condition for (S.1.3) implies that  $\frac{\partial V_s(W'_s)}{\partial W'_s} = u'(C_0(s)) = u'(\bar{C}_0)$ , where the

second equality uses the market clearing condition. As  $u' > 0$ , equation (S.1.4) implies

$$\sum_{s=1}^N \frac{\frac{\partial}{\partial V_s} \mathcal{I}[V(W')]}{\sum_{s=1}^N \frac{\partial}{\partial V_s} \mathcal{I}[V(W')]} R_{A,j}(s) = 1,$$

as in equation (11) of the paper.

S.1.3. *The Example of Recursive Utility*

Here, we provide details of the computation of the A-SDF for the recursive utility in Section 3.2 of the paper. We illustrate that because the announcement in our model leads uncertainty to resolve before the realization of consumption shocks, the computation of utilities and, therefore, marginal utilities differs from that in models in which resolution of uncertainty happens at the same time of the realization of the consumption shocks.

Figure S.1 illustrates a two-period model with announcement and one without announcement. The top panel is the same as that in Figure 2 in our main text, where the announcement at time  $0^+$  fully reveals the true state and leads to early resolution of uncertainty. In the bottom panel of Figure S.1, due to the absence of announcement, the

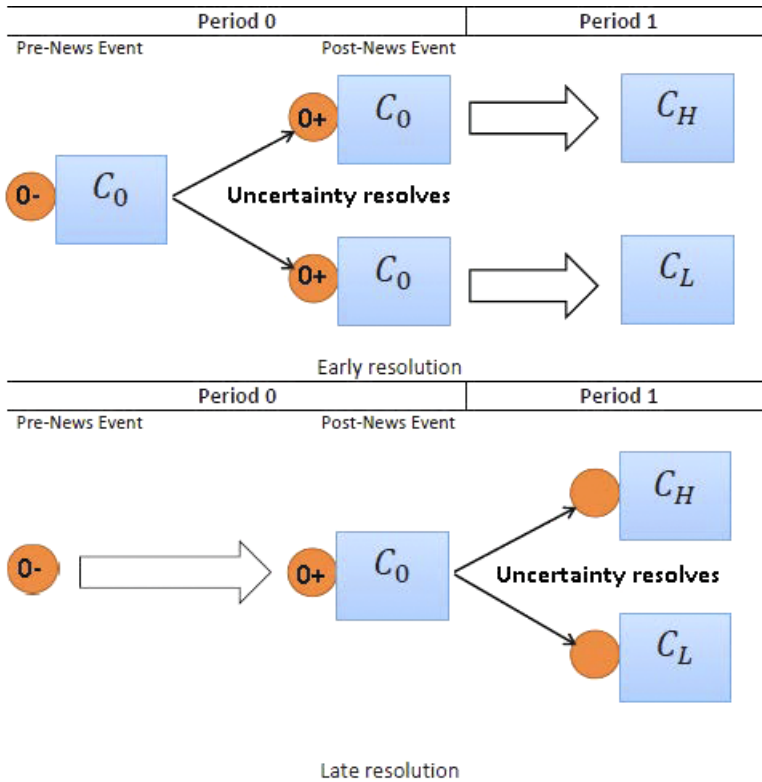


FIGURE S.1.—Early and late resolution of uncertainty. Figure S.1 plots a consumption plan with early resolution of uncertainty (top panel) and a consumption plan with late resolution of uncertainty (bottom panel).

uncertainty is resolved in period 1 when consumption is realized; that is, it is a case of late resolution of uncertainty.<sup>1</sup>

We denote the utility at  $0^-$  in the case of early resolution as  $V^E(\{C_0(s), C_1(s)\}_{s=1}^N)$ . Consistent with our previous notations, in the rest of this section, we will allow  $C_0$  to depend on  $s$  when evaluating utilities, and will impose market clearing,  $C_0(s) = \bar{C}_0$  for all  $s$  when computing stochastic discount factors. In the case of early resolution, because there is no uncertainty in period  $0^+$ , we first aggregate over time to compute the continuation utility as  $V_s = \frac{1}{1-\frac{1}{\psi}} C_0^{1-\frac{1}{\psi}}(s) + \beta \frac{1}{1-\frac{1}{\psi}} C_1^{1-\frac{1}{\psi}}(s)$ , and then aggregate over uncertain realizations of the announcement to compute its certainty equivalent at  $0^-$  as

$$V^E = \left\{ \sum_{s=1}^N \pi(s) \left[ \left\{ C_0^{1-\frac{1}{\psi}}(s) + \beta C_1^{1-\frac{1}{\psi}}(s) \right\}^{\frac{1-\gamma}{1-\frac{1}{\psi}}} \right] \right\}^{\frac{1}{1-\gamma}}. \quad (\text{S.1.5})$$

Clearly,  $\forall s$ ,

$$\frac{\partial V^E}{\partial C_0(s)} = \pi(s) (V^E)^\gamma V_s^{\frac{1}{\psi}-\gamma} C_0^{-\frac{1}{\psi}}(s), \quad \frac{\partial V^E}{\partial C_1(s)} = \beta \pi(s) (V^E)^\gamma V_s^{\frac{1}{\psi}-\gamma} C_1^{-\frac{1}{\psi}}(s).$$

Therefore, the marginal utility of one unit of date-0 state un-contingent consumption can be computed as

$$\begin{aligned} \sum_{s=1}^N \frac{\partial V^E}{\partial C_0(s)} &= \sum_{s=1}^N \pi(s) (V^E)^\gamma V_s^{\frac{1}{\psi}-\gamma} C_0^{-\frac{1}{\psi}}(s) \\ &= \left[ \sum_{s=1}^N \pi(s) V_s^{\frac{1}{\psi}-\gamma} \right] (V^E)^\gamma \bar{C}_0^{-\frac{1}{\psi}}, \end{aligned}$$

where the second equality imposes market clearing. Therefore, the price of one unit of consumption good paid in period 1, state  $s$ , measured in date-0 state un-contingent consumption is

$$\frac{\frac{\partial V^E}{\partial C_1(s)}}{\sum_{s=1}^N \frac{\partial V^E}{\partial C_0(s)}} = \beta \pi(s) \left( \frac{\bar{C}_1(s)}{\bar{C}_0} \right)^{-\frac{1}{\psi}} \frac{V_s^{\frac{1}{\psi}-\gamma}}{\sum_{s=1}^N \pi(s) V_s^{\frac{1}{\psi}-\gamma}}. \quad (\text{S.1.6})$$

In the case of late resolution,  $0^-$  and  $0^+$  have the same utility level, which we denote as  $V^L(C_0, \{C_1(s)\}_{s=1}^N)$ . We first aggregate over uncertain period-1 consumption to compute

<sup>1</sup>The comparison between early and late resolution of uncertainty here is the same as that in Figure 2 of [Kreps and Porteus \(1978\)](#). Our top panel corresponds to node  $d_0(a)$  and the bottom panel corresponds to node  $d_0(b)$  in that figure.

its certainty equivalent:  $\{E[C_1^{1-\gamma}(s)]\}^{\frac{1}{1-\gamma}}$ , and then aggregate over time to compute  $V^L$  as

$$V^L = \left\{ \frac{1}{1 - \frac{1}{\psi}} C_0^{1-\frac{1}{\psi}} + \beta \frac{1}{1 - \frac{1}{\psi}} \left\{ \sum_{s=1}^N \pi(s) [C_1^{1-\gamma}(s)] \right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right\}^{\frac{1}{1-\frac{1}{\psi}}}. \quad (\text{S.1.7})$$

The Arrow–Debreu price for one unit of consumption in period 1 measured in period-0 consumption numeraire can be computed as

$$\frac{\frac{\partial V^L}{\partial C_1(s)}}{\frac{\partial V^L}{\partial C_0}} = \pi(s) \beta \left( \frac{\bar{C}_1(s)}{\bar{C}_0} \right)^{-\frac{1}{\psi}} \left\{ \frac{\bar{C}_1(s)}{\left[ \sum_{s=1}^N \pi(s) \bar{C}_1^{1-\gamma}(s) \right]^{\frac{1}{1-\gamma}}} \right\}^{\frac{1}{\psi} - \gamma}. \quad (\text{S.1.8})$$

Clearly, the SDF for the early resolution case, (S.1.6), can be decomposed into the  $m^*$  in equation (10) and an SDF that discounts period-1 cash flow into period-0+ consumption units:  $\beta \left( \frac{\bar{C}_1(s)}{\bar{C}_0} \right)^{-\frac{1}{\psi}}$ . The SDF in (S.1.8) takes a familiar form as in many consumption-based asset pricing models where uncertainty is assumed to resolve at the same time of the realization of consumption shocks. In general, the term  $\left\{ \frac{\bar{C}_1(s)}{\left[ \sum_{s=1}^N \pi(s) \bar{C}_1^{1-\gamma}(s) \right]^{\frac{1}{1-\gamma}}} \right\}^{\frac{1}{\psi} - \gamma}$  does not integrate to 1 unless in the special case of unit IES.

## S.2. EXAMPLES OF DYNAMIC PREFERENCES AND A-SDF

In this section, we show that most of the non-expected utility proposed in the literature can be represented in the form of (14). We also provide an expression for the implied A-SDF.<sup>2</sup>

• The recursive utility of [Kreps and Porteus \(1978\)](#) and [Epstein and Zin \(1989\)](#). The recursive preference can be generally represented as

$$U_t = u^{-1} \left\{ (1 - \beta) u(C_t) + \beta u \circ h^{-1} E[h(U_{t+1})] \right\}. \quad (\text{S.2.1})$$

For example, the well-known recursive preference with constant IES and constant risk aversion is the special case in which  $u(C) = \frac{1}{1-\frac{1}{\psi}} C^{1-\frac{1}{\psi}}$  and  $h(U) = \frac{1}{1-\gamma} U^{1-\gamma}$ . With a monotonic transformation,

$$V = u(U), \quad (\text{S.2.2})$$

the recursive relationship for  $V$  can be written in the form of (14) with the same  $u$  function in equation (S.2.1) and the certainty equivalent functional

$$\mathcal{I}(V) = \phi^{-1} \left( \int \phi(V) dP \right),$$

<sup>2</sup>Depending on the model, additional conditions may be needed so that the assumptions of Theorem 1 can be verified. We provide the expressions for A-SDF assuming appropriate conditions on the primitive utility functions can be imposed to guarantee its existence.

where  $\phi = h \circ u^{-1}$ . The A-SDF can be written as

$$m^*(V) \propto \phi'(V), \quad (\text{S.2.3})$$

where we suppress the normalizing constant, which is chosen so that  $m^*(V)$  integrates to 1.

- The maxmin expected utility of Gilboa and Schmeidler (1989). The dynamic version of this preference was studied in Epstein and Schneider (2003) and Chen and Epstein (2002). This preference can be represented as the special case of (14) where the certainty equivalent functional is of the form

$$\mathcal{I}(V) = \min_{m \in M} \int mV dP,$$

where  $M$  is a family of probability densities that is assumed to be convex and closed in the weak\* topology. As we show in Section 3.2 of the paper, the A-SDF for this class of preference is the Radon–Nikodym derivative of the minimizing probability measure with respect to  $P$ .

- The variational preferences of Maccheroni, Marinacci, and Rustichini (2006a), the dynamic version of which was studied in Maccheroni, Marinacci, and Rustichini (2006b), features a certainty equivalent functional of the form

$$\mathcal{I}(V) = \min_{E[m]=1} \int mV dP + c(m),$$

where  $c(m)$  is a convex and weak\*-lower semi-continuous function. Similarly to the maxmin expected utility, the A-SDF for this class of preference is minimizing probability density.

- The multiplier preferences of Hansen and Sargent (2008) and Strzalecki (2011) are represented by the certainty equivalent functional

$$\mathcal{I}(V) = \min_{E[m]=1} \int mV dP + \theta R(m),$$

where  $R(m)$  denotes the relative entropy of the density  $m$  with respect to the reference probability measure  $P$ , and  $\theta > 0$  is a parameter. In this case, the A-SDF is also the minimizing probability that can be written as a function of the continuation utility:  $m^*(V) \propto e^{-\frac{1}{\theta}V}$ .

- The second-order expected utility of Ergin and Gul (2009) can be written as (14) with the following choice of  $\mathcal{I}$ :

$$\mathcal{I}(V) = \phi^{-1} \left( \int \phi(V) dP \right),$$

where  $\phi$  is a concave function. In this case, the A-SDF can be written as a function of continuation utility:

$$m^*(V) \propto \phi'(V).$$

- The smooth ambiguity preference of Klibanoff, Marinacci, and Mukerji (2005) and Klibanoff, Marinacci, and Mukerji (2009) can be represented as

$$\mathcal{I}(V) = \phi^{-1} \left( \int_M \phi \left( \int_{\Omega} mV dP \right) d\mu(m) \right), \quad (\text{S.2.4})$$

where  $\mu$  is a probability measure on a set of probability densities  $M$ . The A-SDF can be written as

$$m^*(\omega) \propto \int_M \phi' \left( \int_{\Omega} mV dP \right) m(\omega) d\mu(m). \quad (\text{S.2.5})$$

• The certainty equivalent functional  $\mathcal{I}$  for the disappointment aversion preference is implicitly defined as  $\mathcal{I}[V] = \mu$ , where  $\mu$  is the unique solution to the following equation:

$$\phi(\mu) = \int \phi(V) dP - \theta \int_{\mu \geq V} [\phi(\mu) - \phi(V)] dP,$$

where  $\phi$  is a concave function. The A-SDF can be written as

$$m^*(V) = \begin{cases} \frac{\phi'(V)}{\phi'(\mu)[1 + \theta P(V \leq \mu)]} & \text{if } V > \mu, \\ \frac{(1 - \theta)\phi'(V)}{\phi'(\mu)[1 + \theta P(V \leq \mu)]} & \text{if } V \leq \mu, \end{cases}$$

whenever  $\mathcal{I}[V]$  is differentiable at  $V$ .

• Hayashi and Miao (2011) developed a class of generalized recursive smooth ambiguity model that takes the form<sup>3</sup>

$$\bar{V}_t = u^{-1} \left\{ (1 - \beta)u(C_t) + \beta[u \circ v^{-1}] \left( \int_M [\nu \circ \phi^{-1}] \left( \int m\phi(\bar{V}_{t+1}) dP \right) d\mu(m) \right) \right\}, \quad (\text{S.2.6})$$

where  $u$ ,  $v$ , and  $\phi$  are all smooth and monotone functions. As in the Klibanoff, Marinacci, and Mukerji (2005) model,  $M$  is a set of probability densities that represent ambiguous beliefs, and  $\mu$  is a measure on the set of densities. With a monotonic transformation,  $V_t = u(\bar{V}_t)$ , the above can be written in the form of (14) with

$$\mathcal{I}(V) = [u \circ v^{-1}] \left( \int_M [\nu \circ \phi^{-1}] \left( \int m[\phi \circ u^{-1}](V) dP \right) d\mu(m) \right).$$

The A-SDF for this class of preferences can be written as

$$m^*(\omega) \propto \int_M [\nu \circ \phi^{-1}]' \left( \int m[\phi \circ u^{-1}](V) dP \right) m(\omega) [\phi \circ u^{-1}]'(V(\omega)) d\mu(m).$$

### S.3. DETAILS OF THE CONTINUOUS-TIME MODEL

In this section, we provide details of the solution of the continuous-time model. Section S.3.1 provides the solution to the model with periodic announcement in Sections 5.1 and 5.2 of the main text of the paper. Section S.3.2 provides the omitted proofs for the results on time-non-separable preferences discussed in Section 5.3.

<sup>3</sup>The model in Hayashi and Miao (2011) is more general than (S.2.6) and may not permit a representation of the form  $V_t = u(C_t) + \beta\mathcal{I}[V_{t+1}]$ . However, the applied examples of this preference are often special cases of (S.2.6). See also the related generalized recursive multiple-priors model of Hayashi (2005), which can be obtained as a limiting case of (S.2.6).

S.3.1. *Asset Pricing in the Learning Model**Value Function of the Representative Agent*

Because announcements fully reveal the value of  $x_t$  at  $nT$ ,  $q_{nT}^+ = 0$ . We start from  $q_0 = 0$ . In the interior of  $(0, T)$ , the standard optimal filtering implies that the posterior mean and variance of  $x_t$  are given by equations (30) and (31). Here  $q_t$  has a closed-form solution:

$$q(t) = \frac{\sigma_x^2(1 - e^{-2\hat{a}t})}{(\hat{a} - a_x)e^{-2\hat{a}t} + a_x + \hat{a}}, \quad (\text{S.3.1})$$

where  $\hat{a} = \sqrt{a_x^2 + (\sigma_x/\sigma)^2}$ . In general, we can write  $q_t = q(t \bmod T)$  for all  $t$ .<sup>4</sup>

Using the results from [Duffie and Epstein \(1992\)](#), the representative consumer's preference is specified by a pair of aggregators  $(f, \mathcal{A})$  such that the utility of the representative agent,  $V_t$ , is the solution to the following stochastic differential equation:

$$dV_t = \left[ -f(C_t, V_t) - \frac{1}{2} \mathcal{A}(V_t) \|\sigma_V(t)\|^2 \right] dt + \sigma_V(t) dB_t,$$

for some square-integrable process  $\sigma_V(t)$ . We adopt the convenient normalization  $\mathcal{A}(V) = 0$  ([Duffie and Epstein \(1992\)](#)), and denote  $\bar{f}$  the normalized aggregator, and  $\bar{V}_t$  the corresponding utility process. Under this normalization,

$$\bar{f}(C, \bar{V}) = \rho \{ (1 - \gamma) \bar{V} \ln C - \bar{V} \ln[(1 - \gamma) \bar{V}] \}.$$

Due to homogeneity, the value function is of the form<sup>5</sup>

$$\bar{V}(\hat{x}_t, t, C_t) = \frac{1}{1 - \gamma} H(\hat{x}_t, t) C_t^{1 - \gamma}, \quad (\text{S.3.2})$$

where  $H(\hat{x}, t)$  satisfies the following Hamilton–Jacobi–Bellman (HJB) equation:

$$\begin{aligned} & -\frac{\rho}{1 - \gamma} \ln H(\hat{x}_t, t) H(\hat{x}_t, t) + \left( \hat{x}_t - \frac{1}{2} \gamma \sigma^2 \right) H(\hat{x}_t, t) + \frac{1}{1 - \gamma} H_t(\hat{x}_t, t) \\ & + \left[ \frac{1}{1 - \gamma} a_x (\bar{x} - \hat{x}_t) + q_t \right] H_x(\hat{x}_t, t) + \frac{1}{2} \frac{1}{1 - \gamma} H_{xx}(\hat{x}_t, t) \frac{q_t^2}{\sigma^2} = 0, \end{aligned} \quad (\text{S.3.3})$$

with the boundary condition that for all  $n = 1, 2, \dots$ ,

$$H(\hat{x}_{nT}^-, nT) = E[H(\hat{x}_{nT}^+, nT) \mid \hat{x}_{nT}^-, q_{nT}^-]. \quad (\text{S.3.4})$$

The solution to the partial differential equation (PDE) (S.3.3) together with the boundary condition (S.3.4) is separable and given by

$$H(\hat{x}, t) = e^{\frac{1 - \gamma}{a_x + \rho} \hat{x} + h(t)},$$

<sup>4</sup>We use the notation  $t \bmod T$  for the remainder of  $t$  divided by  $T$ .

<sup>5</sup>As  $\Delta \rightarrow 0$ , the discrete-time approximation, (32), converges to the following monotonic transformation of  $\bar{V}$ :  $V_t = \frac{1}{1 - \gamma} \ln[(1 - \gamma) \bar{V}_t]$ .



where  $h(t)$  satisfies the following ODE:

$$-\rho h(t) + h'(t) + f(t) = 0, \quad (\text{S.3.5})$$

where  $f(t)$  is defined as

$$f(t) = \frac{(1-\gamma)^2}{a_x + \rho} q(t) + \frac{1}{2} \frac{(1-\gamma)^2}{(a_x + \rho)^2} \frac{1}{\sigma^2} q^2(t) - \frac{1}{2} \gamma (1-\gamma) \sigma^2 + a_x \bar{x} \frac{1-\gamma}{a_x + \rho}.$$

The general solution to (S.3.5) is of the following form on  $(0, T)$ :

$$h(t) = h(0)e^{\rho t} - e^{\rho t} \int_0^t e^{-\rho s} f(s) ds.$$

We focus on the steady state in which  $h(t) = h(t \bmod T)$  and use the convention  $h(0) = h(0^+)$  and  $h(T) = h(T^-)$ . Under these notations, the boundary condition (S.3.4) implies  $h(T) = h(0) + \frac{1}{2} \left( \frac{1-\gamma}{a_x + \rho} \right)^2 q(T^-)$ .

### Asset Prices

For  $n = 1, 2, \dots$ , in the interior of  $(nT, (n+1)T)$ , the law of motion of the state price density,  $\pi_t$ , satisfies the stochastic differential equation of the form

$$d\pi_t = \pi_t [-r(\hat{x}_t, t) dt - \sigma_\pi(t) d\tilde{B}_{C,t}],$$

where

$$r(\hat{x}, t) = \rho + \hat{x} - \gamma \sigma^2 + \frac{1-\gamma}{a_x + \rho} q(t)$$

is the risk-free interest rate, and

$$\sigma_\pi(t) = \gamma \sigma + \frac{\gamma - 1}{a_x + \rho} \frac{q(t)}{\sigma}$$

is the market price of the Brownian motion risk.

We denote  $p(\hat{x}_t, t)$  as the price-to-dividend ratio. For  $t \in (nT, (n+1)T)$ , the price of the claim to the dividend process can then be calculated as

$$p(\hat{x}_t, t) D_t = E_t \left[ \int_t^{(n+1)T} \frac{\pi_s}{\pi_t} D_s ds + \frac{\pi_{(n+1)T}}{\pi_t} p(\hat{x}_{(n+1)T}^-, (n+1)T^-) D_{(n+1)T} \right].$$

The above present value relationship implies that

$$\pi_t D_t + \lim_{\Delta \rightarrow 0} \frac{1}{\Delta} \{ E_t [\pi_{t+\Delta} p(\hat{x}_{t+\Delta}, t+\Delta) D_{t+\Delta}] - \pi_t p(\hat{x}_t, t) D_t \} = 0. \quad (\text{S.3.6})$$

Equation (S.3.6) can be used to show that the price-to-dividend ratio function must satisfy the following PDE:

$$1 - p(\hat{x}, t) \varpi(\hat{x}, t) + p_t(\hat{x}, t) - p_x(\hat{x}, t) \nu(\hat{x}, t) + \frac{1}{2} p_{xx}(\hat{x}, t) \frac{q^2(t)}{\sigma^2} = 0, \quad (\text{S.3.7})$$

where the functions  $\varpi(\hat{x}, t)$  and  $\nu(\hat{x}, t)$  are defined by

$$\begin{aligned}\varpi(\hat{x}, t) &= \rho - \mu + \phi\bar{x} + (1 - \phi)\hat{x} + (\phi - 1)\left[\gamma\sigma^2 + \frac{\gamma - 1}{a_x + \rho}q(t)\right], \\ \nu(\hat{x}, t) &= a_x(\hat{x} - \bar{x}) + (\gamma - \phi)q(t) + \frac{\gamma - 1}{a_x + \rho}\left(\frac{q(t)}{\sigma}\right)^2.\end{aligned}$$

Also, equation (S.3.6) can be used to derive the following boundary condition for  $p(\hat{x}, t)$ :

$$p(\hat{x}_T^-, T^-) = \frac{E\left[e^{\frac{1-\gamma}{a_x+\rho}\hat{x}_T^+} p(\hat{x}_T^+, T^+) \mid \hat{x}_T^-, q_T^-\right]}{e^{\frac{1-\gamma}{a_x+\rho}\hat{x}_T^- + \frac{1}{2}\left(\frac{1-\gamma}{a_x+\rho}\right)^2(q_T^- - q_T^+)}}. \quad (\text{S.3.8})$$

Again, we focus on the steady state and denote  $p(\hat{x}, 0) = p(\hat{x}, nT^+)$  and  $p(\hat{x}, T) = p(\hat{x}, nT^-)$ . Under this condition, PDE (S.3.7) together with the boundary condition can be used to determine the price-to-dividend ratio function.

We define  $\mu_{R,t}$  to be the instantaneous risk premium, that is,

$$\mu_{R,t} dt = \frac{1}{p(\hat{x}_t, t)D_t} \{D_t dt + E_t d[p(\hat{x}_t, t)D_t]\}. \quad (\text{S.3.9})$$

In the interior of  $(nT, (n+1)T)$ , the instantaneous risk premium,  $\mu_{R,t} - r(\hat{x}, t)$ , can be computed as

$$[\mu_{R,t} - r(\hat{x}, t)] dt = -\text{Cov}_t \left[ \frac{d[p(\hat{x}_t, t)D_t]}{p(\hat{x}_t, t)D_t}, \frac{d\pi_t}{\pi_t} \right].$$

We have

$$\mu_{R,t} - r(\hat{x}, t) = \left[ \gamma\sigma + \frac{\gamma - 1}{a_x + \rho} \frac{q(t)}{\sigma} \right] \left[ \phi\sigma + \frac{p_x(\hat{x}, t) q(t)}{p(\hat{x}, t) \sigma} \right]. \quad (\text{S.3.10})$$

To gain a better understanding on how the risk premium and the announcement premium depend on the parameters, let  $\varrho(\hat{x}, t) = \ln p(\hat{x}, t)$ ; then equation (S.3.7) can be written as

$$e^{-\varrho(\hat{x}, t)} - \varpi(\hat{x}, t) + \varrho_t(\hat{x}, t) - \varrho_x(\hat{x}, t)\nu(\hat{x}, t) + \frac{1}{2}[\varrho_{xx}(\hat{x}, t) + \varrho_x^2(\hat{x}, t)]\frac{q^2(t)}{\sigma^2} = 0. \quad (\text{S.3.11})$$

Note that  $\hat{x}_t$  is itself an Ornstein–Uhlenbeck process with steady state  $\bar{x}$ . Using a log-linear approximation around  $\hat{x} = \bar{x}$ , we can replace the term  $e^{-\varrho(\hat{x}, t)}$  with  $e^{-\varrho(\hat{x}, t)} \approx e^{-\bar{\varrho}} - e^{-\bar{\varrho}}[\varrho(\hat{x}, t) - \bar{\varrho}]$ , where we denote  $\bar{\varrho} \equiv \varrho(\bar{x}, t)$ , and write

$$\begin{aligned}e^{-\bar{\varrho}}[1 + \bar{\varrho} - \varrho(\hat{x}, t)] - \varpi(\hat{x}, t) + \varrho_t(\hat{x}, t) - \varrho_x(\hat{x}, t)\nu(\hat{x}, t) \\ + \frac{1}{2}[\varrho_{xx}(\hat{x}, t) + \varrho_x^2(\hat{x}, t)]\frac{q^2(t)}{\sigma^2} = 0.\end{aligned} \quad (\text{S.3.12})$$

We conjecture that  $\varrho(\hat{x}, t) = A\hat{x} + B(t)$ , and equation (S.3.12) can be used to solve for  $A$  and  $B(t)$  by the method of undetermined coefficients to get  $A = \frac{\phi-1}{a_x+e^{-\bar{\varrho}}}$ .

Using the log-linearization result to evaluate equation (S.3.10) at  $\hat{x} = \bar{x}$ , we obtain (35). In addition, using  $p(\hat{x}_T^+, T^+) \approx e^{A\hat{x}_T^+ + B(T^+)}$ , we can compute the expectation in (S.3.8) explicitly and obtain (36).

### Numerical Solutions

To solve the PDE (S.3.7) with the boundary condition (S.3.8), we consider the following auxiliary problem:

$$p(x_t, t) = E \left[ \int_t^T e^{-\int_t^s \varpi(x_u, u) du} ds + e^{-\int_t^T \varpi(x_u, u) du} p(x_T, T) \right], \quad (\text{S.3.13})$$

where the state variable  $x_t$  follows the law of motion:

$$dx_t = -\nu(\hat{x}, t) dt + \frac{q(t)}{\sigma} dB_t. \quad (\text{S.3.14})$$

Note that the solution to (S.3.13) and (S.3.8) satisfies the same PDE. Given an initial guess of the pre-announcement price-to-dividend ratio,  $p^-(x_\tau, \tau)$ , we can solve (S.3.13) by the Markov chain approximation method (Kushner and Dupuis (2001)):

(i) We first start with an initial guess of a pre-announcement price-to-dividend ratio function,  $p(x_T, T)$ .

(ii) We construct a locally consistent Markov chain approximation of the diffusion process (S.3.14) as follows: We choose a small  $dx$ , let  $Q = |\nu(\hat{x}, t)| dx + (\frac{q(t)}{\sigma})^2$ , and define the time increment  $\Delta = \frac{dx^2}{Q}$  to be a function of  $dx$ . Define the following Markov chain on the space of  $x$ :

$$\begin{aligned} \Pr(x + dx | x) &= \frac{1}{Q} \left[ -\nu(\hat{x}, t)^+ dx + \frac{1}{2} \left( \frac{q(t)}{\sigma} \right)^2 \right], \\ \Pr(x - dx | x) &= \frac{1}{Q} \left[ -\nu(\hat{x}, t)^- dx + \frac{1}{2} \left( \frac{q(t)}{\sigma} \right)^2 \right]. \end{aligned}$$

One can verify that as  $dx \rightarrow 0$ , the above Markov chain converges to the diffusion process (S.3.14). (In the language of Kushner and Dupuis (2001), this is a Markov chain that is locally consistent with the diffusion process (S.3.14).)

(iii) With the initial guess of  $p(x_T, T)$ , for  $t = T - \Delta, T - 2\Delta$ , etc., we use the Markov chain approximation to compute the discounted problem in (S.3.13) recursively:

$$p(x_t, t) = \Delta + e^{-\varpi(x_t, t)\Delta} E[p(x_{t+\Delta}, t + \Delta)],$$

until we obtain  $p(x, 0)$ .

(iv) Compute an updated pre-announcement price-to-dividend ratio function,  $p(x_T, T)$  using (S.3.8):

$$p(\hat{x}_T^-, T^-) = \frac{E \left[ e^{\frac{1-\gamma}{ax+\rho} \hat{x}_T^+} p(\hat{x}_T^+, 0) \mid \hat{x}_T^-, q_T^- \right]}{e^{\frac{1-\gamma}{ax+\rho} \hat{x}_T^- + \frac{1}{2} \left( \frac{1-\gamma}{ax+\rho} \right)^2 [q_T^- - q_T^+]}}.$$

Go back to step 1 and iterate until the function  $p(x_T, T)$  converges.

### Choice of Parameter Values

The numerical example we presented in the paper uses parameter values in the standard long-run risk model (see Table S.I). All parameters are annual. We assume that announcements are made at the monthly frequency, that is,  $T = \frac{1}{12}$ .

TABLE S.I  
LIST OF PARAMETER VALUES IN THE CONTINUOUS-TIME MODEL

$\rho$	$\gamma$	$\bar{x}$	$a_x$	$\sigma_x$	$\sigma$	$\phi$	$\sigma_S^2$
0.02	10	1.5%	0.10	0.025%	3%	4	0

### Pre-Announcement Drift

The density of communication in the top panel of Figure 4 is generated from a Beta distribution with parameter  $\alpha = 2$ ,  $\delta = 3$  on  $[-6, 0]$  hours before announcement. The density of the Beta distribution is

$$f(y | \alpha, \delta) = B[\sigma, \delta]^{-1} y^{\alpha-1} (1-y)^{\delta-1} \quad \text{for } y \in (0, 1),$$

where  $B[\sigma, \delta]$  is the Beta function. In our example, the density of the occurrence of a communication  $h$  hours before announcement is  $f(1 - \frac{h}{6} | \alpha, \delta)$ .

During a small interval  $dt$ , the expected return of the dividend claim is  $\mu_{R,t} dt$  if the announcement does not occur. The expected return is  $\frac{E[p(\hat{x}_T^+, T^+) \hat{x}_T^-, q_T^-]}{p(\hat{x}_T^-, T^-)}$  if the announcement return occurs during  $dt$ . Given that the probability of an announcement during hour  $(k - h, k)$  is  $\int_{k-h}^k f(1 - \frac{t}{6} | \alpha, \delta) dt$ , the expected return of the dividend claim during hour  $(k - h, k)$  can be written as

$$E \left[ \int_{k-h}^k \left\{ f \left( 1 - \frac{t}{6} \mid \alpha, \delta \right) \frac{E \left[ p \left( \hat{x}_{T+\frac{t}{2880}}^+, \left( T + \frac{t}{2880} \right)^+ \right) \hat{x}_{T+\frac{t}{2880}}^-, q_{T+\frac{t}{2880}}^- \right]}{p \left( \hat{x}_{T+\frac{t}{2880}}^-, \left( T + \frac{t}{2880} \right)^- \right)} \right. \right. \\ \left. \left. + \mu_{R, T+\frac{t}{2880}} \right\} dt \right]. \quad (\text{S.3.15})$$

The above calculation assumes that there are 360 days per year and 8 hours per day. Because  $t$  is measured in hours, it needs to be divided by  $360 \times 8 = 2880$  to translate into annual unit. Numerically, because the pre-announcement drift happens within hours before  $T$ , replacing  $T + \frac{t}{2880}$  with  $T$  does not make any material difference in the evaluation of (S.3.15). In addition, the term  $\int_{k-h}^k \mu_{R, T+\frac{t}{2880}} dt$  is negligible. We can therefore approximate the average return during hour  $(k - h, k)$  as

$$E \left[ \int_{k-h}^k f \left( 1 - \frac{t}{6} \mid \alpha, \delta \right) dt \right] \times E \left[ \frac{E[p(\hat{x}_T^+, T^+) | \hat{x}_T^-, q_T^-]}{p(\hat{x}_T^-, T^-)} \right].$$

### S.3.2. Time-non-separable Utilities

To guarantee that the model is well defined, we make the following assumptions on the weighting function  $\{\xi(t, s)\}_{s=0}^t$ :

$$\int_0^t \xi(t, s) ds \leq 1 \quad \text{for all } t > 0, \quad (\text{S.3.16})$$

$$\int_0^\infty \xi(t+s, t) ds < \infty \quad \text{for all } t > 0, \quad (\text{S.3.17})$$

$$\left(1 - \int_0^t \xi(t, s) ds\right) H_0 + \int_0^t \xi(t, s) C_s ds < C_t \quad \text{for all } t > 0. \quad (\text{S.3.18})$$

The first assumption requires that  $\{\xi(t, s)\}_{s=0}^t$  is an appropriate weighting function, that is, total weight is less than 1. The second assumption implies that the contribution of  $C_t$  to future habit stock is finite, and the last assumption ensures  $C_t - H_t > 0$  so that the utility function is well defined.

### External Habit

Under the assumption of complete markets, the state-price density can be constructed from the marginal utility of the representative agent. In the external habit model,

$$\pi_t = e^{-\beta t} u'(C_t + bH_t).$$

### Internal Habit

In this case, the calculation of the state price density must take into account the impact of  $C_t$  on future habit stock. Therefore, the state price density is given by (39). Because announcement fully reveals  $x_t$ , we need to show that

$$E \left[ \int_0^\infty e^{-\beta s} \xi(t+s, t) u'(C_{t+s} + bH_{t+s}) ds \mid x_t = x \right] \quad (\text{S.3.19})$$

is a decreasing function of  $x$ . Without loss of generality, we assume  $t = 0$  in the following lemma.

LEMMA S.1: *Fixing the path of Brownian motions  $\{B_{C,s}, B_{x,s}\}_{s=0}^\infty$ ,*

$$\frac{\partial}{\partial x_0} [C_t + bH_t] > 0 \quad \text{for all } t > 0. \quad (\text{S.3.20})$$

PROOF: Using the law of motion of  $C_t$ , we have

$$\ln C_t = \ln C_0 - \frac{1}{2} \sigma^2 t + \int_0^t \sigma dB_{C,s} + \int_0^t x_s ds.$$

Since  $x_t$  is an Ornstein–Uhlenbeck process, we can solve  $\int_0^t x_s ds$  explicitly:

$$\int_0^t x_s ds = (x_0 - \bar{x}) \frac{1}{a_x} [1 - e^{-a_x t}] + \bar{x} t + \frac{1}{a_x} \int_0^t [1 - e^{a_x(s-t)}] \sigma_x dB_{x,s}.$$

Therefore, for given realizations of the Brownian motion paths,

$$\frac{\partial}{\partial x_0} C_t = C_t \frac{1}{a_x} [1 - e^{-a_x t}],$$

and

$$\begin{aligned}
 \frac{\partial}{\partial x_0} H_t &= \int_0^t \xi(t, s) \frac{\partial C_s}{\partial x_0} ds \\
 &= \int_0^t \xi(t, s) C_s \frac{1}{a_x} [1 - e^{-a_x s}] ds \\
 &< \int_0^t \xi(t, s) C_s ds \frac{1}{a_x} [1 - e^{-a_x t}] \\
 &< C_t \frac{1}{a_x} [1 - e^{-a_x t}],
 \end{aligned}$$

where the first inequality is true because  $s < t$ , and the second is due to the fact that  $\int_0^t \xi(t, s) C_s ds \leq H_t < C_t$ . The inequality (S.3.20) follows because  $b \in (-1, 0)$ . *Q.E.D.*

Consider two initial conditions,  $x_0 = x$  and  $x_0 = x'$ . The above lemma implies that  $x > x'$  implies that  $C_{t+s} + bH_{t+s}$  first-order stochastically dominates  $C'_{t+s} + bH'_{t+s}$ . Because  $u'(\cdot)$  is a strictly decreasing function, we conclude that (S.3.19) must be a decreasing function of  $x$ .

### Consumption Substitutability

Because (S.3.19) is a decreasing function of  $x$ , with  $b > 0$ , the state price density in (39) must be a decreasing function of  $x_t$  as well. As a result, the announcement premium must be positive for any payoff that is increasing in  $x_t$ .

### REFERENCES

- CHEN, Z., AND L. EPSTEIN (2002): "Ambiguity, Risk, and Asset Returns in Continuous Time," *Econometrica*, 70 (4), 1403–1443. [6]
- DUFFIE, D., AND L. G. EPSTEIN (1992): "Stochastic Differential Utility," *Econometrica*, 60 (2), 353–394. [8]
- EPSTEIN, L., AND M. SCHNEIDER (2003): "Recursive Multiple-Priors," *Journal of Economic Theory*, 113 (1), 1–31. [6]
- EPSTEIN, L., AND S. E. ZIN (1989): "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework," *Econometrica*, 57 (4), 937–969. [5]
- ERGIN, H., AND F. GUL (2009): "A Theory of Subjective Compound Lotteries," *Journal of Economic Theory*, 144, 899–929. [6]
- GILBOA, I., AND D. SCHMEIDLER (1989): "Maxmin Expected Utility With Non-Unique Prior," *Journal of Mathematical Economics*, 18 (2), 141–153. [6]
- HANSEN, L. P., AND T. SARGENT (2008): *Robustness*. Princeton, New Jersey: Princeton University Press. [6]
- HAYASHI, T. (2005): "Intertemporal Substitution, Risk Aversion and Ambiguity Aversion," *Economic Theory*, 25 (4), 933–956. [7]
- HAYASHI, T., AND J. MIAO (2011): "Intertemporal Substitution and Recursive Smooth Ambiguity Preferences," *Theoretical Economics*, 121 (3), 496–520. [7]
- KLIBANOFF, P., M. MARINACCI, AND S. MUKERJI (2005): "A Smooth Model of Decision Making Under Ambiguity," *Econometrica*, 73 (6), 1849–1892. [6,7]
- (2009): "Recursive Smooth Ambiguity Preferences," *Journal of Economic Theory*, 144 (3), 930–976. [6]
- KREPS, D. M., AND E. L. PORTEUS (1978): "Temporal Resolution of Uncertainty and Dynamic Choice Theory," *Econometrica*, 46 (1), 185–200. [4,5]
- KUSHNER, H. J., AND P. DUPUIS (2001): *Numerical Methods for Stochastic Control Problems in Continuous Time* (Second Ed.). Stochastic Modelling and Applied Probability, Vol. 24. New York: Springer-Verlag. [11]
- MACCHERONI, F., M. MARINACCI, AND A. RUSTICHINI (2006a): "Ambiguity Aversion, Robustness, and the Variational Representation of Preferences," *Econometrica*, 74 (6), 1447–1498. [6]

- (2006b): “Dynamic Variational Preferences,” *Journal of Economic Theory*, 128 (1), 4–44. [6]  
STRZALECKI, T. (2011): “Axiomatic Foundations of Multiplier Preferences,” *Econometrica*, 79 (1), 47–73. [6]

---

*Co-editor Giovanni L. Violante handled this manuscript.*

*Manuscript received 5 August, 2016; final version accepted 30 January, 2018; available online 8 February, 2018.*