

SUPPLEMENT TO “ASSET MARKETS WITH
HETEROGENEOUS INFORMATION”
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THIS APPENDIX DESCRIBES (without formally stating all the equilibrium objects) what the equilibrium looks like under two other possible information structures.

FALSE POSITIVES AND FALSE NEGATIVES TOGETHER

Suppose

$$x(i, b) = \mathbb{I}(i \geq b),$$

so buyers with $b < \lambda$ observe some “false-positives” while buyers with $b > \lambda$ observe some “false-negatives.” Suppose further that, for $b > \lambda$, $w(b)$ is strictly increasing and differentiable.

The key equilibrium object is a function $p(i)$, defined for $i \in [\lambda, 1]$, which denotes the lowest price at which asset i is offered for sale. As in the false-negatives case, false-negative buyer b will want to buy in the cheapest market where he can observe good assets. If $p(i)$ is increasing (which will be true in equilibrium because $w(b)$ is increasing), then this means a market where the price is $p(b)$. The same reasoning that leads to equation (31) in the main text implies

$$(68) \quad p(i) = \frac{w(i)}{r(i)},$$

where $r(i)$ is the number of i assets held by distressed sellers that they were unable to sell at prices above $p(i)$.

For false-positive buyers, if they buy in a market where the price is $p(i)$, they will be drawing bad assets in the range $[b, \lambda)$ (coming from all sellers) and good assets in the range $[\lambda, i]$ (only from distressed sellers). They obtain terms of trade

$$(69) \quad \tau(b, i, p(i)) = \frac{1}{p(i)} \frac{\mu(i - \lambda)}{\mu(i - \lambda) + \lambda - b}.$$

In choosing what market to buy from, they face a tradeoff between better selection and lower prices. The marginal rate of substitution is

$$\text{MRS}(b, i, p) \equiv \frac{\frac{\partial \tau(b, i, p)}{\partial i}}{\frac{\partial \tau(b, i, p)}{\partial p}} = - \frac{p(i)}{(i - \lambda)} \frac{\lambda - b}{\mu(i - \lambda) + \lambda - b},$$

which is increasing in b . Hence there is single crossing and false-positive buyers sort into different markets: higher- b buyers (which, among false-positive buyers, means more expert) prefer lower prices and worse selection compared to lower- b buyers. False positive buyers' sorting can be summarized by a function $\tilde{b}(i)$ that says which buyer buys in the market where the price is $p(i)$.

Therefore, the equilibrium is given by the functions $p(i)$, $\tilde{b}(i)$, and $r(i)$. These must satisfy the following conditions. The first is a first-order condition for buyer $\tilde{b}(i)$ to find it optimal to buy from market $p(i)$. Using (69):

$$(70) \quad p'(i) = \frac{p(i)}{i - \lambda} \frac{\lambda - \tilde{b}(i)}{\mu(i - \lambda) + \lambda - \tilde{b}(i)}.$$

The second condition is a market-clearing condition. The purchases of each false-positive buyer are spread over many assets but add up to determine the unsold remainder $r(i)$. In market $p(i)$, false-positive buyers buy $r'(i) \times (i - \lambda)$ good assets and $r'(i) \frac{\lambda - \tilde{b}(i)}{\mu}$ bad assets. This requires spending a total of $p(i) [\frac{\lambda - \tilde{b}(i)}{\mu} + i - \lambda] r'(i)$. These buyers have a total wealth of $-w(\tilde{b}(i)) \tilde{b}'(i)$. Hence, in equilibrium, it must be that

$$(71) \quad \tilde{b}'(i) = - \frac{1}{w(\tilde{b}(i))} p(i) \left[\frac{\lambda - \tilde{b}(i)}{\mu} + (i - \lambda) \right] r'(i).$$

Finally, (68) can be rewritten in differential form as

$$(72) \quad r'(i) = \frac{\frac{\partial w(i)}{\partial i} p(i) - p'(i) w(i)}{[p(i)]^2}.$$

Equations (70)–(72) constitute a system of differential equations. The terminal conditions are

$$(73) \quad \frac{1}{p(i^*)} \frac{\mu(i^* - \lambda)}{\mu(i^* - \lambda) + \lambda - \tilde{b}(i^*)} = 1,$$

$$(74) \quad \tilde{b}(\lambda) = \lambda,$$

$$(75) \quad r(i^*) = \mu,$$

$$(76) \quad p(i^*) = \frac{w(i^*)}{\mu}.$$

Equation (73) is an indifference condition. For some cutoff asset i^* , buyer $\tilde{b}(i^*)$ is indifferent between buying and not buying. As in the false-positives case, buyers with $b < \tilde{b}(i^*)$ find that τ is below 1 in all markets and do not trade. Equation (74) says that all false-positive buyers with $b \in [\tilde{b}(i^*), \lambda]$ buy assets $i \in [\lambda, i^*]$, which requires that buyer $b = \lambda$ buy asset λ . Equation (75) says that, since no false-positive buyers are present in markets where prices are above $p(i^*)$, distressed buyers cannot sell any asset below i^* in those markets, so the unsold remainder of asset i^* equals their entire endowment μ . Finally, equation (76) is equation (68) evaluated at i^* .

Figures S1 and S2 illustrate the equilibrium. Figure S1 shows the solution to the system of differential equations. The cutoff asset for this example is $i^* = 0.85$. The left panel shows the price function for assets in the range $[\lambda, i^*]$. Buyer $\tilde{b}(i^*) = 0.34$ buys asset i^* ; buyers below this cutoff do not buy at all, and buyers between 0.34 and $\lambda = 0.4$ spend all their wealth buying assets. Their collective purchases bring down the unsold remainder from $r(i^*) = \mu = 0.5$ up to $r(\lambda) = 0.41$.

Figure S2 shows what markets buyers buy from, what assets they obtain, and what are the resulting terms of trade. The first panel shows which market each buyer chooses to buy from. Buyer λ , who gets a perfect signal, buys good assets at the lowest possible price. Away from λ in either direction, buyers choose higher-priced markets. To the left of $b = \lambda$, false-positive buyers buy in higher-priced markets because they need to mix in more good assets with the bad assets that they are unable to filter out. To the right of $b = \lambda$, less expert false-negative buyers need to buy at higher prices because they cannot detect the good assets that are on sale at $p(\lambda)$. Those with $b \in (\lambda, i^*)$ buy in markets

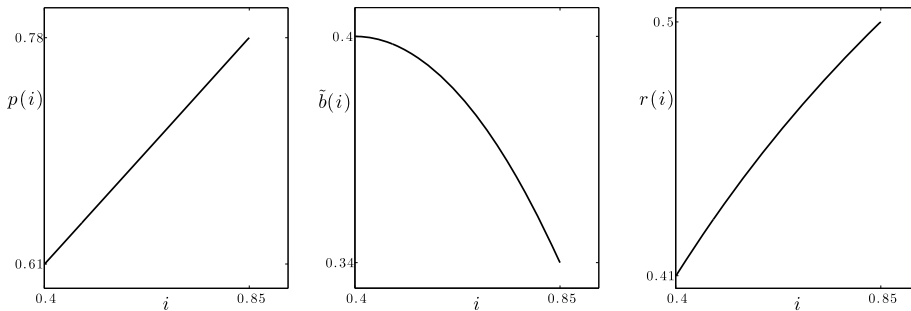


FIGURE S1.—Solution to the differential equations. The example uses $\lambda = 0.4$, $\mu = 0.5$, and $w(b) = \max\{0.05(b + 1), 0.1\}$.

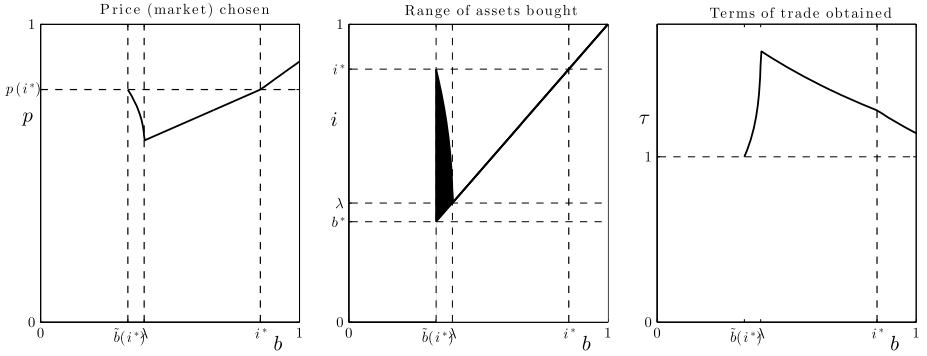


FIGURE S2.—Characterization of the equilibrium. The example uses $\lambda = 0.4$, $\mu = 0.5$, and $w(b) = \max\{0.05(b + 1), 0.1\}$.

where a false-positive buyer is also present, whereas those with $i > i^*$ buy in markets with no other buyers. Notice, however, that even in markets where more than one type of buyer is present, the assets they accept overlap on a zero-measure set, so the allocation they obtain does not depend on the order in which they clear. Hence, the clearing algorithm that is used is indeterminate.

The selection of assets that each buyer obtains is shown in the second panel. By choosing a higher-priced market, lower- b false-positive buyers find higher- i good assets on sale, which makes up for the fact that the lower bound on the assets they accept is lower. False negative buyers, on the other hand, each buy a single asset type: $i = b$, the lowest i that they can tell is a good asset.

The third panel shows the terms of trade that result. Buyer $b = \lambda$ gets the best terms of trade, and they fall off in either direction. For this example, τ is above 1 for all false-negative buyers (because $p(i) < 1$), so they all strictly choose to trade, but τ reaches 1 for $b = 0.34$, so buyers below this point choose not to trade. This need not be the case in general; it depends on the function $w(b)$.

NONNESTED SIGNALS

Suppose

$$x(i, b) = \mathbb{I}(i \in [0, \max\{\lambda b + \Delta - \lambda, 0\}] \cup [\lambda b, \lambda b + \Delta] \cup [\lambda, 1])$$

for some $\Delta < \lambda$, as illustrated in Figure S3. All buyers observe false-positives for Δ assets, but the set of bad assets for which they observe $x(i, b) = 1$ are not nested. For $b \leq \lambda - \Delta$, buyers observe false-positives for an interval of length Δ starting at $i = \lambda b$. For $b > \lambda - \Delta$, buyers observe false-positives for $i \in [\lambda b, \lambda]$ and for $i \in [0, \lambda b + \Delta - \lambda]$.

Further suppose $w(b) = w$ is a constant. As a result, all buyers are symmetric (because they observe the same fraction of false positives and have the same

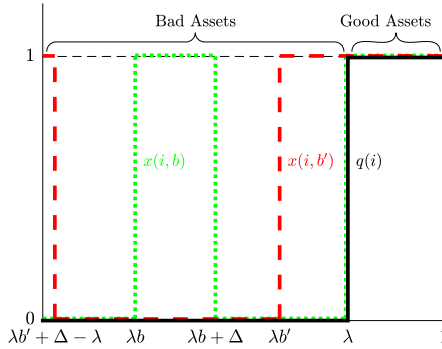


FIGURE S3.—Information of buyers with nonnested signals.

wealth) and all bad assets are symmetric (because they give false positive signals to the same fraction of buyers).

The equilibrium for this case is as follows. All assets trade at the same price p^* but in different markets since all buyers direct their demand to a market where they are guaranteed to trade first. In any market, at price p^* , the supply includes a measure $\mu(1 - \lambda)$ of good assets from distressed sellers and a measure λ of bad assets from all sellers, of which any buyer accepts a measure Δ . Assumption (1) implies that buyers do not spend all their endowment, so they must be indifferent between buying and not buying, which implies

$$p^* = \frac{\mu(1 - \lambda)}{\mu(1 - \lambda) + \Delta}.$$

Adding across all markets with $p = p^*$, all good assets are sold, as well as a fraction $\frac{\Delta}{\lambda}$ of bad assets.

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